Miniproject #1

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{a}

Parameters for the Selected System

```
%% parameters
M = 30
```

```
M = 30
```

```
mw = 2
```

```
mw = 2
```

```
r = 0.1
```

```
r = 0.1000
```

```
d = 0.7
```

```
d = 0.7000
```

```
K_tL = 0.5
```

```
K_tL = 0.5000
```

```
K_tR = 0.5
```

```
K_tR = 0.5000
```

```
B_L = 0.1
```

```
B_L = 0.1000
```

```
B_R = 0.1
```

```
B_R = 0.1000
```

```
J_m = 0.0263
```

```
J_m = 0.0263
```

```
J_M = 2.45
```

```
J_M = 2.4500
```

```

alpha = M*r^2 + 3*mw*r^2 + 3*J_m + J_M*(r^2/d^2)/2 - ...
    2*J_M^2*(r^2/d^2)/(M*r^2) + 3*mw*r^2 + 3*J_m + J_M*(r^2/d^2)
```

```
alpha = -0.1639
```

```

beta = 2*J_M^2*(r^4/d^4)/(M*r^2) + 3*mw*r^2 + 3*J_m + J_M*(r^2/d^2)
```

```
beta = 0.2056
```

```
gama = 2*J_M*(r^2/d^2)/(M*r^2) + 3*mw*r^2 + 3*J_m + J_M*(r^2/d^2)
```

```
gama = 0.5222
```

State-Space Representation of the System

```
%% system matrix
A = [0     1         0         0     0          0;...
     0 -B_L/alpha -1/alpha    0 beta/alpha gama/alpha;...
     0     0         0         0     0          0;...
     0     0         0         0    1/alpha     0;...
     0 beta/alpha gama/alpha 0 -B_R/alpha -1/alpha;...
     0     0         0         0     0          0]
```

```
A =

       0    1.0000         0         0         0         0
       0    0.6103    6.1025         0   -1.2545   -3.1869
       0         0         0         0         0         0
       0         0         0         0   -6.1025         0
       0   -1.2545   -3.1869         0    0.6103    6.1025
       0         0         0         0         0         0
```

```
B = [    0                  0;...
     K_tL/alpha 2*K_tL*gama/alpha;...
         0                  0;...
         0                  0;...
     2*K_tR*gama/alpha K_tR/alpha;...
         0                  0]
```

```
B =

       0         0
 -3.0513   -3.1869
       0         0
       0         0
 -3.1869   -3.0513
       0         0
```

```
C = [1 1 0 1 1 0]
```

```
C =

    1     1     0     1     1     0
```

```
D = [0 0]
```

```
D =

    0     0
```

$$\dot{\mathbf{x}} = A\mathbf{x} + BU$$
$$y = C\mathbf{x} + DU$$

where,

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0.6103 & 6.1025 & 0 & -1.2545 & -3.1869 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -6.1025 & 0 \\ 0 & -1.2545 & -3.1869 & 0 & 0.6103 & 6.1025 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \ B = \begin{pmatrix} 0 & 0 \\ -3.0513 & -3.1869 \\ 0 & 0 \\ 0 & 0 \\ -3.1869 & -3.0513 \\ 0 & 0 \end{pmatrix}$$

$C = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$, $D = \begin{pmatrix} 0 & 0 \end{pmatrix}$

Check the Controllability of the System

Step1: find the C matrix;

Where, $C = (B \quad AB \quad A^2B \quad A^3B \quad A^4B \quad A^5B)$

```
Ct = ctrb(A,B)
```

```
Ct =
         0         0   -3.0513   -3.1869    2.1359    1.8829   -1.0586   -1.5304    1.2738    0.3941    0.2830   -1.3575
   -3.0513   -3.1869    2.1359    1.8829   -1.0586   -1.5304    1.2738    0.3941    0.2830   -1.3575    1.8756   -1.1834
         0         0         0         0         0         0         0         0         0         0         0         0
         0         0   19.4484   18.6204  -11.4904  -13.0344    9.3393    6.4602   -2.4048   -7.7735    8.2841   -1.7270
   -3.1869   -3.0513    1.8829    2.1359   -1.5304   -1.0586    0.3941    1.2738   -1.3575    0.2830   -1.1834    1.8756
         0         0         0         0         0         0         0         0         0         0         0         0
```

$$C = \begin{pmatrix} 0 & 0 & -3.0513 & -3.1869 & 2.1359 & 1.8829 & -1.0586 & -1.5304 & 1.2738 & 0.3941 & 0.2830 & -1.3575 \\ -3.0513 & -3.1869 & 2.1359 & 1.8829 & -1.0586 & -1.5304 & 1.2738 & 0.3941 & 0.2830 & -1.3575 & 1.8756 & -1.1834 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 19.4484 & 18.6204 & -11.4904 & -13.0344 & 9.3393 & 6.4602 & -2.4048 & -7.7735 & 8.2841 & -1.7270 \\ -3.1869 & -3.0513 & 1.8829 & 2.1359 & -1.5304 & -1.0586 & 0.3941 & 1.2738 & -1.3575 & 0.2830 & -1.1834 & 1.8756 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Step2: find the rank of C matrix and check if C is full row rank or not

```
rnk = rank(Ct)
```

```
rnk = 4
```

```
controllable = rank(Ct) == size(Ct,1)
```

```
controllable = logical
   0
```

So, $\rho(C) \neq r$ and thats why the system is not controllable.

