HSB
Hochschule Bremen
City University of Applied Sciences

UNDERWATER ACOUSTICS AND SONAR SIGNAL PROCESSING

SS 2018

ASSIGNMENT 5

SONAR EQUATION

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by

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Contents

Introduction

The sonar equation is a systematic way of estimating the expected signal-to-noise ratios for sonar systems. The signal-to-noise ratio determines whether or not a sonar will be able to detect a signal in the presence of background noise in the ocean. It takes into account the source level, sound spreading, sound absorption, reflection losses, ambient noise, and receiver characteristics. The sonar equation is used to estimate the expected signal-to-noise ratios for all types of sonar systems.

In this assignment we first develop a MATLAB program for determining the signal to noise ratio. The calculations for various parameters are then carried out. We then discuss the impact of bottom type (mud, sand and gravel) and wind speed on the signal to noise ratio.

Theory

0.1 Sonar performance prediction

The basic problem in sonar is to measure some signal (possibly an echo) against a background of noise (or reverberation). So that the signal may be detected above the background, the ratio of the measured signal to the measured background (signal-to-noise ratio) must be at least some minimum value that is determined by the system. The various terms in the sonar equations are called sonar parameters. These parameters may be grouped according to whether they are determined by the equipment, medium, or target. Equipment parameters are source level, detection threshold, directivity index, self-noise level, and array gain (also determined by medium). Medium parameters are transmission loss, ambient noise level, and reverberation level (also determined by equipment). And target parameters are target strength, and target source level. Oceanographic interest in marine acoustics is concentrated mostly in the parameters determined by the medium.

Sonar may be either active or passive. Active sonar provide their own sound source and listen for echoes as they are reflected from the target which they are trying to detect. Passive sonar have no such source. They are simply listening devises that rely on the target to emit their own noise source (e.g. engine noise from an enemy ship or communication noises from a whale).

0.1.1 Performance parameters

To assess the capabilities of a sonar system, parameters that measure the performance have to be defined, e.g.

EL	Echo Level
EE	Echo Excess
SN	Signal to noise ratio
SE	Signal Excess

Echo level is the intensity of the echo as measured in the water at the hydrophone. Echo excess is obtained by removing the noise from the echo level. The amount by which the signal to noise ratio, SN exceeds the detection threshold, DT, is called the signal excess (SE). If the SE is greater than 0 dB, then the decision is made that the target is present. If the SE is negative, the decision is that the target is not present. The signal to noise ratio, SN is that measured at the output of the receiver, where the decision of whether or not a signal is present is made.

0.1.2 Sound propagation related parameters

The parameters described in the following sections are transmission loss (TL), isotropic noise level (NL), bottom reverberation strength (RS_B), surface reverberation strength (RS_S) and volume reverberation strength (RS_V).

0.1.2.1 Transmission loss

The intensity of an acoustic signal reduces with range. This observed reduction in the acoustic signal with distance from the source is due to the combined effects of spreading and attenuation and is accounted for by the transmission loss term.

The Transmission loss is given by

$$TL = \text{spreading loss} + \text{attenuation} \tag{1}$$

where TL is defined to be 0 dB on a sphere around the source of radius $r = 1m$.

For a constant sound velocity profile and spherical spreading the transmission loss can be determined by

$$TL(r,\ z) = 20log_{10}(R) + \alpha(R - 1m) \tag{2}$$

with

$$R = \sqrt{(r - r_0)^2 + (z - z_0)^2} \tag{3}$$

where r_0, z_0 denote the horizontal and vertical coordinates of the source locations and the receiver position, respectively.

In case of depth and range dependent cylinder symmetric sound velocity profiles the TL can be calculated by

$$TL(r,\ z) = 20log_{10}(R) + 10log_{10}(F(r,\ z)) + \alpha(R - 1m) \tag{4}$$

where $F(r,\ z)$ denotes the so called focusing factor given by

$$F(r,\ z) = \frac{\text{actual spreading at } r,\ z}{spherical spreading at r,\ z} \tag{5}$$

0.1.2.2 Isotropic noise level

This is an essentially a steady state, isotropic (equal in all directions) sound which is generated by amongst other things wind, waves, biological activity and shipping. The isotropic noise level $NL(f{,}v_w{,}r{,}s{,}v_v)$ describes for particular v_w, r, s and v_v the noise power within a 1Hz band around frequency f.

