2. Spectral Analysis of Sinusoidal Signals and Nonstationary Signals

2.1 DFT of a sinusoidal sequence

Write a MATLAB script to compute and plot the DFT of a sinusoidal signal given by $x(t) = \cos(2\pi(10\text{ Hz})t)$ in the following 3 different cases:

a) Sampling frequency Fs = 64 Hz, window length $R = 32$, DFT length $N = 32$

b) Sampling frequency Fs = 24 Hz, window length $R = 32$, DFT length $N = 32$

c) Sampling frequency Fs = 24 Hz, window length $R = 32$, DFT length $N = 128$

Analyze the plots of the corresponding DFTs.

2.2 Minimization of leakage using a Kaiser window

The performance of the DFT spectral analysis depends on the window being used.

To improve the frequency resolution, use a window with very small main lobe width

To reduce the leakage, the window must have a very small relative sidelobe level.

At this point, a Kaiser window with adjustable relative sidelobe attenuation (see DSP I) should be used.

A Kaiser window of length L is given by

$$w_K[n] = \begin{cases} I_0[\beta(1-([(n-\alpha)/\alpha]^2)^{1/2})]/I_0(\alpha), & 0 \le n \le L-1 \\ 0, \text{ otherwise} & \end{cases}, \text{ where } \alpha = \frac{L-1}{2}$$

and $I_0(\cdot)$ is the modified zeroth-order Bessel function of first kind.

The magnitude responses of different Kaiser windows are shown below.

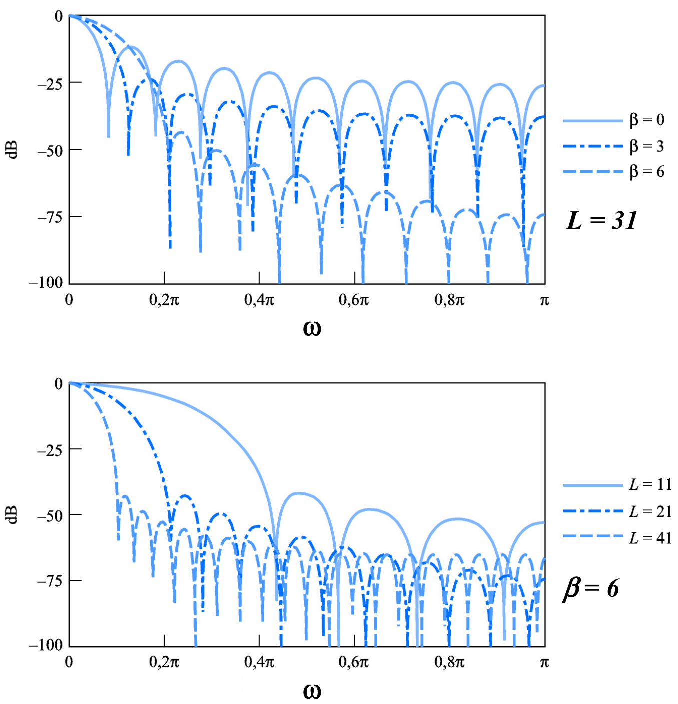

Fig. 2.1: Magnitude responses of different Kaiser windows

As can be seen in the figure, the parameter β controls the relative sidelobe attenuation. Furthermore, the main lobe width is inversely proportional to the length L of the window.

The required window length L depends on the main lobe width Δ_{ML} and the relative sidelobe attenuation A_{sl}, and can be estimated using the following formula (Kaiser and Schafer, 1980):

$$L \cong \frac{24\pi(A_{sl} + 12)}{155 \cdot \Delta_{ML}} + 1$$

The parameter β depending on the relative sidelobe attenuation A_{sl} can be estimated using the foolowing empirical relation (Kaiser and Schafer, 1980) given below.

$$\beta \cong \begin{cases} 0, & A_{sl} < 13.26 \\ 0.76609 \cdot (A_{sl} - 13.26)^{0.4} + 0.09834 \cdot (A_{sl} - 13.26), & 13.26 \le A_{sl} < 60 \\ 0.12438 \cdot (A_{sl} + 6.3), & 60 \le A_{sl} < 120 \end{cases}$$

Investigate the effects of DFT spectral analysis for a sum of two sinusoids with closely spaced frequencies given by

$$x[n] = \cos(2\pi \cdot n/14) + 0.1 \cdot \sin(4\pi \cdot n/15)$$

Compute and plot the DFT of $x[n]$ windowed by a Kaiser window $w[n]$, $0 \le n \le R-1$ (M-File ***kaiser***). First, determine the parameter β for $A_{sl} = 40$ dB, 70 db and 100 dB and the window length being a power of 2 (for the evaluation with the FFT). The DFT length N should be equal to the window length.

Verify the frequency separation and compare the result of DFT analysis with Kaiser window with that obtained using a rectangular window.

Make also use of the Window Design & Analysis Tool (***wintool***) to analyze the window.

2.3 Fourier Analysis of a tone sequence

Write a MATLAB script to analyze the tone sequence given in the M-File Tonleiter. The M-File generates a sequence of 7 tones. Evaluate the M-File and listen to the output of your sound card.

Analyze the magnitude spectrum using a DFT.

Generate the STFT to analyze the time-dependant frequency content.

2.4 Short-time Fourier Transform of a superposition of two chirp signals

Analyze the superposition of two chirp signals using a STFT. To this end, generate two chirp signals using the function ***chirp*** with the method '**linear**' for the range from 0 to 1 s. The first and the last frequency of the first chirp signal is 0 Hz and 500 Hz, and the first and the last frequency of the second chirp signal is 50 Hz and 450 Hz, respectively.

Select a suitable set of arguments to obtain a satisfactory frequency and time resolution.

Compute and plot the magnitude spectrum using a DFT.