



Mechanical and Aerospace Engineering

# THE LABORATORY FOR AUTONOMY IN DATA-DRIVEN AND COMPLEX SYSTEMS

ARC The ARC logo, consisting of the letters 'ARC' in a stylized font with a small red star-like icon to the right.

## Some New Challenges in Chance-Constrained Path Planning for UAS in Unstructured Uncertainty: Part 1 (Fusion)

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THE OHIO STATE  
UNIVERSITY

# LADDCS TEAM

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<b>Current Ph.D.</b> Andrew VanFossen Indranil Nayak Sriram Narayanan Kyle Sharkey	Anil Rao MAE, UNIV. FLORIDA	Greg Guess: ODNR	Matthew Bell POINTPRO
<b>Current MS</b> Anna Lebron Alan Cortez			
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# LADDCS (Lab) TEAM



Alex Soderlund (May, 2020)



Rachit Aggarwal (May, 2021)



Bander Jabr (August, 2021)



Indranil Nayak



Andrew VanFossen



Sriram Narayanan



Anna Lebron



Alan Cortez



Bryce Ford



Joey Caley



Alexandra Mangel



Mrinal Kumar

# LADDCS PROFILE

The collage includes:

- A flight deck display showing a map with flight paths, altitude, and speed information.
- A 3D wireframe model of a complex structure or terrain with a path planning visualization.
- Two firefighters in a forest setting, one holding a hose.
- A red quadcopter drone on a reflective surface.
- A white quadcopter drone flying over a grassy field.
- A heatmap showing detection exposure and cumulative loading in a 'NO FLY ZONE'.
- A dark, textured image of a satellite or sensor array.
- A circular heatmap visualization.
- A map of wind turbines labeled CA-1 through CA-13.
- A 3D simulation of a space mission with multiple satellites and a cityscape background.
- A 3D simulation of a plasma or particle flow.
- A 3D simulation of a storm or weather pattern.

forecasting

missions in adversity

data-driven systems

- closed-loop MC
- probability of failure
- threat assessment
- asset sustainment
- black-box models

- chance-constrained path planning
- integral constraints
- evidential sensor fusion
- platform dev
- wildfire missions

- DMD + Koopman theory
- learning dynamics
- situational awareness
- space explosions/ plasma mechanics



Ohio

Third Frontier  
Innovation Creating Opportunity

• autonomy

• space



# TALK OVERVIEW

## 1. UAS Missions in Unstructured Uncertainty

- A couple scenarios

## 2. Obstacle Characterization: Evidential Sensor Fusion

- Traditional Bayes' formalism
- The Dempster-Shafer alternative
- Sensor belief modeling – temperature & vision sensors (examples)
- Application to real fire: case study
- Summary

## 3. UAS Path-Planning

- Chance-constraints
- Path-dependent resource/loading constraints
- Graph Search – Hybrid A\*
- Backtracking
- Numerical studies
- Summary

## 4. Wrap up and lookahead

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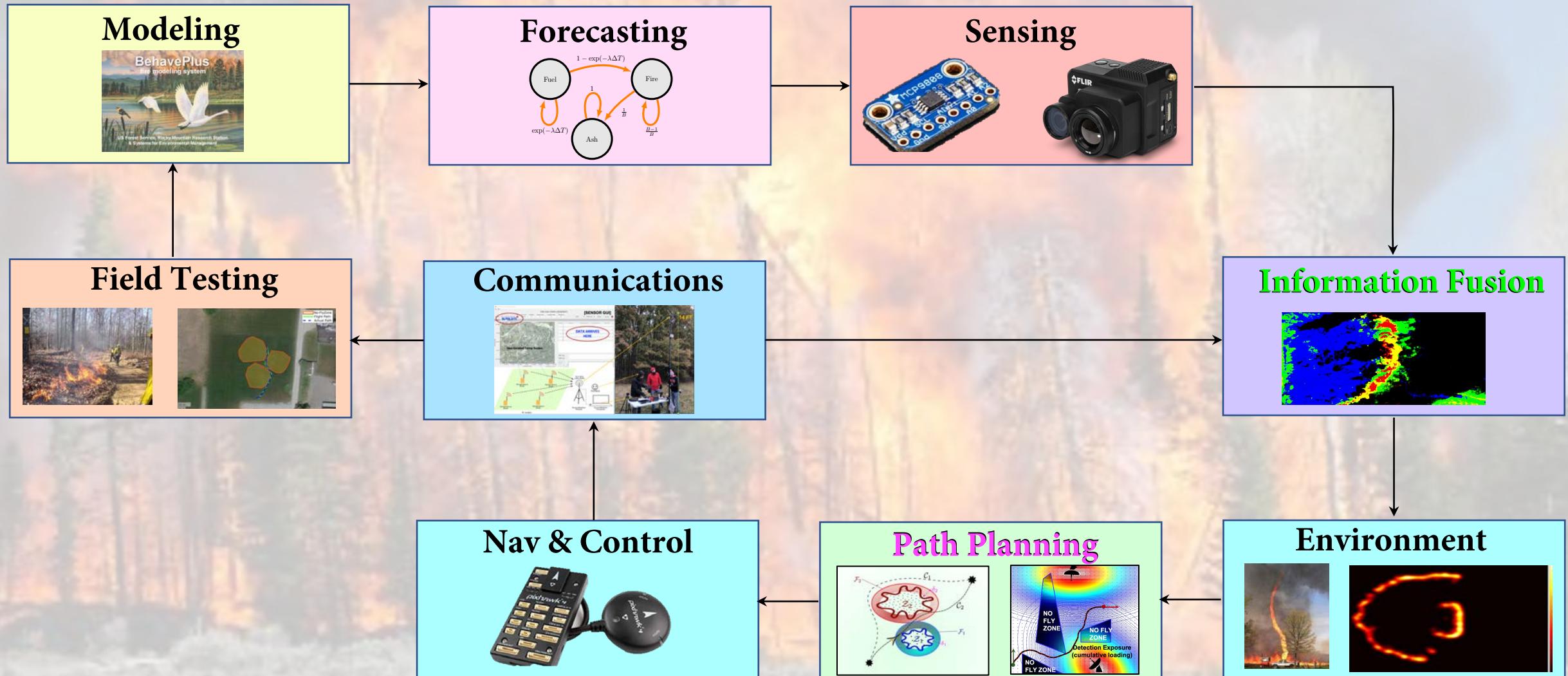
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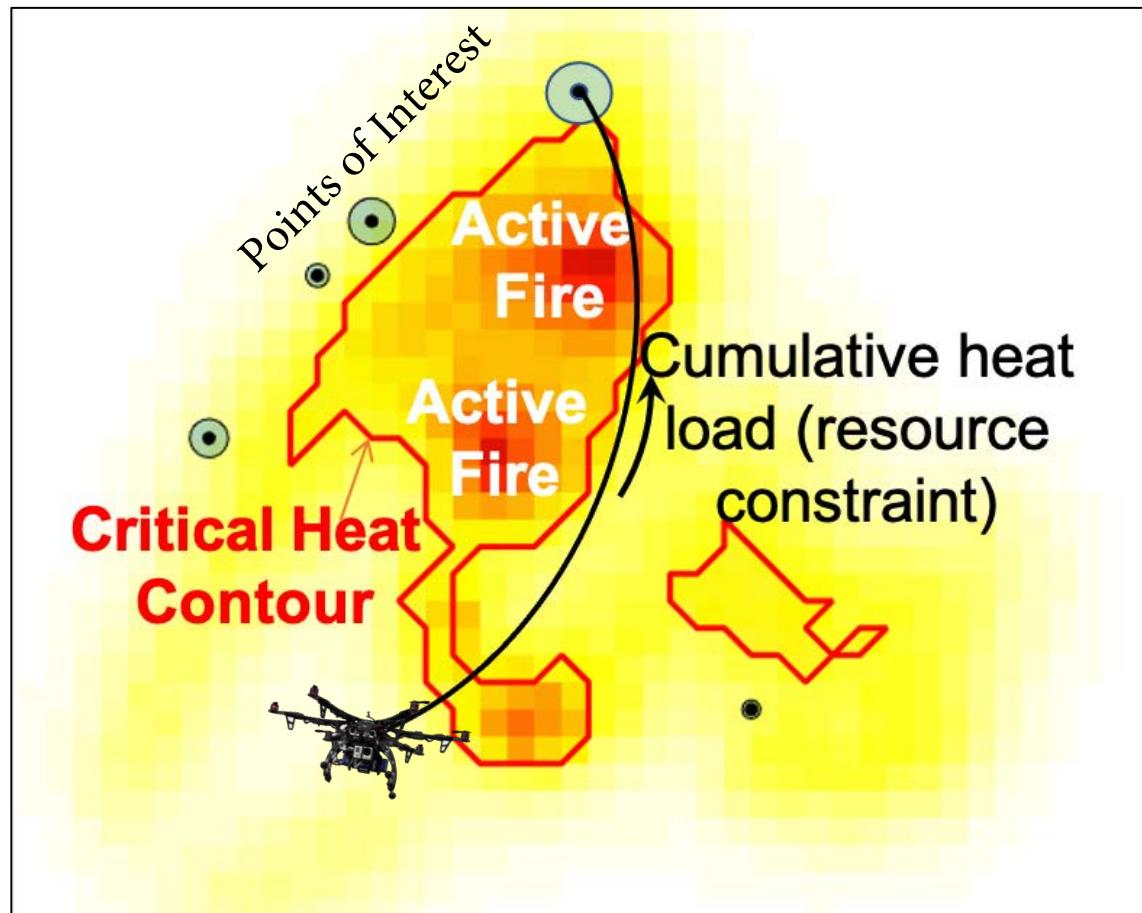
# UAS MISSION PLANNING

