Matrix Algebra

A matrix $A = (a_{ij})$ of order $m \times n$ is a set of data arranged in m rows and n coulmns.

$$A = (a_{ij}) = \begin{pmatrix} 2 & 4 & 5 \\ 0 & -6 & -1 \\ 1 & -2 & 8 \end{pmatrix}_{3 \times 3}$$

For an element a_{ij} of a matrix A
→ i tells us the row
→ j tells us the column

$a_{11} = 2$, $a_{12} = 4$, $a_{13} = 5$

$a_{21} = 0$, $a_{22} = -6$, $a_{23} = -1$

$a_{31} = 1$, $a_{32} = -2$, $a_{33} = 8$.

Types :- Column Matrix (Order 3×1) :- 1 column allowed

$$\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$$

Row matrix

$$\begin{pmatrix} 1 & 2 & -6 \end{pmatrix}$$

Symmetric Matrix :- $a_{ij} = a_{ji}$

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$$

a_{12}

a_{21}

Example: $a_{21} = a_{12}$
$\Rightarrow 2 = 2$

Anti-Symmetric Matrix:

$$\begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & 6 \\ 2 & -6 & 0 \end{pmatrix}$$

$a_{ij} = -a_{ji}$

$a_{21} = -a_{12}$

$\Rightarrow 1 = -1$

* Diagonal elements will be zero.

* An antisymmetric matrix is a square matrix where the elements satisfy the condition that if a_{ij} is non-zero, then a_{ji} must be zero, and vice versa. In other words, the matrix is skew-symmetric with respect to the main diagonal, where the elements below the diagonal are the negatives of the corresponding elements above the diagonal.

Null matrix:

$$O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Identity matrix:

every identity matrix is a square matrix. Ex:

$$I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}_{3 \times 3}$$

Diagonal matrix:

$$\begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -4 \end{pmatrix}$$

Upper triangular matrix: An upper triangular matrix in which all the entries below the main diagonal are zero.

$$\begin{pmatrix} 9 & 7 & 6 \\ 0 & 3 & 8 \\ 0 & 0 & -1 \end{pmatrix}$$

Lower triangular matrix: A lower triangular matrix is a square matrix in which all those entries above the main diagonal are zero.

$$\begin{pmatrix} -8 & 0 & 0 \\ 6 & 1 & 0 \\ -4 & 7 & -2 \end{pmatrix}$$

Scalar Matrix: It's a type of matrix where all diagonal elements are equal, and all the off-diagonal elements are zero.

$$\begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

Matrix Transpose

$$A = \begin{pmatrix} -2 & -1 \\ -1 & 0 \\ -3 & -8 \end{pmatrix}_{3\times 2} \rightarrow \begin{pmatrix} -2 & -1 & -3 \\ -1 & 0 & -8 \end{pmatrix}_{2\times 3}$$

Addition/subtraction: The result of adding two matrices is another matrix of the same order, whose elements are the sum of the elements that occupy the same position.

$$\begin{pmatrix} 7 & 4 & 5 \\ -7 & 5 & 2 \\ 0 & -7 & 0 \end{pmatrix} + \begin{pmatrix} -9 & 1 & 0 \\ 0 & 9 & -1 \\ -7 & -2 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} (7)+(-9) & (4)+(1) & (5)+(0) \\ \vdots & \vdots & \vdots \end{pmatrix} = \begin{pmatrix} -2 & 5 & 5 \\ -7 & 14 & 1 \\ -7 & -9 & -3 \end{pmatrix}$$

* In ~~other~~ order to add matrices, they must have the same dimension, meaning they must have the same number of rows and columns.

Product by Number :- Scalar multiplication

$$7 \times \begin{pmatrix} 9 & 8 & -4 \\ 5 & 7 & -5 \\ 9 & -4 & -4 \end{pmatrix}_{3 \times 3} \Rightarrow \begin{pmatrix} 7\cdot(9) & 7\cdot(8) & 7\cdot(-4) \\ \vdots & \vdots & \vdots \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 63 & 56 & -28 \\ 35 & 49 & -35 \\ 63 & -28 & -28 \end{pmatrix}$$

Product :-

Matrix Row by Matrix column :- In order to multiply these two matrices, the number of columns in the first one must be equal to the number of rows in the second.

The result of multiplying them is a number that is obtained by adding the products of the elements that occupy the same position.

$$\rightarrow \begin{pmatrix} 9 & 8 & -4 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 5 \\ 9 \end{pmatrix} = (9)\cdot(-3) + (8)\cdot(5) + (-4)\cdot(9)$$

$$= -23$$