Mechanical and Aerospace Engineering

THE LABORATORY FOR AUTONOMY IN DATA-DRIVEN AND COMPLEX SYSTEMS

Some New Challenges in Chance-Constrained Path Planning for UAS in Unstructured Uncertainty (Part II: Path Planning)

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THE OHIO STATE UNIVERSITY

LADDCS TEAM

Lab Team	Collaborators: Academia	Collaborators: Labs/Govt	Collaborators: Industry
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Current MS Anna Lebron Alan Cortez			
Current UG Alexandra Mangel Bryce Ford Joey Caley			

LADDCS (Lab) TEAM


Alex Soderlund (May, 2020)


Rachit Aggarwal (May, 2021)


Bander Jabr (August, 2021)


Indranil Nayak


Andrew VanFossen


Sriram Narayanan


Anna Lebron


Alan Cortez

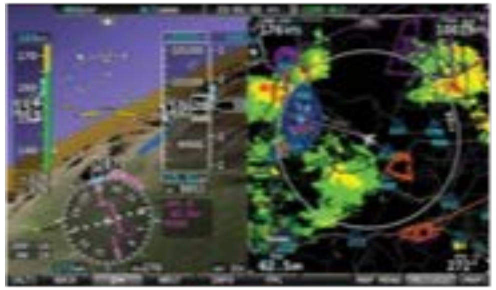

Bryce Ford

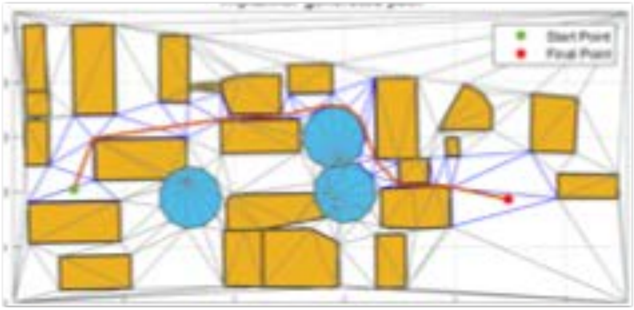

Joey Caley

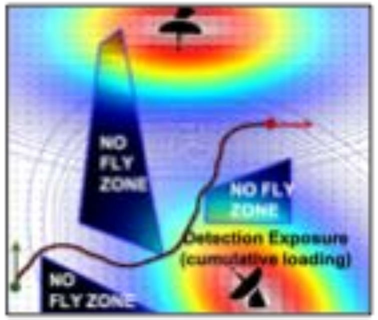

Alexandra Mangel

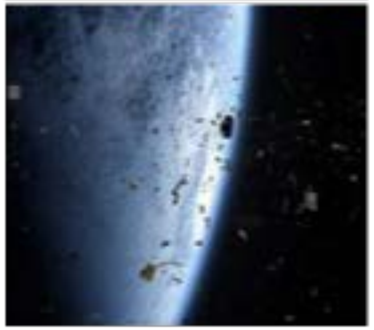

Mrinal Kumar

LADDCS PROFILE









forecasting

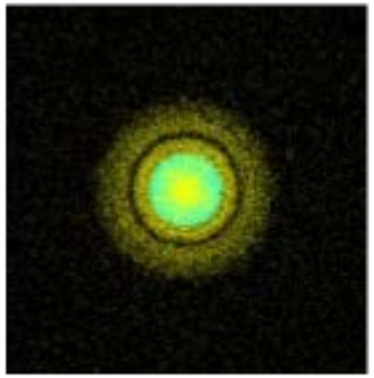
- closed-loop MC
- probability of failure
- threat assessment
- asset sustainment
- black-box models

missions in adversity

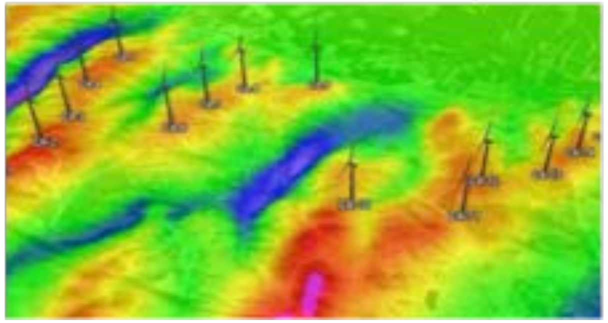
- chance-constrained path planning
- integral constraints
- evidential sensor fusion
- platform dev
- wildfire missions

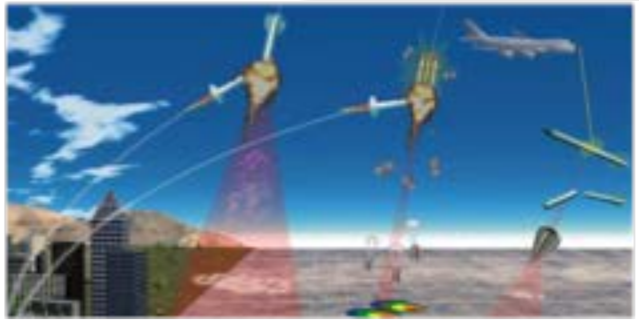
data-driven systems

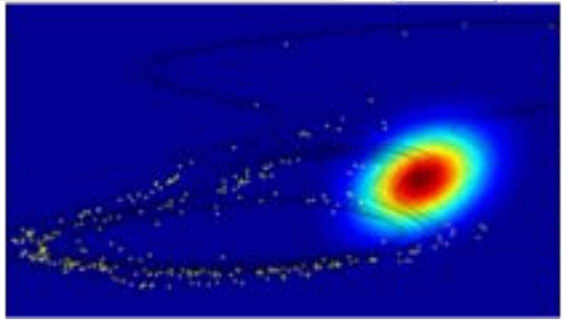
- DMD + Koopman theory
- learning dynamics
- situational awareness
- space explosions/ plasma mechanics

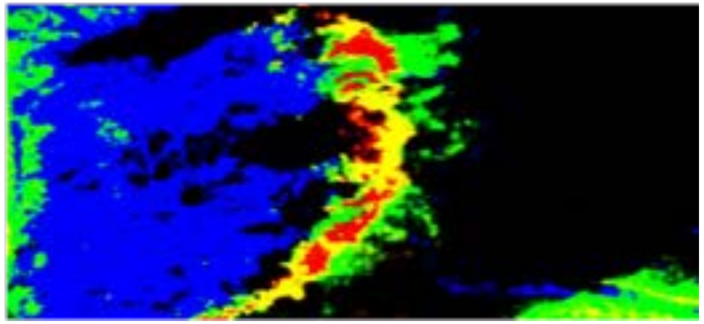


- autonomy
- space









TALK OVERVIEW

1. UAS Missions in Unstructured Uncertainty
   - A couple scenarios

2. Obstacle Characterization: Evidential Sensor Fusion
   - Traditional Bayes' formalism
   - The Dempster-Shafer alternative
   - Sensor belief modeling – temperature & vision sensors (examples)
   - Application to real fire: case study
   - Summary

3. UAS Path-Planning
   - Chance-constraints
   - Path-dependent resource/loading constraints
   - Graph Search – Hybrid A*
   - Backtracking
   - Numerical studies
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4. Wrap up and lookahead

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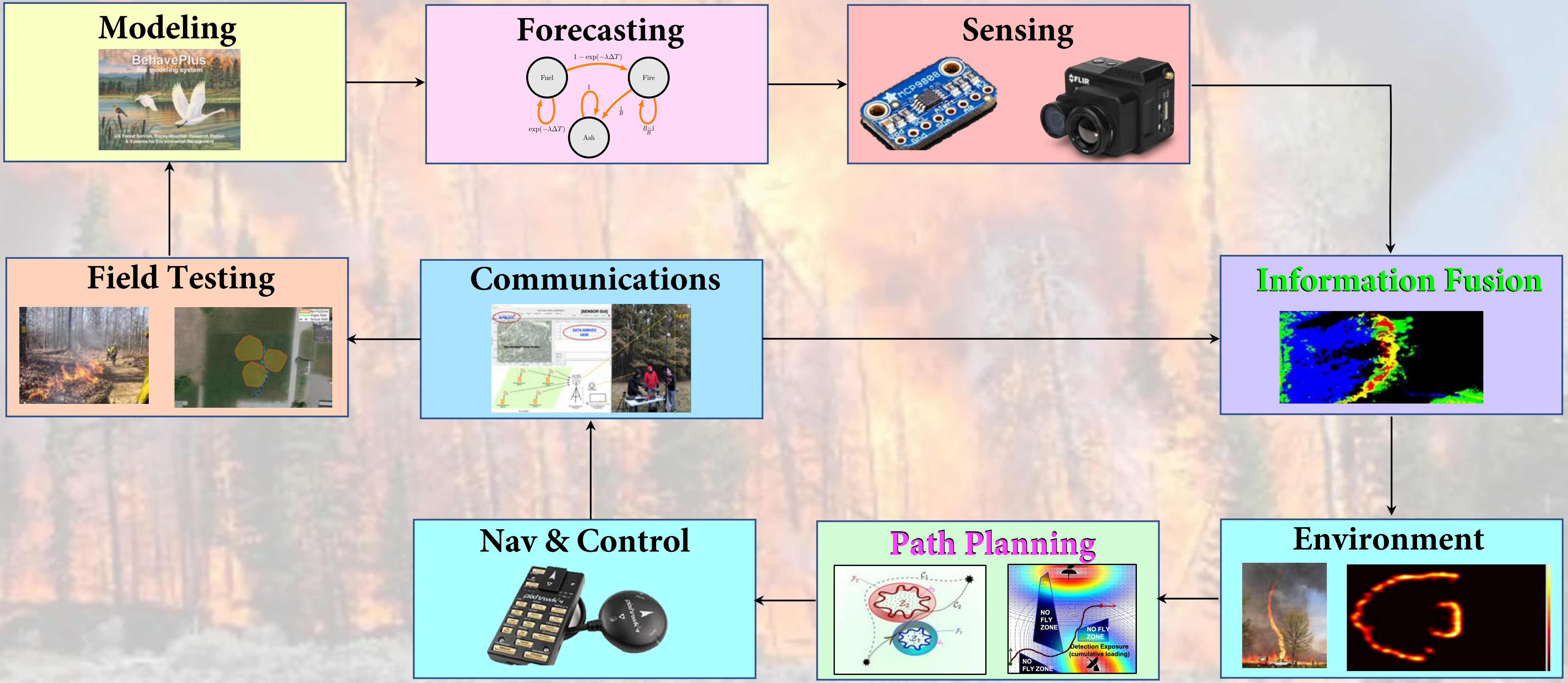
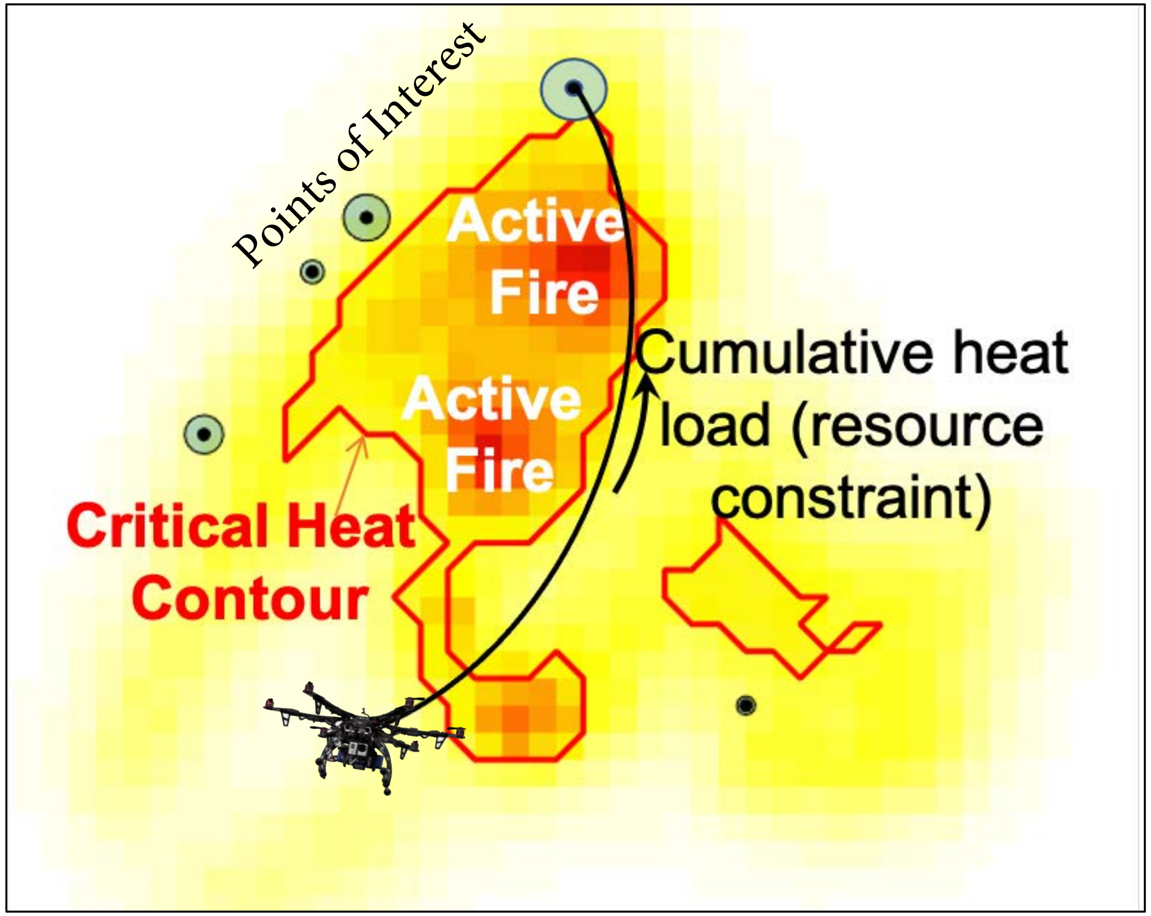
UAS MISSION PLANNING

