Miniproject #1

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{a}

Parameters for the Selected System

```
%% parameters
M = 30
```

```
M = 30
```

```
mw = 2
```

```
mw = 2
```

```
r = 0.1
```

```
r = 0.1000
```

```
d = 0.7
```

```
d = 0.7000
```

```
K_tL = 0.5
```

```
K_tL = 0.5000
```

```
K_tR = 0.5
```

```
K_tR = 0.5000
```

```
B_L = 0.1
```

```
B_L = 0.1000
```

```
B_R = 0.1
```

```
B_R = 0.1000
```

```
J_m = 0.0263
```

```
J_m = 0.0263
```

```
J_M = 2.45
```

```
J_M = 2.4500
```

```

alpha = M*r^2 + 3*mw*r^2 + 3*J_m + J_M*(r^2/d^2)/2 - ...
    2*J_M^2*(r^2/d^2)/(M*r^2) + 3*mw*r^2 + 3*J_m + J_M*(r^2/d^2)
```

```
alpha = -0.1639
```

```

beta = 2*J_M^2*(r^4/d^4)/(M*r^2) + 3*mw*r^2 + 3*J_m + J_M*(r^2/d^2)
```

```
beta = 0.2056
```

```
gama = 2*J_M*(r^2/d^2)/(M*r^2) + 3*mw*r^2 + 3*J_m + J_M*(r^2/d^2)
```

```
gama = 0.5222
```

State-Space Representation of the System

```
%% system matrix
A = [0     1        0        0     0          0;...
     0 -B_L/alpha -1/alpha   0 beta/alpha gama/alpha;...
     0     0        0        0     0          0;...
     0     0        0        0    1/alpha     0;...
     0 beta/alpha gama/alpha 0 -B_R/alpha -1/alpha;...
     0     0        0        0     0          0]
```

```
A =

         0    1.0000         0         0         0         0
         0    0.6103    6.1025         0   -1.2545   -3.1869
         0         0         0         0         0         0
         0         0         0         0   -6.1025         0
         0   -1.2545   -3.1869         0    0.6103    6.1025
         0         0         0         0         0         0
```

```
B = [    0                  0;...
     K_tL/alpha 2*K_tL*gama/alpha;...
         0                  0;...
         0                  0;...
     2*K_tR*gama/alpha K_tR/alpha;...
         0                  0]
```

```
B =

         0         0
   -3.0513   -3.1869
         0         0
         0         0
   -3.1869   -3.0513
         0         0
```

```
C = [1 1 0 1 1 0]
```

```
C =

     1     1     0     1     1     0
```

```
D = [0 0]
```

```
D =

     0     0
```

$$\dot{\mathbf{x}} = A\mathbf{x} + BU$$
$$y = C\mathbf{x} + DU$$

where,

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0.6103 & 6.1025 & 0 & -1.2545 & -3.1869 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -6.1025 & 0 \\ 0 & -1.2545 & -3.1869 & 0 & 0.6103 & 6.1025 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \; B = \begin{pmatrix} 0 & 0 \\ -3.0513 & -3.1869 \\ 0 & 0 \\ 0 & 0 \\ -3.1869 & -3.0513 \\ 0 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix},\ D = \begin{pmatrix} 0 & 0 \end{pmatrix}$$

Check the Controllability of the System

Step1: find the C matrix;

Where, $C = \begin{pmatrix} B & BA & BA^2 & BA^3 & BA^4 & BA^5 \end{pmatrix}$

```
Ct = ctrb(A,B)
```

```
Ct =
         0         0   -3.0513   -3.1869    2.1359    1.8829   -1.0586   -1.5304    1.2738    0.3941    0.2830   -1.357
   -3.0513   -3.1869    2.1359    1.8829   -1.0586   -1.5304    1.2738    0.3941    0.2830   -1.3575    1.8756   -1.183
         0         0         0         0         0         0         0         0         0         0         0
         0         0   19.4484   18.6204  -11.4904  -13.0344    9.3393    6.4602   -2.4048   -7.7735    8.2841   -1.727
   -3.1869   -3.0513    1.8829    2.1359   -1.5304   -1.0586    0.3941    1.2738   -1.3575    0.2830   -1.1834    1.875
         0         0         0         0         0         0         0         0         0         0         0
```

$$C = \begin{pmatrix} 0 & 0 & -3.0513 & -3.1869 & 2.1359 & 1.8829 & -1.0586 & -1.5304 & 1.2738 & 0.3941 & 0.2830 & -1.3575 \\ -3.0513 & -3.1869 & 2.1359 & 1.8829 & -1.0586 & -1.5304 & 1.2738 & 0.3941 & 0.2830 & -1.3575 & 1.8756 & -1.1834 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 19.4484 & 18.6204 & -11.4904 & -13.0344 & 9.3393 & 6.4602 & -2.4048 & -7.7735 & 8.2841 & -1.7270 \\ -3.1869 & -3.0513 & 1.8829 & 2.1359 & -1.5304 & -1.0586 & 0.3941 & 1.2738 & -1.3575 & 0.2830 & -1.1834 & 1.8756 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Step2: find the rank of C matrix and check if C is full row rank or not

```
rnk = rank(Ct)
```

```
rnk = 4
```

```
controllable = rank(Ct) == size(Ct,1)
```

```
controllable = logical
   0
```

So, $\rho(C) \neq r$ **and thats why the system is not controllable.**

Check if the system is Stable or not

```
[num,den] = ss2tf(A,B,C,D,2)
```

```
num =
         0   -6.2382   27.0660  -29.9881   -0.0000         0         0

den =
    1.0000   -1.2205   -1.2013         0         0         0         0
```

```
sys = tf(num,den)
```

```
sys =


  -6.238 s^5 + 27.07 s^4 - 29.99 s^3

                      - 8.792e-15 s^2

  -------------------------------------

       s^6 - 1.221 s^5 - 1.201 s^4


Continuous-time transfer function.
```

```
stability = isstable(sys)
```

```
stability = logical
   0
```

```
pl = pole(sys)
```

```
pl =
         0
         0
         0
         0
    1.8647
   -0.6442
```

