Vector Algebra

direction + magnitude = vector

Null vectors : ~~Direction of a vector and~~ Magnitude of the vector is zero.

~~Magni~~ Magnitude of the vector :-

$$4\hat{i} + 3\hat{j}$$

$$= \sqrt{4^2 + 3^2}$$

$$= \sqrt{25} = 5.$$

Scaling Vectors : multiplying each component of the vector by a scalar value, which changes the magnitude (length) of the vector but not its direction.

Ex :- $v = [3, 4]$

scaling it by a factor of 2 would result in the vector ~~$2v = [4, 6]$~~ $2v = [6, 8]$.

It also referred as "vector multiplication" or "scalar multiplication".

Vector Addition :- two vectors are added together to produce a resultant vector.

Ex :-

$$v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \quad \& \quad w = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

Add these two vectors together, you simply add their corresponding components:-

$$v + w = \begin{bmatrix} v_1 + w_1 \\ v_2 + w_2 \\ \vdots \quad \vdots \\ v_n + w_n \end{bmatrix}$$

So, the resultant vector will have the same no. of components as the original vectors, and each component of the resultant vector is the sum of the corresponding components of the original vectors.

Vector addition represents the process of joining vectors tip-to-tail, where the resultant vector is the vector from the tail of the first vector to the tip of the second vector.

$u = 3\hat{i} + 4\hat{j}$

$v = 7\hat{i} + 5\hat{j}$

$u + v = 10\hat{i} + 9\hat{j}$

Dot product Insight : It's also known as the scalar product or inner product, is a fundamental operation in linear algebra. It takes two vectors and produces a scalar quantity. The dot product of two vectors is calculated by multiplying corresponding components of the vectors and summing the results.

$$v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \quad \& \quad w = \begin{bmatrix} w_1 \\ w_2 \\ w_n \end{bmatrix}$$

The dot product $v \cdot w$ is calculated

$$v \cdot w = v_1 \cdot w_1 + v_2 \cdot w_2 + \cdots v_n \cdot w_n$$

Alternatively, the dot product can be expressed as the product of the magnitude of the vector and the cosine of the angle between them:

$$v \cdot w = |v| \cdot |w| \cdot \cos(\theta) \longrightarrow \text{imp.}$$

where $|v|$ and $|w|$ are the magnitudes (lengths) of vectors v and w resp. and θ is the angle between the two vectors.

* Vector → ~~magn~~ magnitude + direction
* Scalar → magnitude

Orthogonality: ~~Orthogonal~~ Orthogonal: In three dimensional space ~~are orthogonal~~ (and more generally in vector spaces), two vectors are orthogonal if their dot product is zero. Geometrically, this means the vectors are ~~perped~~ perpendicular to each other.

Orthonormal: describes a set of vectors where each vector:

1. Has a length of 1 (normalized)
2. Is at right angles (perpendicular) to every other vector in the set.

Cross Product: Also known as the vector product, is a mathematical operation between two vectors in three-dimensional space. Unlike the dot product, which results in a scalar quantity, the cross product yields a vector that is perpendicular to the plane containing the original two vectors.

The two vectors v & w, their cross product $v \times w$ is a vector defined as:

$$v \times w = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \times \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{pmatrix}$$

x - component (first entry) is given by $v_2 w_3 - v_3 w_2$
y - component (second entry) is given by $v_3 w_1 - v_1 w_3$
z - component (third entry) is given by $v_1 w_2 - v_2 w_1$

Vector Norms :

L1 Norm :- (Taxicab Norm or Manhattan Norm)

* Denoted as $||v||_1$; it calculates the sum of the absolute vector values.
* Represent the Manhattan distance from the origin in the vector space.
* Example: Suppose we have a vector $v = [3, -4, 2]$. The L_1 norm is

$$||v||_1 = |3| + |-4| + |2|$$
$$= 3 + 4 + 2 = 9.$$

∴, the L1 norm of vector v is 9.

L2 Norm :- (~~Ecu~~ Euclidean Norm)

* Denoted as $||v||_2$, it calculates the square root of the sum of the squared vector values.
* Represent the ~~Ecu~~ Euclidean distance from the origin.
* Example: For the same vector v, the L2 norm is:

$$||v||_2 = \sqrt{3^2 + (-4)^2 + 2^2}$$
$$= \sqrt{9 + 16 + 4} = \sqrt{29}$$
$$= 5.38$$

∴, the L2 norm of vector v is approximately 5.38

* Max Norm (Infinity Norm):
* Denoted as $||V||_{+\infty}$, it calculates the maximum absolute value of the vector elements.
* ~~Useful~~ Useful for constraining weights in neural networks.

* $V = [3, -4, 2]$. The max norm is

$$||v||_{+\infty} = \max(|3|, |-4|, |2|)$$
$$= 4$$

∴ The max norm of vector v is 4.