UNDERWATER ACOUSTICS AND SONAR SIGNAL PROCESSING

SS 2018

ASSIGNMENT 6

IMAGE SOURCE APPROACH

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by

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Introduction

Image source approach is a systematic way for sound propagation modeling. The primary advantage of image source methods is their robustness. They guarantee that all specular paths up to a given order or reverberation time are found. However, image source methods model only specular reflection, and their expected computational complexity grows exponentially.

In this assignment we first develop a MATLAB program for determining the pressure distribution. It is then depicted in colour coded two dimensional diagrams. We then discuss the impact of different frequency values on the pressure distribution.

Theory

0.1 Sound Propagation Modeling

Sound propagation in the ocean is mathematically formulated by the wave equation, whose parameters and boundary conditions are descriptive of the ocean environment. Modeling acoustic propagation conditions is an important issue in underwater acoustics and there exist several mathematical/numerical models based on different approaches. Some of the most used approaches are based on ray theory, modal expansion and wave number integration techniques. In this assignment we will discuss the image source approach in detail.

0.2 Image Source Approach

The wave field within a homogeneous waveguide can be interpreted as the superposition of infinitely many spherical waves that are reflected at the boundaries.

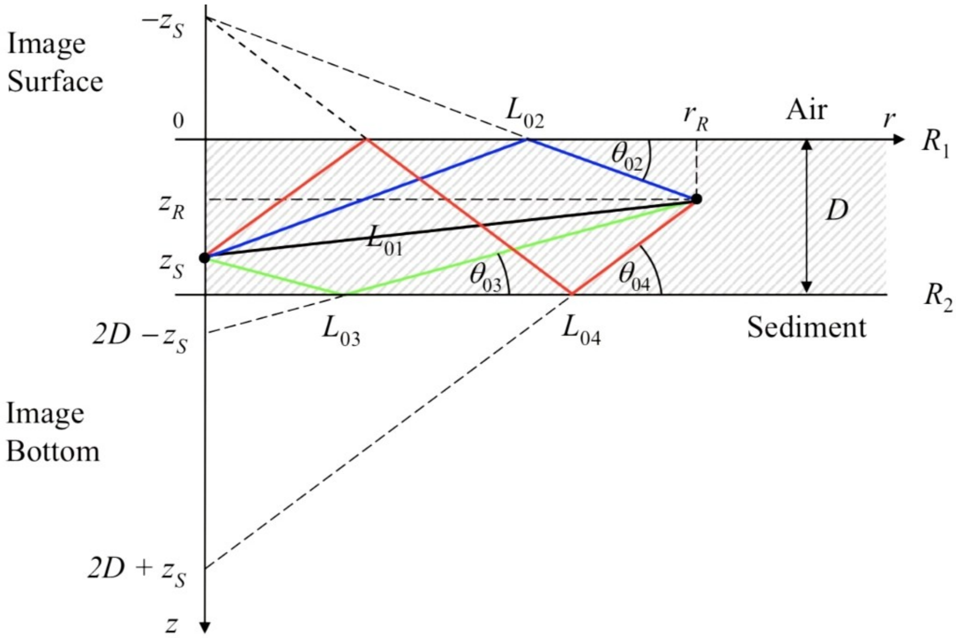


Figure 1: Image source technique

As a first approximation, the sound pressure in the waveguide can be determined by superimposing the four contributions indicated in the Figure 1, i.e.

$$P(r_R, z_R, \omega) = A(\omega)\left(\frac{e^{-jkL_{01}}}{L_{01}} + R_1(\theta_{02}, \omega)\frac{e^{-jkL_{02}}}{L_{02}} + R_3(\theta_{03}, \omega)\frac{e^{-jkL_{03}}}{L_{03}} + R_4(\theta_{04}, \omega)\frac{e^{-jkL_{04}}}{L_{04}}\right) \tag{1}$$

with

$$\begin{aligned} L_{01} &= \sqrt{r_R^2 + (z_R - z_S)^2} \\ L_{02} &= \sqrt{r_R^2 + (z_R + z_S)^2} \\ L_{03} &= \sqrt{r_R^2 + (2D - z_S - z_R)^2} \\ L_{04} &= \sqrt{r_R^2 + (2D + z_S - z_R)^2} \end{aligned} \tag{2}$$

and

$$\begin{aligned} \theta_{02} &= arctan((z_S + z_R)/r_R) \\ \theta_{03} &= arctan((2D - z_S - z_R)/r_R) \\ \theta_{04} &= arctan((2D + z_S - z_R)/r_R) \end{aligned} \tag{3}$$