Remark: The System is not stable

Check the observability of the System

Step1: Check if (A, C) is observable or not

find the O matrix. where, $O = \begin{pmatrix} C \\ CA \\ CA^2 \\ CA^3 \\ CA^4 \\ CA^5 \end{pmatrix}$

```
O = obsv(A,C)
```

```
O =
    1.0000    1.0000         0    1.0000    1.0000         0
         0    0.3558    2.9156         0   -6.7467    2.9156
         0    8.6807   23.6726         0   -4.5635  -42.3060
         0   11.0223   67.5181         0  -13.6747  -55.5140
         0   23.8809  110.8441         0  -22.1722 -118.5773
         0   42.3878  216.3952         0  -43.4886 -211.4132
```

Step2: Check if O is full column rank or not

```
rk = rank(O)
```

```
rk = 4
```

```
observability = rk == size(O,2)
```

```
observability = logical
   0
```

Remark: As $\rho(O) \neq c$ the system is not observable

Decompose the System into Controllable and Uncontrollolable part

If the controllability matrix of (*A*, *B*) has rank *r* ≤ *n*, where *n* is the size of *A*, then there exists a similarity transformation such that

$\bar{A} = TAT^T, \bar{B} = TB, \bar{C} = CT^T$

where *T* is unitary, and the transformed system has a *staircase* form, in which the uncontrollable modes, if there are any, are in the upper left corner.

$$\bar{A} = \begin{pmatrix} A_{\bar{c}} & 0 \\ A_{21} & A_c \end{pmatrix}, \bar{B} = \begin{pmatrix} 0 \\ B_c \end{pmatrix}, \bar{C} = \begin{pmatrix} C_{\bar{c}} & C_c \end{pmatrix}$$

where (A_c, B_c) is controllable, all eigenvalues of $A_{\bar{c}}$ are uncontrollable, and $C_c(sI - A_c)^{-1}B_c = C(sI - A)^{-1}B$.

`[Abar,Bbar,Cbar,T,k] = ctrbf(A,B,C)` decomposes the state-space system represented by `A`, `B`, and `C` into the controllability staircase form, `Abar`, `Bbar`, and `Cbar`, described above. `T` is the similarity transformation matrix and `k` is a vector of length *n*, where *n* is the order of the system represented by `A`. Each entry of `k` represents the number of controllable states factored out during each step of the transformation matrix calculation. The number of nonzero elements in `k` indicates how many iterations were necessary to calculate `T`, and `sum(k)`is the number of states in A_c, the controllable portion of `Abar`.

```
[Abar,Bbar,Cbar,T,kbar] = ctrbf(A,B,C)
```

```
Abar =
         0         0         0         0         0         0
    0.0000   -0.0000   -0.0000   -0.0000   -0.0000    0.0000
   -0.0000    0.0000    0.0000    0.0000    0.7071   -0.7071
    0.0000    0.0000   -0.0000    0.0000   -4.3151   -4.3151
   -6.5686    6.5686    0.0000    0.0000    1.8647    0.0000
    2.0616    2.0616    0.0000    0.0000         0   -0.6442
Bbar =
         0         0
    0.0000    0.0000
   -0.0000   -0.0000
    0.0000    0.0000
   -0.0959    0.0959
   -4.4111   -4.4111
Cbar =
         0    0.0000   -1.0000    1.0000    0.0000    1.4142
T =
         0         0    1.0000         0         0         0
    0.0000    0.0000         0         0   -0.0000    1.0000
   -1.0000   -0.0000         0    0.0000    0.0000    0.0000
    0.0000    0.0000         0    1.0000   -0.0000    0.0000
    0.0000   -0.7071         0         0    0.7071         0
   -0.0000    0.7071         0         0    0.7071         0
kbar =
     2     2     0     0     0     0
```

Extract the Controllable and Uncontrollable part from Abar, Bbar and Cbar:

```
Auc = Abar(1:size(Abar,1)-sum(kbar),1:size(Abar,1)-sum(kbar))
```

```
Auc =
   1.0e-29 *

         0         0
    0.6537   -0.6860
```

```
Buc = Bbar(1:size(Bbar,1)-sum(kbar),1:2)
```

```
Buc =
   1.0e-30 *

         0         0
    0.4438    0.2482
```

```
Cuc = Cbar(1:size(Abar,1)-sum(kbar))
```

```
Cuc =
   1.0e-15 *

         0    0.9992
```

```
Duc = zeros(1,size(Buc,2))
```

```
Duc =
     0     0
```

Now Check if the uncontrollable part is stable or not:

```
[num,den] = ss2tf(Auc,Buc,Cuc,Duc,2)
```

```
num =
   1.0e-45 *

         0    0.2480         0

den =
    1.0000    0.0000         0
```

```
sys = tf(num,den)
```

```
sys =

     2.48e-46 s
  ----------------
  s^2 + 6.86e-30 s

Continuous-time transfer function.
```

```
stability = isstable(sys)
```

```
stability = logical
   0
```

```
pl = round(pole(sys),4)
```

```
pl =
     0
     0
```

Remark: The uncontrollable part has poles which values are zero, means on imaginary axis. So, the system is marginally stable

Decompose the System into Controllable and Observable part

```
MSYS = minreal(ss(A,B,C,D))
```

```
3 states removed.

MSYS =

  A =
                x1          x2
   x1       0.6103      -1.254
   x2       -1.254      0.6103
   x3       0.7071      -4.315

                x3
   x1   -9.872e-32
   x2    2.482e-31
   x3   -4.862e-32

  B =
              u1         u2
   x1     -3.051     -3.187
   x2     -3.187     -3.051
   x3   7.11e-31  7.363e-31

  C =
          x1     x2     x3
   y1      1      1  1.414

  D =
       u1  u2
   y1   0   0

Continuous-time state-space model.
```

