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A new I–V model for light-emitting devices with a quantum well

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Abstract

This letter reports a new current versus voltage model for light-emitting devices with a quantum well where electrons and holes are injected and recombine. The current is entirely caused by the recombination of electrons and holes. Historically, the equation used for light-emitting diodes (LEDs) and laser diodes (LDs) has been the renowned Sah–Noyce–Shockley (SNS) diode equation. In this equation at typical forward bias condition, most of the current is caused by the diffusion of carriers over the depletion region. It is clear that this condition is different from what actually happen in LEDs and LDs. We thus looked into the fundamental of carrier transport and developed a new model for devices with a quantum well. Based on the new model, calculated I–V curves agree well with measurement results of GaN/sapphire LEDs with GaInN quantum wells. In calculation, junction temperature T_j rather than case temperature T_c is used to achieve better agreement.

Keywords: Light-emitting diodes; Diodes; Semiconductor lasers; p–n junctions; Quantum well devices

1. Introduction

For decades, the diode equation for p–n junctions has been used to describe the I–V characteristic of p–n junction LEDs [1,2]. This equation is by [1–4],

$$I = I_0[\exp\left(\frac{qV}{\eta k T}\right) - 1], \tag{1}$$

where I_0 is the reverse saturation current, η is the ideality factor, and T is the temperature. If the diode is ideal, $\eta = 1$ and the equation is known as the Shockley ideal diode Eq. (5). It was derived under the assumption of no carrier recombination in the depletion region. Thus, only diffusion current was considered. The current due to recombination in the depletion region was later added. This leads to $1<\eta\leqslant 2$ in Eq. (1) [6].The improved equation is referred to as Sah–Noyce–Shockley (SNS) equation. The recombination current is significant only under very small forward bias condition and can be ignored under typical forward bias operation. Eq. (1) is often extended to LEDs and LDs with quantum wells (QWs) [1,7–11]. The most popular approach is to use the SNS equation and adjust the ideality factor η to fit the measured curve.

In our recent LED study [12,13], we looked into the fundamental mechanism of current flow in LEDs with QWs versus that in a typical p–n junction diode. We believe that, for LEDs and LDs with QWs, the use of Eq. (1) is fundamentally inappropriate. We explain this as follows. LEDs and LDs are forward biased heavily in typical operation. Under this bias condition, the current in Eq. (1) is contributed almost entirely by diffusion, as illustrated by the carrier concentrations in Fig. 1(a), where w is the depletion width, and n_p and p_n the minority carrier concentrations. The hole current is evaluated at $x = w$ after the holes have diffused over the depletion region without recombination and arrived at the quasi-neutral n-region. Likewise, the electron current is evaluated at $x = 0$. The total current density equals $J_p(w) + J_n(0)$, which is the diffusion current. A term is added to account for the recombination of carriers in the depletion region [6]. However, the current caused by recombination is only significant when the forward bias voltage is small, i.e., in the sub-threshold range. In typical forward bias condition, the recombination current can be entirely ignored.

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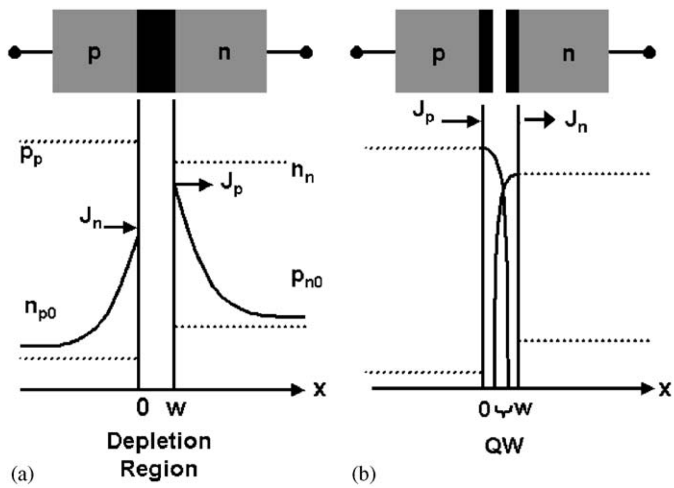

Fig. 1. (a) Carrier concentrations of a p–n junction diode. (b) Carrier concentrations of a p–n junction light-emitting device with a quantum well (QW).

