Hochschule Bremen
City University of Applied Sciences

UNDERWATER ACOUSTICS AND SONAR SIGNAL PROCESSING

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ASSIGNMENT 8

AMBIGUITY FUNCTION

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by

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Introduction

For directional reception of wave energy, it is necessary to use equidistantly arranged linear or planar arrays on extended antennas. The sonar transmitting/receiving antenna characteristics can be determined by the geometry and shading of the antenna aperture and also the properties of individual transducers. If simultaneous detection of signals from many directions of incidence is required, beam forming must be carried out in parallel for many direction channels. It is also possible to steer antenna in defined direction and range zones and thus process near-field signals selectively with regard to bearing and range of their source. The term antenna beam pattern (or radiation pattern) most commonly refers to the directional (angular) dependence of radiation from the antenna. It is the geometric pattern of the relative strengths of the field emitted by the antenna.

In this assignment we first develop a MATLAB program that determines the beam pattern for a linear array. Then the graphs are plotted for beam pattern using various parameters. The parameters used in this assignment are beam forming, amplitude shading, beam shaping and electronic sheering.

Theory

0.1 Antenna Beam Pattern

A radiation pattern or the antenna beam pattern defines the variation of the power radiated by an antenna as a function of the direction away from the antenna. Very often, only the relative amplitude is plotted, normalized either to the amplitude on the antenna boresight. As a consequence of the reciprocity theorem, the receiving pattern (sensitivity as a function of direction) is identical to the relative power density of the wave transmitted by the same antenna (power density as a function of direction). The pattern of an antenna may be determined experimentally at an antenna range, or alternatively, deduced by computation. The plots of antenna pattern can be used to benchmark a given radar antenna. They also tell you how much degradation you can expect if the antenna is not aimed properly.

The complex beam pattern for a line array is considered in this assignment. It is defined by

$$\tilde{b}(\beta) = \frac{1}{\hat{Q}} \sum_{n=0}^{N-1} Q_n e^{j y'_n sin\beta}$$

with

$$Q_n = \hat{Q}_n e^{j\alpha_n}$$

and

$$\hat{Q} = \sum_{n=0}^{N-1} \hat{Q}_n$$

For $Q_0 = Q_1 = ... = Q_{N-1}$, i.e., $\hat{Q}_n = 1$, $\alpha_n = 0$ and $y_n{}' = -nd$, the complex beam pattern simplifies to

$$\tilde{b}(\beta) = \frac{1}{N} \sum_{n=0}^{N-1} e^{jkndsin\beta}$$

where $k = 2\pi/\lambda$, d is the element spacing, n is the number of point sources and β is the angle between vector r and axis x.

The squared magnitude of complex beam pattern in dB is called beam pattern and it is expressed as

$$\tilde{B}(\varphi, \theta) = 10 \log_{10} |\tilde{b}(\varphi, \theta)|^2 = 20 \log_{10} |\tilde{b}(\varphi, \theta)|$$

In the following sections, we will discuss in brief the parameters that alter the antenna beam pattern. The parameters discussed in the assignment are beam forming, amplitude shading, parabolic phase shading and linear phase shading.

0.1.1 Beam Forming

Beam forming or spatial filtering is the process by which an array of large number of spatially separated sensors discriminate the signal arriving from a specified direction from a combination of isotropic random noise called ambient noise and other directional signals.

0.1.2 Amplitude Shading

Shading is most commonly used to suppress side lobes (responses away from the main response lobe) or to suppress responses in noisy direction (known as Adaptive Beam forming). Having all coefficients equal, offers the maximum array gain in an isotropic noise field. Shading increases the width of the main lobe, decreases side lobes and reduces array gain. With shading, we are trading off the main lobe width and side lobe level.

0.1.3 Parabolic Phase Shading (Beam Shaping)

According to our requirements, the beam pattern and main lobe can be designed this is known as beam shaping. Time delaying a signal is the time-domain analog to phase shading of the signal in frequency domain. The phase shading provides broadening of the main lobe.

0.1.4 Linear Phase Shading (Electronic Steering)

Electronic steering is about changing the direction of the main lobe of a radiation pattern electronically by changing magnitude and phase of the hydrophone. Since there are no moving parts in Electronic Steering as compared to Mechanical steering, this method of beam steering is more efficient.