The system becomes:

$$\dot{X} = \begin{pmatrix} 0.6103 & -1.254 & -9.872e-32 \\ -1.254 & 0.6103 & 2.482e-31 \\ 0.7071 & -4.315 & -4.862e-32 \end{pmatrix} X + \begin{pmatrix} -3.051 & -3.187 \\ -3.187 & -3.051 \\ 7.11e-31 & 7.363e-31 \end{pmatrix} u$$

$$y = (1 \quad 1 \quad 1.414)X + (0 \quad 0)u$$

Extract the System Matrix:

```
Ac = MSYS.A
```

```
Ac =
    0.6103   -1.2545   -0.0000
   -1.2545    0.6103    0.0000
    0.7071   -4.3151   -0.0000
```

```
Bc = MSYS.B
```

```
Bc =
   -3.0513   -3.1869
   -3.1869   -3.0513
    0.0000    0.0000
```

```
Cc = MSYS.C
```

```
Cc =
    1.0000    1.0000    1.4142
```

```
Dc = MSYS.D
```

```
Dc =
     0     0
```

Design of State-Feedback Controller for the Controllable Part

Step1: Check if (A_c, B_c) is controllable or not

```
Ct = ctrb(Ac,Bc)
```

```
Ct =
   -3.0513   -3.1869    2.1359    1.8829   -1.0586   -1.5304
   -3.1869   -3.0513    1.8829    2.1359   -1.5304   -1.0586
    0.0000    0.0000   11.5945   10.9131   -6.6146   -7.8853
```

```
controllable = rank(Ct) == size(Ct,1)
```

```
controllable = logical
```

```
    1
```

So, (A_c, B_c) is controllable

Step2: Check if eigen values of A_c are distinct or not

```
ev = round(eig(Ac), 4)
```

```
ev =

    1.8647
   -0.6442
         0
```

```
fprintf('%d\n',ev)
```

```
1.864700e+00
-6.442000e-01
0
```

```
iseigdistinct = length(ev) == length(unique(ev))
```

```
iseigdistinct = logical
   1
```

So, All eigen values of A_c are distinct

Step3: Now let, select $K_1 = 0, A_1 = A_c$

```
K1 = zeros(size(Bc'))
```

```
K1 =

     0     0     0
     0     0     0
```

```
A1 = Ac
```

```
A1 =

    0.6103   -1.2545   -0.0000
   -1.2545    0.6103    0.0000
    0.7071   -4.3151   -0.0000
```

Step4: Now randomly select q

```
q = [1 0]'
```

```
q =

     1
     0
```

Step5: let $B_1 = B_c q$

```
B1 = Bc*q
```

```
B1 =

   -3.0513
   -3.1869
    0.0000
```

Step6: Check if $(A1, B1)$ is controllable or not

```
Ct = ctrb(A1,B1)
```

```
Ct =
   -3.0513    2.1359   -1.0586
   -3.1869    1.8829   -1.5304
    0.0000   11.5945   -6.6146
```

```
controllable = rank(Ct) == size(Ct,1)
```

```
controllable = logical
   1
```

So, (A_1, B_1) is controllable

Step7: Now find K^T by using Ackermann's Formula

First, define the desired poles using ITAE prototype

```
poles = [-0.7081  -0.5210+1.068i -0.5210-1.068i]
```

```
poles =
  -0.7081 + 0.0000i  -0.5210 + 1.0680i  -0.5210 - 1.0680i
```

Now, find the Characteristic polynomial $\Delta_d(\lambda)$using these poles

```
del_d_lambda = poly(poles)
```

```
del_d_lambda =
    1.0000    1.7501    2.1499    0.9999
```

So, $\Delta_d(\lambda) = \lambda^3 + 1.7501\lambda^2 + 2.1499\lambda + 0.9999$

Now, find $\Delta_d(A_1)$

```
del_d_A1 = zeros(size(A1));
for i=1:length(del_d_lambda)-1
    del_d_A1 = del_d_A1 + del_d_lambda(i)*A1^(length(del_d_lambda)-i);
end
del_d_A1 = del_d_A1 + del_d_lambda(end)*eye(size(A1))
```

```
del_d_A1 =
    8.8261   -8.7523   -0.0000
   -8.7523    8.8261    0.0000
   19.7321  -24.9185    0.9999
```

Now, find K^T using Ackermann's Formula

```
K_T = [zeros(1,size(A1,2)-1) 1]*pinv(Ct)*del_d_A1
```

```
K_T =
   27.8846  -27.6295   -0.0482
```

Step8: Now find Feedback gain K

```
K = K1 + q*K_T
```

```
K =
   27.8846  -27.6295   -0.0482
         0         0         0
```

Step9: Find the FeedForword gain F using the formula given below:

For asymptotic tracking, $C(-A + BK^T)^{-1}BF = 1$

where, $F = \begin{pmatrix} F_1 \\ F_2 \end{pmatrix}$

```
coeff = Cc*pinv(-Ac+Bc*K)*Bc
```

```
coeff =
   -29.3707   90.8695
```

So, $-29.3707F_1 + 90.8695F_2 = 1$

let, $F_1 = 1$

```
F1 = 1
```

```
F1 = 1
```

```
F2 = (1-coeff(1))/coeff(2)
```

```
F2 = 0.3342
```

```
F=[F1;F2]
```

```
F =
      1.0000
      0.3342
```

So, the Controller input is : $u(t) = -K^T x(t) + Fr(t)$

where, $K^T = 10^{13}\begin{pmatrix} 27.8846 & -27.6295 & -0.0482 \\ 0 & 0 & 0 \end{pmatrix}$ **and** $F = \begin{pmatrix} 1 \\ 0.3342 \end{pmatrix}$