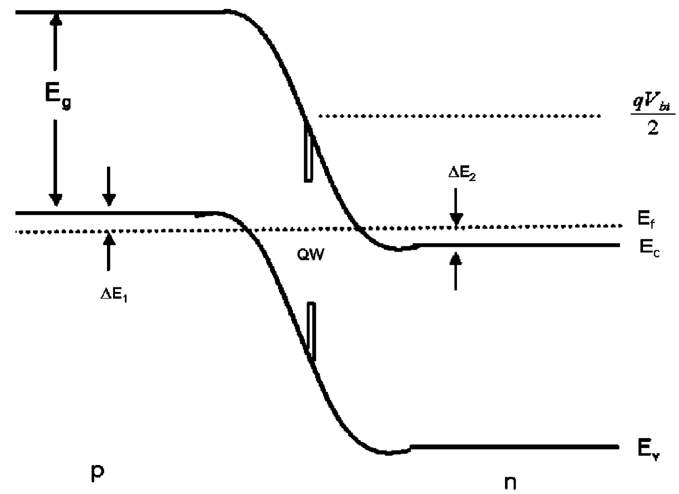

Fig. 2. Energy band diagram of the light-emitting device with a QW at thermal equilibrium condition.

On the other hand, for LEDs and LDs with QWs, the electrons and holes are injected into the QW and are expected to recombine entirely there [1,2,7]. Fig. 1(b) exhibits the carrier concentrations of a LED or LD with a QW in our model under forward bias. Holes and electrons are injected into the QW where one electron recombines with one hole, releasing a photon. Thus, $J_n(w)$ equals $J_p(0)$, which is the recombination current. We see that the current in the SNS equation is almost entirely the diffusion current while in reality the current in LEDs and LDs is the recombination current. Thus, it is clear that the SNS equation cannot be used for LEDs and LDs with QWs because it is derived under the assumption that is untrue for LEDs and LDs with QWs. And yet, the SNS equation has been used for LEDs and LDs for decades probably for its convenience and simplicity.

2. The new *I–V* model for leds with quantum wells

To introduce the model for LEDs with QW, we first quantitatively describe the energy band diagram of LEDs with QW at thermal equilibrium, depicted in Fig. 2. The QW is sandwiched in a p–n junction. This model is applicable not only to LEDs but also to laser diodes that have the prescribed structure. For GaN semiconductor, the energy band gap E_g is 3.42 eV at $T = 300$ K. The LEDs that we studied have 10 InGaN/GaN quantum wells and the total width is 0.003 μm, which is typical of GaN-based LEDs. This width is much smaller than depletion width of the p–n junction. Thus, for simplicity in this initial effort, the quantum well region is approximated by a single quantum well. The opening of the quantum well is assumed to locate at half of the height of the built-in energy, qV_{bi}.

For typical LED structures, the doping concentration is very high and both *n*-GaN and *p*-GaN become degenerate. The Fermi level moves into the valence band in the p-side and conduction band in the n-side. The built-in potential energy qV_{bi} is given by

$$qV_{bi} = E_g + \Delta E_1 + \Delta E_2, \tag{2}$$

where $\Delta E_1 = E_v - E_f$ on p-side and $\Delta E_2 = E_f - E_c$ on n-side, also illustrated in Fig. 2. Notice that qV_{bi} is larger than E_g. For the LEDs studied, the doping concentrations of n-GaN and p-GaN are $2 \times 10^{18}\,\mathrm{cm}^{-3}$ [13]. Based on our calculation, $\Delta E_1 = 0.95$ KT and $\Delta E_2 = 0.95$ KT. We wish to point out that our model is valid even if the semiconductor is non-degenerate.