Check if the system is Stable or not

```
[num,den] = ss2tf(A,B,C,D,2)
```

```
num =
         0   -6.2382   27.0660  -29.9881   -0.0000         0         0
den =
    1.0000   -1.2205   -1.2013         0         0         0         0
```

```
sys = tf(num,den)
```

```
sys =

  -6.238 s^5 + 27.07 s^4
          - 29.99 s^3
             - 8.792e-15 s^2
  --------------------------
  s^6 - 1.221 s^5
                 - 1.201 s^4

Continuous-time transfer function.
```

```
stability = isstable(sys)
```

```
stability = logical
   0
```

```
pl = pole(sys)
```

```
pl =
         0
         0
         0
         0
    1.8647
   -0.6442
```

Remark: The System is not stable

Check the observability of the System

Step1: Check if (A, C) is observable or not

find the O matrix. where, $O = \begin{pmatrix} C \\ CA \\ CA^2 \\ CA^3 \\ CA^4 \\ CA^5 \end{pmatrix}$

```
O = obsv(A,C)
```

```
O =
    1.0000    1.0000         0    1.0000    1.0000         0
         0    0.3558    2.9156         0   -6.7467    2.9156
         0    8.6807   23.6726         0   -4.5635  -42.3060
         0   11.0223   67.5181         0  -13.6747  -55.5140
         0   23.8809  110.8441         0  -22.1722 -118.5773
         0   42.3878  216.3952         0  -43.4886 -211.4132
```

Step2: Check if O is full column rank or not

```
rk = rank(O)
```

```
rk = 4
```

```
observability = rk == size(O,2)
```

```
observability = logical
   0
```

Remark: As $\rho(O) \neq c$ the system is not observable

Decompose the System into Controllable and Uncontrollolable part

If the controllability matrix of (*A*, *B*) has rank *r* ≤ *n*, where *n* is the size of *A*, then there exists a similarity transformation such that

$$\bar{A} = TAT^T, \bar{B} = TB, \bar{C} = CT^T$$

where *T* is unitary, and the transformed system has a *staircase* form, in which the uncontrollable modes, if there are any, are in the upper left corner.

$$\bar{A} = \begin{pmatrix} A_{\bar{c}} & 0 \\ A_{21} & A_c \end{pmatrix}, \bar{B} = \begin{pmatrix} 0 \\ B_c \end{pmatrix}, \bar{C} = \begin{pmatrix} C_{\bar{c}} & C_c \end{pmatrix}$$

where (A_c, B_c) is controllable, all eigenvalues of $A_{\bar{c}}$ are uncontrollable, and $C_c(sI - A_c)^{-1}B_c = C(sI - A)^{-1}B$.

`[Abar,Bbar,Cbar,T,k] = ctrbf(A,B,C)` decomposes the state-space system represented by `A`, `B`, and `C` into the controllability staircase form, `Abar`, `Bbar`, and `Cbar`, described above. `T` is the similarity transformation matrix and `k` is a vector of length *n*, where *n* is the order of the system represented by `A`. Each entry of `k` represents the number of controllable states factored out during each step of the transformation matrix calculation. The number of nonzero elements in `k` indicates how many iterations were necessary to calculate `T`, and `sum(k)`is the number of states in A_c, the controllable portion of `Abar`.

```
[Abar,Bbar,Cbar,T,kbar] = ctrbf(A,B,C)
```

```
Abar =
         0         0         0         0         0         0
    0.0000   -0.0000   -0.0000   -0.0000   -0.0000    0.0000
   -0.0000    0.0000    0.0000    0.0000    0.7071   -0.7071
    0.0000    0.0000   -0.0000    0.0000   -4.3151   -4.3151
   -6.5686    6.5686    0.0000    0.0000    1.8647    0.0000
    2.0616    2.0616    0.0000    0.0000         0   -0.6442
Bbar =
         0         0
    0.0000    0.0000
   -0.0000   -0.0000
    0.0000    0.0000
   -0.0959    0.0959
   -4.4111   -4.4111
Cbar =
         0    0.0000   -1.0000    1.0000    0.0000    1.4142
T =
         0         0    1.0000         0         0         0
    0.0000    0.0000         0         0   -0.0000    1.0000
   -1.0000   -0.0000         0    0.0000    0.0000    0.0000
    0.0000    0.0000         0    1.0000   -0.0000    0.0000
    0.0000   -0.7071         0         0    0.7071         0
   -0.0000    0.7071         0         0    0.7071         0
kbar =
     2     2     0     0     0     0
```

Extract the Controllable and Uncontrollable part from Abar, Bbar and Cbar:

```
Auc = Abar(1:size(Abar,1)-sum(kbar),1:size(Abar,1)-sum(kbar))
```

```
Auc =
   1.0e-29 *

         0         0
    0.6537   -0.6860
```

```
Buc = Bbar(1:size(Bbar,1)-sum(kbar),1:2)
```

```
Buc =
   1.0e-30 *

         0         0
    0.4438    0.2482
```

```
Cuc = Cbar(1:size(Abar,1)-sum(kbar))
```

```
Cuc =
   1.0e-15 *

         0    0.9992
```

```
Duc = zeros(1,size(Buc,2))
```

```
Duc =
```

```
     0     0
```

Now Check if the uncontrollable part is stable or not:

```
[num,den] = ss2tf(Auc,Buc,Cuc,Duc,2)
```

```
num =
   1.0e-45 *

         0    0.2480         0

den =
    1.0000    0.0000         0
```

```
sys = tf(num,den)
```

```
sys =

     2.48e-46 s
  ----------------
  s^2 + 6.86e-30 s

Continuous-time transfer function.
```

```
stability = isstable(sys)
```

```
stability = logical
   0
```

```
pl = round(pole(sys),4)
```

```
pl =
     0
     0
```

Remark: The uncontrollable part has poles which values are zero, means on imaginary axis. So, the system is marginally stable