Thus, assuming NL approximately white over the frequency band B of interest, the noise level is given by

$$NL_B = NL(f{,}v_w{,}r{,}s{,}v_v) + 10log_{10}B \tag{6}$$

0.1.2.3 Bottom reverberation strength

The bottom reverberation coefficient $RS_B(f, bt, \beta)$ describes the reverberation strength of an insonified area of $1m^2$.

With

c	sound speed
τ	pulse length
$2\theta_h$	$\min(2\theta_{h,T}, 2\theta_{h,R})$
$2\theta_{h,T}$	horizontal $3dB$ beam width of transmitter
$2\theta_{h,R}$	horizontal $3dB$ beam width of receiver
r_0/z_0	coordinates of transmitter/ receiver configuration
$r/\ z$	coordinates of a particular point on the sea floor

The bottom reverberation strength can be determined for given f and bt as a function of β by

$$RS_B = S_B(f, bt, \beta) + 10log_{10}(A_B) \tag{7}$$

where A_B denotes the insonified bottom area.

$$A_B = 2\theta_h R \frac{c\tau}{2cos\beta} \tag{8}$$

with

$$R = \sqrt{(r - r_0)^2 + (z - z_0)^2} \tag{9}$$

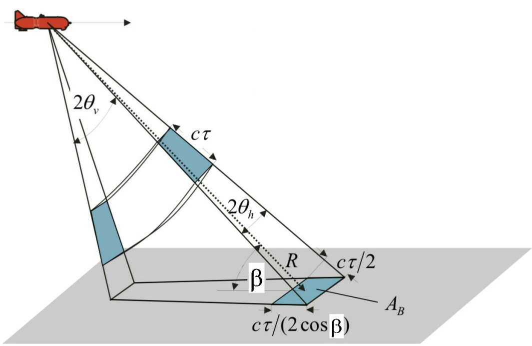


Figure 1: Surface reverberation versus grazing angle for frequency dependency

0.1.2.4 Surface reverberation strength

Analog to the bottom reverberation strength, the surface reverberation strength is provided by

$$RS_S = S_S(f, bt, \beta) + 10log_{10}(A_S) \tag{10}$$

where A_S denotes the insonified sea surface area.

$$A_S = 2\theta_h\sqrt{(r-r_0)^2 + (z-z_0)^2}\frac{c\tau}{2cos\beta} \tag{11}$$

With

c	sound speed
τ	pulse length
$2\theta_h$	$\min(2\theta_{h,T}, 2\theta_{h,R})$
$2\theta_{h,T}$	horizontal $3dB$ beam width of transmitter
$2\theta_{h,R}$	horizontal $3dB$ beam width of receiver
r_0/z_0	coordinates of transmitter/ receiver configuration
$r/\ z$	coordinates of a particular point on the sea surface

0.1.2.5 Volume reverberation strength

The volume reverberation coefficient $RS_V(f, bt, \beta)$ describes the reverberation strength of an insonified area of $1m^3$. Thus, the volume reverberation strength can be calculated by

$$RS_V = S_V(S_p, f) + 10log_{10}(V) \tag{12}$$

where V denotes the insonified volume (isovelocity).

$$V = \frac{c\tau}{2cos\beta} 2\theta_h 2\theta_v R^2 \tag{13}$$

and

$$R = \sqrt{(r - r_0)^2 + (z - z_0)^2} \tag{14}$$

With

c	sound speed
τ	pulse length
$2\theta_h$ / $2\theta_v$	$\min(2\theta_{h,T}, 2\theta_{h,R})$ / $\min(2\theta_{v,T}, 2\theta_{v,R})$
$2\theta_{h,T}$ / $2\theta_{v,T}$	horizontal / vertical $3dB$ beam width of transmitter
$2\theta_{h,R}$ / $2\theta_{v,R}$	horizontal / vertical $3dB$ beam width of receiver
r_0/z_0	coordinates of transmitter/ receiver configuration
$r/\ z$	coordinates of a particular point on water volume

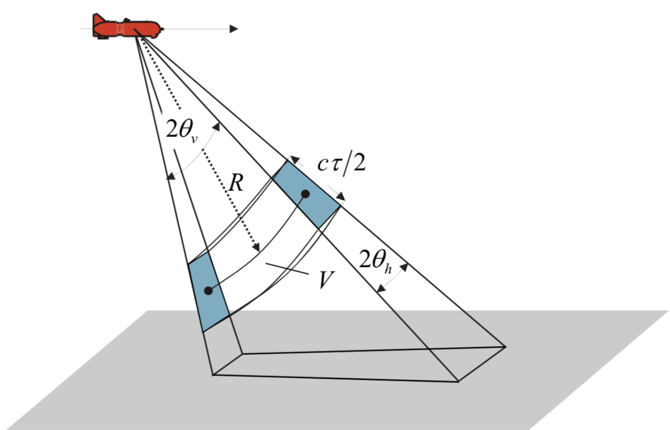


Figure 2: Surface reverberation versus grazing angle for frequency dependency

0.1.3 Sonar equation

To determine the aforementioned reverberation levels the following parameters have to be specified.