- Missions in *unstructured uncertainty:* • *poor models* • *sensing conflict and anomalies* • *"broken", dynamic environment*



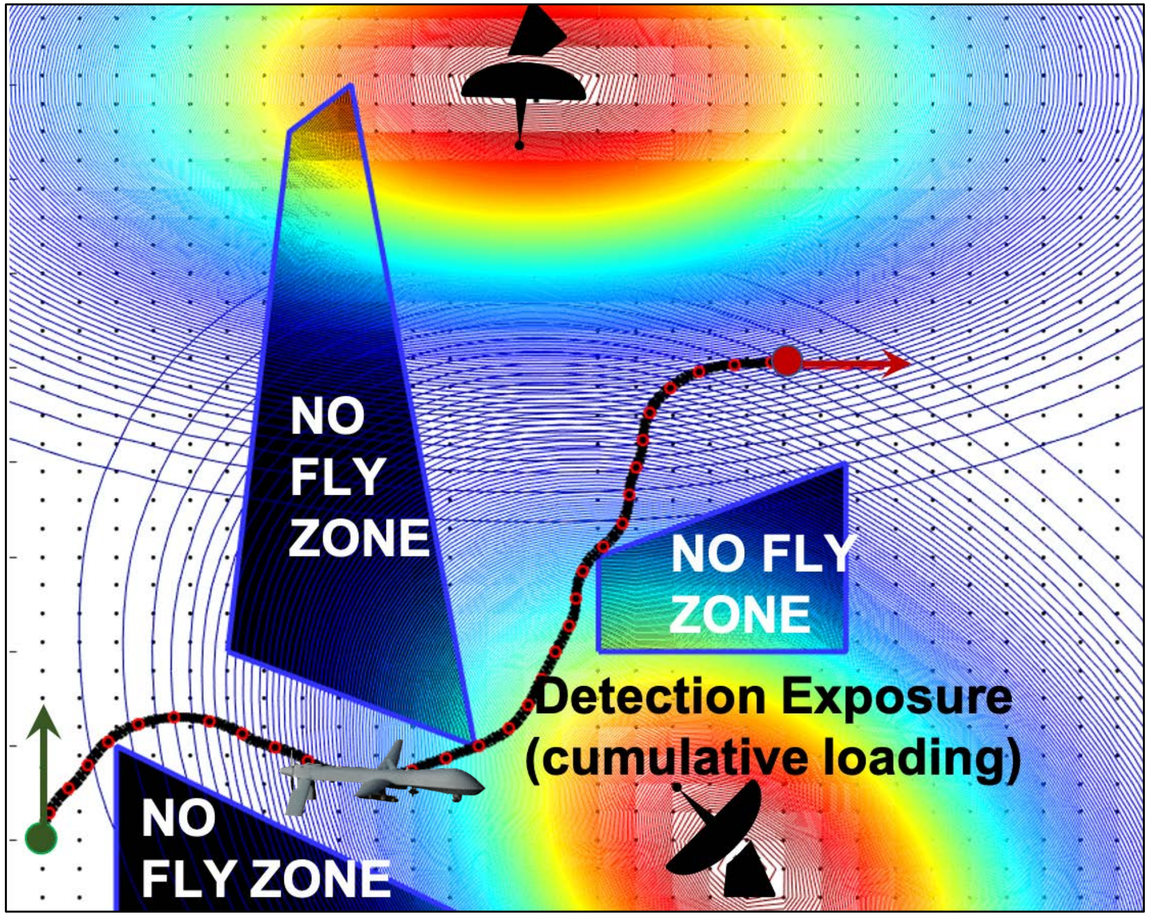
UAS MISSION PLANNING: Intangible Obstacle Scenarios

Scenario 1.



Flight above active fire: UAS safety constraints

Scenario 2.



Flight through enemy territory: constraint on radar detection, tracking and eventual attack

TALK OVERVIEW

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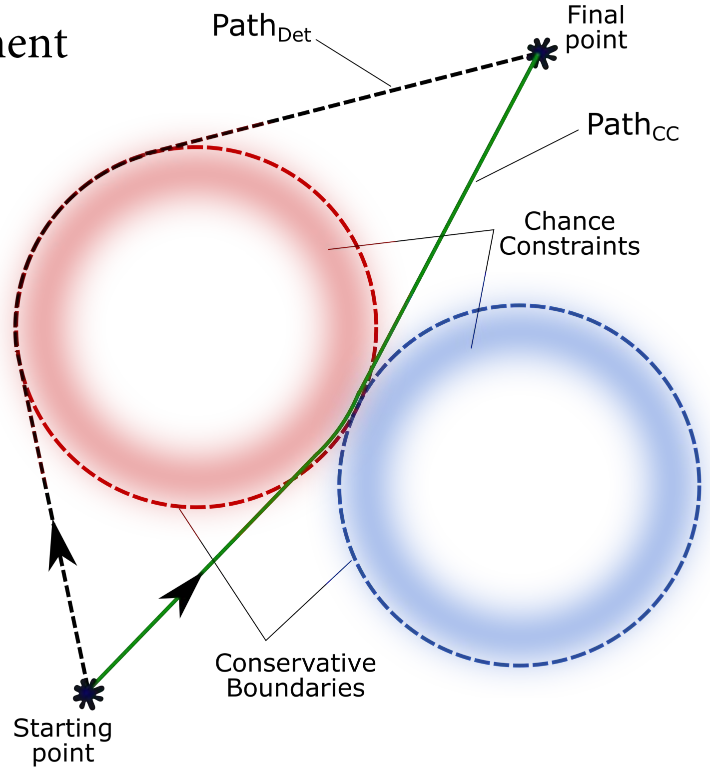
Chance-Constraints

CONTEXT Rapid path planning for UAS in an unstructured, uncertain environment to reach identified goal in minimum time

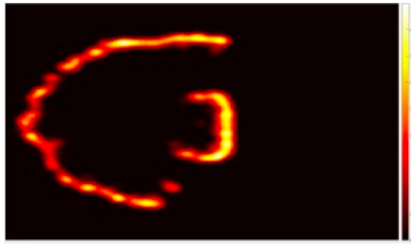


Where chance-constraints make sense

Challenges

- Poorly understood obstacles in the environment
- Deterministic characterization may require "robust approach" (worst case) – which may crowd out solution space or cause lengthy diversions

1. Notionally, it is a better idea to model such obstacles as probabilistic entities and assimilate the uncertainty into path optimization
2. This seems like a good idea for wildfires because the "obstacles" are intangible, e.g., heat-flux contours
3. Allows the decision maker to *assume* mission-appropriate risk



Chance-Constraints

Allocated or Assumed Risk

Structure of a chance-constraint:

$$P\left[\underbrace{F(t,\mathbf{y},\mathbf{u},\boldsymbol{\xi}) \in \psi}_{\text{Failure event}}\right] < \delta,\ \ \boldsymbol{\xi} \sim \mathcal{W}_{\boldsymbol{\xi}}$$

Uncertain parameter(s)

Failure threshold

Chance-Constrained Optimal Control Problem:

Objective Function

$$\min_{u} J = \underbrace{\Phi(\mathbf{y}(t_0), t_0, \mathbf{y}(t_f), t_f)}_{\substack{\text{Mayer Form} \\ \text{Terminal Cost}}} + \underbrace{\int_{t_0}^{t_f} L(\mathbf{y}(t), \mathbf{u}(t), t)dt}_{\substack{\text{Lagrange Form} \\ \text{Path Cost}}}$$

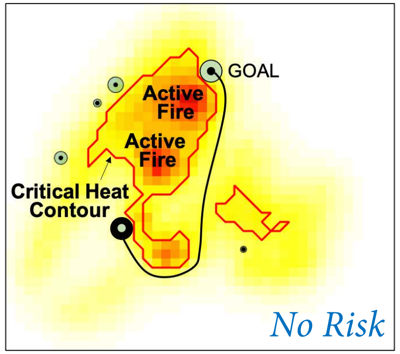
subject to:

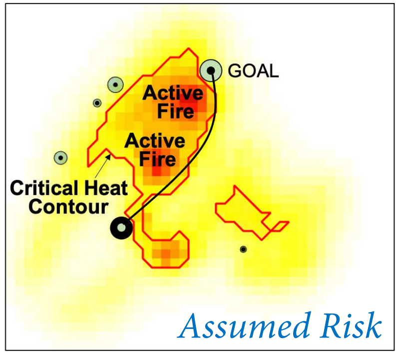
Dynamics $\dot{\mathbf{y}} = \mathbf{f}(\mathbf{y}(t), \mathbf{u}(t), t)$

Path Constraints $\mathbf{c}_{\min} \leq \mathbf{c}(\mathbf{y}(t), \mathbf{u}(t), t) \leq \mathbf{c}_{\max}$

Boundary Constraints $\mathbf{b}_{\min} \leq \mathbf{b}(\mathbf{y}(t_0), t_0, \mathbf{y}(t_f), t_f) \leq \mathbf{b}_{\max}$

Chance-Constraints $P_j(\mathbf{g}_{\min} \leq \mathbf{g}(\mathbf{y}(t), \mathbf{u}(t), t; \xi) \leq \mathbf{g}_{\max}) \geq 1 - \epsilon_j, \quad j = 1, \ldots, n$



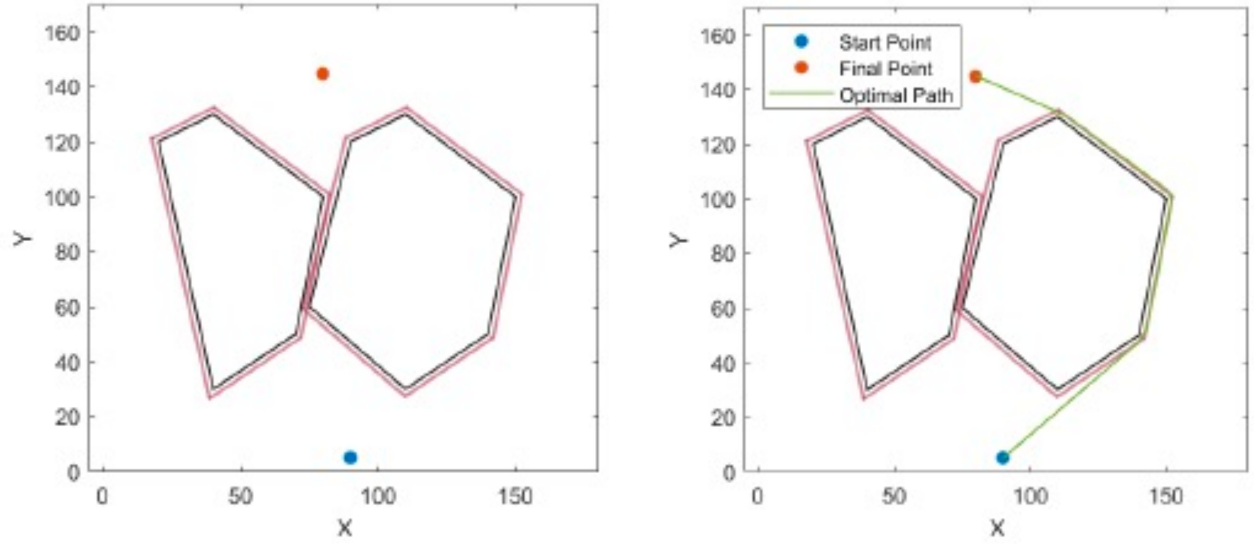


Aggarwal, R, Soderlund, A., and Kumar, M. "*Multi-UAV Path Planning in a Spreading Wildfire*". In Guidance, Navigation and Control Conference at Scitech Forum, (Virtual), Jan 11-15, 2021.
Finalist: Best Paper of 2021 GNC)

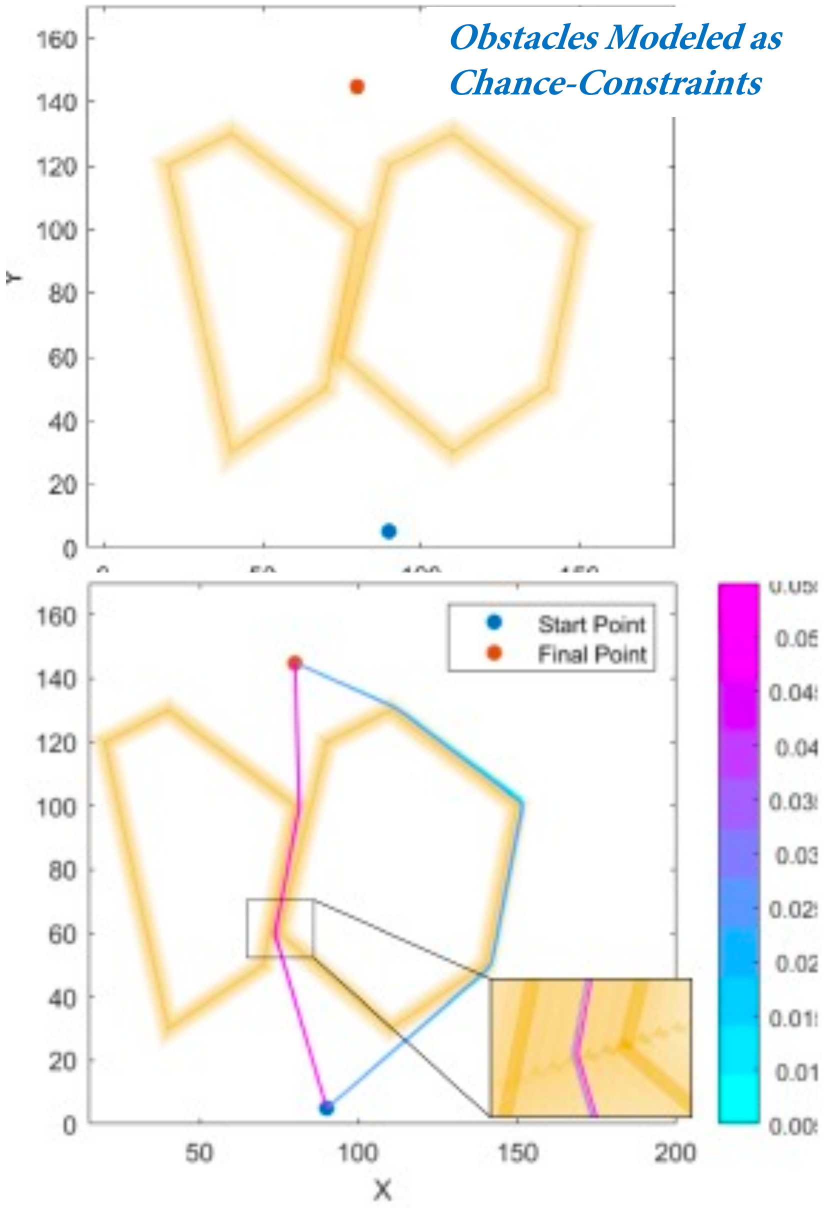
Chance-Constrained Optimal Control: Pseudospectral Solution

Recent work: Toy Problem

- Compare against "robust" (worst-case) formulation
- Transcribe CCOCP to an NLP via LGR pseudospectral collocation
- Use MCMC/other sampling to evaluation CC
- Use Triangulation methods (Kallman et al.) to determine initial guess
- Solve using IPOPT (GPOPS II (Rao et al.))
- ***Keyhole trajectories emerge: <u>a risk versus reward decision must be made</u>***



Worst Case Representation of Obstacles



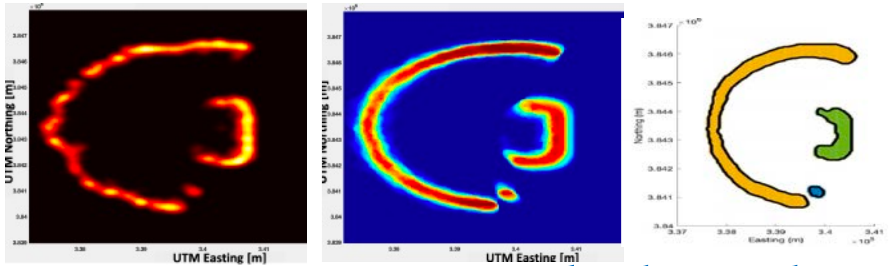
Obstacles Modeled as Chance-Constraints

Keil, R., Kumar, M. and Rao, A. *Warm Start Method for Solving Chance Constrained Optimal Control Problems Using Biased Kernel Density Estimators*, J. Dyn. Sys, Meas. & Control, Vol. 143, Issue 12, Dec 2021