Extract the Controllable System Matrix:

```
Ac = Abar(size(Abar,1)-sum(kbar)+1:end,1+size(Abar,1)-sum(kbar):end) %MSYS.A
```

```
Ac =
    0.0000    0.0000    0.7071   -0.7071
   -0.0000    0.0000   -4.3151   -4.3151
    0.0000    0.0000    1.8647    0.0000
    0.0000    0.0000         0   -0.6442
```

```
Bc = Bbar(size(Bbar,1)-sum(kbar)+1:end,1:2)%MSYS.B
```

```
Bc =
   -0.0000   -0.0000
    0.0000    0.0000
   -0.0959    0.0959
   -4.4111   -4.4111
```

```
Cc = Cbar(1+size(Abar,1)-sum(kbar):end)%MSYS.C
```

```
Cc =
   -1.0000    1.0000    0.0000    1.4142
```

```
Dc = zeros(size(Cc,1),size(Bc,2)) %MSYS.D
```

```
Dc =
     0     0
```

Design of State-Feedback Controller for the Controllable Part

Step1: Check if (A_c, B_c) is controllable or not

```
Ct = ctrb(Ac,Bc)
```

```
Ct =
   -0.0000   -0.0000    3.0513    3.1869   -2.1359   -1.8829    1.0586    1.5304
    0.0000    0.0000   19.4484   18.6204  -11.4904  -13.0344    9.3393    6.4602
   -0.0959    0.0959   -0.1789    0.1789   -0.3336    0.3336   -0.6221    0.6221
   -4.4111   -4.4111    2.8417    2.8417   -1.8307   -1.8307    1.1794    1.1794
```

```
controllable = rank(Ct) == size(Ct,1)
```

```
controllable = logical
   1
```

So, (A_c, B_c) is controllable

Step2: Check if eigen values of A_c are distinct or not

```
ev = round(eig(Ac), 4)
```

```
ev =
    1.8647
   -0.6442
         0
         0
```

```
fprintf('%d\n',ev)
```

```
1.864700e+00
-6.442000e-01
0
0
```

```
iseigdistinct = length(ev) == length(unique(ev))
```

```
iseigdistinct = logical
   0
```

So, All eigen values of A_c are not distinct

Step3: Now let,

```
K1 = zeros(size(Bc'));
K1(1,1) = 1;
K1(2,3) = 0
```

```
K1 =
     1     0     0     0
     0     0     0     0
```

```
A1 = Ac-Bc*K1
```

```
A1 =
    0.0000    0.0000    0.7071   -0.7071
   -0.0000    0.0000   -4.3151   -4.3151
    0.0959    0.0000    1.8647    0.0000
    4.4111    0.0000         0   -0.6442
```

```
ev = round(eig(A1), 4)
```

```
ev =
  -0.3330 + 1.7307i
```

```
  -0.3330 - 1.7307i
   1.8865 + 0.0000i
   0.0000 + 0.0000i
```

```
iseigdistinct = length(ev) == length(unique(ev))
```

```
iseigdistinct = logical
   1
```

So, All eigen values of A_1 are distinct

```
Ct = ctrb(A1,Bc)
```

```
Ct =
   -0.0000   -0.0000    3.0513    3.1869   -2.1359   -1.8829   -8.2516   -8.1938
    0.0000    0.0000   19.4484   18.6204  -11.4904  -13.0344  -50.0028  -55.5207
   -0.0959    0.0959   -0.1789    0.1789   -0.0409    0.6394   -0.2811    1.0116
   -4.4111   -4.4111    2.8417    2.8417   11.6286   12.2271  -16.9130  -16.1826
```

```
controllable = rank(Ct) == size(Ct,1)
```

```
controllable = logical
   1
```

Step4: Now randomly select q

```
q = [1 1]'
```

```
q =
     1
     1
```

Step5: let $B_1 = B_c q$

```
B1 = Bc*q
```

```
B1 =
   -0.0000
    0.0000
         0
   -8.8221
```

Step6: Check if $(A1, B1)$ is controllable or not

```
Ct = ctrb(A1,B1)
```

```
Ct =
   -0.0000    6.2382   -4.0188  -16.4454
    0.0000   38.0688  -24.5248 -105.5235
         0   -0.0000    0.5985    0.7305
   -8.8221    5.6834   23.8558  -33.0956
```

```
controllable = rank(Ct) == size(Ct,1)
```

```
controllable = logical
   1
```

So, (A_1, B_1) is controllable

Step7: Now find K^T by using Ackermann's Formula

First, define the desired poles using ITAE prototype

```
poles = [-0.424+1.263i -0.424-1.263i -0.6260+0.4141i -0.6260-0.4141i]
```

```
poles =
  -0.4240 + 1.2630i  -0.4240 - 1.2630i  -0.6260 + 0.4141i  -0.6260 - 0.4141i
```

Now, find the Characteristic polynomial $\Delta_d(\lambda)$ using these poles

```
del_d_lambda = poly(poles)
```

```
del_d_lambda =
    1.0000    2.1000    3.4000    2.7000    0.9999
```

So, $\Delta_d(\lambda) = \lambda^4 + 2.1\lambda^3 + 3.4\lambda^2 + 2.7\lambda + 0.9999$

Now, find $\Delta_d(A_1)$

```
del_d_A1 = zeros(size(A1));
for i=1:length(del_d_lambda)-1
    del_d_A1 = del_d_A1 + del_d_lambda(i)*A1^(length(del_d_lambda)-i);
end
del_d_A1 = del_d_A1 + del_d_lambda(end)*eye(size(A1))
```

```
del_d_A1 =
    3.3626    0.0000    9.0967    0.8431
    8.0083    0.9999 -144.8965    7.0889
    1.2342    0.0000   44.8272   -0.3801
   -5.2592    0.0000   17.4755    3.7506
```

Now, find K^T using Ackermann's Formula

```
K_T = [zeros(1,size(A1,2)-1) 1]*pinv(Ct)*del_d_A1
```

```
K_T =
    2.4224   -0.1936   38.8003   -0.3764
```

Step8: Now find Feedback gain K

```
K = K1 + q*K_T
```

```
K =
    3.4224   -0.1936   38.8003   -0.3764
    2.4224   -0.1936   38.8003   -0.3764
```

Step9: Find the FeedForword gain F using the formula given below:

For asymptotic tracking, $C(-A + BK^T)^{-1}BF = 1$

where, $F = \begin{pmatrix} F_1 \\ F_2 \end{pmatrix}$

```
coeff = Cc*pinv(-Ac+Bc*K)*Bc
```

```
coeff =
   11.5132  -16.6787
```

So, $-29.3707F_1 + 90.8695F_2 = 1$

let, $F_1 = 1$

```
F1 = 1
```

```
F1 = 1
```

```
F2 = (1-coeff(1))/coeff(2)
```

```
F2 = 0.6303
```

```
F=[F1;F2]
```

```
F =
    1.0000
    0.6303
```

So, the Controller input is : $u(t) = -K^T x(t) + Fr(t)$

where, $K^T = \begin{pmatrix} 3.4224 & -0.1936 & 38.8003 & -0.3764 \\ 2.4224 & -0.1936 & 38.8003 & -0.3764 \end{pmatrix}$ **and** $F = \begin{pmatrix} 1 \\ 0.6303 \end{pmatrix}$

{b}

State space representation of the closed loop system

```
AcL = Ac-Bc*K
```

```
AcL =
    0.0000   -0.0000    0.7071   -0.7071
   -0.0000    0.0000   -4.3151   -4.3151
    0.0959    0.0000    1.8647    0.0000
   25.7821   -1.7079  342.3021   -3.9647
```

```
BcL = Bc*F
```

```
BcL =
   -0.0000
    0.0000
   -0.0355
   -7.1915
```

```
CcL = Cc
```

```
CcL =
   -1.0000    1.0000    0.0000    1.4142
```

```
DcL = [0]
```

```
DcL = 0
```

State Space Representation of Closed-loop System:

$$\dot{\mathbf{x}} = \begin{pmatrix} 0 & 0 & 0.7071 & -0.7071 \\ 0 & 0 & -4.3151 & -4.3151 \\ 0.0959 & 0 & 1.8647 & 0 \\ 25.7821 & -1.7079 & 342.3021 & -3.9647 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 \\ 0 \\ -0.0355 \\ -7.1915 \end{pmatrix} r$$
$$y = \begin{pmatrix} -1 & 1 & 0 & 1.4142 \end{pmatrix} \mathbf{x}$$

Plot the step response of the closed loop system:

```
sys = tf(ss(AcL,BcL,CcL,DcL))
```

```
sys =


  -10.17 s^3 + 27.92 s^2

          - 4.471 s + 0.9999

  --------------------------
```

```
  s^4 + 2.1 s^3 + 3.4 s^2

            + 2.7 s + 0.9999


Continuous-time transfer function.
```

Define the time vector:

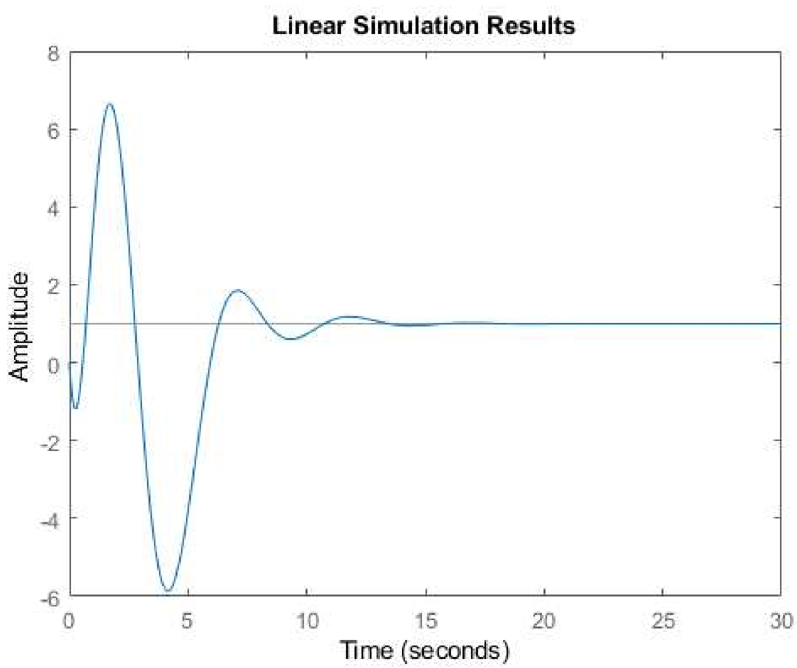
```
dt = 0.01;
tFinal = 30;
t = 0:dt:tFinal;
n = length(t);
```