Environmental Parameters	
bt	Bottom type
v_w	Wind speed
S	Salinity
T	Water temperature
c	Sound speed profile

Source level is the ratio of the source intensity to a reference intensity, converted to dB. The reference intensity is the intensity of a sound wave having a root mean square (rms) pressure of $1\mu Pa$.Directivity index is the amount by which the antenna array has to reject the omni-directional noise dB. It is obtained by forming the logarithm of the directivity factor D.

Sonar Parameters	
SL	Source level in dB at 1m
f	Center frequency of the sound signal
BP_T	Transmitter beam pattern (vertical)
BP_R	Receiver beam pattern (vertical)
$2\theta_h$	Horizontal 3 dB beamwidth $\min(2\theta_{h,T}, 2\theta_{h,R})$
$2\theta_v$	Vertical 3 dB beamwidth $\min(2\theta_{v,T}, 2\theta_{v,R})$

Target strength is the sonar analog of radar cross section. Target strength is the ratio of the intensity of a reflected signal at 1 m from a target to the incident intensity, converted to dB. Using the conservation of energy or, equivalently, power, the incident power on a target equals the reflected power. The incident power is the incident signal intensity multiplied by an effective cross-sectional area. The reflected power is the reflected signal intensity multiplied by the area of a sphere of radius R centered on the target.

Target Parameters	
TS	Target strength
L_l	Target extent in lateral direction
L_r	Target extent in radial direction

The performance parameters can be determined by

$$\begin{aligned} EL(r,z) &= 10log_{10}(el(r,z)) = 10log_{10}(sl.bp_{T,E}.bp_{R,E}.ts/tl_E^2) \\ &= 10log_{10}(10^{0.1SL}.10^{0.1BP_{T,E}}.10^{0.1BP_{R,E}}.10^{-0.2TL_E}.10^{0.1TS}) \\ &= SL + BP_{T,E} + BP_{R,E} - 2TL_E + TS \end{aligned} \tag{15}$$

$$\begin{aligned} EE(r,z) &= 10log_{10}(ee(r,z)) = 10log_{10}(el.di/nl_B) \\ &= 10log_{10}(10^{0.1EL}.10^{-0.1(NL_B-DI)}) = EL - (NL_B - DI) \\ &= SL + BP_{T,E} + BP_{R,E} - 2TL_E + TS - (NL_B - DI) \end{aligned} \tag{16}$$

$$\begin{aligned} SN(r,z) &= 10log_{10}(sn(r,z)) = 10log_{10}(el/til) \\ &= 10log_{10}(10^{0.1EL}.10^{-0.1TIL}) = EL - TIL \\ &= SL + BP_{T,E} + BP_{R,E} - 2TL_E + TS - TIL \end{aligned} \tag{17}$$

and

$$\begin{aligned} SE(r,z) &= 10log_{10}(se(r,z)) = 10log_{10}(sn/dt) \\ &= 10log_{10}(10^{0.1SN}.10^{-0.1DT}) = SN - DT \\ &= SL + BP_{T,E} + BP_{R,E} - 2TL_E + TS - TIL - DT \end{aligned} \tag{18}$$

where DT denotes the detection threshold, TIL the total inference level

$$\begin{aligned} TIL(r,z) &= 10log_{10}(til(r,z)) = 10log_{10}(nl_B/dt + rl_B + rl_S + rl_V) \\ &= 10log_{10}(10^{0.1(NL_B-DI)} + 10^{0.1RL_B} + 10^{0.1RL_S} + 10^{0.1RL_V}) \end{aligned} \tag{19}$$

and RL_B, RL_S, and RL_V the reverberation level of the bottom, surface and volume, respectively, i.e.