- Missions in *unstructured uncertainty*:
  - poor models
  - sensing conflict and anomalies
  - “broken”, dynamic environment



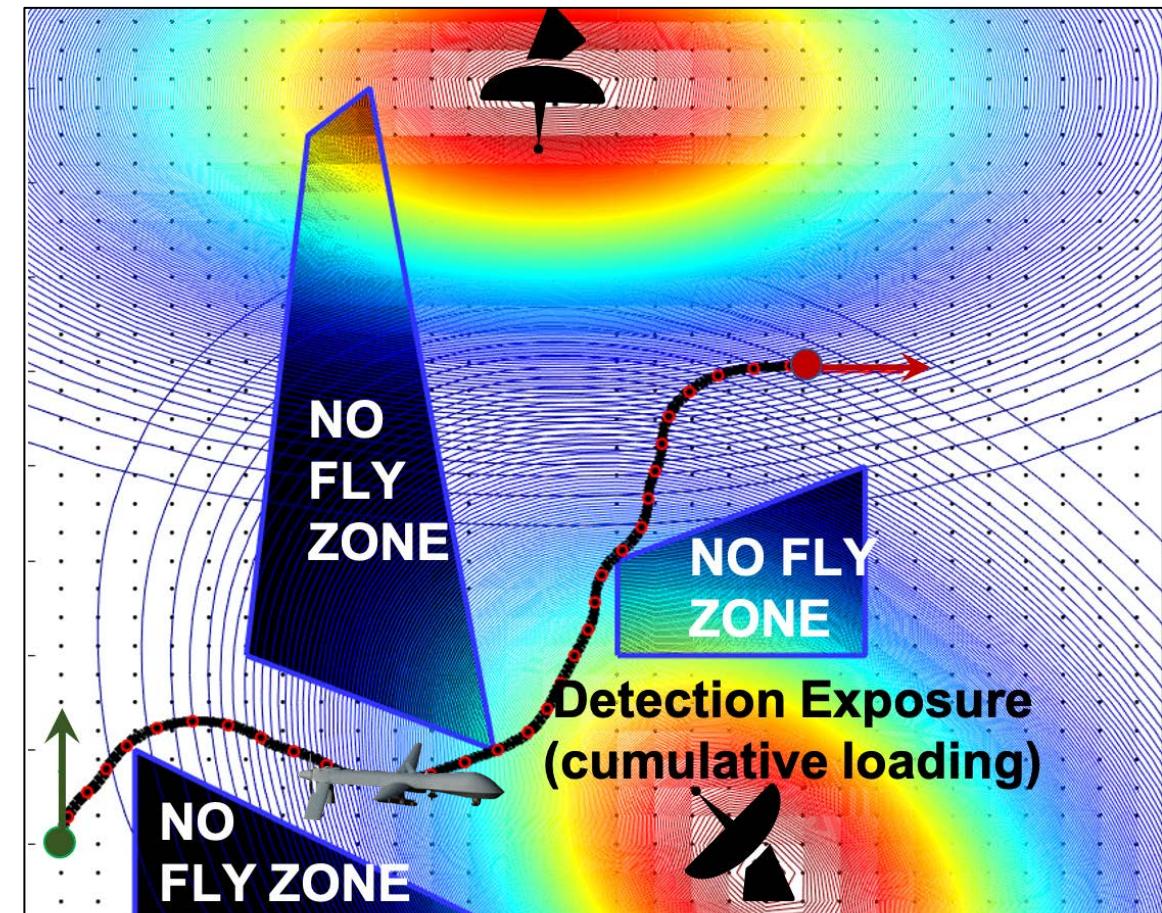
# UAS MISSION PLANNING: Intangible Obstacle Scenarios

Scenario 1.