Define the reference signal:

```
r = ones(1,n);
```

Plot the system response:

```
y_Ff = lsim(sys,r,t);
lsim(sys,r,t)
```



Remark: $y(t)$ **asymptotically tracks the reference signal**

{c}

Simulation of the system by adding disturbance

Adding disturbance after 10 second:

```
n1 = find(t==10);
```

Define the distubance:

```
w = [zeros(1,n1) ones(1,n-n1)]';
w =[w w];
```

Define the u matrix:

```
u = [w r'];
```

Define new B matrix:

```
Bd = [Bc Bc*F]
```

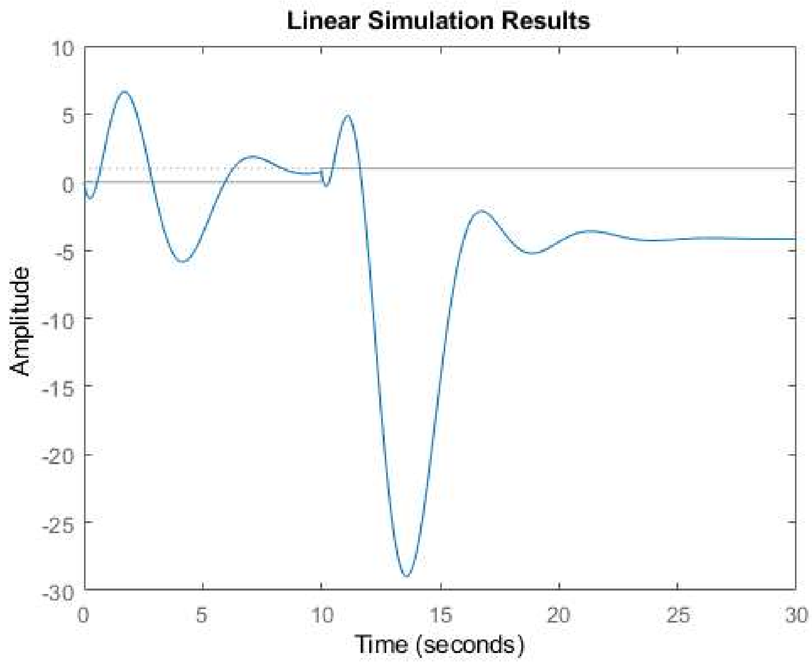
```
Bd =
   -0.0000   -0.0000   -0.0000
    0.0000    0.0000    0.0000
   -0.0959    0.0959   -0.0355
   -4.4111   -4.4111   -7.1915
```

```
Dd = [0 0 0];
sys = tf(ss(AcL,Bd,CcL,Dd))
```

```
sys =

  From input 1 to output:

  -6.238 s^3 - 18.41 s^2

           + 87.67 s + 11.51

  -------------------------

  s^4 + 2.1 s^3 + 3.4 s^2

            + 2.7 s + 0.9999


  From input 2 to output:

  -6.238 s^3 + 73.51 s^2

           - 146.2 s - 16.68

  -------------------------

  s^4 + 2.1 s^3 + 3.4 s^2

            + 2.7 s + 0.9999


  From input 3 to output:

  -10.17 s^3 + 27.92 s^2

          - 4.471 s + 0.9999

  -------------------------

  s^4 + 2.1 s^3 + 3.4 s^2

            + 2.7 s + 0.9999


Continuous-time transfer function.
```

Plot the step responce in the presence of disturbance:

```
y_Ff_w = lsim(sys,u,t);
lsim(sys,u,t)
```



Remark: $y(t)$ does not asymptotically tracks the reference signal

{d}

Design of an Integral Controller

Add a new pole for the integral controller:

```
poles = [-0.975 -0.424+1.263i -0.424-1.263i -0.6260+0.4141i -0.6260-0.4141i]
```

```
poles =
  -0.9750 + 0.0000i  -0.4240 + 1.2630i  -0.4240 - 1.2630i  -0.6260 + 0.4141i  -0.6260 - 0.4141i
```

Calculate new State Space matrix:

$$\dot{\bar{X}} = \begin{pmatrix} A_c & 0 \\ -C_c & 0 \end{pmatrix} \bar{X} + \begin{pmatrix} B_c \\ 0 \end{pmatrix} u + \begin{pmatrix} B_c \\ 0 \end{pmatrix} w + \begin{pmatrix} 0 \\ 1 \end{pmatrix} r$$

$$y = \begin{pmatrix} C_c & 0 \end{pmatrix} \bar{X}$$

$$u = -\begin{pmatrix} K^T & -K_I \end{pmatrix} \bar{X}$$

```
Ai = zeros(size(Ac,1)+1);
Ai(1:size(Ac,1),1:size(Ac,2)) = Ac;
Ai(size(Ac,1)+1,1:length(Cc)) = -Cc;
Bi = zeros(size(Bc,1)+1,2);
Bi(1:size(Bc,1),1:size(Bc,2)) = Bc;
Ci = zeros(1,size(Cc,2)+1);
Ci(1,1:length(Cc)) = Cc
```

```
Ci =
   -1.0000    1.0000    0.0000    1.4142         0
```

```
Di = zeros(size(Ci,1),size(Bi,2));
k = place(Ai,Bi,poles)
```

```
k =
```

```
   -3.7621    2.5288  -18.5399    0.6886   -2.0719
    3.8663   -2.5277   18.5363   -0.8560    2.0957
```

Calculate the new Closed loop Syatem when W=0:

```
K_T = k(1:2,1:size(Ac,2))
```

```
K_T =
   -3.7621    2.5288  -18.5399    0.6886
    3.8663   -2.5277   18.5363   -0.8560
```

```
K_I = -k(:,end)
```

```
K_I =
    2.0719
   -2.0957
```

```
Aicl=Ai
```

```
Aicl =
    0.0000    0.0000    0.7071   -0.7071         0
   -0.0000    0.0000   -4.3151   -4.3151         0
    0.0000    0.0000    1.8647    0.0000         0
    0.0000    0.0000         0   -0.6442         0
    1.0000   -1.0000   -0.0000   -1.4142         0
```

```
Aicl(1:size(Ac,1),1:size(Ac,2)) = Ac - Bc*K_T;
Aicl(1:size(Bc,1),end) = Bc*K_I
```

```
Aicl =
   -0.0000    0.0000    0.7071   -0.7071   -0.0000
   -0.0000    0.0000   -4.3151   -4.3151    0.0000
   -0.7319    0.4851   -1.6924    0.1482   -0.3998
    0.4594    0.0049   -0.0158   -1.3826    0.1049
    1.0000   -1.0000   -0.0000   -1.4142         0
```

```
Bi=[zeros(1,size(Bc,1)) 1]';
Di = zeros(size(Ci,1),size(Bi,2));
```

New closed-loop system w=0:

$$\dot{\bar{X}} = \begin{pmatrix} A_c - BK^T & BK_I \\ -C_c & 0 \end{pmatrix} \bar{X} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} r$$

$$y = \begin{pmatrix} C_c & 0 \end{pmatrix} \bar{X}$$

where,

$$K^T = \begin{pmatrix} -3.7621 & 2.5288 & -18.5399 & 0.6886 \\ 3.8663 & -2.5277 & 18.5363 & -0.8560 \end{pmatrix}$$