Chance-Constrained Optimal Control: Pseudospectral Solution

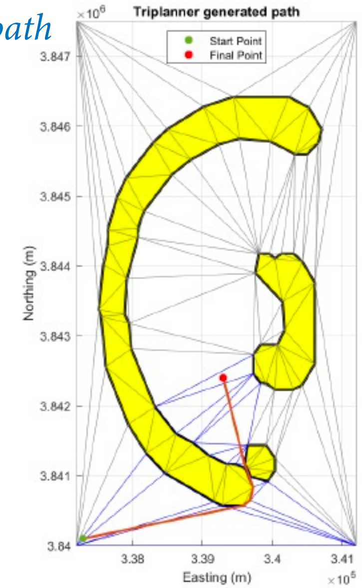
Recent work: Wildfire

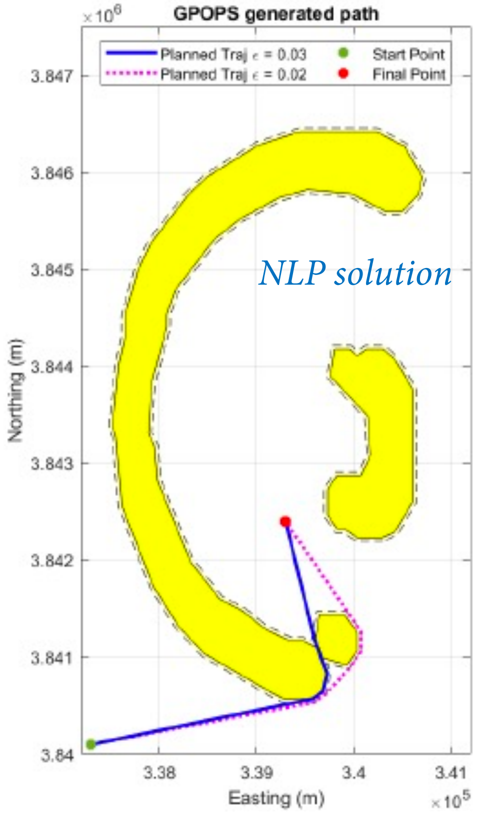
- Use clustering to identify heat-flux contours
- Transcribe CCOCP to an NLP via LGR pseudospectral collocation
- Use MCMC/other sampling to evaluation CC
- Use Triangulation methods (Kallman et al.) to determine initial guess
- Solve using IPOPT (GPOPS II (Rao et al.)
- ***Keyhole trajectories emerge: risk v reward***
- Repeat as obstacles are updated



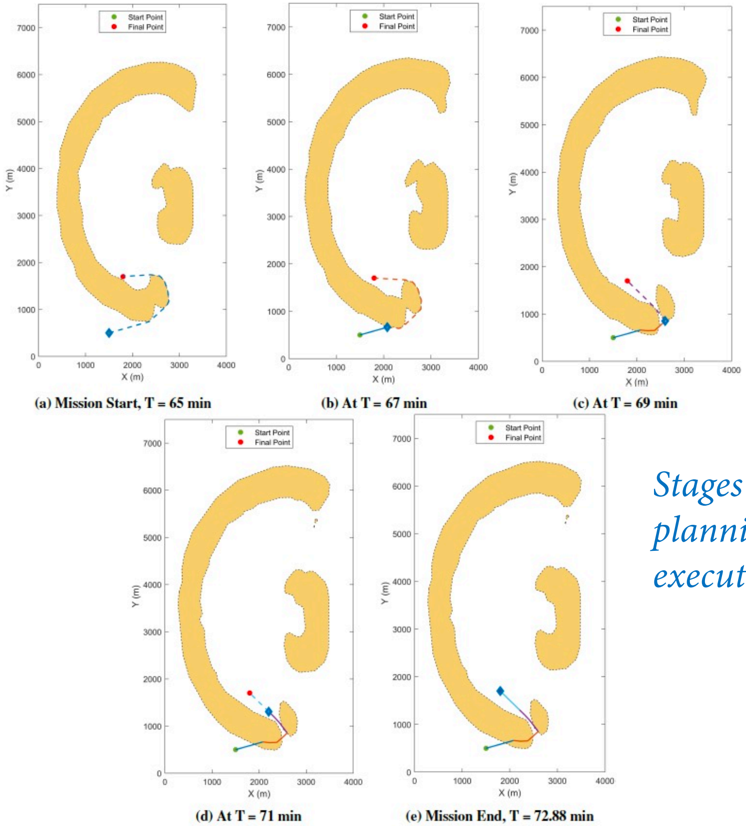
Obstacle ID via clustering

Initial guess path





NLP solution



Stages of mission planning and execution

Aggarwal, R., Soderlund, A., Kumar, M. and Grymin, D. *Risk-Aware Path Planning for Unmanned Aerial Systems in a Spreading Wildfire*. AIAA Journal of Guidance, Control and Dynamics, under review. Also in 2020 ACC (Virtual Conference)

Path Dependent Integral Chance-Constraints 1/3

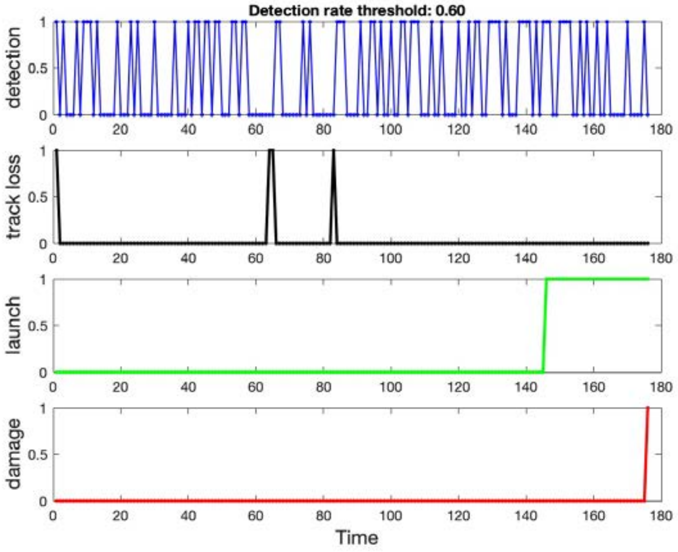
Scenario 1. Flight through enemy territory with uncertain radar locations*

$$P(\delta_e(k) = 1) \leq \epsilon_{\mathrm{detect}}$$ ← pointwise CC

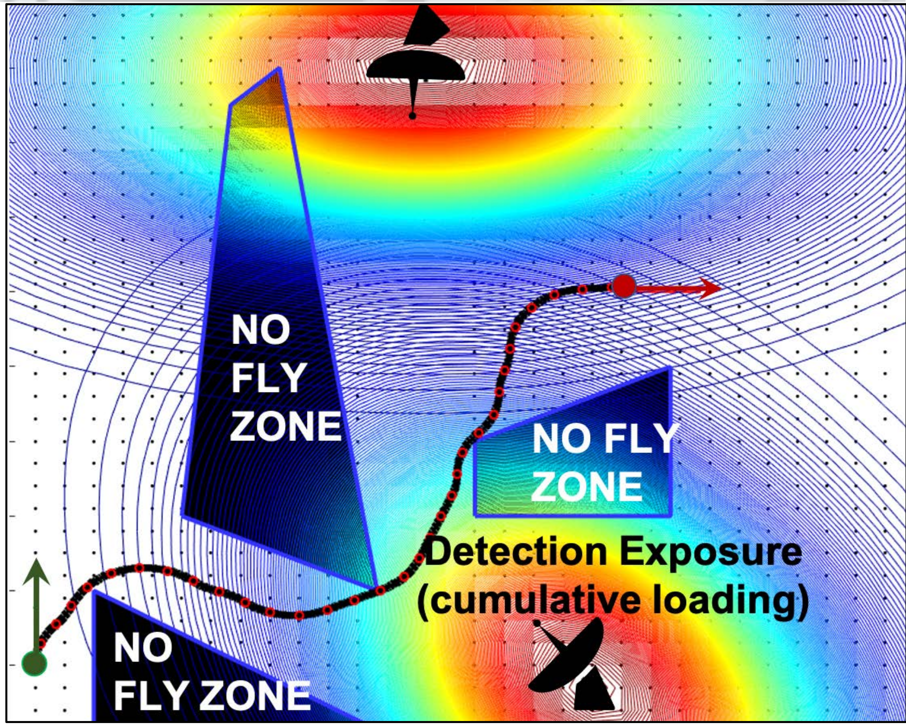
$$P(\delta_l(k) = 1) \leq \epsilon_{\mathrm{launch}}$$

$$P(\delta_d(k) = 1) \leq \epsilon_{\mathrm{damage}}$$

"accumulating" (loading) CC



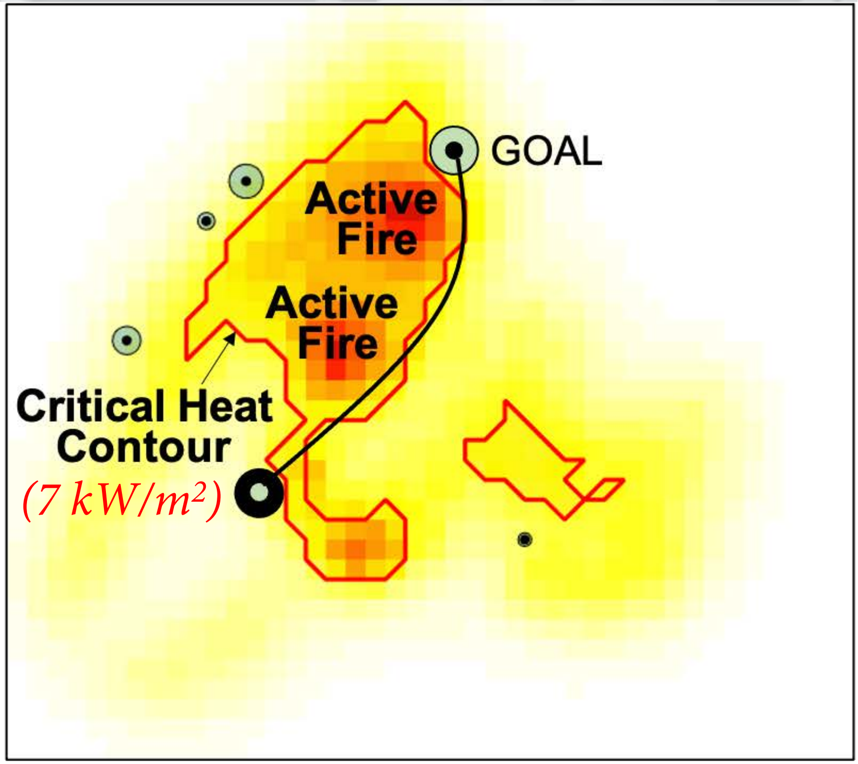
Example detection – launch – damage cascade of events



*Atif Chaudhry, Kathy Misovec, and Raffaello D'Andrea. *Low observability path planning for an unmanned air vehicle using mixed integer linear programming*. In 43rd CDC, Bahamas, December 14-17, 2004

Path Dependent Integral Chance-Constraints 2/3

Scenario 2. Flight over active wildfire*



Deterministic version of path dependent loading constraint

$$\underbrace{\text{“}\delta(t) = 1\text{”}}_{\text{“damage”}} \equiv \underbrace{\int_0^t \underbrace{\mathcal{F}_{\mathcal{L}}(\tau, \mathbf{s}(\tau), u(\tau))}_{\text{rate of damage}}\, d\tau}_{\text{path load}} \leq \underbrace{\mathcal{L}^\star}_{\text{loading limit}}$$

where,

$$\mathcal{F}_{\mathcal{L}}(t, \mathbf{s}(t), \mathbf{u}(t)) = \frac{A}{mc_p}\phi(\mathbf{s}(t)) \qquad \text{(normalized heat-flux)}$$