Flight above active fire: UAS safety constraints

Scenario 2.



Flight through enemy territory: constraint on radar detection, tracking and eventual attack

# TALK OVERVIEW

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# Evidential Information Fusion: Obstacle Characterization

**CONTEXT** Timely and precise characterization of an **evolving wildfire** is critical to the success of its suppression and its propensity to threaten communities.

1. Current wildland fire containment efforts require improvement in two areas: i.) Early detection of fire presence and, ii.) Preventing the spread of the wildfire to highly volatile fuel, such as dry grassland
2. Decision-making regarding wildfire containment can be aided with knowledge of the probabilistic belief states of the fire's presence as it spreads



## Challenges

- An adverse environment is typified by high incidence of poor priors (in the Bayesian sense),
- Trust in sensor data is an issue,
- Sensor conflict is common.

## HOW TO CONDUCT OBSTACLE CHARACTERIZATION?

# Traditional Bayes' Information Fusion

- “*Traditional*” estimation proceeds via the **Bayes’ rule**:

$$\text{posterior} \quad \mathbb{P}(A|B) = \frac{\text{likelihood} \quad \mathbb{P}(B|A)\mathbb{P}(A)}{\sum_{i=1}^n \mathbb{P}(B|A_i)\mathbb{P}(A_i)} \quad \text{prior}$$

- **Criticisms** of the Bayesian framework, especially pertinent in *unstructured uncertainty*:

- Bayesian view does not allow belief to be withheld from one proposition without according that belief to its complement:

$$\mathbb{P}(A) + \mathbb{P}(\bar{A}) = 1$$

- The prior probability  $\mathbb{P}(A)$  may not be adequately known.
- The certainty of event  $B$  is required.

# The Dempster-Shafer Alternative 1/3

- The Dempster-Shafer theory of probable reasoning (DST) is an alternative belief framework to conventional probability theory.
- Key Aspects:
  1. Beliefs in events are represented as **intervals**, as opposed to point probabilities.
  2. Beliefs can be assigned to **sets** of mutually-exclusive events.
  3. Beliefs are assigned based on the available **supporting evidence**.
- Consider a question with multiple finite possible hypotheses:
$$\Theta = \{\theta_1, \theta_2, \dots, \theta_i, \dots, \theta_n\}$$
- A **proposition**  $A$  is a set of mutually-exclusive hypotheses.

did it not not snow?

did it snow?

- The numerical “amount” of belief for any proposition  $A \subseteq \Theta$  supplied by an expert  $E$  is quantified through the **mass number**  $m_E(A)$ .
- The mass numbers that are assigned over the entire frame of discernment  $\Theta$  is the mass number array  $\mathbf{m}_E$  defined as:

$$\mathbf{m}_E = (m_E(A_1), \dots, m_E(A_i), \dots, m_E(A_{|2^\Theta|}))$$

where  $\sum_{A_i \in 2^\Theta} m_E(A_i) = 1$  and  $m_E(\emptyset) = 0$

- The mass assigned to a proposition composed of multiple hypotheses is not, in general, divisible into the masses of its constituent parts - e.g.
$$m(\Theta) \neq m(\{\theta_1\}) + m(\{\theta_2\}) + \dots + m(\{\theta_n\})$$
- **Ignorance** is defined as  $m(\Theta)$

# The Dempster-Shafer Alternative 2/3

- The **belief** in  $A$  from expert  $E$ ,  $\text{Bel}_E(A)$ , is interpreted as the sum of evidence directly implying  $A$ :

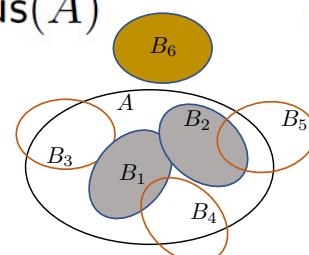
$$\text{Bel}_E(A) = \sum_{B \subseteq A} m_E(B)$$

- The **plausibility** in  $A$  from expert  $E$ ,  $\text{Plaus}_E(A)$ , is interpreted as the sum of evidence that does not contradict  $A$ :

$$\text{Plaus}_E(A) = \sum_{B \cap A \neq \emptyset} m(B)$$

- The inequality  $\text{Bel}(A) \leq \text{Plaus}(A)$  always holds.

plausibility  
\_\_\_\_\_  
belief  
\_\_\_\_\_



- The mass numbers of two independent experts  $E1$  and  $E2$  can be combined into a new mass number array through Dempster's **Rule of Combination**:

$$m_C(A) = K \sum_{A_i \cap B_j = A} m_{E1}(A_i)m_{E2}(B_j)$$

- The normalization constant  $K = \frac{1}{1-\kappa}$ , is computed from the *degree of conflict*  $\kappa \in [0, 1]$  between the masses of the two experts  $E1$  and  $E2$ :

$$\kappa = \sum_{A_i \cap B_j = \emptyset} m_{E1}(A_i)m_{E2}(B_j)$$

- The combination of two belief functions is valid where two experts do not completely contradict, i.e.  $\kappa < 1$ .