{b}

State space representation of the closed loop system

```
AcL = Ac-Bc*K
```

```
AcL =
    85.6933  -85.5594   -0.1469
    87.6119  -87.4434   -0.1535
     0.7071   -4.3151   -0.0000
```

```
BcL = Bc*F
```

```
BcL =
    -4.1164
    -4.2067
     0.0000
```

```
CcL = Cc
```

```
CcL =
      1.0000      1.0000      1.4142
```

```
DcL = [0]
```

```
DcL = 0
```

State Space Representation of Closed-loop System:

$$\dot{\mathbf{x}} = \begin{pmatrix} 85.6933 & -85.5594 & -0.1469 \\ 87.6119 & -87.4434 & -0.1535 \\ 0.7071 & -4.3151 & 0 \end{pmatrix}\mathbf{x} + \begin{pmatrix} -4.1164 \\ -4.2067 \\ 0 \end{pmatrix} r$$

$$y = \begin{pmatrix} 1.0000 & 1.0000 & 1.4142 \end{pmatrix}\mathbf{x}$$

Plot the step response of the closed loop system:

```
sys = tf(ss(AcL,BcL,CcL,DcL))
```

```
sys =
 
    -8.323 s^2 + 21.37 s + 0.9999
  --------------------------------
  s^3 + 1.75 s^2 + 2.15 s + 0.9999
 
Continuous-time transfer function.
```

Define the time vector:

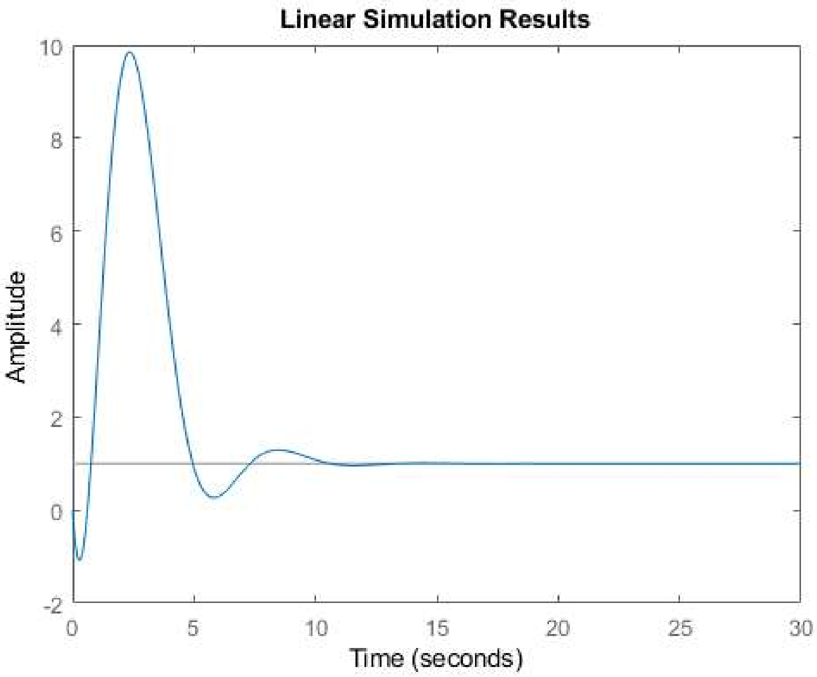
```
dt = 0.01;
tFinal = 30;
t = 0:dt:tFinal;
n = length(t);
```

Define the reference signal:

```
r = ones(1,n);
```

Plot the system response:

```
y_Ff = lsim(sys,r,t);
lsim(sys,r,t)
```



Remark: $y(t)$ **asymptotically tracks the reference signal**

{c}

Simulation of the system by adding disturbance

Adding disturbance after 10 second:

```
n1 = find(t==10);
```

Define the distubance:

```
w = [zeros(1,n1) ones(1,n-n1)]';
w =[w w];
```

Define the u matrix:

```
u = [r' w];
```

Define new B matrix:

```
Bd = [Bc*F Bc]
```

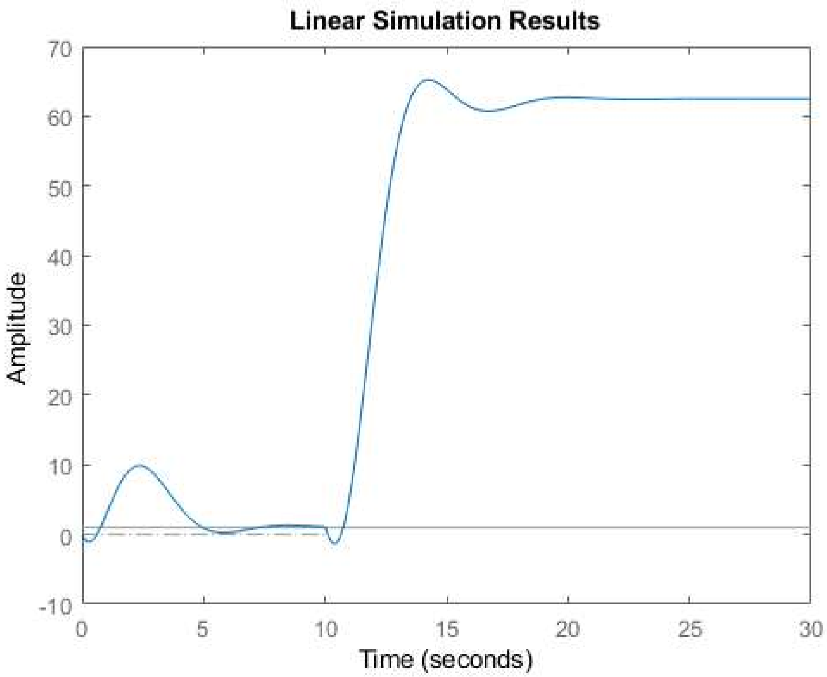
```
Bd =

   -4.1164   -3.0513   -3.1869
   -4.2067   -3.1869   -3.0513
    0.0000    0.0000    0.0000
```

```
Dd = [0 0 0];
sys = tf(ss(AcL,Bd,CcL,Dd))
```

```
sys =

  From input 1 to output:
   -8.323 s^2 + 21.37 s + 0.9999
  --------------------------------
  s^3 + 1.75 s^2 + 2.15 s + 0.9999

  From input 2 to output:
    -6.238 s^2 + 28.03 s - 29.37
  --------------------------------
  s^3 + 1.75 s^2 + 2.15 s + 0.9999

  From input 3 to output:
    -6.238 s^2 - 19.92 s + 90.86
  --------------------------------
  s^3 + 1.75 s^2 + 2.15 s + 0.9999

Continuous-time transfer function.
```