Integral chance-constraint (path dependent)

$$P\left(\delta(t) = 1\right) < \epsilon$$

*Bryce Ford, Rachit Aggarwal, Mrinal Kumar, S. Gupta Manyam and David Casbeer. *Backtracking Hybrid A* for Resource Constrained Path Planning*. In GNC @ Scitech Forum, San Diego CA, January 3-7, 2022

Path Dependent Integral Chance-Constraints 3/3

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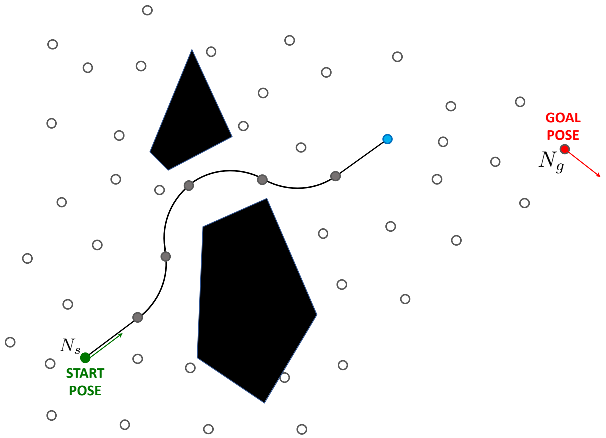
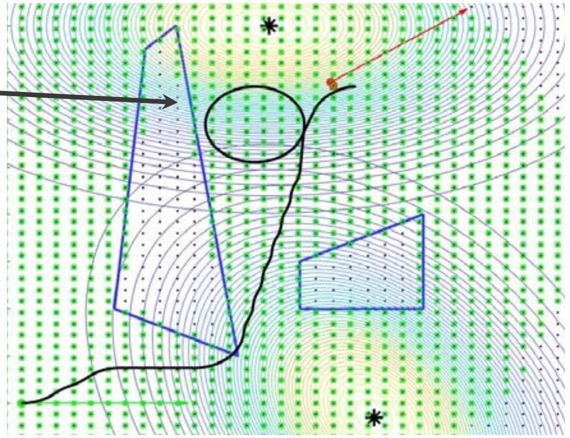
- Many problems can be posed as a planning problem with accumulating/loading constraints, e.g., cumulative noise, heating, concentration, spending
- These are generally studied in the literature as *resource constrained optimization problems* (mainly in the operational research community)
- Resource constrained optimization is known to be NP hard
- MILP is the state-of-the-art and can only solve “small scale” problems
- We will set up **graph search** for faster, suboptimal solutions

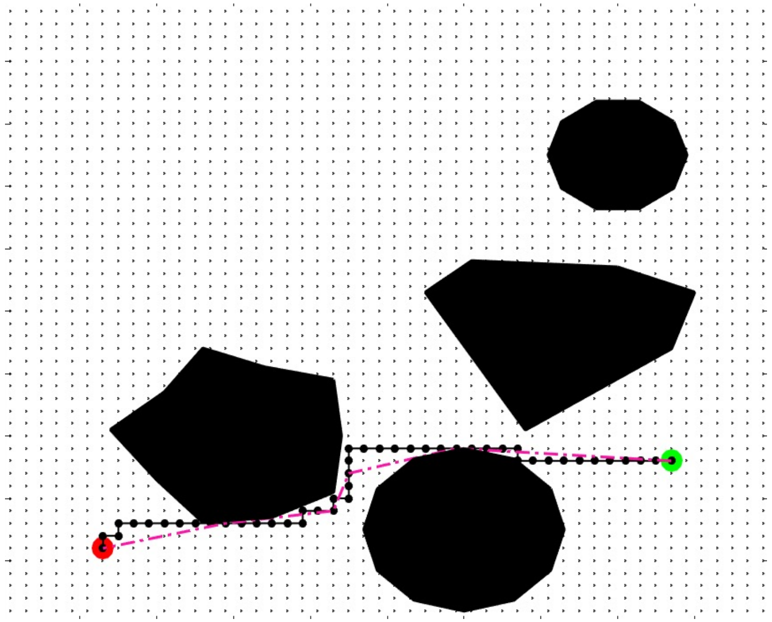


Hybrid A* Graph Search

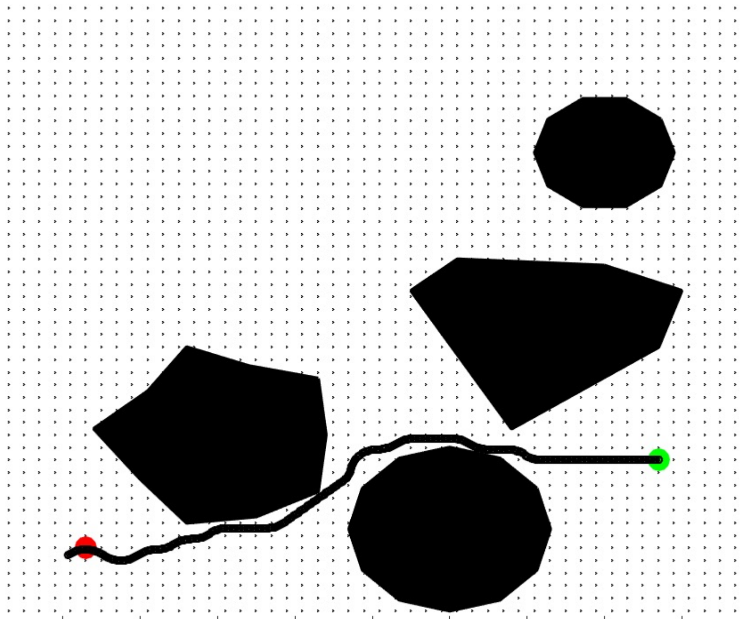
- Graph search is **fun** – but we must consider real life vehicle challenges (non-holonomic constraints, min turn radius)

- Starting point of this work: Hybrid A* Search
  - search over pose-space (x, y, θ): thus, computationally burdensome
  - candidate paths are the primitives of a Dubins vehicle: thus, automatically account for min turn radius and kinematic constraints
  - results in smoother paths than A*
  - satisfies pose constraints: important for fixed wing aircraft





$\mathscr{A}^*$



$\mathscr{H}\,\mathscr{A}^*$

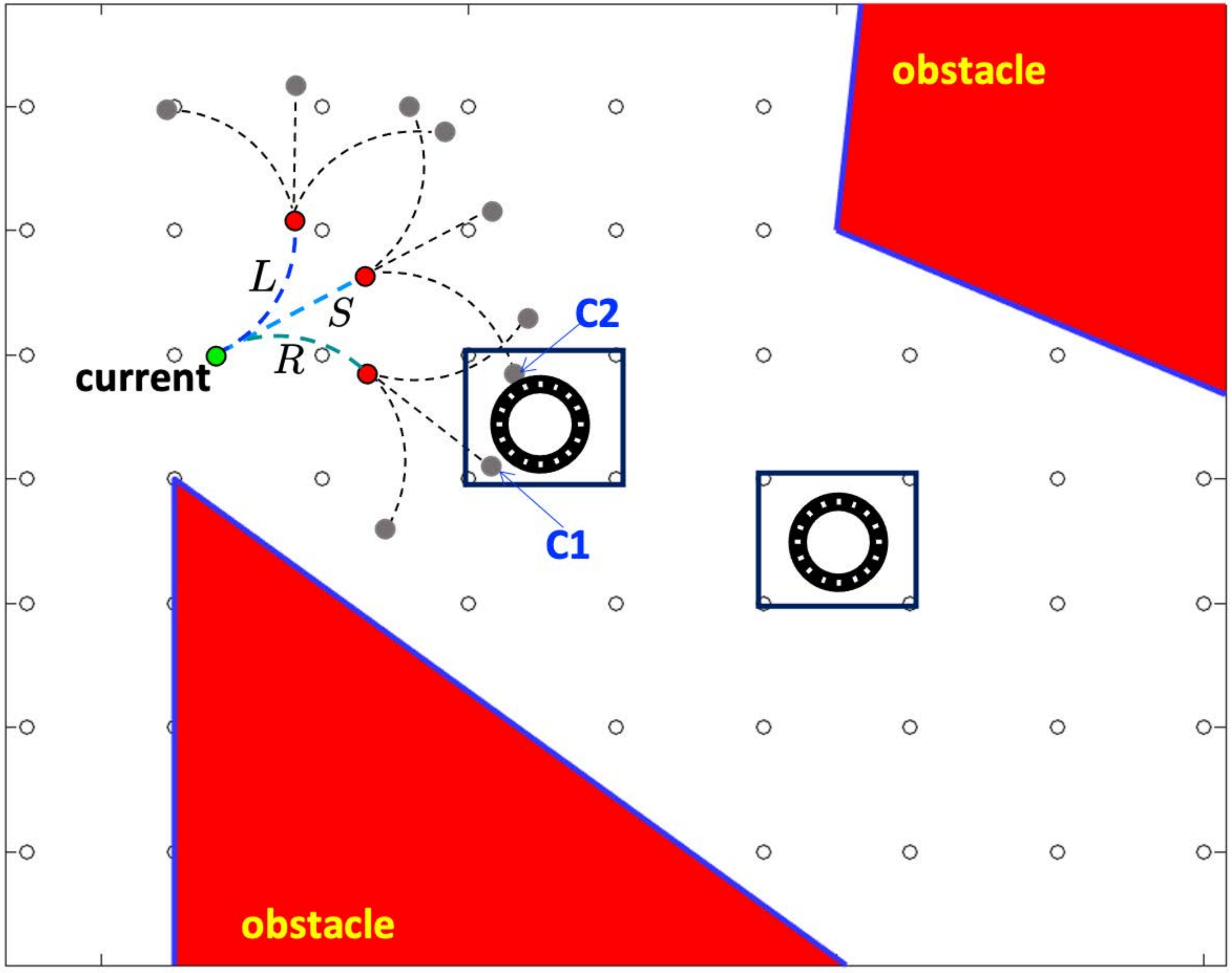
Hybrid-A* Graph Search

How does HA* Work?