# The Dempster-Shafer Alternative 3/3

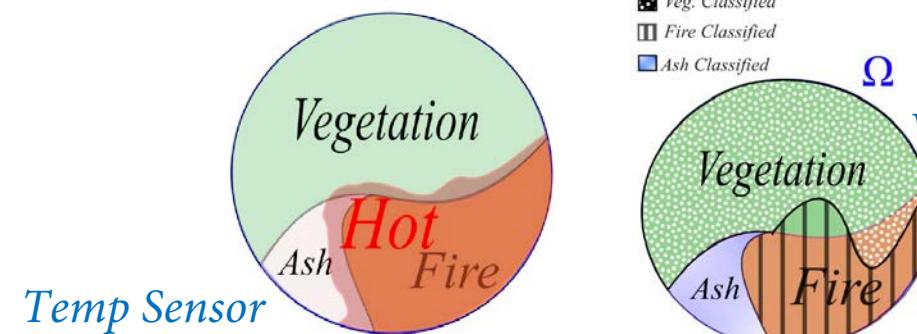
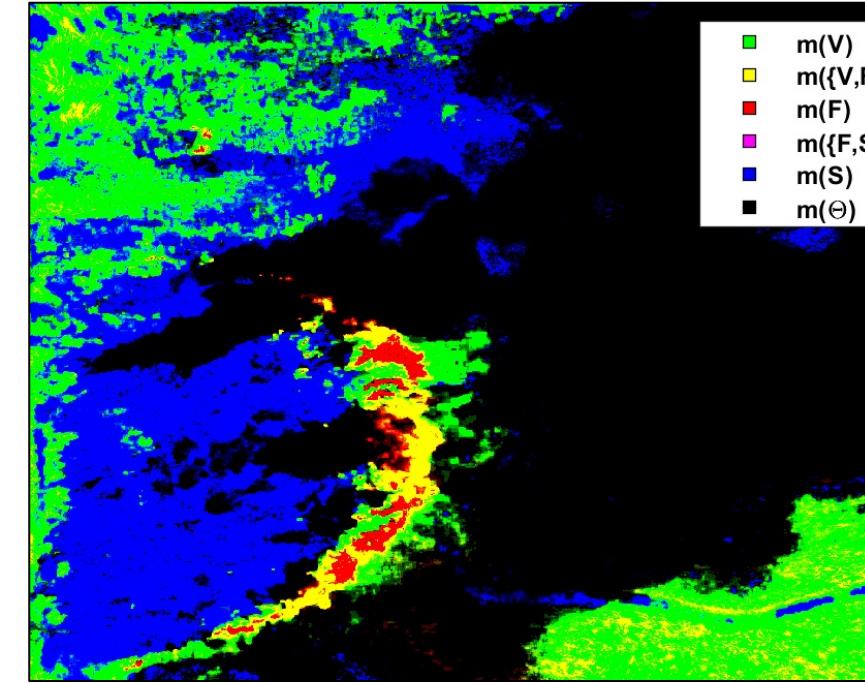
The Process



The Experts/Witnesses



The Evidential Estimate



*Vision Sensor: more complex!*

# Propositions for Evidential Fusion: Wildfire

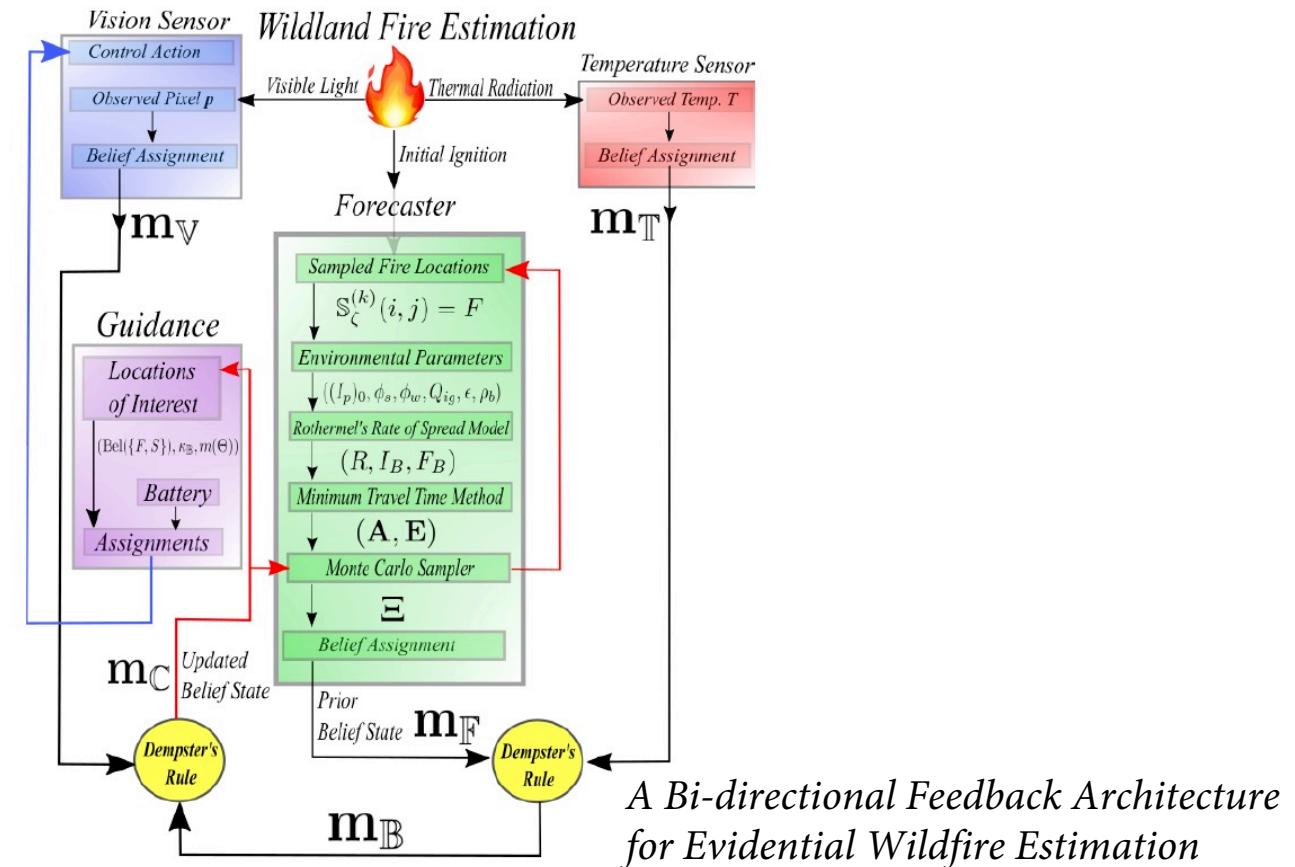
Wildfire frame of discernment:  $\Theta = \{ \text{Vegetation}(V), \text{Fire}(F), \text{Nonburnable}(S) \}$

Finite power set of propositions:  $2^\Theta = \{\emptyset, V, \{V, F\}, F, \{F, S\}, S, \{V, S\}, \Theta\}$

*Feasible* set of propositions:  $\mathcal{F} = \{V, \{V, F\}, F, \{F, S\}, S, \Theta\}$

Sources of information (will expand):