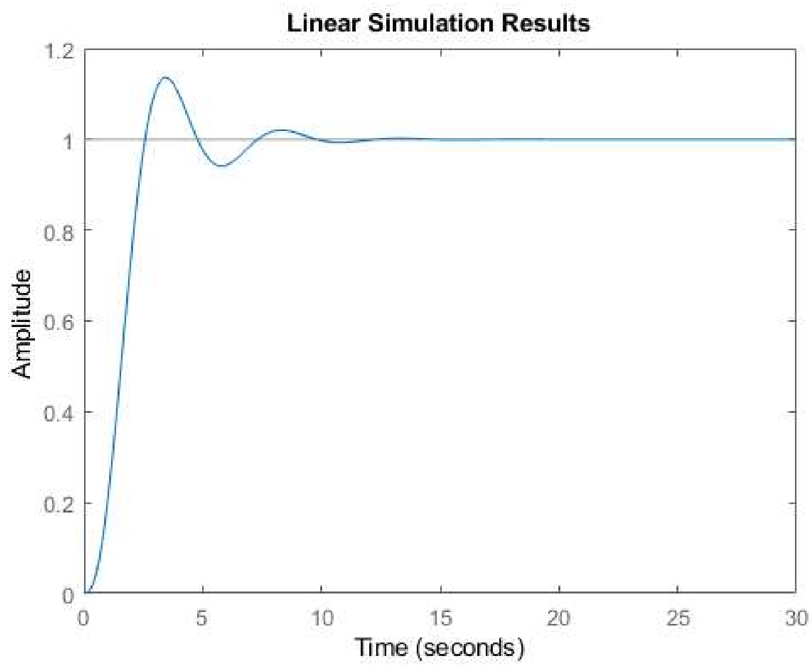
$$K_I = \begin{pmatrix} 2.0719 \\ -2.0957 \end{pmatrix}$$

```
syscl = tf(ss(Aicl,Bi,Ci,Di))
```

```
syscl =


  0.1484 s^3 + 1.89 s^2

           + 2.25 s + 0.9749

  -------------------------

  s^5 + 3.075 s^4

          + 5.447 s^3
```

```
         + 6.015 s^2

         + 3.632 s + 0.9749


Continuous-time transfer function.
```

```
lsim(syscl,r,t);
```



{e}

Simulation of the Integral controller Closed- loop system by adding disturbance

```
Bi=[Bc zeros(size(Bc,1),1);0 0 1]
```

```
Bi =
   -0.0000   -0.0000         0
    0.0000    0.0000         0
   -0.0959    0.0959         0
   -4.4111   -4.4111         0
         0         0    1.0000
```

```
Di = zeros(size(Ci,1),size(Bi,2));
```

New Closed-loop system when w = 1 for t>10:

$$\dot{\bar{X}} = \begin{pmatrix} A_c - BK^T & BK_I \\ -C_c & 0 \end{pmatrix} \bar{X} + \begin{pmatrix} B \\ 0 \end{pmatrix} w + \begin{pmatrix} 0 \\ 1 \end{pmatrix} r$$

$$y = \begin{pmatrix} C_c & 0 \end{pmatrix} \bar{X}$$

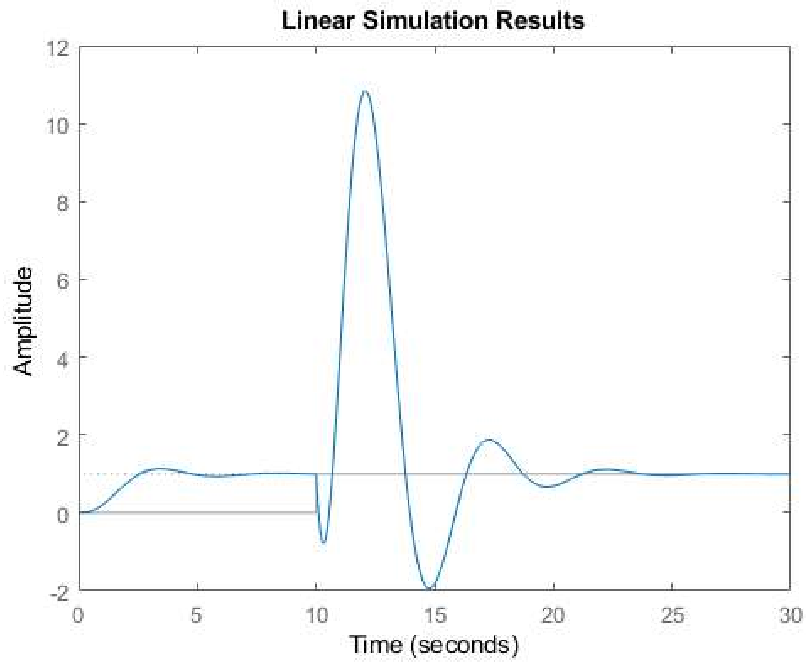
```
syscl = tf(ss(Aicl,Bi,Ci,Di))
```

```
syscl =

  From input 1 to output:
```

```
  -6.238 s^4 + 5.842 s^3

          + 14.55 s^2

                   + 6.914 s

  --------------------------

  s^5 + 3.075 s^4

          + 5.447 s^3

          + 6.015 s^2

          + 3.632 s + 0.9749


  From input 2 to output:

  -6.238 s^4 + 4.874 s^3

          + 13.31 s^2

                    + 6.37 s

  --------------------------

  s^5 + 3.075 s^4

          + 5.447 s^3

          + 6.015 s^2

          + 3.632 s + 0.9749


  From input 3 to output:

  0.1484 s^3 + 1.89 s^2

           + 2.25 s + 0.9749

  --------------------------

  s^5 + 3.075 s^4

          + 5.447 s^3

          + 6.015 s^2

          + 3.632 s + 0.9749


Continuous-time transfer function.
```

Plot the step responce in the presence of disturbance:

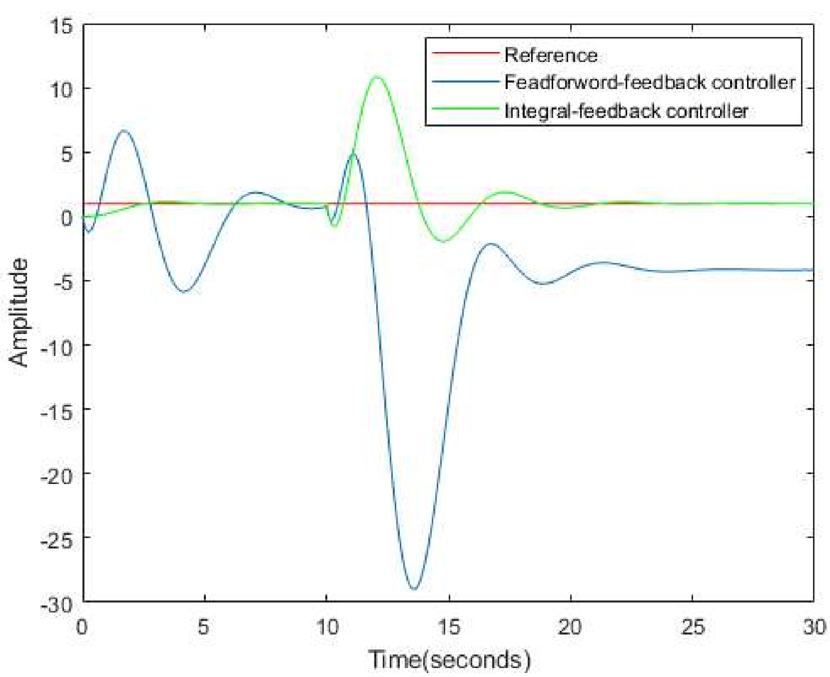
```
y_I_w = lsim(syscl,u,t);
lsim(syscl,u,t)
```



Remark: $y(t)$ **asymptotically tracks the reference signal**

{f}

```
figure
plot(t,ones(size(t)),'r');
hold on;
plot(t,y_Ff_w);
hold on;
plot(t,y_I_w,'g');
xlabel('Time(seconds)')
ylabel('Amplitude')
legend('Reference', 'Feadforword-feedback controller','Integral-feedback controller','Location','best')
```



```
figure
```

Remark: The Integral controller can track reference signal asymptotically in the presence of constant disturbance but Feedforward gain controller can not track the reference signal asymptotically.

{g}

Check the observability of the System

Step1: Check if (A_c, C_c) is observable or not

find the O matrix. where, $O = \begin{pmatrix} C_c \\ C_cA_c \\ C_cA_c^2 \\ C_cA_c^3 \end{pmatrix}$

```
O = obsv(Ac,Cc)
```

```
O =
   -1.0000    1.0000    0.0000    1.4142
   -0.0000    0.0000   -5.0222   -4.5191
   -0.0000   -0.0000   -9.3651    2.9113
   -0.0000   -0.0000  -17.4634   -1.8755
```

Step2: Check if O is full column rank or not

```
rk = rank(O)
```

```
rk = 3
```

```
observability = rk == size(O,2)
```

```
observability = Logical
   0
```

Remark: The system is not observable

Decompose the System into Observable and UnObservable part

If the observability matrix of (*A*, C) has rank *r* ≤ *n*, where *n* is the size of *A*, then there exists a similarity transformation such that

$$\bar{A} = TAT^T, \bar{B} = TB, \bar{C} = CT^T$$

where *T* is unitary, and the transformed system has a *staircase* form, in which the unobsrvable modes, if there are any, are in the upper left corner.

$$\bar{A} = \begin{pmatrix} A_{\bar{o}} & 0 \\ A_{21} & A_o \end{pmatrix}, \bar{B} = \begin{pmatrix} B_{\bar{0}} \\ B_o \end{pmatrix}, \bar{C} = \begin{pmatrix} 0 & C_o \end{pmatrix}$$

where (C_o, A_o) is observable, all eigenvalues of $A_{\bar{o}}$ are unobservlable modes.

`[Abar,Bbar,Cbar,T,k] = obsvf(A,B,C)` decomposes the state-space system represented by `A`, `B`, and `C` into the observability staircase form, `Abar`, `Bbar`, and `Cbar`, described above. `T` is the similarity transformation matrix and `k` is a vector of length *n*, where *n* is the order of the system represented by `A`. Each entry of `k` represents the number of observable states factored out during each step of the transformation matrix calculation. The number of nonzero elements in `k` indicates how many iterations were necessary to calculate `T`, and `sum(k)`is the number of states in A_o, the controllable portion of `Abar`.

```
[Abar,Bbar,Cbar,T,kbar] = obsvf(Ac,Bc,Cc)
```

```
Abar =
   -0.0000    0.7491   -3.5005    2.5111
    0.0000    0.0788   -1.3748    0.8045
   -0.0000    1.1363    2.7395   -0.5119
    0.0000   -0.0000    2.9763   -1.5977

Bbar =
    0.0000    0.0000
   -2.5801   -2.6831
    1.7553    1.5934
   -3.1191   -3.1191

Cbar =
    0.0000    0.0000   -0.0000    2.0000

T =
   -0.7071   -0.7071    0.0000   -0.0000
    0.4218   -0.4218   -0.5368    0.5966
   -0.2684    0.2684   -0.8437   -0.3796
   -0.5000    0.5000    0.0000    0.7071

kbar =
     1     1     1     0
```

```
Ao = Abar(size(Abar,1)-sum(kbar)+1:end,1+size(Abar,1)-sum(kbar):end)
```

```
Ao =
    0.0788   -1.3748    0.8045
    1.1363    2.7395   -0.5119
   -0.0000    2.9763   -1.5977
```

```
Bo = Bbar(size(Bbar,1)-sum(kbar)+1:end,1:2)
```

```
Bo =
   -2.5801   -2.6831
    1.7553    1.5934
   -3.1191   -3.1191
```

```
Co = Cbar(1+size(Abar,1)-sum(kbar):end)
```

```
Co =
    0.0000   -0.0000    2.0000
```

```
Do = zeros(size(Co,1),size(Bo,2))
```

```
Do =
     0     0
```