- 3D Grid (x,y,θ)
- Dubins motion primitives
- Graph evolution proceeds as in A*
- Obstacles and constraints enforced through node testing

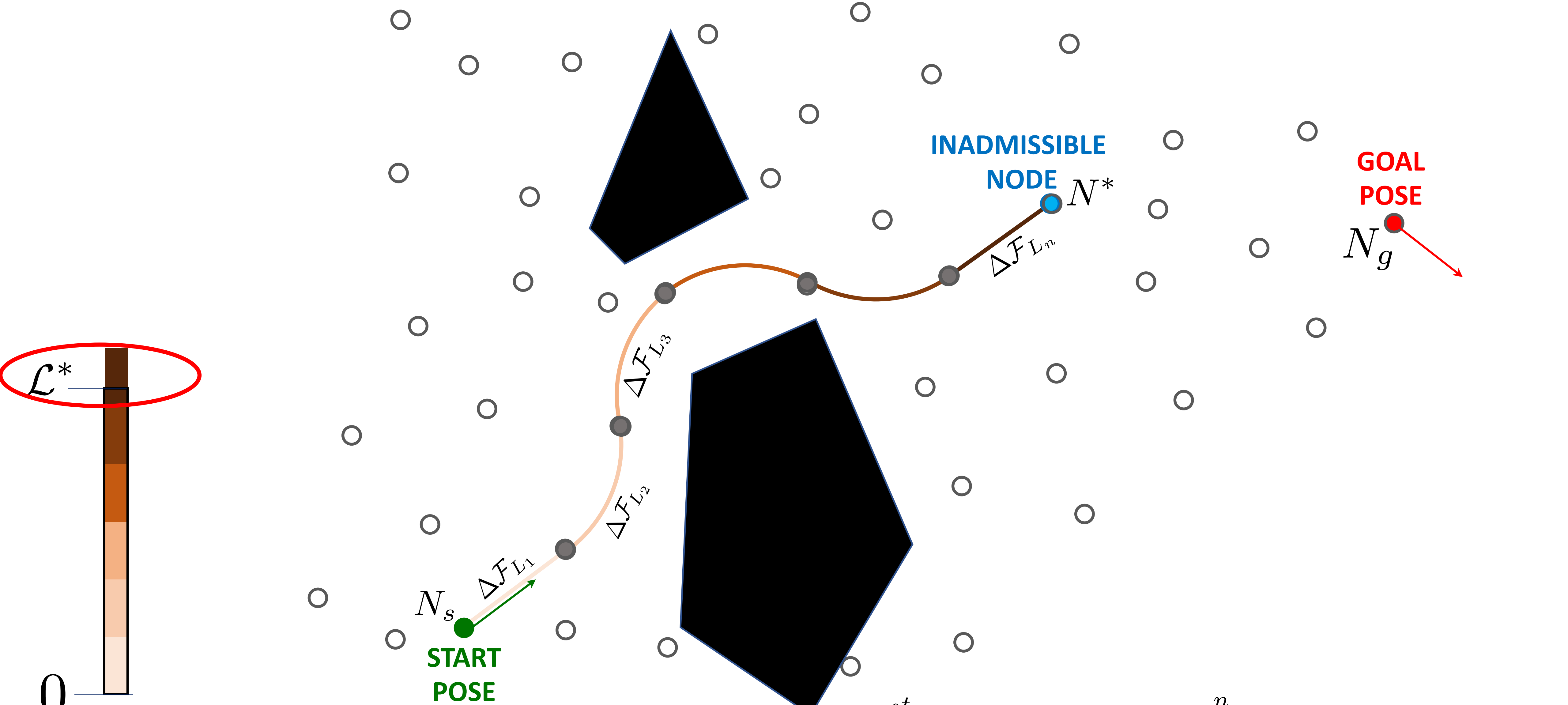
Algorithm 1 $\mathscr{H}\,A^*$ Search with Integral (Path-Load) Constraints

```
Require: O, G, v, R, ΔT, DOM, OBS, DAMAGE
δx ← v ∗ Δt, δy ← v ∗ Δt, δψ ← v ∗ Δt/R
Gx ← DOM_xmin: −δx: O.x : δx: DOM_xmax
Gy ← DOM_ymin: −δy: O.y : δy: DOM_ymax
Gψ ← DOM_ψmin: −δψ: O.ψ : δψ: DOM_ψmax
𝒟 ← Gx ⊗ Gy ⊗ Gψ
Current ← O
Front ← O, Closed ← Empty, Parent ← O
Cost ← 0, PathLoad ← 0
while not(Empty(Front)) do
    if Size(Front) > 1 and Visited(Front(1)) then
        Front ← POP(Front)
    else
        Current ← Front(1)
    end if
    if Reached(G) then
        End
    end if
    Front ← POP(Front)
    GETACTIONS(Current, v, R, ΔT)
    for k = 1 to NumActions do
        Candidate ←Applyaction(k)
        Candidate-Adm ← CHECK(DOM, OBS)
        Candidate-Cost← Cost(Current) + ActionCost(k)
        Candidate-CostTG←HEURISTIC(NewNode, G)
        Candidate-Load←LOAD(NewNode, DAMAGE)
        if Candidate-Adm and Candidate-Load then
            Front ← PUSH(Candidate)
            Closed ← Closed ∪ Front(1)
        end if
    end for
end while
```



$\mathscr{H}\mathscr{A}^*$ Frontier Evolution

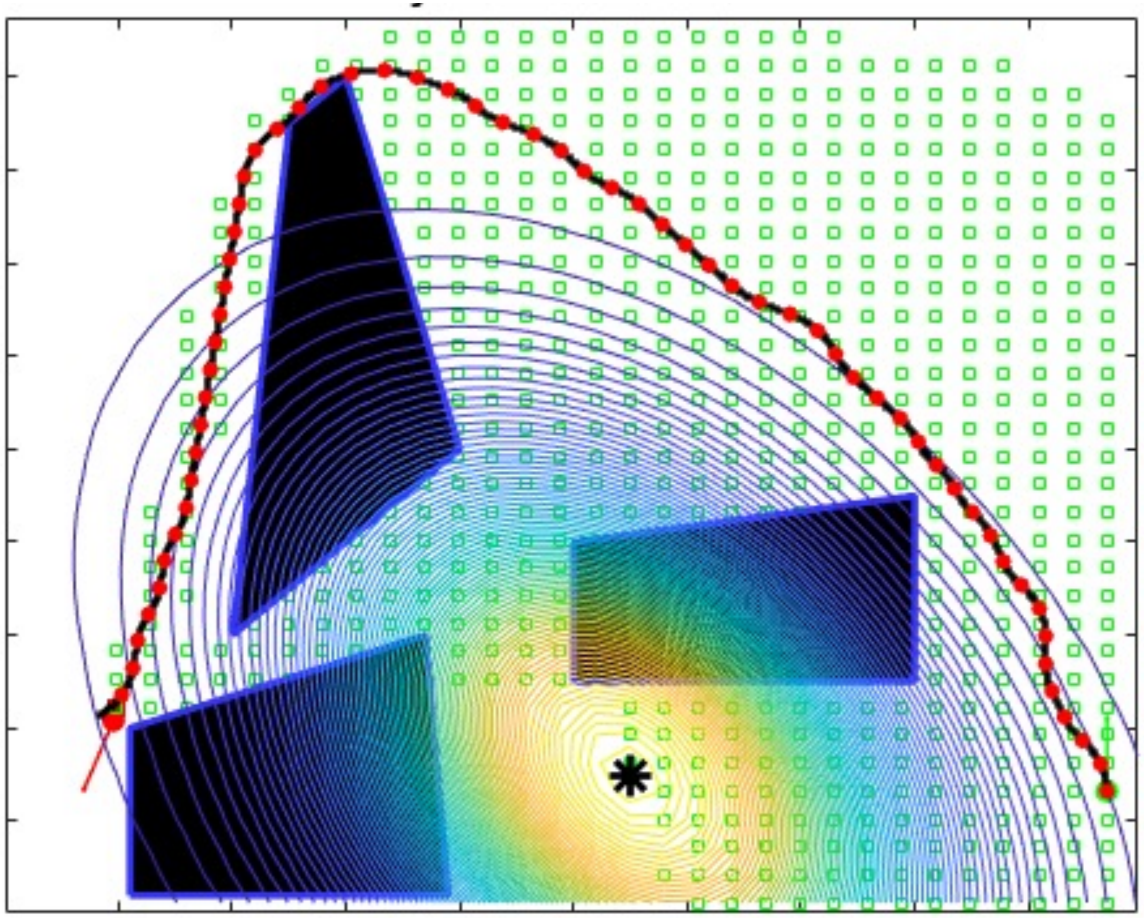
Hybrid-A* Graph Search with Integral Constraints

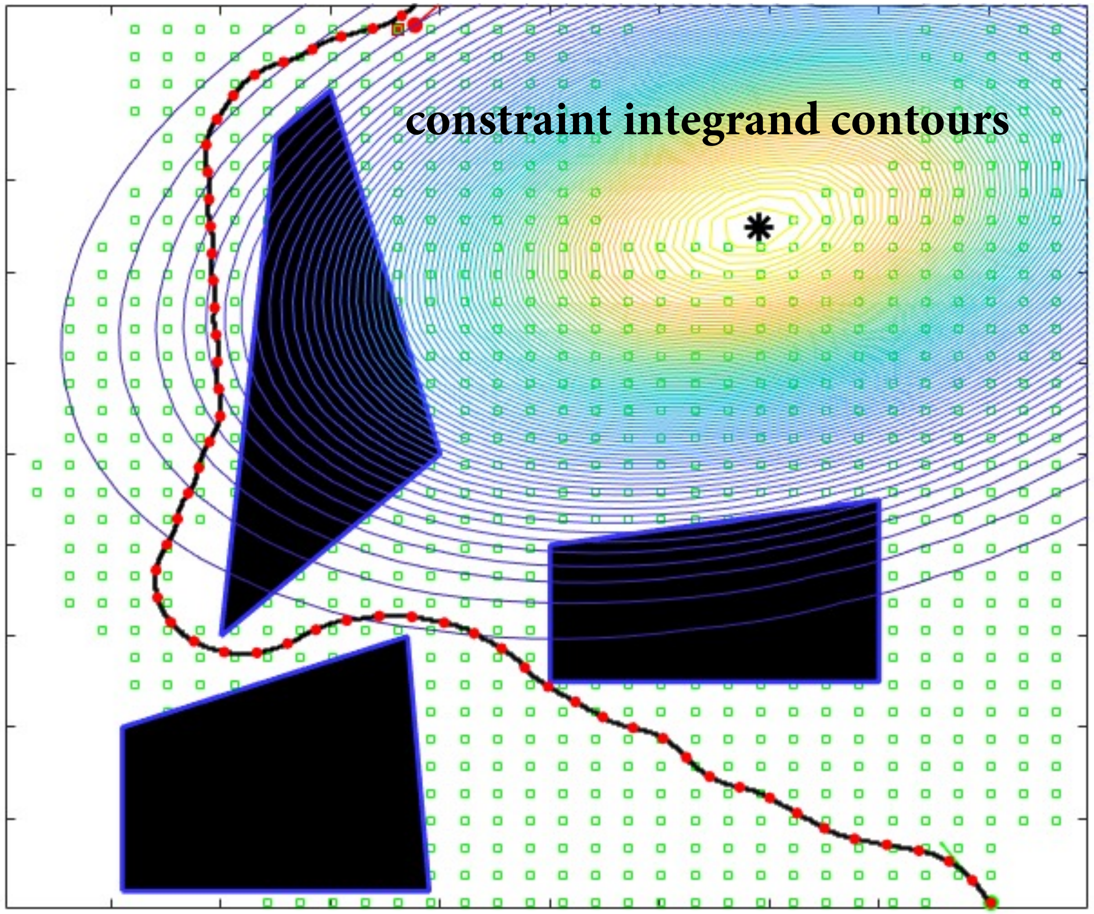


$$\int_0^t \mathcal{F}_{\mathcal{L}}(\tau, \mathbf{s}(\tau), u(\tau)) \approx \sum_{k=1}^{n} \Delta\mathcal{F}_{\mathcal{L}_k}(t_k, \mathbf{s}(t_k), u(t_k))$$