1. **Computational Forecaster:** based on Monte Carlo sims
2. **Thermal sensors:** indirect fire detection through increase in ambient temperature readings
3. **Airborne vision sensors:** pattern recognition to classify feasible propositions



# Belief Construction: Temperature Sensor 1/2

- A series of temperature readings over an activation period of  $\tau$  timesteps  $\mathbf{T}_h = [T_h^1, \dots, T_h^{(k)}, \dots, T_h^\tau]$  can be generated for each headfire spread rate within a given range  $R_h \in [R_{min}, R_{max}]$ .
- By cataloging the actual environmental states of the sensor's location  $\mathbb{S}^{(k)}(i, j)$  for each spread rate  $R_h$  and activated time  $T_h^{(k)}$ , a frequentist chance value of proposition  $A$  at each time can be obtained as

$$q_A^{(k)} = \frac{\sum_{i \in m} \sum_{j \in n} \{1 \mid \mathbb{S}^{(k)}(i, j) \subseteq A\}}{\sum_{i \in m} \sum_{j \in n} \{\mathbb{T}^{(k)}(i, j)\}}$$

- The array of mass numbers for each temperature sub-expert at each time can be constructed as

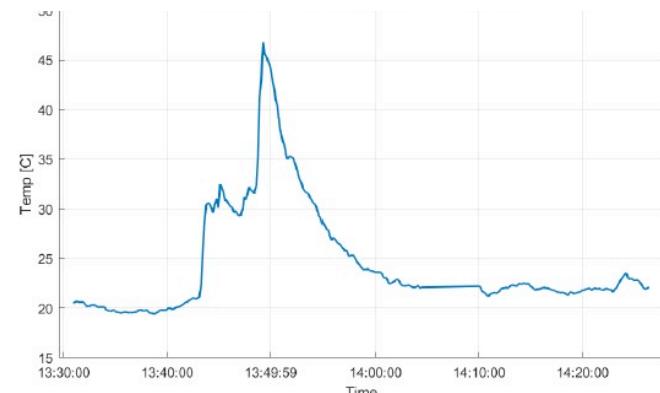
$$m_{\mathbb{T}-B}^{(k)}(A) = \begin{cases} q_B^{(k)}, & \text{if } A = B \\ 1 - q_B^{(k)}, & \text{if } A = \Theta \\ 0, & \text{otherwise} \end{cases}$$

- The internally-combined set of beliefs for the temperature agent  $\mathbf{Bel}_{\mathbb{T}}^{(k)}$  are computed as:

$$\mathbf{Bel}_{\mathbb{T}}^{(k)} = \mathbf{Bel}_{\mathbb{T}-V}^{(k)} \oplus \mathbf{Bel}_{\mathbb{T}-\{V,F\}}^{(k)} \oplus \mathbf{Bel}_{\mathbb{T}-F}^{(k)} \oplus \mathbf{Bel}_{\mathbb{T}-\{F,S\}}^{(k)} \oplus \mathbf{Bel}_{\mathbb{T}-S}^{(k)}$$