Continuation of the image source technique in multiples m = 1,2,... of groups of four contributions provides

$$
\begin{gathered}
P(r_R, z_R, \omega) = A(\omega) \sum_{m=0}^{\infty} \Bigg(R_1^m(\theta_{m1}, \omega) R_2^m(\theta_{m1}, \omega) \frac{e^{-jkL_{m1}}}{L_{m1}} + \\
+ R_1^{m+1}(\theta_{m2}, \omega) R_2^m(\theta_{m2}, \omega) \frac{e^{-jkL_{m2}}}{L_{m2}} + R_1^m(\theta_{m3}, \omega) R_2^{m+1}(\theta_{m3}, \omega) \frac{e^{-jkL_{m3}}}{L_{m3}} + \\
+ R_1^{m+1}(\theta_{m4}, \omega) R_2^{m+1}(\theta_{m4}, \omega) \frac{e^{-jkL_{m4}}}{L_{m4}} \Bigg)
\end{gathered}
\tag{4}
$$

with

$$
\begin{aligned}
L_{m1} &= \sqrt{r_R^2 + (2Dm - z_S + z_R)^2} \\
L_{m2} &= \sqrt{r_R^2 + (2Dm + z_S + z_R)^2} \\
L_{m3} &= \sqrt{r_R^2 + (2D(m+1) - z_S + z_R)^2} \\
L_{m4} &= \sqrt{r_R^2 + (2D(m+1) + z_S + z_R)^2}
\end{aligned}
\tag{5}
$$

and

$$
\begin{aligned}
\theta_{m1} &= arctan\Big((2Dm - z_S + z_R)/r_R\Big) \\
\theta_{m2} &= arctan\Big((2Dm + z_S + z_R)/r_R\Big) \\
\theta_{m3} &= arctan\Big((2D(m+1) - z_S - z_R)/r_R\Big) \\
\theta_{m4} &= arctan\Big((2D(m+1) + z_S - z_R)/r_R\Big)
\end{aligned}
\tag{6}
$$

Taking into account that the reflection coefficients at the ocean surface and bottom can be approximated by

$$R \approx -1, water - air - interface$$

$$R \approx 1, water - hardbottom - interface$$

the calculation of the sound pressure simplifies to

$$
P(r_R, z_R, \omega) = A(\omega) \sum_{m=0}^{\infty} (-1)^m \left(\frac{e^{-jkL_{m1}}}{L_{m1}} - \frac{e^{-jkL_{m2}}}{L_{m2}} + \frac{e^{-jkL_{m3}}}{L_{m3}} - \frac{e^{-jkL_{m4}}}{L_{m4}} \right)
\tag{7}
$$

Experimental Research

0.3 Pressure distribution

The MATLAB program for determining the pressure distribution was developed. The following parameter values were considered.

Signal Parameters	
Waveform type	Sinusoidal
Amplitude, A	1
Frequency, f	10 Hz, 100 Hz, 1 kHz, 10 kHz and 100 kHz

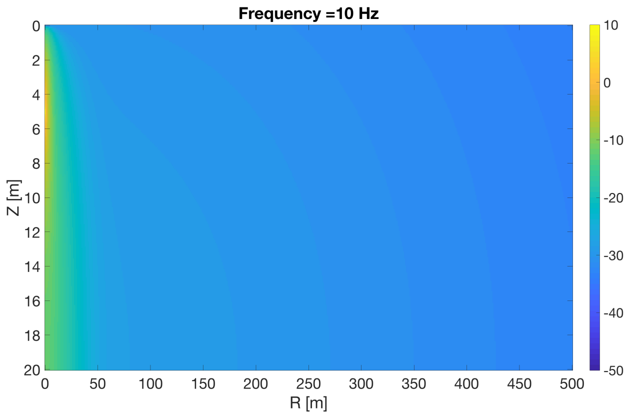
Waveguide Parameters	
Water depth, D	20 m
Source location	$r_S = 0$ m and $z_S = 5$ m
Receiver location	$(r_R, z_R)^T \in [0, 500] \times [0, D]$
Bottom type	Hard, i.e. $R_2 = 1$
Sound speed, c	1480 m/s

