1. Computation of the Discrete Fourier Transform

1.1 Goertzel algorithm

Implement the *2nd*-order IIR digital filter (Goertzel filter) given below using the function ***filter*** to implement the Goertzel algorithm.

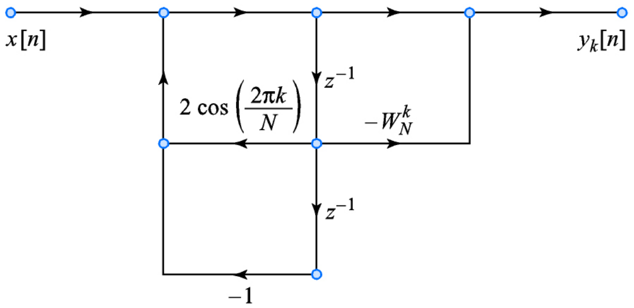


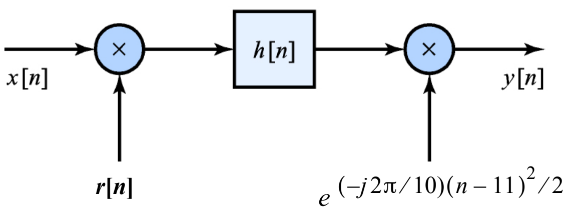
Fig. 1.1: Goertzel filter

Use the length-8 input sequence **x[n] = [1 2 3 4 5 6 7 8]** and determine all DFT values **X[k]** using the above Goertzel filter.

Verify your results by comparing the DFT samples with those obtained by the M-file ***fft***.

1.2 Chirp transform algorithm

Consider the block diagram representation of the chirp transform algorithm (CTA) given below.



$$h[n] = \begin{cases} e^{(j2\pi/10)(n-11)^2/2}, & n = 0, 1, \ldots, 15 \\ 0, & otherwise \end{cases}$$

Fig. 1.2: Block diagram representation of the chirp transform algorithm

Note the modification of the algorithm in fig.1.2 to obtain a causal system.

The impulse response $h[n]$ of the FIR chirp filter and the demodulation sequence are shifted by $N - 1 = 11$ samples, respectively. Therefore, the desired values at the output are given by $y[n+11]$.

Determine the input modulation sequence $r[n]$ such that the DTFT values are given by

$$y[n+11] = X(e^{j\omega_n}) \text{ for } n = 0, 1, \ldots, 4 \text{ where } \omega_n = 2\pi/19 + 2\pi n/10 .$$

Implement the 3-step procedure (modulation of the input sequence, convolution and demodulation of the filter output) in a MATLAB script. Evaluate your result by comparing the values with that obtained by the function ***czt***.

As input sequence, use the impulse response sequence of a FIR filter generated by
`h=fir1(11,125/500,rectwin(12))`

1.3 Verification of signal flow graphs for DIT and DIF FFT algorithm

a) Verify the radix-2 DIT 8-point FFT algorithm given by the signal flow graph below.

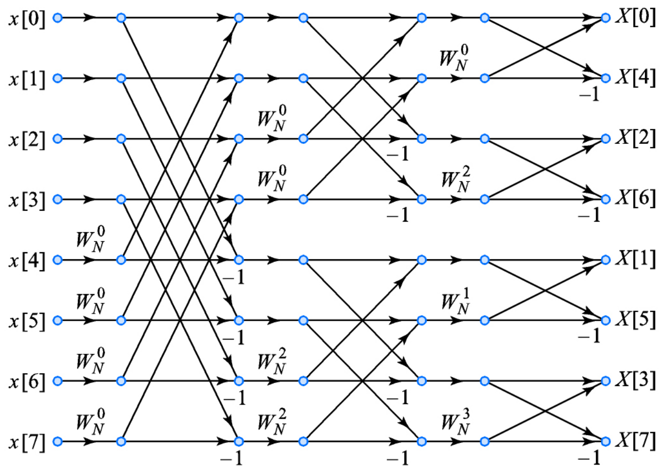


Fig. 1.3: Flow graph of a decimation-in-time 8-point FFT algorithm

To do this end, define the matrices for the representation of the 3 butterfly stages, the diagonal matrices for the multiplications by the twiddle factors and a matrix for the bit reversal, respectivey. Show that the product of all submatrices results in the DFT matrix.

Verify also that the transpose of the matrix factorization results in the DFT matrix.

b) Verify the signal flow graph of the DIF FFT 8-point algorithm given below.

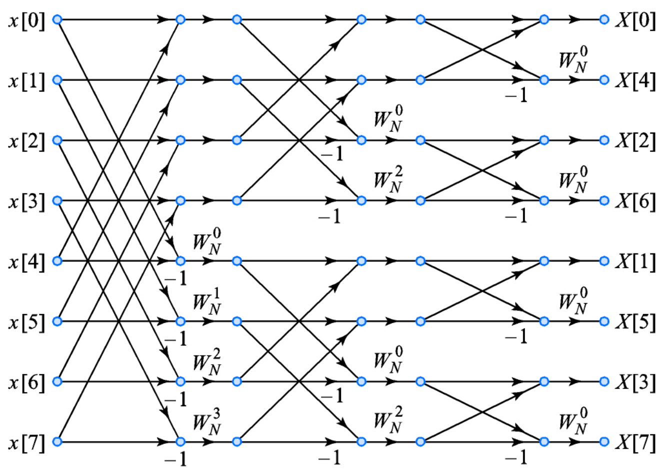


Fig. 1.4: Flow graph of a decimation-in-frequency 8-point FFT algorithm

1.4 Execution times for DFT and FFT computation

Write a script to compare the execution times in MATLAB for the computation of the DFT by using the matrix vector product and the the function ***fft***, respectively.

The matrix vector product computation of the DFT should be implemented by using the function ***dftmtx*** and a random sequence.

Hint: Use the **tic** and **toc** functions to mneasure the execution times.

1.5 Alternative IDFT computations

Write a MATLAB script to evaluate two alternative IDFT approaches given by the formula

$$x[n] = \frac{1}{N}\left\{\sum_{k=0}^{N-1} X^*[k] W_N^{nk}\right\}^*$$

and the figure below, respectively.

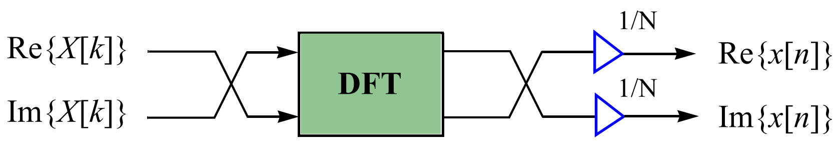


Fig. 1.5: Alternative approach to the inverse DFT computation using a DFT algorithm

Hint: To compare the numerical results, use the function ***norm*** for the difference vector.