Plot the step responce in the presence of disturbance:

```
y_Ff_w = lsim(sys,u,t);
lsim(sys,u,t)
```



Remark: $y(t)$ does not asymptotically tracks the reference signal

{d}

Design of an Integral Controller

Add a new pole for the integral controller:

```
poles = [-0.424+1.263i -0.424-1.263i -0.6260+0.4141i -0.6260-0.4141i]
```

```
poles =
  -0.4240 + 1.2630i  -0.4240 - 1.2630i  -0.6260 + 0.4141i  -0.6260 - 0.4141i
```

Calculate new State Space matrix:

$$\dot{\bar{X}} = \begin{pmatrix} A_c & 0 \\ -C_c & 0 \end{pmatrix}\bar{X} + \begin{pmatrix} B_c \\ 0 \end{pmatrix} u + \begin{pmatrix} B_c \\ 0 \end{pmatrix} w + \begin{pmatrix} 0 \\ 1 \end{pmatrix} r$$

$$y = \begin{pmatrix} C_c & 0 \end{pmatrix}\bar{X}$$

$$u = -\begin{pmatrix} K^T & -K_I \end{pmatrix}\bar{X}$$

```
Ai = zeros(size(Ac,1)+1);
Ai(1:size(Ac,1),1:size(Ac,2)) = Ac;
Ai(size(Ac,1)+1,1:length(Cc)) = -Cc;
Bi = zeros(size(Bc,1)+1,2);
Bi(1:size(Bc,1),1:size(Bc,2)) = Bc;
Ci = zeros(1,size(Cc,2)+1);
Ci(1,1:length(Cc)) = Cc
```

```
Ci =
    1.0000    1.0000    1.4142         0
```

```
Di = zeros(size(Ci,1),size(Bi,2));
k = place(Ai,Bi,poles)
```

```
k =
   12.2165  -16.2802    4.4214   -2.1085
  -12.2995   16.5458   -4.4992    2.1843
```

Calculate the new Closed loop Syatem when W=0:

```
K_T = k(1:2,1:size(Ac,2))
```

```
K_T =
   12.2165  -16.2802    4.4214
  -12.2995   16.5458   -4.4992
```

```
K_I = -k(:,end)
```

```
K_I =
    2.1085
   -2.1843
```

```
Aicl=Ai
```

```
Aicl =
    0.6103   -1.2545   -0.0000         0
   -1.2545    0.6103    0.0000         0
    0.7071   -4.3151   -0.0000         0
   -1.0000   -1.0000   -1.4142         0
```

```
Aicl(1:size(Ac,1),1:size(Ac,2)) = Ac - Bc*K_T;
Aicl(1:size(Bc,1),end) = Bc*K_I
```

```
Aicl =
   -1.3117    1.8008   -0.8481    0.5277
    0.1499   -0.7883    0.3623   -0.0547
    0.7071   -4.3151    0.0000   -0.0000
   -1.0000   -1.0000   -1.4142         0
```

```
Bi=[zeros(1,size(Bc,1)) 1]';
Di = zeros(size(Ci,1),size(Bi,2));
```

New closed-loop system w=0:

$$\dot{\bar{X}} = \begin{pmatrix} A_c - BK^T & BK_I \\ -C_c & 0 \end{pmatrix}\bar{X} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} r$$

$$y = \begin{pmatrix} C_c & 0 \end{pmatrix}\bar{X}$$

where,

$$K^T = \begin{pmatrix} 12.2165 & -16.2802 & 4.4214 \\ -12.2995 & 16.5458 & -4.49927 \end{pmatrix}$$

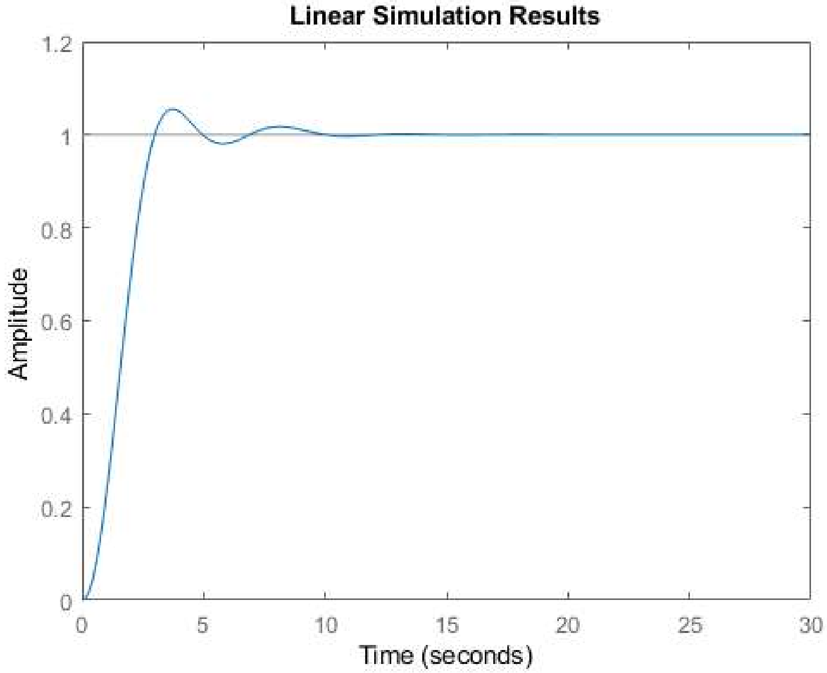
$$K_I = \begin{pmatrix} 2.1085 \\ -2.1843 \end{pmatrix}$$

```
syscl = tf(ss(Aicl,Bi,Ci,Di))
```

```
syscl =
 

      0.473 s^2 + 1.186 s + 0.9999
 
  ------------------------------------
 
  s^4 + 2.1 s^3 + 3.4 s^2 + 2.7 s
 
                             + 0.9999
```

```
Continuous-time transfer function.
```

```
lsim(syscl,r,t);
```