$$\begin{aligned} RL_B &= SL + BP_{T,B} + BP_{R,B} - 2TL_B + RS_B \\ RL_S &= SL + BP_{T,S} + BP_{R,S} - 2TL_S + RS_S \\ RL_V &= SL + \overline{BP_{T,V} + BP_{R,V} - 2TL_V} + RS_V \end{aligned} \tag{20}$$

For $c = const.$, we can write

$$TL_E = TL_B = TL_S = TL_V \tag{21}$$

Furthermore, the following abbreviations have been used.

$$\left.\begin{array}{lll} BP_{T,E} & : & \text{Transmitter} \\ BP_{R,E} & : & \text{Receiver} \end{array}\right\}\text{Beampattern value for the ray directed toward the target position}$$

$$\left.\begin{array}{lll} BP_{T,B} & : & \text{Transmitter} \\ BP_{R,B} & : & \text{Receiver} \end{array}\right\}\text{Beampattern value for the ray directed toward the insonified bottom area}$$

$$\left.\begin{aligned} BP_{T,S} &\quad : \quad \text{Transmitter} \\ BP_{R,S} &\quad : \quad \text{Receiver} \end{aligned}\right\}\text{Beampattern value for the ray directed toward the insonified surface area}$$

$$\left.\begin{aligned} TL_E \\ TL_B \\ TL_S \\ TL_V \end{aligned}\right\}\text{Tranmission loss for the echo, bottom, surface and volume reverberation, respectively}$$