For α as an incident angle there would be a delay in receiving the wave by different elements, which corresponds to a phase shift given by

$$\varphi = kndsin(\alpha)$$

$$Q_n = e^{j\varphi} = e^{kndsin(\alpha)}$$

where Q_n denote the amplitude of the n^{th} point source with $n = 1, ..., N$. The complex beam pattern is defined as

$$\tilde{b}(\beta) = \frac{1}{\hat{Q}} \sum_{n=0}^{N-1} Q_n e^{jy_n' sin\beta}$$

Experimental Research

Rectangular Pulse

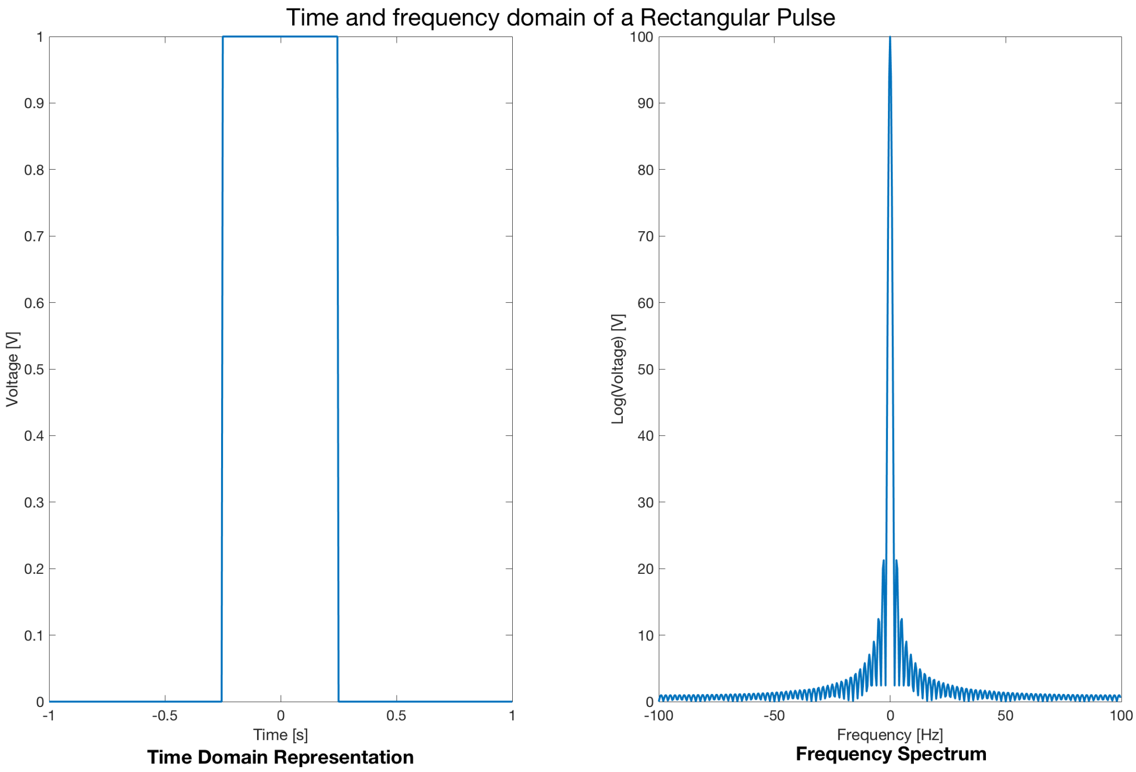

Figure 1: A rectangular pulse in time domain (left) along with its spectrum (right)