When a forward bias voltage is applied, the potential energy across the depletion region is reduced to $q(V_{bi} - V_a)$, as illustrated Fig. 3. Some electrons now have energy higher than the opening of the QW and can be injected into the QW easily. There is a net diffusion of electrons across the barrier from n-side. Similarly, since the applied voltage V_a also reduces the barrier to the holes, there is a net amount of holes injected into the QW. For ease of illustration, it is assumed that half of applied energy $qV_a/2$ incurs in the n-side depletion region and the other half in the p-side depletion. The electron current density J_n equals the hole current density J_p because one electron recombines with one hole in the QW, releasing one photon. The opening of the QW is still at half of the height of the potential energy barrier, that is, $q/2(V_{bi} - V_a)$. For illustration purpose, the electron concentration profile per energy interval is superimposed in the conduction band in Fig. 3. The shaded area equals the electron concentration that has energy higher than the opening of the quantum well and can diffuse and fall into the QW. This electron concentration is expressed as

$$n_{inj} = \int_a^b f(E) g_c(E)\,\mathrm{d}E, \tag{3}$$

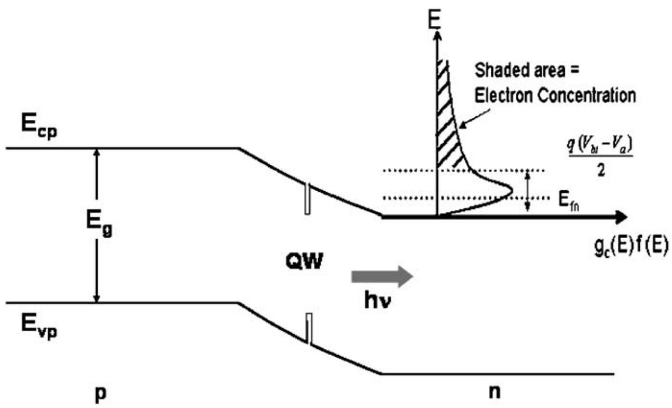

Fig. 3. Energy band diagram of the light-emitting device with a QW under forward bias voltage V_a. Electron concentration per energy interval is superimposed in the conduction band of the n-side where the shaded area shows the portion of electrons that can be injected into the QW.

where the density of states $g_c(E)$ per energy interval at E in the conduction band is given as [3,4]

$$g_c(E) = \frac{m_n^* \sqrt{2m_n^*(E-E_c)}}{\pi^2 \hbar^3}, \tag{4}$$

here m_n^* is the electron effective mass, E_c is the conduction edge and $\hbar$ is the Plank's constant. For GaN semiconductor, the electron effective mass $m_n^*$is 0.3 m_0 where $m_0 = 9.1 \times 10^{-31}$ kg is the electron rest mass [14]. The Fermi–Dirac function $f(E)$ is given by [3,4]

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}} \approx \frac{1}{e^{(E-E_f)/kT}}, \tag{5}$$

where E_f is the Fermi level. When energy E is higher than the Fermi level E_f by several kT, the exponential term in the denominator is much larger than 1, and the Fermi–Dirac function $f(E)$ can be approximated by the Boltzman distribution indicated in Eq. (5). The upper integration limit of Eq. (3) can be set at infinity because the integrand is very small at the tail of the Fermi–Dirac function. The lower limit is $q/2(V_{bi} - V_a)$, the location of the opening the QW. Thus, Eq. (3) becomes,

$$n_{inj} = \frac{m_n^* \sqrt{2m_n^*}}{\pi^2 \hbar^3} \int_{q/2(V_{bi}-V_a)}^{\infty} \frac{\sqrt{E-E_c}\, dE}{e^{(E-E_f)/kT}}. \tag{6}$$

So far, we have not been able to find an analytical expression for the above integration. Numerical integration can be easily performed.

With the injected electron concentration information available, we now can discuss how the electron injection is related to current flow in the LED with QW. We assume that electrons are injected into the QW entirely by diffusion. Similarly, holes are injected into the QW by diffusion. Since diffusion is the dominant mechanism of current flow, the diffusion current density equation is used (3,4). For the LEDs studied, the depletion width on p-side and n-side is calculated to be 1 μm. The depletion region is much smaller than the diffusion length. Thus a linear approximation of the current density equation within the depletion is taken,

$$J_n(x) = qD_n \frac{dn}{dx} \cong qD_n \frac{n}{x_n}, \tag{7}$$

where D_n is the electron diffusion coefficient, in units of cm^2/s, x_n is the depletion width on n-side. For GaN, the electron diffusion coefficient is 25 cm^2/s at room temperature [14]. If electrons and holes recombine completely in the QW, the electron current density J_n must be equal to the hole current density J_p that is injected from the p-side, i.e.