Experimental Research

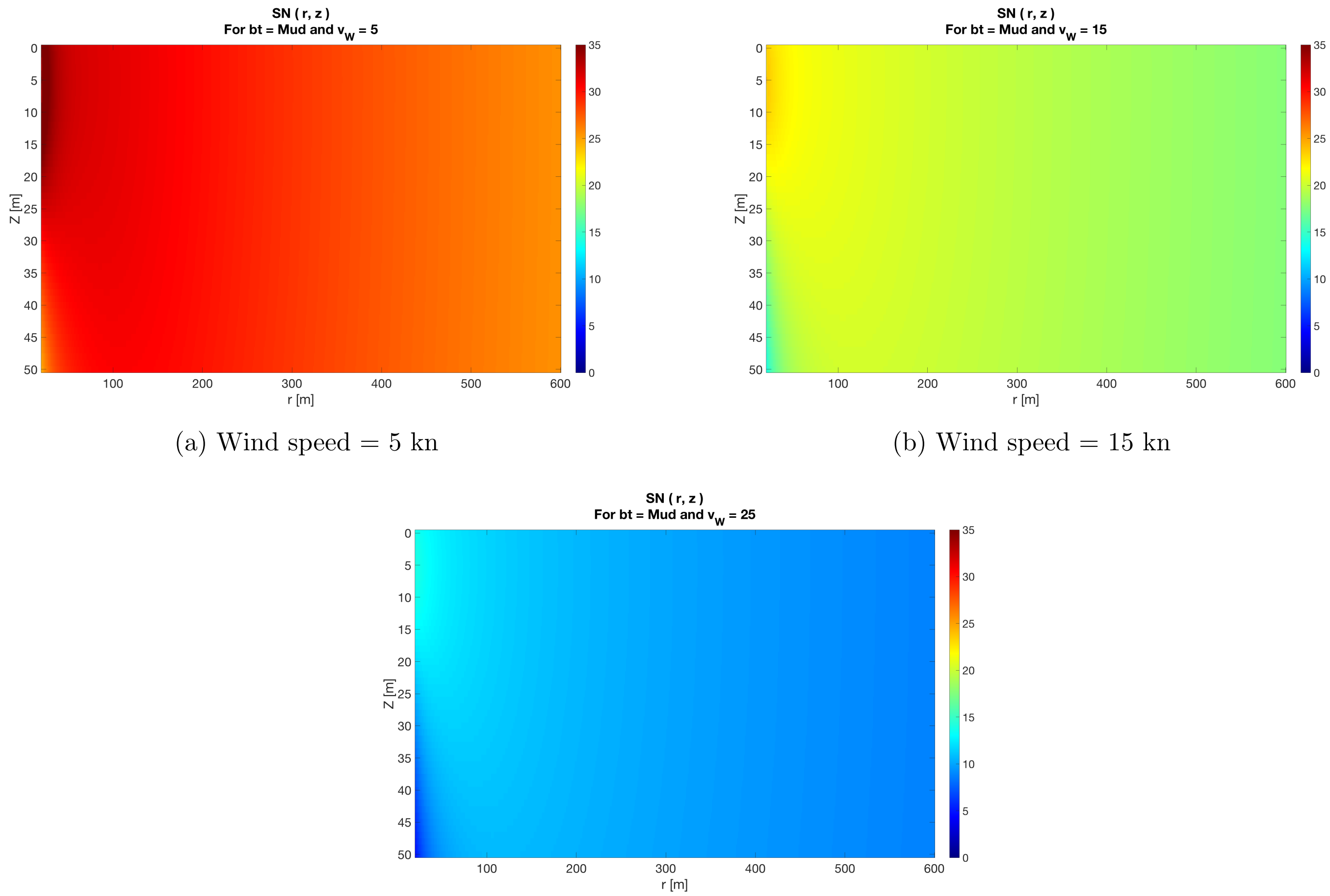
0.2 Signal to Noise Ratio

The MATLAB program for determining the signal to noise ratio was developed. The following parameter values were considered.

z / r	uptp 50 m / 600 m
bt	mud, sand and gravel
v_W	5, 15 and 25 knots
S	33 ppt
T	15°
c	1480 m/s
SL	220 dB
f	100 kHz
τ	100μs
DI	30 dB

B	10 kHz
BP_T	0 dB (±90°)
BP_R	0 dB (±90°)
$2\theta_{h,R}$	0.5°
$2\theta_{h,T}$	90°
$2\theta_{v,R}$	180°
$2\theta_{v,T}$	180°
r_S	0 m
z_S	5 m
TS	$-15dB$

Signal to noise ratio (SNR) is a measure of how strong the signal of interest is with respect to the noise environment. The higher the SNR, the better the detection. The Figure 1, Figure 2 and Figure 3 show SNR as a function of depth and range for this particular sonar/environment scenario. In the images, colour is mapped to SNR where red is high and blue is low.



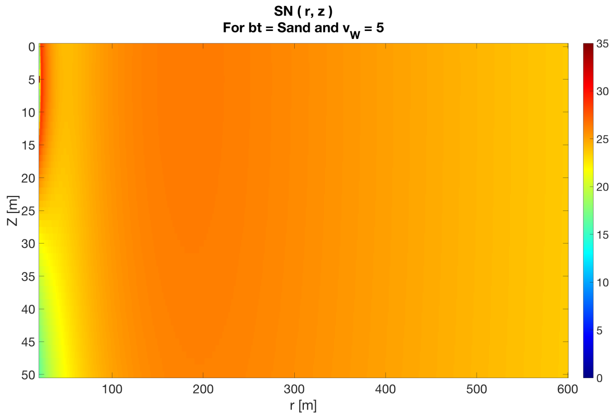
(a) Wind speed = 5 kn

(b) Wind speed = 15 kn

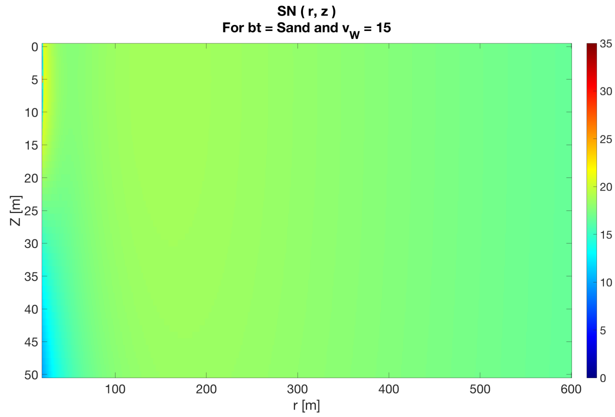
(c) Wind speed = 25 kn

Figure 3: Impact of mud as the bottom type and wind speed of 5, 15 and 25 knots on the signal to noise ratio

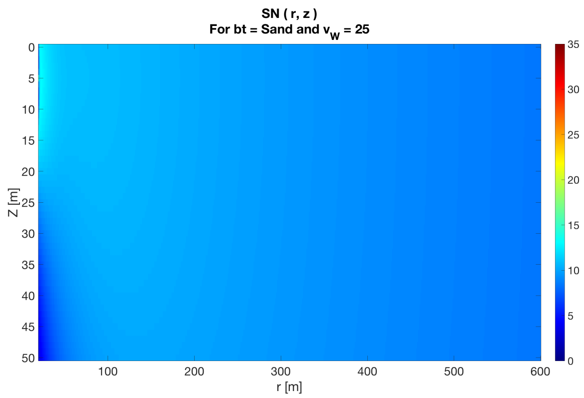
The Figure 1 shows the impact of wind speed on signal to noise ratio in the range of $r(50, 600)$. The figure has been plotted for fixed value of wind speeds (5 kn, 15 kn and 25 kn) and the bottom type is considered to be mud. We can see from the Figure 1 that as the wind speed is increased from 5 kn to 25 kn, the signal to noise ratio decreases.