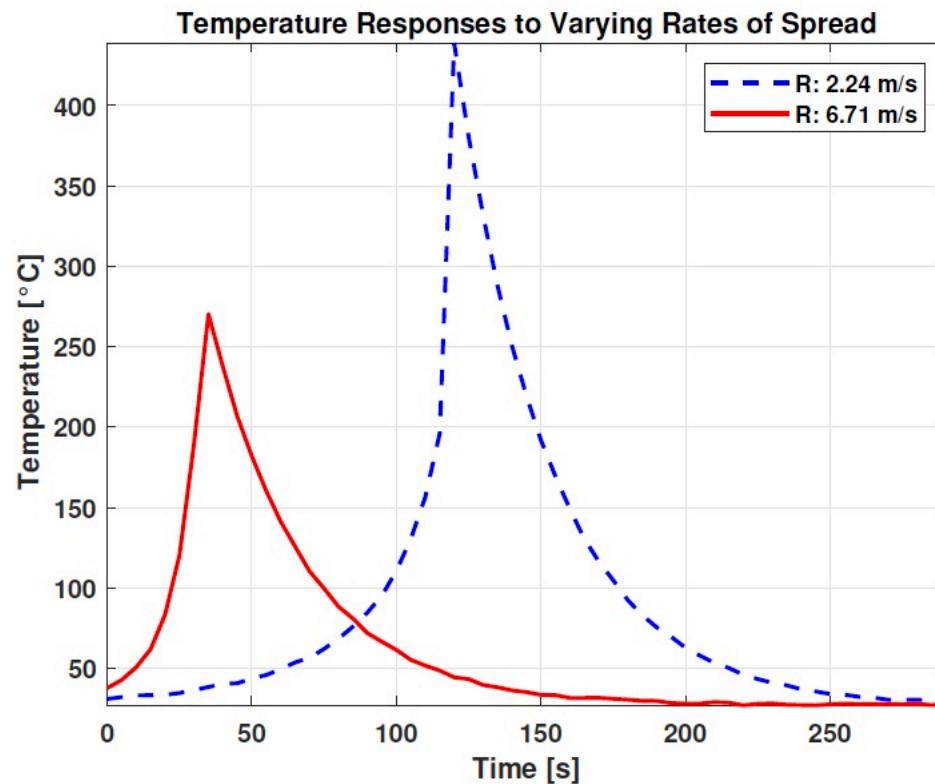


*MCP9808 Sensor posted shortly before a prescribed burn in Marion, Ohio.*

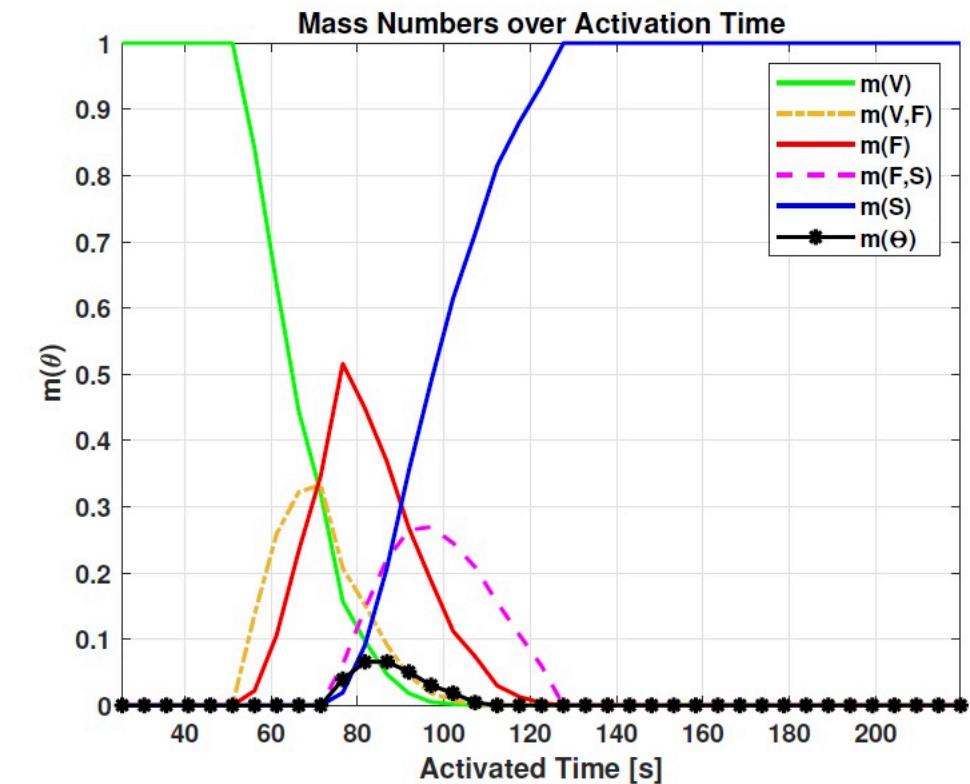


*Temperature reading of a sensor during the passing of a fire front.*

# Belief Construction: Temperature Sensor 2/2



(a)



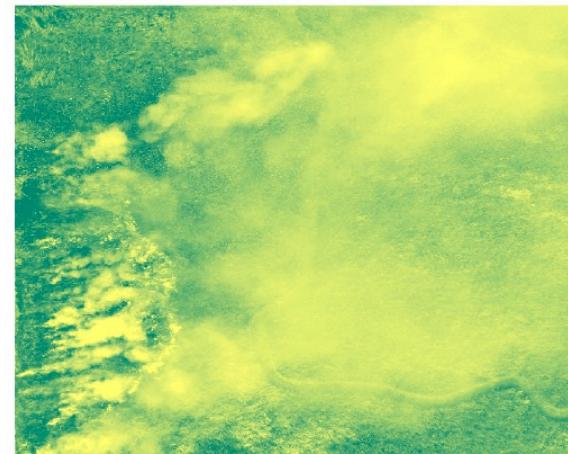
(b)

- (a) The temperature response when encountering a flame front with the highest and lowest spread rates.  
(b) The mass numbers for each proposition over the temperature agent's activation time.

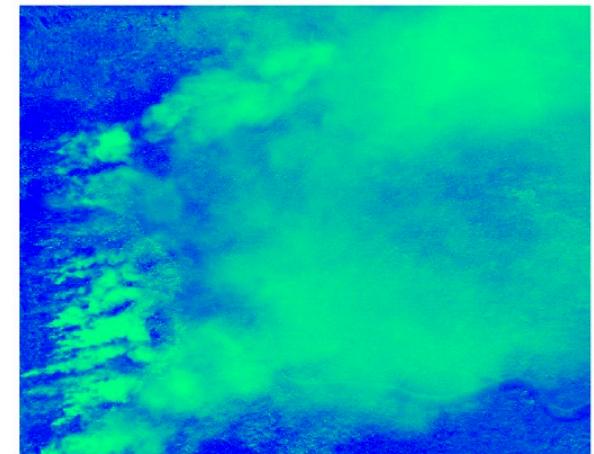
# Belief Construction: Vision Sensor 1/3



(a)

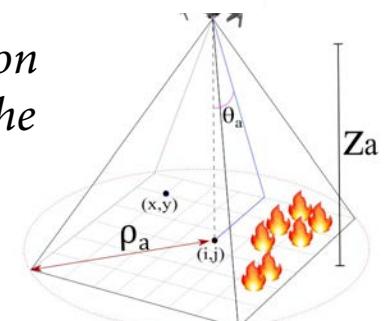


(b)



(c)

*The digital camera housed on each drone captures and stores the received visible light information through the additive RGB color model where every pixel at image location and time k contains the array of red, green, and blue intensity values*



# Belief Construction: Vision Sensor 2/3

- The pixel observation  $\mathbf{p}^{(k)}(x, y)$  can be cast as a “test point” computed to be some distance away from each state proposition distribution  $\mathcal{N}_A(\hat{\mathbf{p}}_A, \mathbf{C}_A)$  through the Mahalanobis distance metric:

$$\mathcal{M}_A^{(k)}(x, y) = \sqrt{(\mathbf{p}^{(k)}(x, y) - \hat{\mathbf{p}}_A)^T \mathbf{C}_A^{-1} (\mathbf{p}^{(k)}(x, y) - \hat{\mathbf{p}}_A)}$$

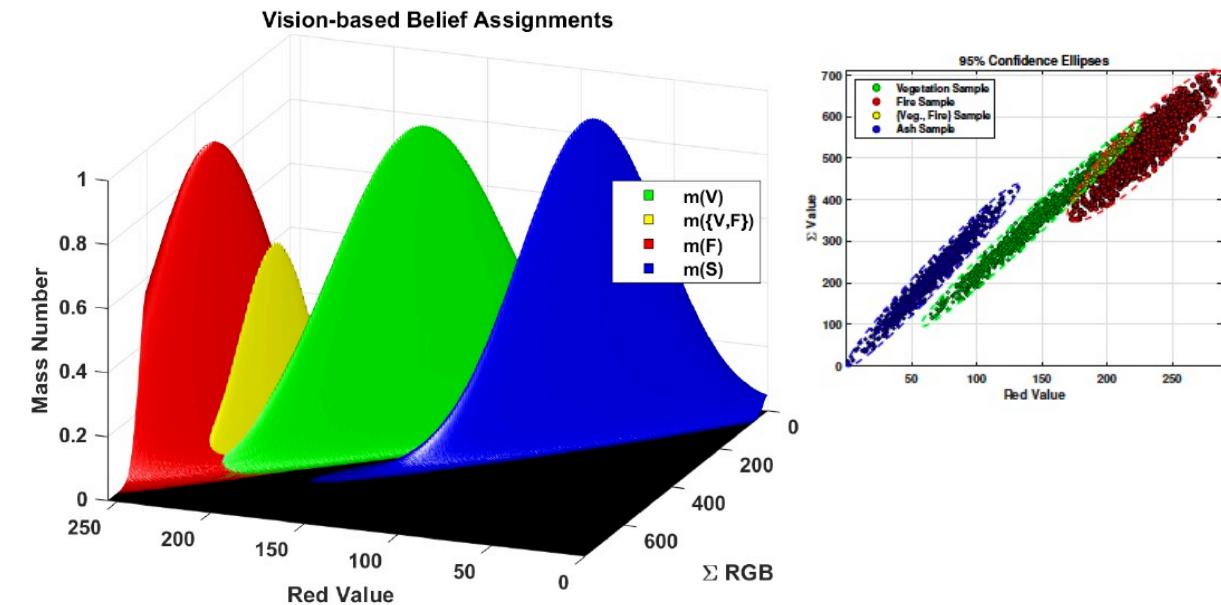
- This can be used to assign the mass number for each proposition  $A$ :