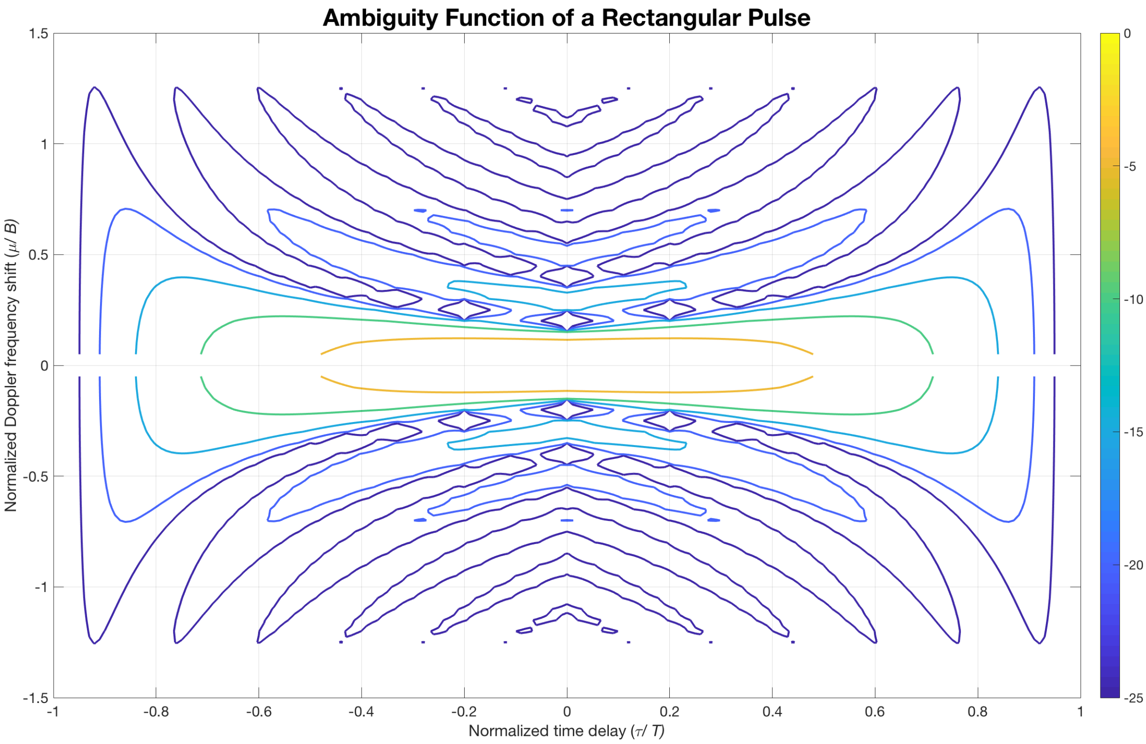

Figure 2: Ambiguity function of a rectangular pulse

Linear Frequency Modulated pulse with a Rectangular Envelope

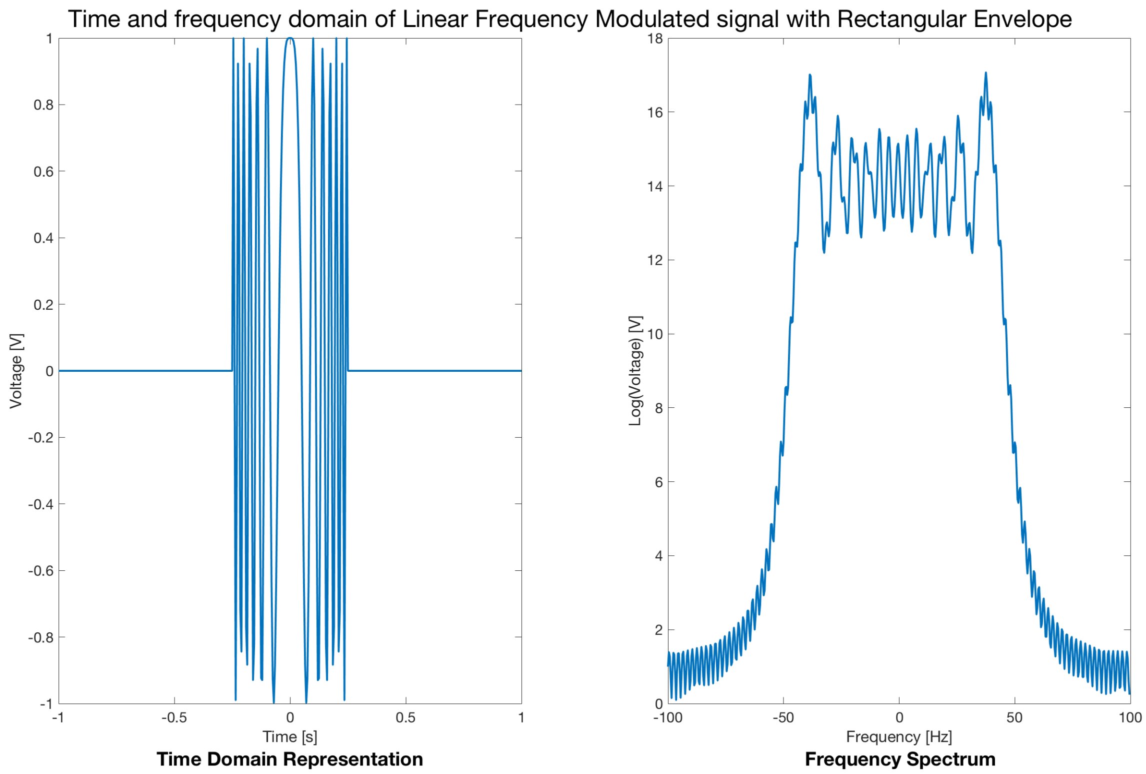

Figure 3: A Linear Frequency Modulated (LFM) pulse with rectangular envelope in time domain (left) along with its spectrum (right)

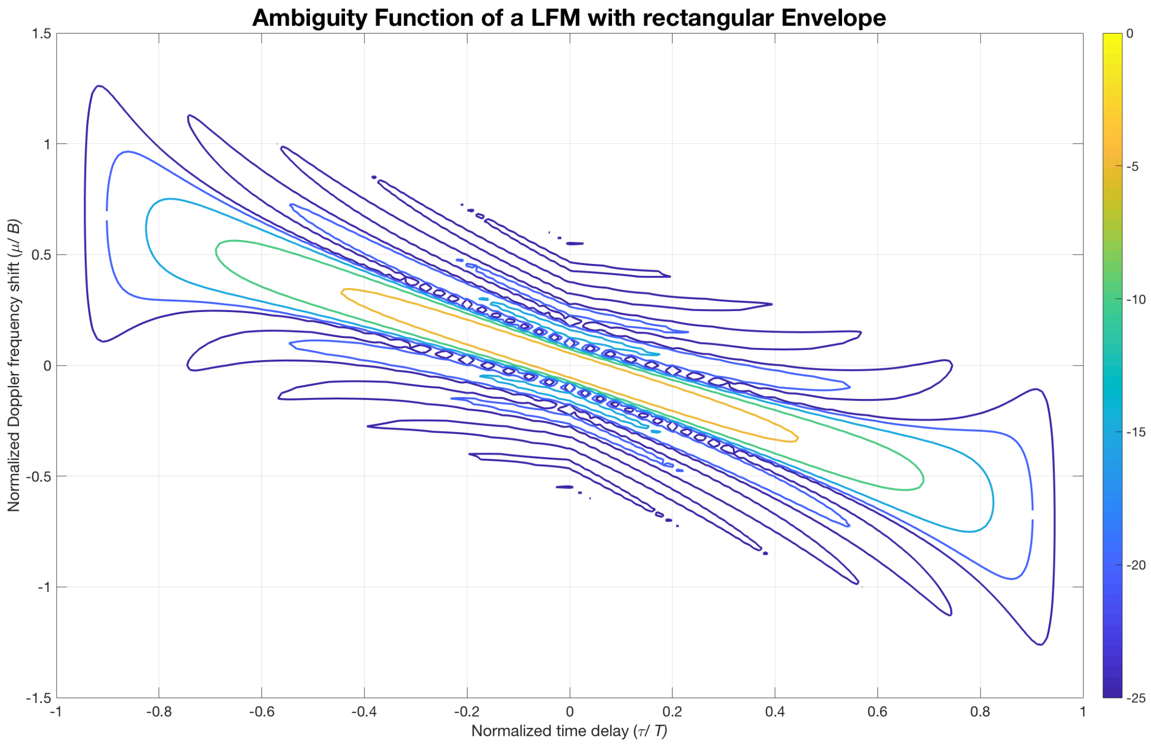

Figure 4: The ambiguity function of a Linear Frequency Modulated (LFM) pulse with rectangular envelope