$$J_n = J_p. \tag{8}$$

Thus the electron current density can be taken as the total current density. For the LEDs studied, the junction area is 370 × 380 μm. In calculating the I–V curve using Eq. (7), junction temperature rather than case temperature is used because it is the temperature of the active region where carrier action takes place. T_j is previously measured with the nematic liquid crystal thermography [12]. The LEDs measured have a thermal resistance $\Theta_{jc} = 166$ K/W. The parasitic series resistance of the LED is 13 Ω, mainly caused by the current spreading layer. T_j is related to the LED drive power by the well-known expression

$$T_j = \Theta_{jc} P_e + T_c, \tag{9}$$

where T_c is the case temperature set at 300 K during the measurement and P_e is the electrical power. Fig. 4 shows the calculated I–V curve (crossed) using Eqs. (6) and (7). Measured I–V curve (solid) after extracting the effect of the series resistance is also plotted on Fig. 4. The turn-on voltage V_{on} of LED is about 3.45 V and diode current rises sharply at the turn-on voltage. The calculated I–V curve with the $T = T_j$ agrees quite well with the measured I–V curve except at sub-threshold.

At sub-threshold, the two curves do not agree as well. We then looked at the measured optical power. We found that the optical power is linearly proportional to the LED

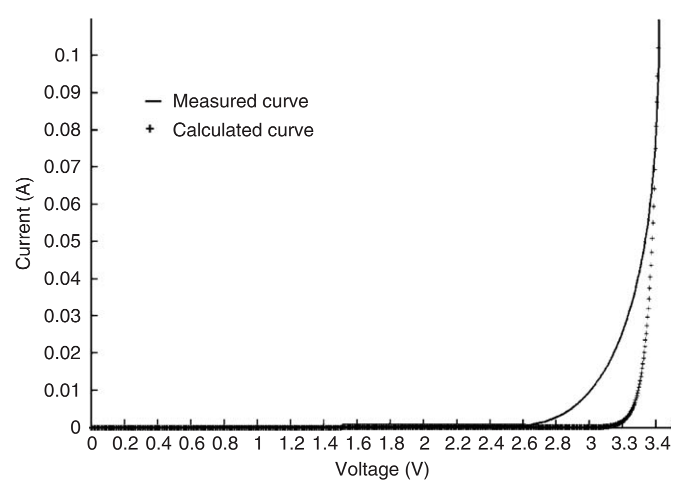

Fig. 4. The current versus voltage curve calculated based on the new model is compared with that extracted from measurement performed on GaN/shappire with GaInN QWs. In the calculation, junction temperature rather than case temperature is used.

current from 0 to 0.08 A which include the sub-threshold range. This implies that the current at sub-threshold is caused by the electrons injected into the quantum well as prescribed in our new model. The more gradual current increase in the measured curve at sub-threshold suggests that the shaded area in Fig. 3 does not increase so sharply with bias voltage as predicted by the new model. This is indeed possible because the density of states function used, Eq. (4), is for non-degenerate semiconductors while p-GaN and n-GaN of the LEDs studied are both degenerate. The use of more elaborated density of states function for degenerate semiconductors [7] in the model probably will achieve better match with measured results.

3. Summary

A new model of current versus voltage relationship for LEDs with a QW is represented. The resulting *I–V* curve agrees quite well with the curve extracted from measurement preformed on GaN/sapphire LEDs with GaInN QWs. In the new model, injected electrons and holes recombine entirely in the QW located at the p–n junction. In the renowned SNS diode model, few electrons and holes recombine in the depletion region at typical forward bias condition [3,4,6]. This is the fundamental difference between our model and the SNS diode model. The existence of this difference is natural because the SNS model is derived for p–n junctions without a QW while our model includes a QW in the p–n junction. It is therefore not appropriate to apply the SNS model to devices with QWs where electrons and holes are injected and recombine. Our model is applicable to any light emitting device with a quantum well located at the p–n junction.

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