(a) third



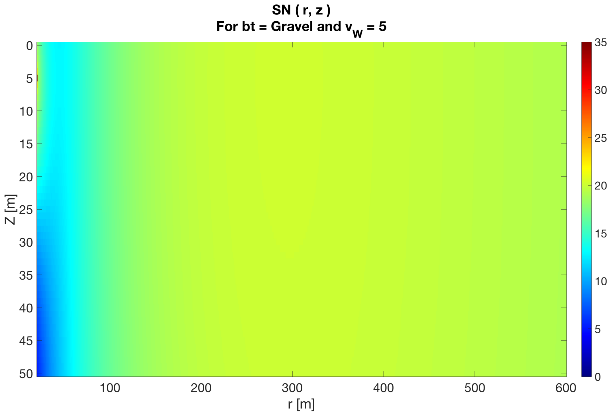
(b) forth



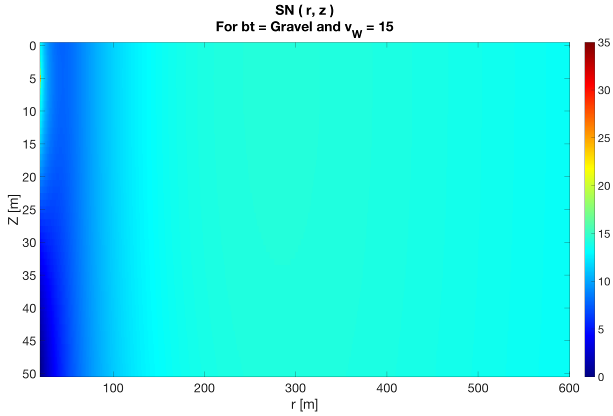
(c) fifth

Figure 4: Impact of sand as the bottom type and wind speed of 5, 15 and 25 knots on the signal to noise ratio

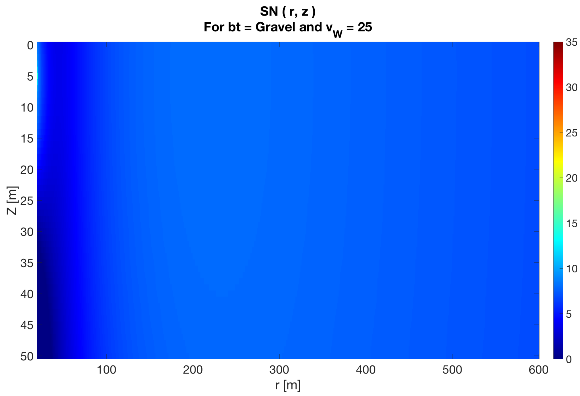
The figure 2 consists of plots with their bottom type as sand. When we compare Figure 2 to Figure 1, we noticed that the signal to noise ratio (SNR) is decreased as the bottom type changed from mud to sand. Also, the dependence if wind speed is similar to that of Figure 1.



(a) first



(b) second



(c) fifth

Figure 5: Impact of gravel as the bottom type and wind speed of 5, 15 and 25 knots on the signal to noise ratio

As the bottom type changes from mud to gravel (soft to hard), signal to noise ratio is decreases gradually. At low wind speed values, the bottom type plays a vital role in determining SNR. For the bottom type of mud, SNR is decreasing with increasing wind speeds. For the hard bottom types with the high wind speeds, SNR will be worst. The SNR has medium values for the highest values of the bottom type mud and sand. For the bottom type gravel, the SNR has the smallest values i.e. SNR is getting worse and we can observe highest noise.

Conclusion

The MATLAB code was written for determining the signal to noise ratio and carry out calculations for the following parameters. Signal to Noise ratio (SNR) due to wind at various speeds ranging from 5 knots to 25 knots and bottom type of mud, sand and gravel is analysed and observed. The signal to noise ratio is higher at lower winds speeds and on soft surfaces (mud) and thus can be well detected.