Linear Frequency Modulated pulse with a Gaussian Envelope

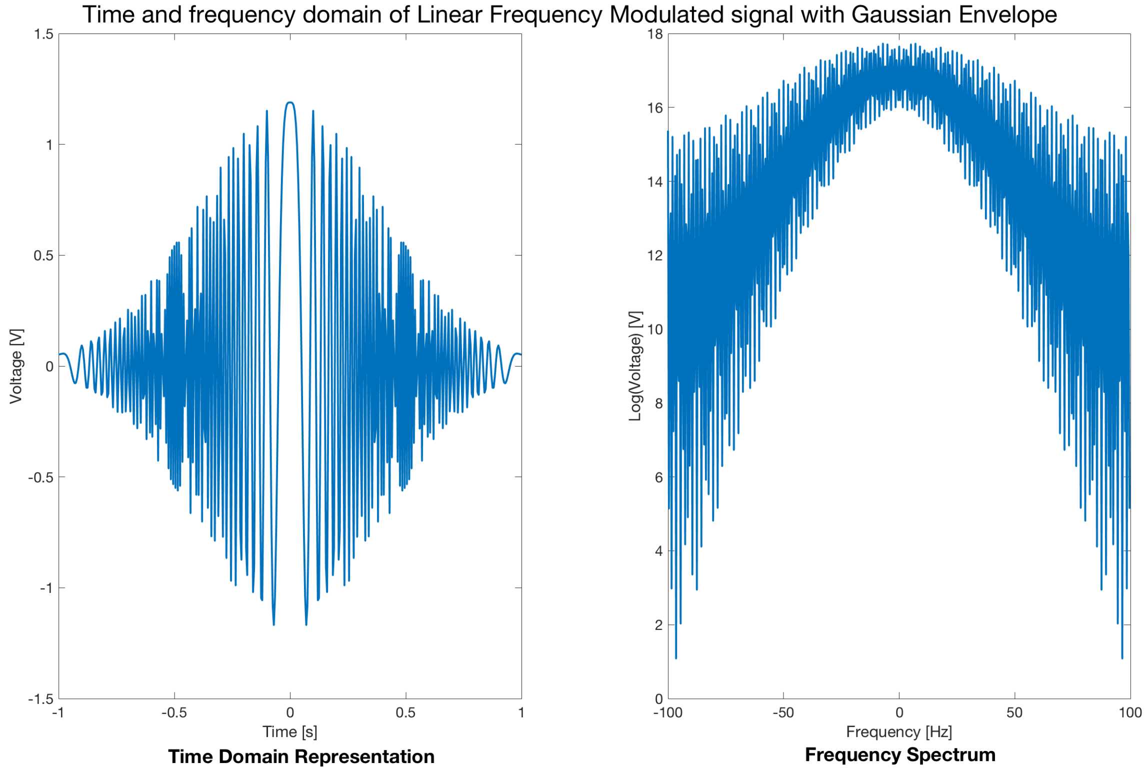

Figure 5: A Linear Frequency Modulated (LFM) pulse with Gaussian envelope in time domain (left) along with its spectrum (right)