Hybrid-A* with Integral Constraints: Example

- EXAMPLES



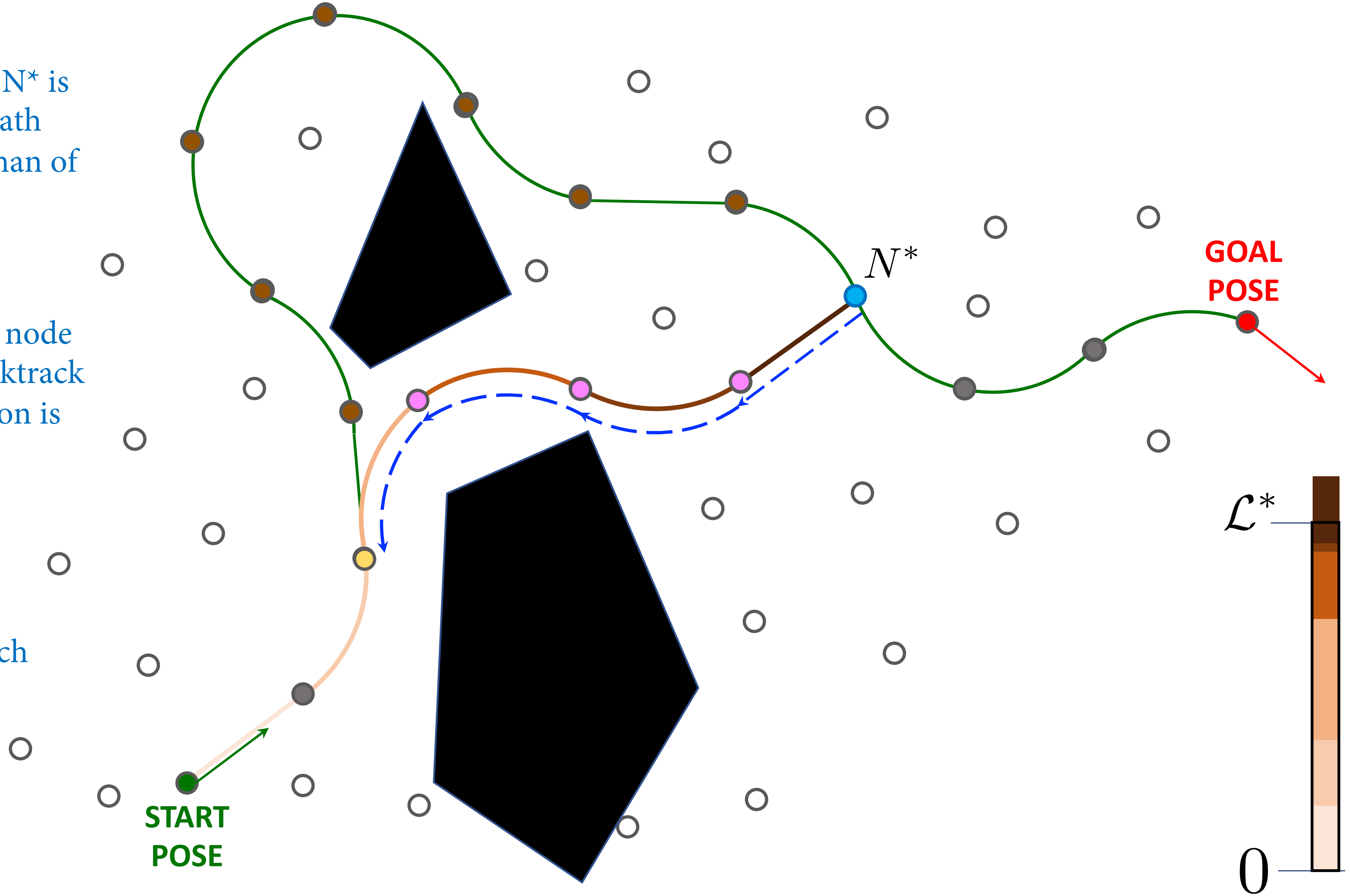


$\mathscr{HA}^*$ with obstacles and integral (loading) constraint

VISITED NODE NODE IN SOLUTION PATH STARTING POSE GOAL POSE

Backtracking Hybrid-A*

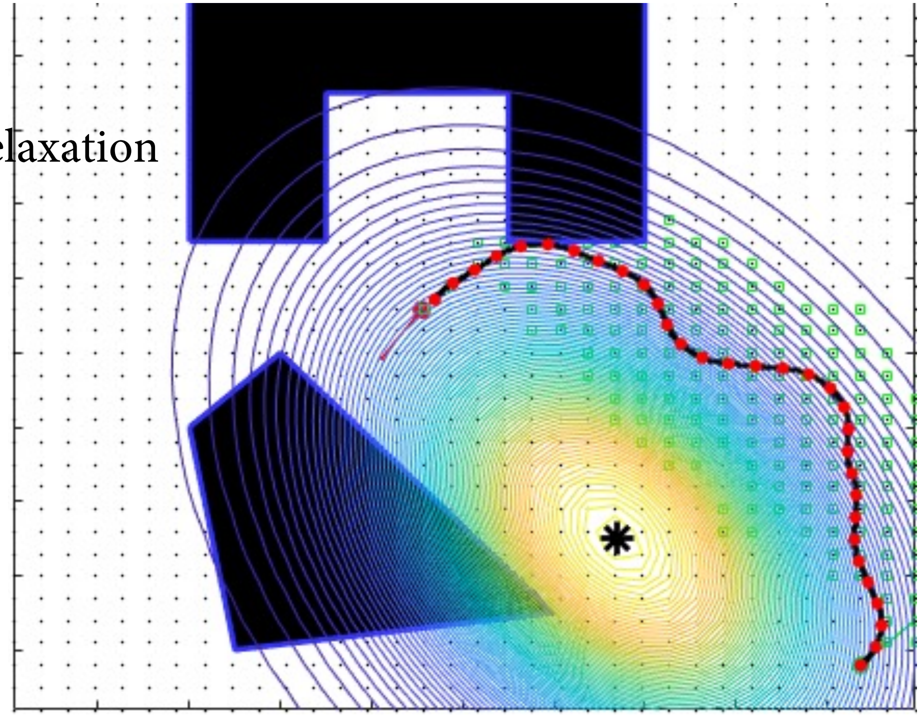
- The inadmissibility of N* is more the fault of the path taken to reach there, than of "itself"
- When an inadmissible node is encountered, we backtrack until a stopping criterion is met
- HA* resumes at this point, causing a redirection of the search



Backtracking Hybrid-A*

The Mechanics of Backtracking

- All backtracked nodes are in the “visited” set: they must be released back into the open set.
- All children of the backtracked nodes that are in the visited set must be released back into the open set.
- All children of the backtracked nodes that are in the frontier must be released back into the open set.
- STOPPING CRITERIA:
  - Naïve: a fixed or random number of backsteps
  - Based on prior optimization of the constraint function (costly): load relaxation
  - Based on various estimates of “load to go”
  - Based on reaching an optimal point in the constraint contours
  - Topic of current research