Appendix

0.3 MATLAB functions required to plot dependence of bottom type and wind speed on signal to noise ratio

0.3.1 MATLAB function to compute Transmission Loss

```
% Function file saved as 'Atten_Schulkin_Marsh.m'
function [A] = Atten_Schulkin_Marsh(f,S,T,Zmax)
A = 2.34e-6;
B = 3.38e-6;
f = f./1000;
ft = 21.9.*10.^(6-(1520./(T+273)));
P = 1.01.*(1+Zmax.*0.1);
A = 8.686e3.*((S.*A.*ft.*f.^2)./(ft^2+f.^2)+(B.*f.^2)./(ft)) ...
    *(1-6.54e-4.*P);
end
```

0.3.2 MATLAB function to compute the noise level

```
% Function file saved as 'Noise_Level.m'
function NL = Noise_Level(f,Vw)
f = f./1000;
NLTraffic = 10.*log10(3e8./(1+1e4.*f.^4)); % Shipping noise (traffic)
NLTurb = 30-30.*log10(f);  % Turbuleance noise
NLVessel = -999; % Self noise of sonar platform (vessel)
NLBio = -999; % Biological noise (fishes, scrimps, etc.)
NLSS = 40+10.*log10(Vw.^2./(1+f.^(5/3))); % Sea state noise
NLThermal = -15+20.*log10(f); % Thermal noise
```

```
% Total isotropic noise level
NL = 10.*log10(10.^(0.1.*NLTraffic)+10.^(0.1.*NLTurb)+10.^ ...
    (0.1.*NLVessel)+10.^(0.1.*NLBio)+10.^(0.1.*NLSS)+10.^ ...
    (0.1.* NLThermal));
end
```

0.3.3 MATLAB function to compute the bottom reverberation

```
% Function file saved as 'Bottom_Reverberation.m'
function [Sb] = Bottom_Reverberation(f,bt,g)
f = f./1000;
k = 1+125.*exp(-2.64.*(bt-1.75).^2-50./bt.*(cot(g)).^2);
b = k.*(sin(g)+0.19).^(bt.*(cos(g)).^16);
Sb = 10.*log10(3.03.*b.*(f.^(3.2-0.8.*bt).* ...
    10.^(2.8.*bt-12)+10.^(-4.42)));
end
```

0.3.4 MATLAB function to compute the surface reverberation

```
% Function file saved as 'Surface_Reverberation.m'
function [Ss] = Surface_Reverberation(f,Vw,g)
f = f./1000;
b = (4.*(Vw+2)./(Vw+1))+(2.5.*(f+0.1).^(-1./3)-4).* ...
    (abs(cos(g))).^(1/8);
Ss = 10.*log10(10.^(-5.05).*(1+Vw).^2.*(f+0.1).^(Vw./150).* ...
    (tan(g)).^(b));
end
```

0.3.5 MATLAB function to compute the volume reverberation

```
% Function file saved as 'Volume_Reverberation.m'
function [Sv] = Volume_Reverberation(f,Pd)
f = f./1000;
if Pd == '1'
    SP = -50;
else
    if Pd == '0.5'
        SP = -70;
else
SP = -90;
   end
end
Sv = SP+7.*log10(f);
end
```