The pressure distribution is a function of source location, receiver location and angular frequency. For the given point source coordinates $(0, z_S)$ the pressure shall be determined at an arbitrary receiver location (r_R, z_R). More the pressure value, better is the prorogation of sound as hydrophones work on the phenomenon of pressure variation.

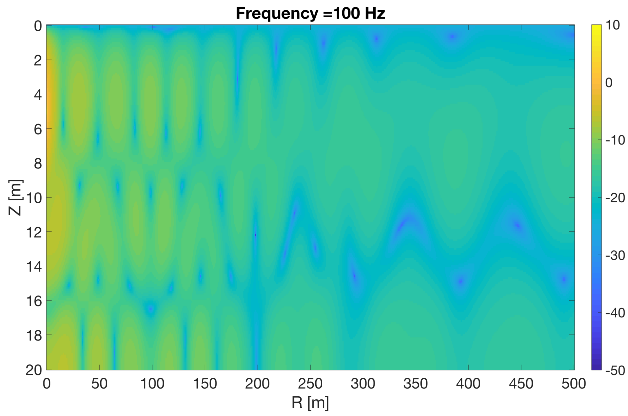
At low frequencies, when the transmission is close to the horizontal, sound wave reflection yields an interesting effect because the reflected signal interferes with the direct path signal. The coherent summation of two signals produces the spatial interference fringes. When the frequency is 10 Hz, the wave length will be very large compared to the wavelength at 1 kHz. Thus if the wavelength is more the graph for pressure variation will be smoother since the probability variation of maxima (constructive interference) or minima (destructive interference) will be less. Hence in Figure 2(a), there is a smooth drop in pressure as we go further away from the source.

Figure 2(b) and Figure 2(c) shows that the pressure distributions along range, r and depth, z, at frequency 100 Hz and 1 kHz. Here we can observe that at 100 Hz, pressure distribution is not smoother as in the case of 10 Hz. Both in Figure 2(b) and Figure 2(c) we can see that there are certain pockets formed in the far field where there is a sudden decrease in pressure. As the frequency increases, the size of these pressure pockets decreases.

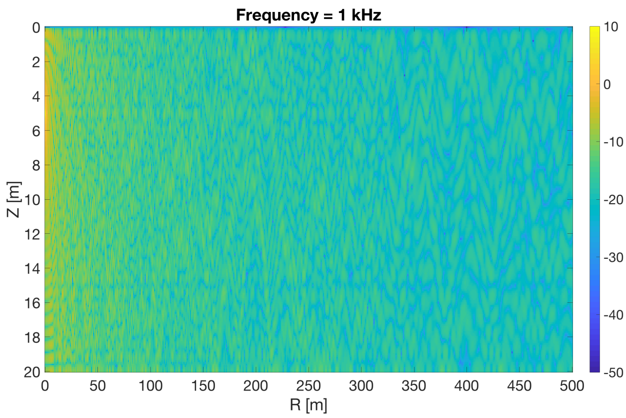
As we move far from the source, the pressure value decreases. By increasing the frequency from 1 kHz to 100 kHz, the distribution of pressure gradually becomes more intense and also more sensitive to the point of observation. A small movement in the horizontal range can lead to a sudden variation in pressure. This can be observed in Figure 2(d) and Figure 2(e), there is a minute blue/teal drop in pressure in between the yellow/green.