$$m_{V-B}^{(k)}(A) = \begin{cases} \exp(-C_B \mathcal{M}_A^{(k)}), & \text{if } A = B \\ 1 - \exp(-C_B \mathcal{M}_A^{(k)}), & \text{if } A = \Theta \\ 0, & \text{otherwise} \end{cases}$$

where  $C_B$  is some proposition-dependent positive scaling parameter.

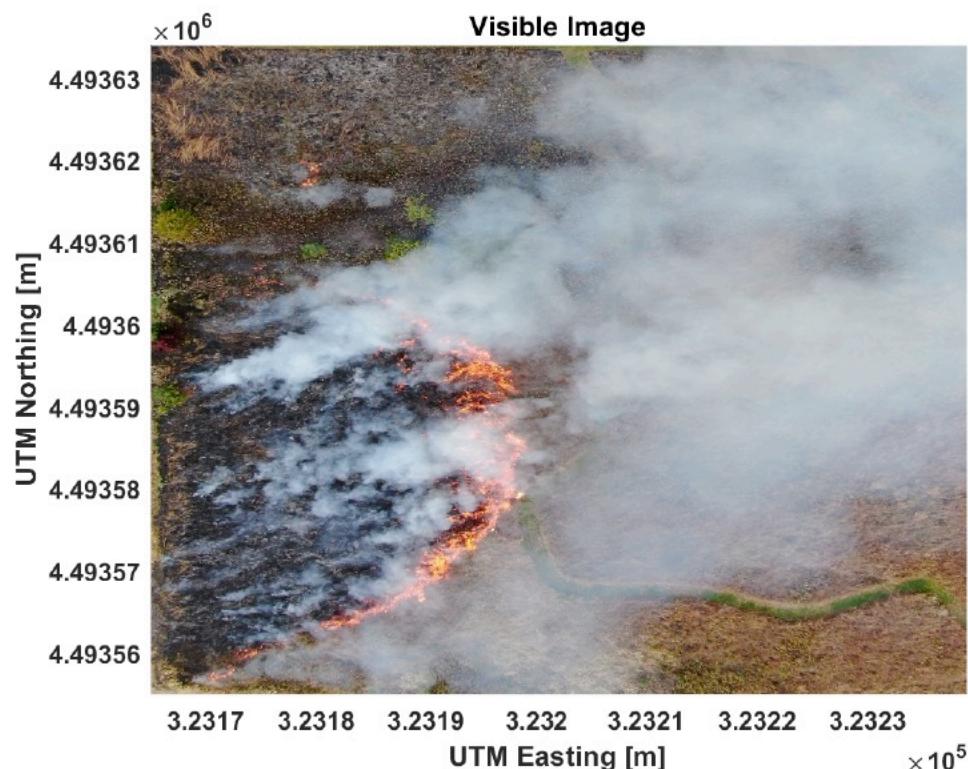
- Each mass number set  $\mathbf{m}_{V-A}^{(k)}$  are mapped to the associated belief set  $\mathbf{Bel}_{V-A}^{(k)}$  and combined through Dempster’s rule to yield the total belief set:

$$\mathbf{Bel}_V^{(k)} = \mathbf{Bel}_{V-V}^{(k)} \oplus \mathbf{Bel}_{V-\{V, F\}}^{(k)} \oplus \mathbf{Bel}_{V-F}^{(k)} \oplus \mathbf{Bel}_{V-S}^{(k)}$$



*Mass number surfaces for the four vision-based propositions*

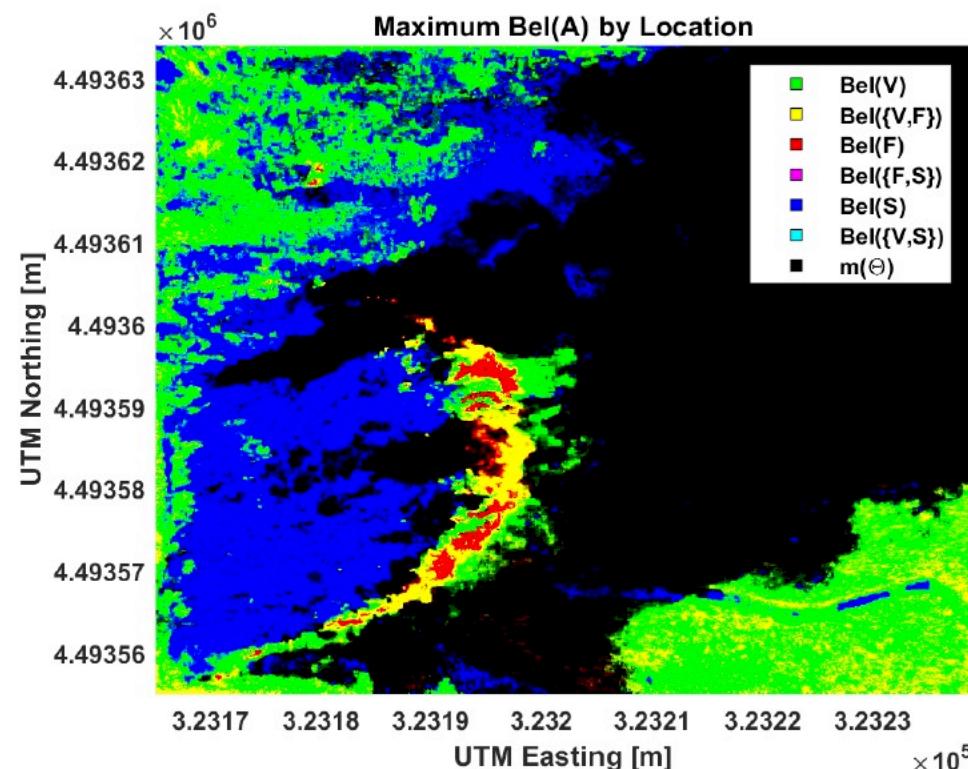
# Belief Construction: Vision Sensor 3/3



(a)

(a) Visible range image a fire front is captured with a DJI Mavic 2 drone at an altitude of 120 m during a prescribed burn event in Marion, Ohio.