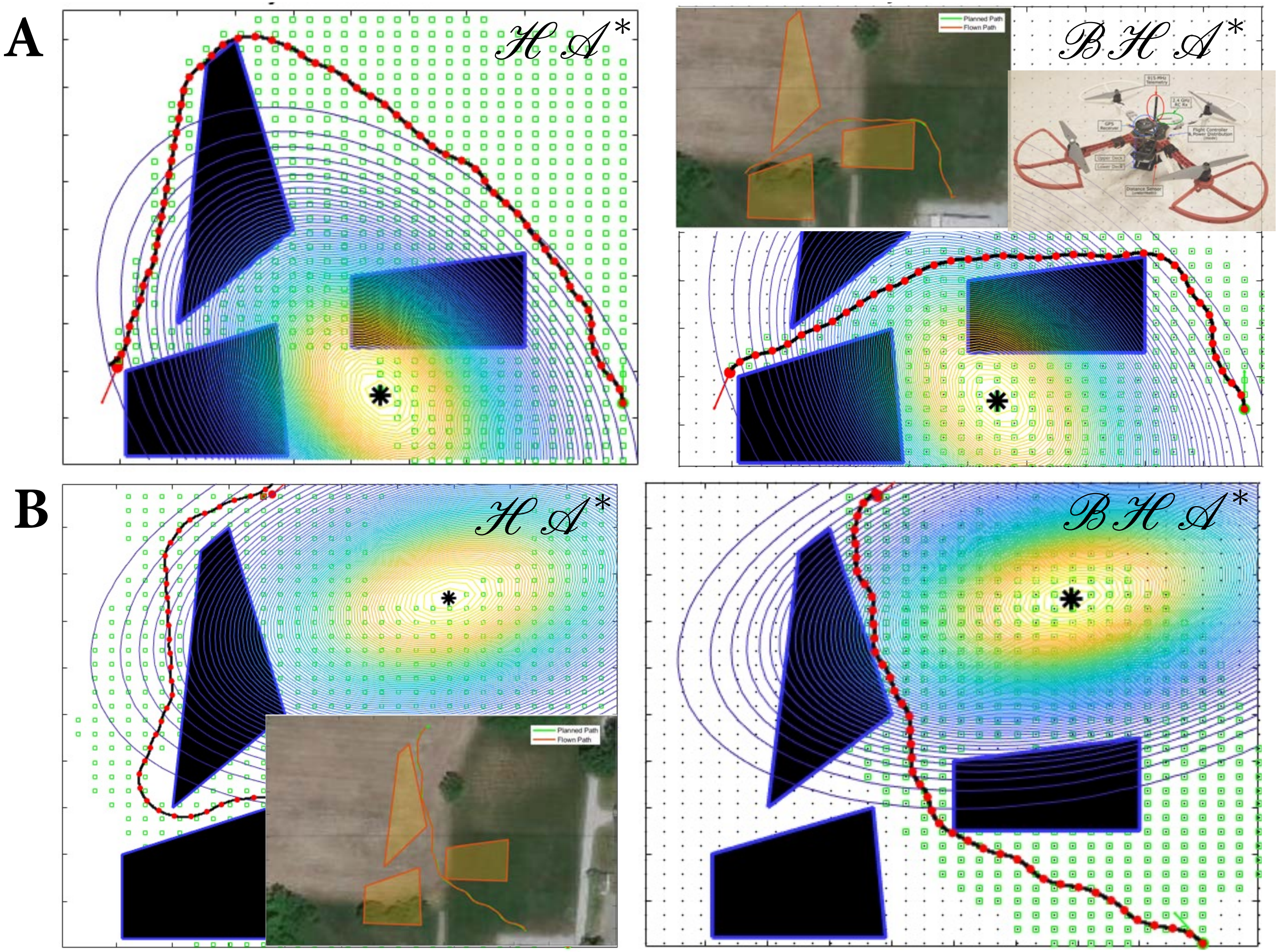


Backtracking Hybrid-A*: Results

- EXAMPLES*

Scenario A	COST	TIME TO SOLUTION
$\mathcal{H}\mathcal{A}^*$	180	6.10 s
$\mathcal{B}\mathcal{H}\mathcal{A}^*$	117 (35%)	4.11 s (33%)

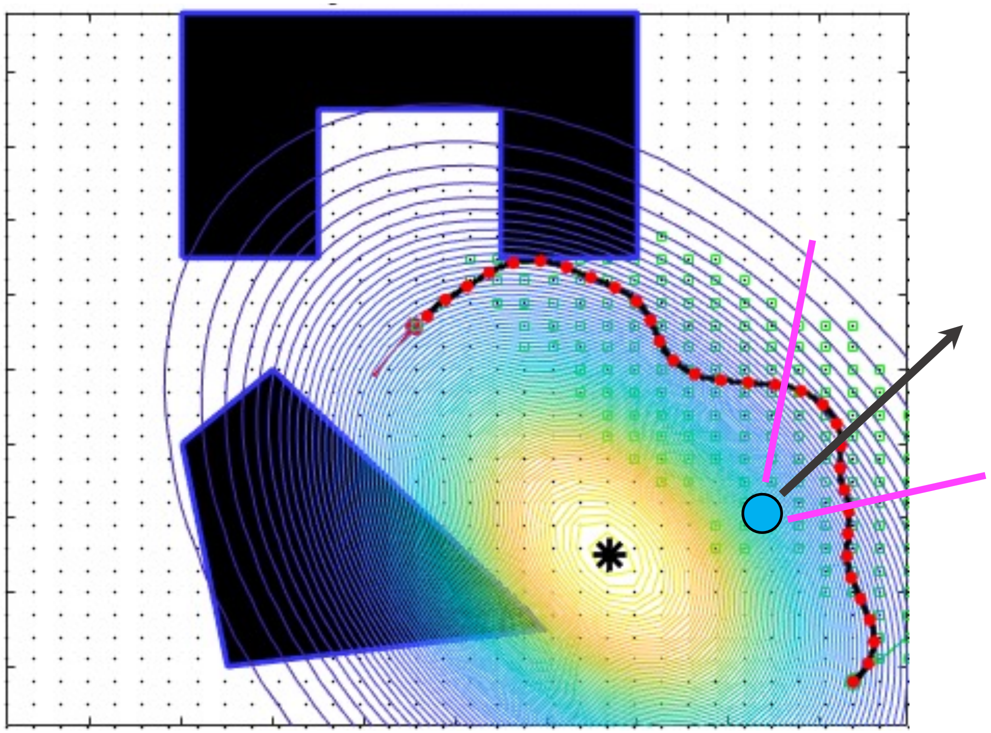
Scenario B	COST	TIME TO SOLUTION
$\mathcal{H}\mathcal{A}^*$	165	5.33 s
$\mathcal{B}\mathcal{H}\mathcal{A}^*$	126 (24%)	5.27 s



*Bryce Ford, Rachit Aggarwal, Mrinal Kumar, S. Gupta Manyam and David Casbeer. *Backtracking Hybrid A* for Resource Constrained Path Planning*. In GNC @ Scitech Forum, San Diego CA, January 3-7, 2022

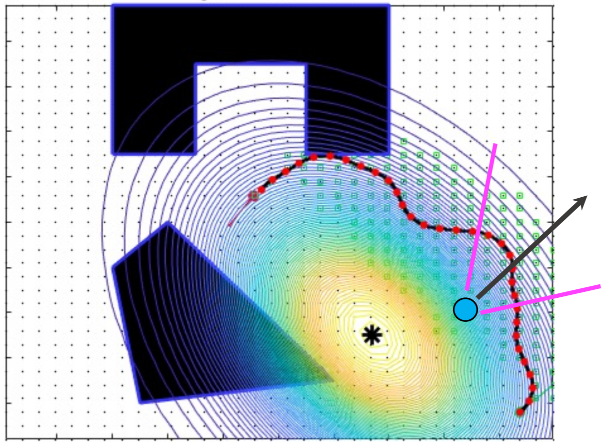
Backtracking Hybrid-A*: Stopping Criteria

- The temptation to include the constraint in the cost function: must be careful!
- Backtracking naturally seeks to move along the gradient of the constraint (integrand) contours
- Steepest descent backtracking – ***finding the cone of favorability***



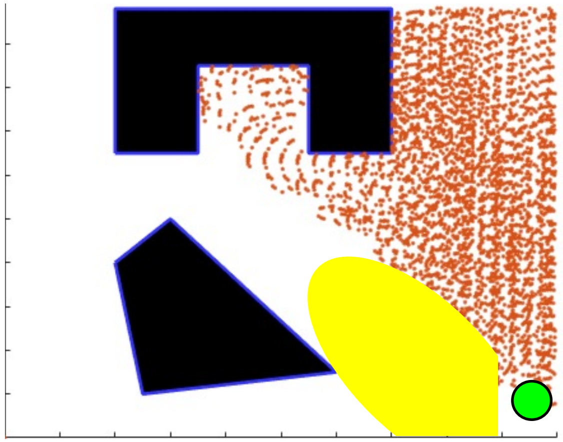
Backtracking Hybrid-A*: Stopping Criteria

- The temptation to include the constraint in the cost function: must be careful!
- Backtracking naturally seeks to move along the gradient of the constraint (integrand) contours
- Steepest descent backtracking – ***finding the cone of favorability***
- What if there is ***load-shedding?*** I.E., periods of time where the platform can off-load? (e.g., pass through a very hot region followed by cooling off)
- Generally, backtracking opens-up the search space

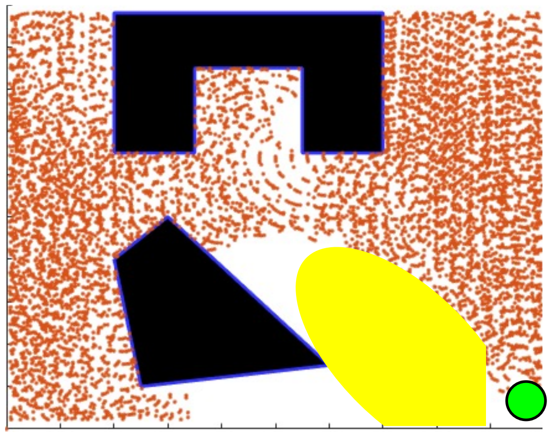


Temperature Evolution Constraint: $\dot{T}(t) = \dfrac{A}{mc_p}\dot{\phi}(x(t), y(t))$

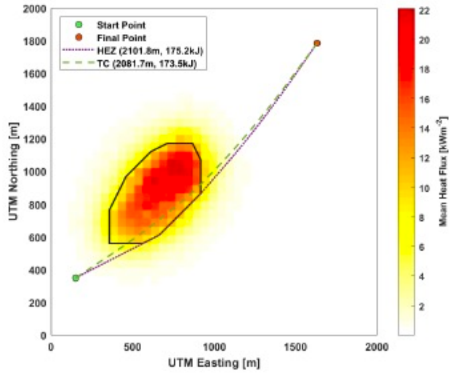
where, $\dot{\phi}(x(t), y(t)) = \begin{cases} \dot{\phi}_{\text{rad}}(x(t), y(t)) = \dot{q}'', & \text{if } \dot{\phi}_{\text{rad}}(x(t), y(t)) > 0 \\ \dot{\phi}_{\text{cool}}(x(t), y(t)) = -h(T - T_{\text{min}}), & \text{otherwise} \end{cases}$

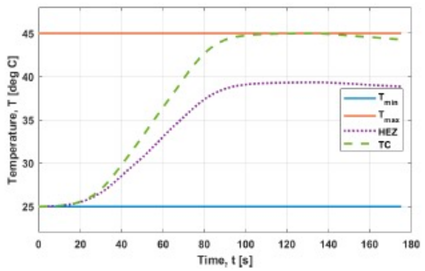


HA* without backtracking



HA* with backtracking





Summary and Lookahead

- Unstructured uncertainty presents special challenges to mission planning and execution
- Use cases in remote, hazardous, "broken" environments present the right context for autonomy
- Learning the environment requires analysis of new modalities of uncertainty, such as ***ignorance*** and ***ambiguity***
  - The evidential framework allows us to perform information fusion despite sensor conflict and bad priors
  - There is need to develop additional mathematical constructs beyond Kolmogorov's axiomatic framework
  - There is scope to combine evidential reasoning with learning tools to discover new belief functions
- Path planning in a dangerous, unstructured environment presents **path dependent "loading" constraints**, e.g., critical exposure, temperature rise, etc.
- Resource constrained path planning is NP hard. Chance-resource constrained problems have not been considered.
- Graph search offers a suboptimal solution in less time, and allows enforcement of kinematic constraints
  - A new backtracking hybrid A* algorithm has shown excellent results in early tests
  - Current research focuses on stopping criteria and alternate methods for constraint enforcement
- UAS mission planning in unstructured uncertainty (e.g., wildfire) requires many coupled problems to be solved together: much more work is needed on many fronts!

Q&A

Thank You!