{e}

Simulation of the Integral controller Closed- loop system by adding disturbance

```
Bi=[Bc zeros(size(Bc,1),1);0 0 1]
```

```
Bi =
   -3.0513   -3.1869         0
   -3.1869   -3.0513         0
    0.0000    0.0000         0
         0         0    1.0000
```

```
Di = zeros(size(Ci,1),size(Bi,2));
```

New Closed-loop system when w = 1 for t>10:

$$\dot{\bar{X}} = \begin{pmatrix} A_c - BK^T & BK_I \\ -C_c & 0 \end{pmatrix} \bar{X} + \begin{pmatrix} B \\ 0 \end{pmatrix} w + \begin{pmatrix} 0 \\ 1 \end{pmatrix} r$$

$$y = \begin{pmatrix} C_c & 0 \end{pmatrix} \bar{X}$$

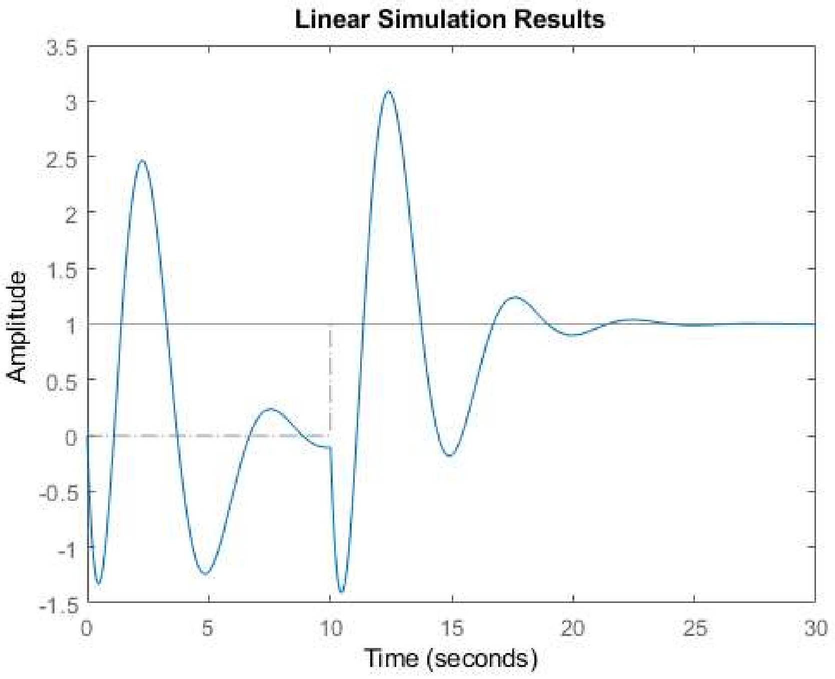
```
syscl = tf(ss(Aicl,Bi,Ci,Di))
```

```
syscl =

  From input 1 to output:

    -6.238 s^3 + 3.615 s^2 + 1.032 s

  -------------------------------------
```

```
    s^4 + 2.1 s^3 + 3.4 s^2 + 2.7 s

                                 + 0.9999


    From input 2 to output:

      -6.238 s^3 + 2.946 s^2 + 0.5383 s

    -------------------------------------

    s^4 + 2.1 s^3 + 3.4 s^2 + 2.7 s

                                 + 0.9999


    From input 3 to output:

        0.473 s^2 + 1.186 s + 0.9999

    -------------------------------------

    s^4 + 2.1 s^3 + 3.4 s^2 + 2.7 s

                                 + 0.9999


  Continuous-time transfer function.
```

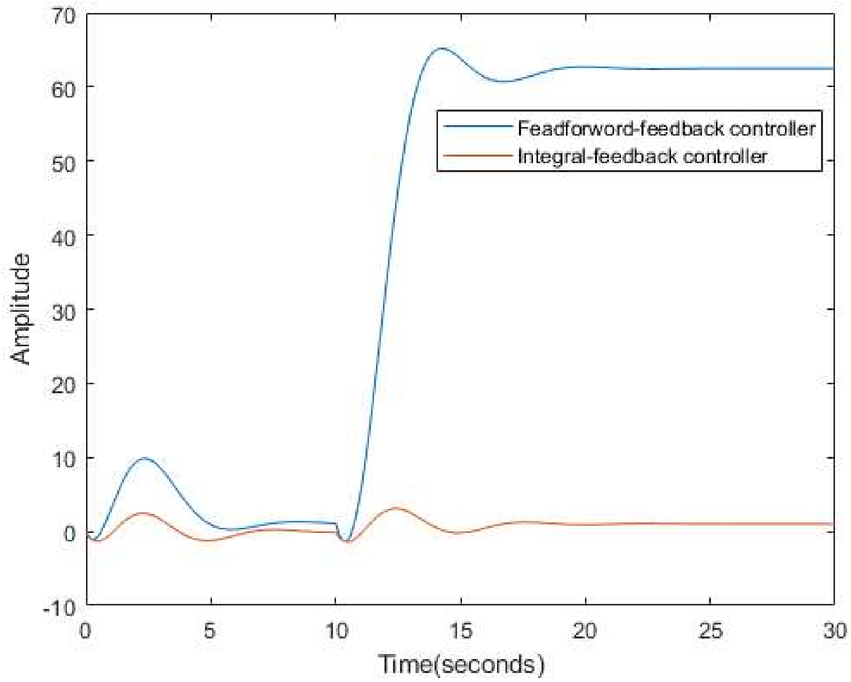
Plot the step responce in the presence of disturbance:

```
y_I_w = lsim(syscl,u,t);
lsim(syscl,u,t)
```