(b) The proposition of highest belief that each pixel contains based on the vision agent's assignments using the color intensity data.

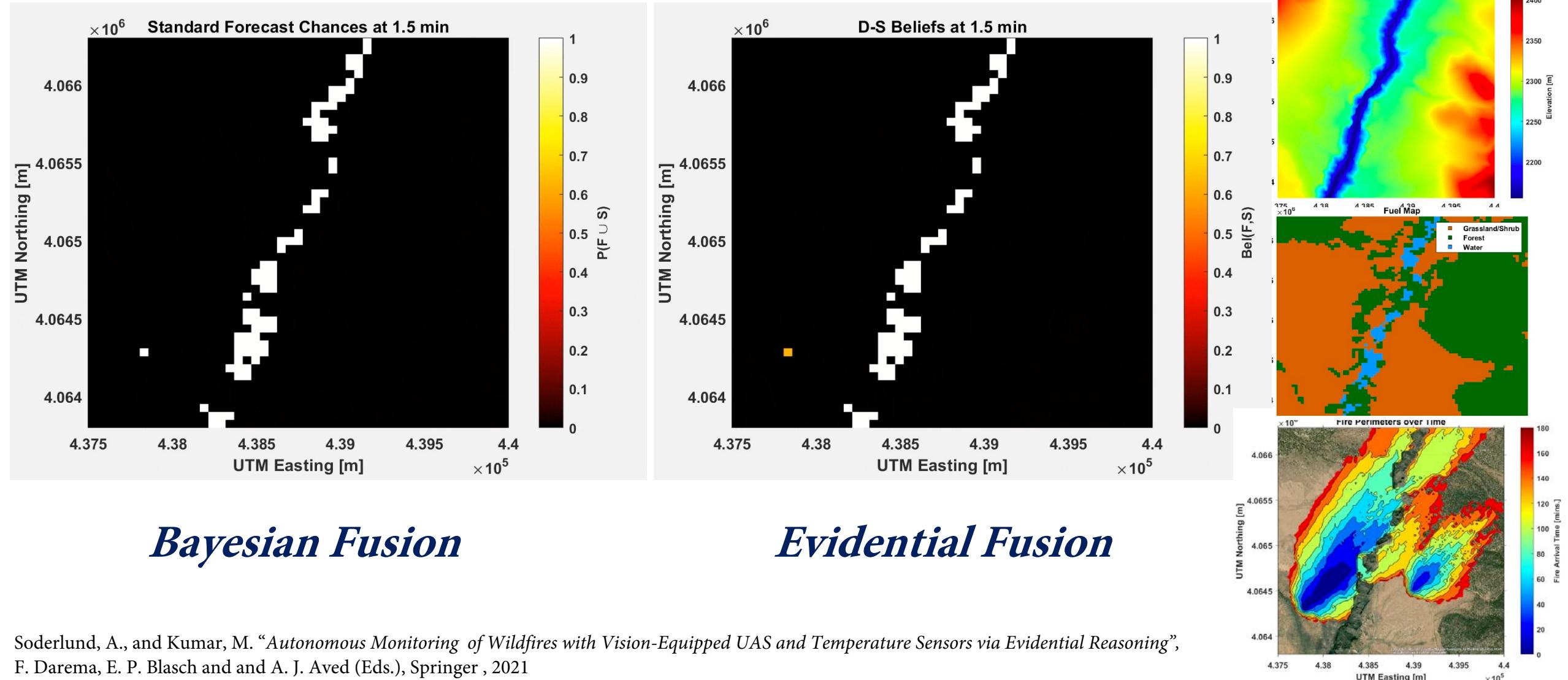


(b)

KEYS

- Recognition (ash, fire, grasslands)
- Ambiguity (interfaces)
- Ignorance (smoke)

# Case Study: Fire in Taos, New Mexico



*Bayesian Fusion*

*Evidential Fusion*

# Evidential Fusion: Summary

- An *evidential-expert system* attempts to emulate the decision-making of human beings.
- Decision-making requires a *knowledge base*. When our models of a domain are incomplete, we are forced to represent our uncertainty through epistemic beliefs. DST is one of a myriad of frameworks to do this - the choice is up to the designers.
- The overall approach given here is to i.) form agents' belief models, ii.) combine agents' beliefs and iii.) make decisions regarding those beliefs.
- Why DST was adopted to represent fire belief:
  1. The inputs required for precise forecasts are often not available.
  2. Gathered evidence is ambiguous. This is a limitation of our sensors and the interpretation of their data.
  3. Agent conflict is beneficial to the fire problem. Resolution leads to better forecasts in the future!
- Informed decision-making relies on data and models that may be incomplete. The idea of trust in an autonomous system becomes important here. As an example, trust is quantified through in the DST framework via a "discount factor" prior to evidence combination. More work is needed!

# Summary and Lookahead

- Unstructured uncertainty presents special challenges to mission planning and execution
- Use cases in remote, hazardous, “broken” environments present the right context for autonomy
- Learning the environment requires analysis of new modalities of uncertainty, such as **ignorance** and **ambiguity**
  - The evidential framework allows us to perform information fusion despite sensor conflict and bad priors
  - There is need to develop additional mathematical constructs beyond Kolmogorov’s axiomatic framework
  - There is scope to combine evidential reasoning with learning tools to discover new belief functions
- Path planning in a dangerous, unstructured environment presents **path dependent “loading” constraints**, e.g., critical exposure, temperature rise, etc.
- Resource constrained path planning is NP hard. Chance-resource constrained problems have not been considered.
- Graph search offers a suboptimal solution in less time, and allows enforcement of kinematic constraints
  - A new backtracking hybrid A\* algorithm has shown excellent results in early tests
  - Current research focuses on stopping criteria and alternate methods for constraint enforcement
- UAS mission planning in unstructured uncertainty (e.g., wildfire) requires many coupled problems to be solved together: much more work is needed on many fronts!

Thank You!