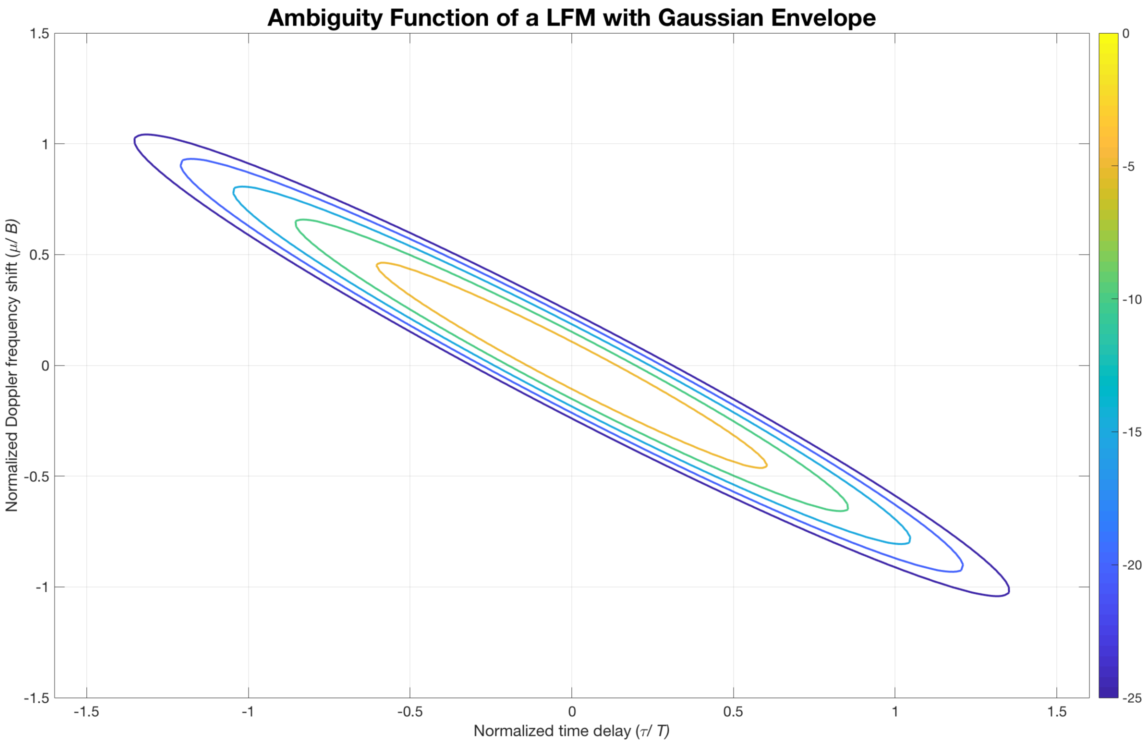

Figure 6: The ambiguity function of a Linear Frequency Modulated (LFM) pulse with Gaussian envelope

Conclusion

Appendix

MATLAB code to plot the spectrums for different waveforms

```
Fs = 200; %Sampling frequency
t = -1:1/Fs:1;
T = max(t); %pulse-width
B = 200; %bandwidth
k = B/T;
width = 0.5;
pulse = rectpuls(t,width);
figure('Color',[1 1 1]);
suptitle('\fontsize{25}Time and frequency domain of a Rectangular Pulse')
s1 = (1/sqrt(max(t))).*pulse;
subplot(1,2,1)
plot(t,s1,'LineWidth',2)
hold on
xlabel('Time [s]')
ylabel('Voltage [V]')
ax = gca; % current axes
ax.FontSize = 15;
a = title('Time Domain Representation','FontSize',20);
set(a,'Position',[0.00,-0.10],...
    'VerticalAlignment','bottom', ...
    'HorizontalAlignment','center')
[f, fft_s1] = FFT(s1, Fs);
subplot(1,2,2)
```

```
plot(f, fft_s1 ,'LineWidth',2);
xlabel('Frequency [Hz]')
ylabel('Log(Voltage) [V]')
ax = gca; % current axes
ax.FontSize = 15;
a = title('Frequency Spectrum','FontSize',20);
set(a,'Position',[0.5,-9.5],...
    'VerticalAlignment','bottom', ...
    'HorizontalAlignment','center')
figure('Color',[1 1 1]);
suptitle('\fontsize{25}Time and frequency domain of Linear Frequency Modulated signal with
    Rectangular Envelope')
s2 = (1/sqrt(max(t))).*pulse.*exp(1i*pi*k.*t.*t);
subplot(1,2,1)
plot(t,s2,'LineWidth',2)
xlabel('Time [s]')
ylabel('Voltage [V]')
ax = gca; % current axes
ax.FontSize = 15;
a = title('Time Domain Representation','FontSize',20);
set(a,'Position',[0.00,-1.20],...
    'VerticalAlignment','bottom', ...
    'HorizontalAlignment','center')
[f, fft_s2] = FFT(s2, Fs);
subplot(1,2,2)
plot(f,fft_s2 ,'LineWidth',2);
xlabel('Frequency [Hz]')
ylabel('Log(Voltage) [V]')
ax = gca; % current axes
ax.FontSize = 15;
a = title('Frequency Spectrum','FontSize',20);
set(a,'Position',[0.5,-1.8],...
    'VerticalAlignment','bottom', ...
    'HorizontalAlignment','center')
figure('Color',[1 1 1]);
```

```
suptitle('\fontsize{25}Time and frequency domain of Linear Frequency Modulated signal with Gaussian
    Envelope')
sig = T/sqrt(2*pi);
s3 = (1/sqrt(sqrt(pi*sig*sig)))*exp(-((t.*t)/(2*sig*sig))  ...
    + (1i*pi*k.*t.*t));
subplot(1,2,1)
plot(t,s3,'LineWidth',2)
xlabel('Time [s]')
ylabel('Voltage [V]')
ax = gca; % current axes
ax.FontSize = 15;
a =title('Time Domain Representation','FontSize',20);
set(a,'Position',[0.05,-1.81],...
    'VerticalAlignment','bottom', ...
    'HorizontalAlignment','center')
[f, fft_s3] = FFT(s3, Fs);
subplot(1,2,2)
plot(f, fft_s3,'LineWidth',2);
xlabel('Frequency [Hz]')
ylabel('Log(Voltage) [V]')
ax = gca; % current axes
ax.FontSize = 15;
a = title('Frequency Spectrum','FontSize',20);
set(a,'Position',[0.5,-1.81],...
    'VerticalAlignment','bottom', ...
    'HorizontalAlignment','center')

function [ f, fft_sig_val ] = FFT( sig, Fs )
% Discrete Fourier Transform
L = length(sig);
N = 512; fft_sig=fft(sig,N);
f = (-N/2:(N/2) - 1)*(Fs/N);
fft_sig_val = fftshift(fft_sig);
fft_sig_val = abs(fft_sig_val);
end
```