Remark: $y(t)$ asymptotically tracks the reference signal

```
figure
plot(t,y_Ff_w);
hold on;
plot(t,y_I_w);
xlabel('Time(seconds)')
ylabel('Amplitude')
legend('Feadforword-feedback controller','Integral-feedback controller','Location','best')
```



```
figure
```

Remark: The Integral controller can track reference signal asymptotically in the presence of constant disturbance but Feedforward gain controller can not track the reference signal asymptotically.

{g}

Check the observability of the System

Step1: Check if (A_c, C_c) **is observable or not**

find the O matrix. where, $O = \begin{pmatrix} C_c \\ C_c A_c \\ C_c A_c^2 \end{pmatrix}$

```
O = obsv(Ac,Cc)
```

```
O =
    1.0000    1.0000    1.4142
    0.3558   -6.7467    0.0000
    8.6807   -4.5635   -0.0000
```

Step2: Check if O is full column rank or not

```
rk = rank(O)
```

```
rk = 3
```

```
observability = rk == size(O,2)
```

```
observability = logical
   1
```

Remark: The system is observable

```
Ao = Ac;
Bo = Bc;
Co = Cc;
Do = Dc;
```

Now Define the desired poles:

Using ITAE prototype:

```
multiplier = 5;
poles = [multiplier*-0.7081  multiplier*-0.5210+1.068i multiplier*-0.5210-1.068i]
```

```
poles =
  -3.5405 + 0.0000i  -2.6050 + 1.0680i  -2.6050 - 1.0680i
```

Now, find the Characteristic polynomial $\Delta_d(\lambda)$ **using these poles**

```
del_d_lambda = poly(poles)
```

```
del_d_lambda =
    1.0000    8.7505   26.3727   28.0643
```

So, $\Delta_d(\lambda) = \lambda^3 + 8.7505\lambda^2 + 26.3727\lambda + 28.0643$

Now, find $\Delta_d(A_o)$

```
del_d_Ao = zeros(size(Ao));
for i=1:length(del_d_lambda)-1
    del_d_Ao = del_d_Ao + del_d_lambda(i)*Ao^(length(del_d_lambda)-i);
end
del_d_Ao = del_d_Ao + del_d_lambda(end)*eye(size(Ao))
```

```
del_d_Ao =
   64.2961  -49.8574   -0.0000
  -49.8574   64.2961    0.0000
   77.7757 -154.0869   28.0643
```

Now, find L^T **using Ackermann's Formula**

```
L_T = [zeros(1,size(Ao,2)-1) 1]*pinv(obsv(Ao,Co))*del_d_Ao
```

```
L_T =
   30.4048  -11.0900   -2.4752
```

```
L = L_T'
```

```
L =
   30.4048
  -11.0900
   -2.4752
```

So, The designed observer is given below:

$$\dot{\hat{X}} = (A_o - LC_o)\hat{X} + Bu + Ly$$

$$\hat{y} = C_o\hat{X}$$

where,

$$L = \begin{pmatrix} 30.4048 \\ -11.0900 \\ -2.4752 \end{pmatrix}$$

{h}

Put the Controller and Observer together:

Calculate the Closed-loop system matrix for the designed controller and observer:

$$\begin{pmatrix} \dot{X} \\ \dot{\bar{X}} \end{pmatrix} = \begin{pmatrix} A - BK^T & BK^T \\ 0 & A - LC \end{pmatrix} \begin{pmatrix} X \\ \bar{X} \end{pmatrix} + \begin{pmatrix} BF \\ 0 \end{pmatrix} r$$

$$y = (c \quad 0) \begin{pmatrix} x \\ \bar{x} \end{pmatrix}$$

```
A=Ac;% or Ao;
B=Bc;% or Bo;
C=Cc;% or Co;
D=Dc;% or Do;
Aco_cL = zeros(2*size(A));
Aco_cL(1:size(A,1),1:size(A,2)) = A-B*K;
Aco_cL(size(A,1)+1:end,size(A,2)+1:end) = A-L*C;
Aco_cL(1:size(A,1),size(A,2)+1:end) = B*K
```

```
Aco_cL =

   85.6933  -85.5594   -0.1469  -85.0831   84.3049    0.1469
   87.6119  -87.4434   -0.1535  -88.8664   88.0537    0.1535
    0.7071   -4.3151   -0.0000    0.0000   -0.0000   -0.0000
         0         0         0  -29.7946  -31.6593  -42.9989
         0         0         0    9.8355   11.7002   15.6836
         0         0         0    3.1823   -1.8399    3.5005
```

```
Bco_cL = [B*F; zeros(size(B,1),1)]
```

```
Bco_cL =

   -4.1164
   -4.2067
    0.0000
         0
         0
         0
```

```
Cco_cL = [C zeros(1,length(C))]
```

```
Cco_cL =

    1.0000    1.0000    1.4142         0         0         0
```

```
Dco_cL = [0]
```

```
Dco_cL = 0
```