0.4 MATLAB program to plot dependence of bottom type and wind speed on signal to noise ratio

```
% main program saved as 'USP5.m'
bt = 1:1:3;
v_W = 5:10:25;
Pd = 'low';
S = 33;
T = 15;
C = 1480;
SL = 220;
TS = -15;
r_S = 0;
z_S = 5;
Zmax = 25;
f = 100000;
tau = 100e-6;
B = 10e3;
Thetah = deg2rad(0.5);
Thetav = deg2rad(60);
% Area
r1 = 20:1:600;
z1 = 0:1:50;
[r,z,Vw,bt] = ndgrid(r1,z1,v_W,bt);
% Distance
RR = sqrt((r-r_S).^2+(z-z_S).^2);
% Transmission loss
alpha = Atten_Schulkin_Marsh(f,S,T,Zmax).*(RR-1)./1000;
TL = 20.*log10(RR) + alpha;
% for isovelocity
TLe = TL;
TLb = TL;
TLs = TL;
TLv = TL;
% Isotropic Noise Level
NL = Noise_Level(f,Vw);
NLb = NL + 10.*log10(B);
% Angles
Thetab = atan((Zmax-z_S)./sqrt(RR.^2-(Zmax-z_S).^2));
Thetas = atan(z_S./sqrt(RR.^2-(z_S).^2));
Thetae = atan((z-z_S)./(r-r_S));
% Bempattern values
BPtb = 10*log10(cos(Thetab));
BPrb = 10*log10(cos(Thetab));
BPts = 10*log10(cos(Thetas));
BPrs = 10*log10(cos(Thetas));
BPte = 10*log10(cos(Thetae));
```

```
BPre = 10*log10(cos(Thetae));
% Bottom reverberation strength
Ab = Thetah.*RR.*C.*tau./cos(Thetab);
Sb = Bottom_Reverberation(f,bt,Thetab);
RSb = Sb + 10.*log10(Ab);
% Surface reverberation strength
As = Thetah.*RR.*C.*tau./cos(Thetas);
Ss = Surface_Reverberation(f,Vw,Thetas);
RSs = Ss + 10.*log10(As);
% Volume reverberation strength
V = 2.*Thetah.*Thetav.*RR.^2.*C.*tau;
Sv = Volume_Reverberation(f,Pd);
RSv = Sv + 10.*log10(V);
% Directivity index
DI = 40 - 10.*log10(Thetav*Thetah*180.^2./(pi.^2));
% SNR
Rlb = SL+BPtb+BPrb-2.*TLb+RSb;
Rls = SL+BPts+BPrs-2.*TLs+RSs;
Rlv = SL-2.*TLv+RSv;
TIL = 10.*log10(10.^(0.1.*(NL-DI))+10.^(0.1.*Rlb)+ ...
    10.^(0.1.*Rls)+10.^(0.1.*Rlv));
SNR = SL+BPte+BPre-2.*TL+TS-(NL-DI)-TIL;
% Plot
set(0,'DefaultAxesFontSize',25)
figure(1);
CLIM = [0 35];
imagesc(r1,z1,SNR(:,:,1,1)',CLIM);
colormap(jet(256));
colorbar('vert');
title({'SN ( r, z )'; 'For bt = Mud and v_W = 5'});
xlabel('r [m]');
ylabel('Z [m]');
figure(2);
CLIM = [0 35];
imagesc(r1,z1,SNR(:,:,2,1)',CLIM);
colormap(jet(256));
colorbar('vert');
title({'SN ( r, z )'; 'For bt = Mud and v_W = 15'});
xlabel('r [m]');
ylabel('Z [m]');
figure(3);
CLIM = [0 35];
imagesc(r1,z1,SNR(:,:,3,1)',CLIM);
colormap(jet(256));
colorbar('vert');
title({'SN ( r, z )'; 'For bt = Mud and v_W = 25'});
xlabel('r [m]');
ylabel('Z [m]');
figure(4);
```

```
CLIM = [0 35];
imagesc(r1,z1,SNR(:,:,1,2)',CLIM);
colormap(jet(256));
colorbar('vert');
title({'SN ( r, z )'; 'For bt = Sand and v_W = 5'});
xlabel('r [m]');
ylabel('Z [m]');
figure(5);
CLIM = [0 35];
imagesc(r1,z1,SNR(:,:,2,2)',CLIM);
colormap(jet(256));
colorbar('vert');
title({'SN ( r, z )'; 'For bt = Sand and v_W = 15'});
xlabel('r [m]');
ylabel('Z [m]');
figure(6);
CLIM = [0 35];
imagesc(r1,z1,SNR(:,:,3,2)',CLIM);
colormap(jet(256));
colorbar('vert');
title({'SN ( r, z )'; 'For bt = Sand and v_W = 25'});
xlabel('r [m]');
ylabel('Z [m]');
figure(7);
CLIM = [0 35];
imagesc(r1,z1,SNR(:,:,1,3)',CLIM);
colormap(jet(256));
colorbar('vert');
title({'SN ( r, z )'; 'For bt = Gravel and v_W = 5'});
xlabel('r [m]');
ylabel('Z [m]');
figure(8);
CLIM = [0 35];
imagesc(r1,z1,SNR(:,:,2,3)',CLIM);
colormap(jet(256));
colorbar('vert');
title({'SN ( r, z )'; 'For bt = Gravel and v_W = 15'});
xlabel('r [m]');
ylabel('Z [m]');
figure(9);
CLIM = [0 35];
imagesc(r1,z1,SNR(:,:,3,3)',CLIM);
colormap(jet(256));
colorbar('vert');
title({'SN ( r, z )'; 'For bt = Gravel and v_W = 25'});
xlabel('r [m]');
ylabel('Z [m]');
```