MATLAB code to plot the ambiguity functions for different waveforms

```
tau = -5:0.05:5;
mu = -1.5:0.05:1.5;
T = max(tau);
B = max(mu);
sig = T/sqrt(2*pi);
k = B/T;
[tau,mu] = meshgrid(tau,mu);
amb_sig = abs(exp(1i*pi.*mu.*tau) .* (1 - abs(tau/T)) ...
    .* (sin(pi.*mu.*(T - abs(tau)))./(pi.*mu.*(T - abs(tau)))));
amb_sig = amb_sig.';
figure,contour(tau/5,mu,20*log10(amb_sig/max(max(amb_sig)))',-(0:5:25), ...
    'LineWidth',2)
colorbar
grid on
xlabel('Normalized time delay (\tau/\it T)','Interpreter','tex')
ylabel('Normalized Doppler frequency shift (\mu/\it B)','Interpreter','tex')
ax = gca; % current axes
ax.FontSize = 15;
title('\fontsize{25}Ambiguity Function of a Rectangular Pulse')
%LFM with rectangular envelope
tau = -5:0.05:5;
mu = -3:0.05:3;
[tau,mu] = meshgrid(tau,mu);
amb_sig = abs(exp(1i*pi.*mu.*tau) .* (1 - abs(tau/T)) ...
    .* (sin(pi.*(k.*tau + mu) .* (T - abs(tau)))./(pi.*(k.*tau + mu) ...
    .*(T - abs(tau)))));
amb_sig = amb_sig.';
figure,contour(tau/5,mu/2,20*log10(amb_sig/max(max(amb_sig)))',-(0:5:25), ...
    'LineWidth',2)
colorbar
grid on
xlabel('Normalized time delay (\tau/\it T)','Interpreter','tex')
```

```
ylabel('Normalized Doppler frequency shift (\mu/\it B)','Interpreter','tex')
ax = gca; % current axes
ax.FontSize = 15;
title('\fontsize{25}Ambiguity Function of a LFM with rectangular Envelope')
tau = -8:0.05:8;
mu = -3:0.05:3;
[tau,mu] = meshgrid(tau,mu);
amb_sig = abs(exp(1i*pi.*mu.*tau) .* exp((-(tau.^2)/(4*sig^2)) ...
    - (pi*sig^2.*(k.*tau + mu).^2)));
amb_sig = amb_sig.';
figure,contour(tau/5,mu/2,20*log10(amb_sig/max(max(amb_sig)))' ...
    ,-(0:5:25),'LineWidth',2)
colorbar
grid on
xlabel('Normalized time delay (\tau/\it T)','Interpreter','tex')
ylabel('Normalized Doppler frequency shift (\mu/\it B)','Interpreter','tex')
ax = gca; % current axes
ax.FontSize = 15;
title('\fontsize{25}Ambiguity Function of a LFM with Gaussian Envelope')
```