So, the augmented Closed-loop system is given below:

$$\begin{pmatrix}\dot{X}\\ \dot{\bar{X}}\end{pmatrix}=\begin{pmatrix}85.6933 & -85.5594 & -0.1469 & -85.0831 & 84.3049 & 0.1469\\ 87.6119 & -87.4434 & -0.1535 & -88.8664 & 88.0537 & 0.1535\\ 0.7071 & -4.3151 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & -29.7946 & -31.6593 & -42.9989\\ 0 & 0 & 0 & 9.8355 & 11.7002 & 15.6836\\ 0 & 0 & 0 & 3.1823 & -1.8399 & 3.5005\end{pmatrix}\begin{pmatrix}X\\ \bar{X}\end{pmatrix}+\begin{pmatrix}-4.1164\\ -4.2067\\ 0\\ 0\\ 0\\ 0\end{pmatrix}r$$

$$y=(1\quad 1\quad 1.4142\quad 0\quad 0\quad 0)\begin{pmatrix}x\\ \bar{x}\end{pmatrix}$$

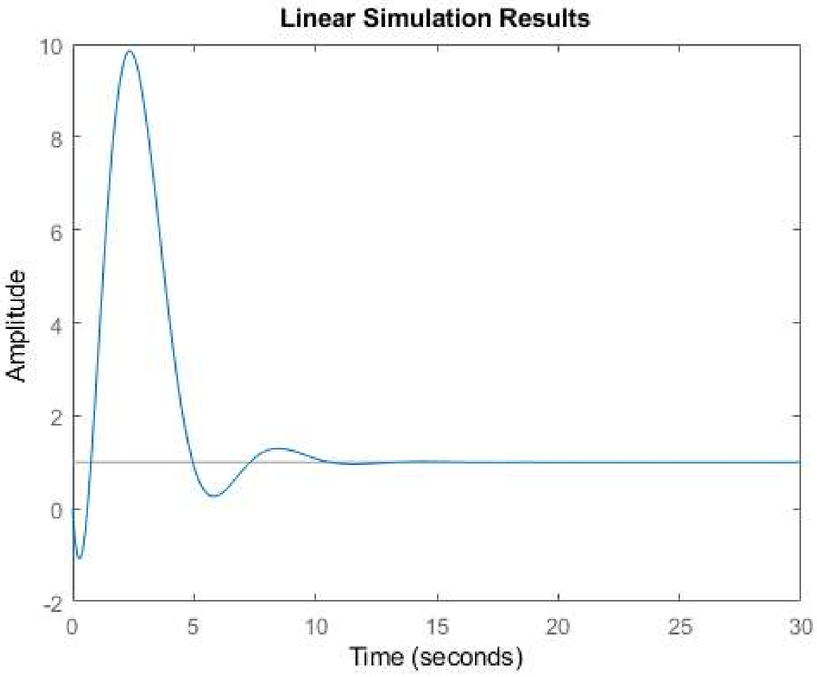
{i}

Response of the Augmented Closed-loop System

```
aug_sys = tf(ss(Aco_cL, Bco_cL, Cco_cL, Dco_cL))
```

```
aug_sys =

   -8.323 s^2 + 21.37 s + 0.9999
  --------------------------------
  s^3 + 1.75 s^2 + 2.15 s + 0.9999

Continuous-time transfer function.
```

```
y_aug = lsim(aug_sys,r,t);
lsim(aug_sys,r,t);
```

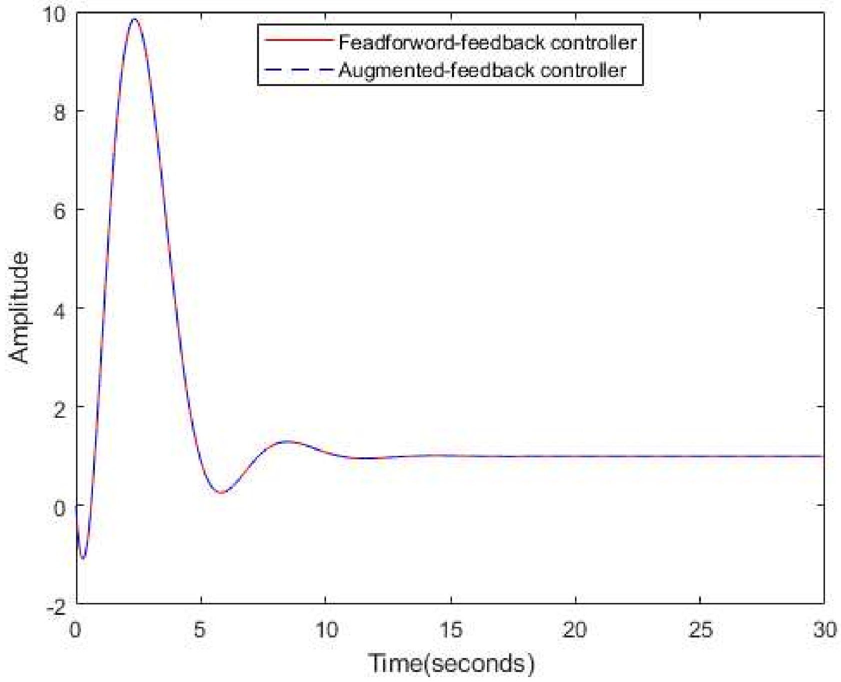


Remark: $y(t)$ **asymptotically tracks the reference signal**

{j}

```
figure
plot(t,y_Ff,'r');
hold on;
```

```
plot(t,y_aug,'b--');
xlabel('Time(seconds)')
ylabel('Amplitude')
legend('Feadforword-feedback controller','Augmented-feedback controller','Location','best')
```



```
figure
```

Remark: The Augmented-feedback controller works as good as the Feedforword-feedback controller

############################## END ##############################