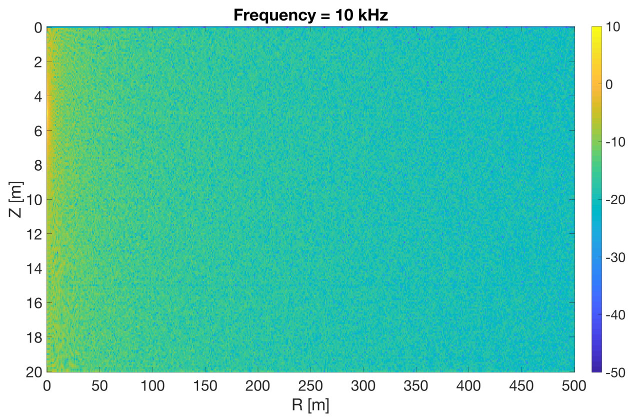
(a) Frequency, $f = 10Hz$



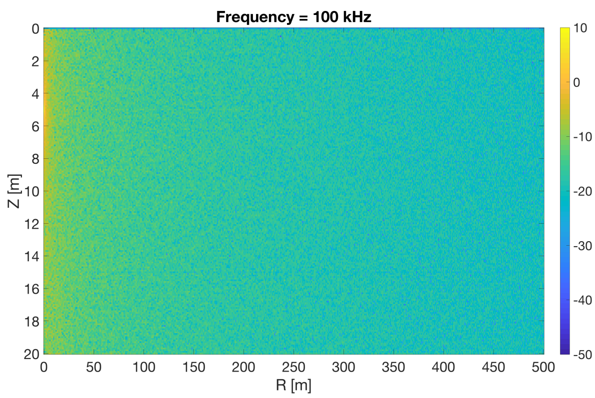
(b) Frequency, $f = 100Hz$



(c) Frequency, $f = 1kHz$



(d) Frequency, $f = 10kHz$



(e) Frequency, $f = 100kHz$

Figure 2: Pressure distribution observed at various frequencies

Conclusion

The MATLAB code was written for determining the pressure distribution for the mentioned parameters. Variation in pressure was observed at different frequency values. In general we can say that the sound pressure near the source is more than the pressure to the far field points. The sound pressure decreases with increase in distance from the source.

Appendix

0.4 MATLAB code to depict the pressure distribution

```
A = 1; % Amplitude
D = 20; % Water depth [m]
r_S = 0; % Source location
z_S = 5; % Source location
c = 1480; % Sound speed
z_R = 0:0.1:D; % Receiver location
r_R = 0:1:500; % Receiver location
for i = 1:1:5
    f = 10.^i;
    k = 2.*pi.*f./c; % Wave number
    [z,r] = ndgrid(z_R,r_R);
    P = zeros(length(z_R),length(r_R),2);
    p = zeros(length(z_R),length(r_R),2);
    for m = 0:1:10 % more than 5 iterations
        C1 = 2.*D;
        C2 = z_S+z;
        C3 = z_S-z;
        l = m+1;
        C4 = C1.*m;
        C5 = C1.*l;
        L(:,:,1) = abs(r+1i.*(C4-C3));
        L(:,:,2) = abs(r+1i.*(C4+C2));
        L(:,:,3) = abs(r+1i.*(C5-C2));
```

```
            L(:,:,4) = abs(r+1i.*(C5+C3));
            P(:,:,2) = A*(-1)^m .* (exp(-1i.*k.*L(:,:,1))./L(:,:,1) ...
                      -exp(-1i.*k.*L(:,:,2))./L(:,:,2) ...
                      + exp(-1i.*k.*L(:,:,3))./L(:,:,3) ...
                      -exp(-1i.*k.*L(:,:,4))./L(:,:,4));
            P = sum(P,3);
        end
    p(:,:,i) = P;
    CLIM = [-50 10];
    set(0,'DefaultAxesFontSize',30)
    figure(i)
    imagesc(r_R,z_R,10.*log10(abs(p(:,:,i))),CLIM);
    colorbar('vert');
    xlabel('R [m]');
    ylabel('Z [m]');
    if i < 3
    title(strcat('Frequency = ',num2str(10.^i),' Hz '));
    else
        if i == 3
        title(strcat('Frequency = 1 kHz '));
        else
            if i == 4
                title(strcat('Frequency = 10 kHz '));
            else
                title(strcat('Frequency = 100 kHz '));
            end
        end
    end
end
```