Now Define the desired poles:

Using ITAE prototype:

```
multiplier = 5;
poles = [multiplier*-0.7081  multiplier*-0.5210+1.068i multiplier*-0.5210-1.068i]
```

```
poles =
  -3.5405 + 0.0000i  -2.6050 + 1.0680i  -2.6050 - 1.0680i
```

Now, find the Characteristic polynomial $\Delta_d(\lambda)$ using these poles

```
del_d_lambda = poly(poles)
```

```
del_d_lambda =
    1.0000    8.7505   26.3727   28.0643
```

So, $\Delta_d(\lambda) = \lambda^3 + 8.7505\lambda^2 + 26.3727\lambda + 28.0643$

Now, find $\Delta_d(A_o)$

```
del_d_Ao = zeros(size(Ao));
for i=1:length(del_d_lambda)-1
    del_d_Ao = del_d_Ao + del_d_lambda(i)*Ao^(length(del_d_lambda)-i);
end
del_d_Ao = del_d_Ao + del_d_lambda(end)*eye(size(Ao))
```

```
del_d_Ao =
   14.7218  -52.6663   17.0151
   63.2650  147.6633  -10.8262
   33.7229  115.9517   -5.7286
```

Now, find L^T using Ackermann's Formula

```
L_T = [zeros(1,size(Ao,2)-1) 1]*pinv(obsv(Ao,Co))*del_d_Ao
```

```
L_T =
    7.3609  -26.3331    8.5076
```

```
L = L_T'
```

```
L =
    7.3609
  -26.3331
    8.5076
```

So, The designed observer is given below:

$$\dot{\hat{X}} = (A_o - LC_o)\hat{X} + Bu + Ly$$
$$\hat{y} = C_o\hat{X}$$
$$where,$$
$$L = \begin{pmatrix} 7.3609 \\ -26.3331 \\ 8.5076 \end{pmatrix}$$

{h}

Put the Controller and Observer together:

Calculate the Closed-loop system matrix for the designed controller and observer:

$$\begin{pmatrix} \dot{X} \\ \dot{\tilde{X}} \end{pmatrix} = \begin{pmatrix} A - BK^T & BK^T \\ 0 & A - LC \end{pmatrix} \begin{pmatrix} X \\ \tilde{X} \end{pmatrix} + \begin{pmatrix} BF \\ 0 \end{pmatrix} r$$
$$y = (c \quad 0)\begin{pmatrix} x \\ \tilde{x} \end{pmatrix}$$

```
Abar(size(Abar,1)-size(Ao,1)+1:end, size(Abar,2)-size(Ao,2)+1:end) = Ao-L*Co;
Aco_cL = zeros(size(Ac,1)+size(Abar,1));
Aco_cL(1:size(Ac,1),1:size(Ac,2)) = Ac-Bc*K;
Aco_cL(size(Ac,1)+1:end,size(Ac,2)+1:end) = T'*Abar*T;
Aco_cL(1:size(Ac,1),size(Ac,2)+1:end) = Bc*K
```

```
Aco_cL =
```

```
    0.0000   -0.0000    0.7071   -0.7071   -0.0000    0.0000   -0.0000    0.0000
   -0.0000    0.0000   -4.3151   -4.3151    0.0000   -0.0000    0.0000   -0.0000
    0.0959    0.0000    1.8647    0.0000   -0.0959         0         0         0
   25.7821   -1.7079  342.3021   -3.9647  -25.7821    1.7079 -342.3021    3.3205
         0         0         0         0    5.9195   -5.9195    0.7071   -9.0785
         0         0         0         0   -5.9195    5.9195   -4.3151    4.0562
         0         0         0         0   18.2658  -18.2658    1.8647  -25.8317
         0         0         0         0   20.4029  -20.4029   -0.0000  -29.4983
```

```
Bco_cL = round([Bc*F; zeros(size(Bc,1),1)],4)
```

```
Bco_cL =
         0
         0
   -0.0355
   -7.1915
         0
         0
         0
         0
```

```
Cco_cL = [Cc zeros(1,length(Cc))]
```

```
Cco_cL =
   -1.0000    1.0000    0.0000    1.4142         0         0         0         0
```

```
Dco_cL = [0]
```

```
Dco_cL = 0
```

So, the augmented Closed-loop system is given below:

$$\begin{pmatrix} \dot{X} \\ \dot{\bar{X}} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0.7071 & -0.7071 & 0 & 0 & 0 & 0 \\ 0 & 0 & -4.3151 & -4.3151 & 0 & 0 & 0 & 0 \\ 0.0959 & 0 & 1.8647 & 0 & -0.0959 & 0 & 0 & 0 \\ 25.7821 & -1.7079 & 342.3021 & -3.9647 & -25.7821 & 1.7079 & -342.3021 & 3.3205 \\ 0 & 0 & 0 & 0 & 5.9195 & -5.9195 & 0.7071 & -9.0785 \\ 0 & 0 & 0 & 0 & -5.9195 & 5.9195 & -4.3151 & 4.0562 \\ 0 & 0 & 0 & 0 & 18.2658 & -18.2658 & 1.8647 & -25.8317 \\ 0 & 0 & 0 & 0 & 20.4029 & -20.4029 & 0 & -29.4983 \end{pmatrix} \begin{pmatrix} X \\ \bar{X} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -0.0355 \\ -7.1915 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} r$$

$$y = \begin{pmatrix} -1 & 1 & 0 & 1.4142 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ \bar{x} \end{pmatrix}$$

{i}

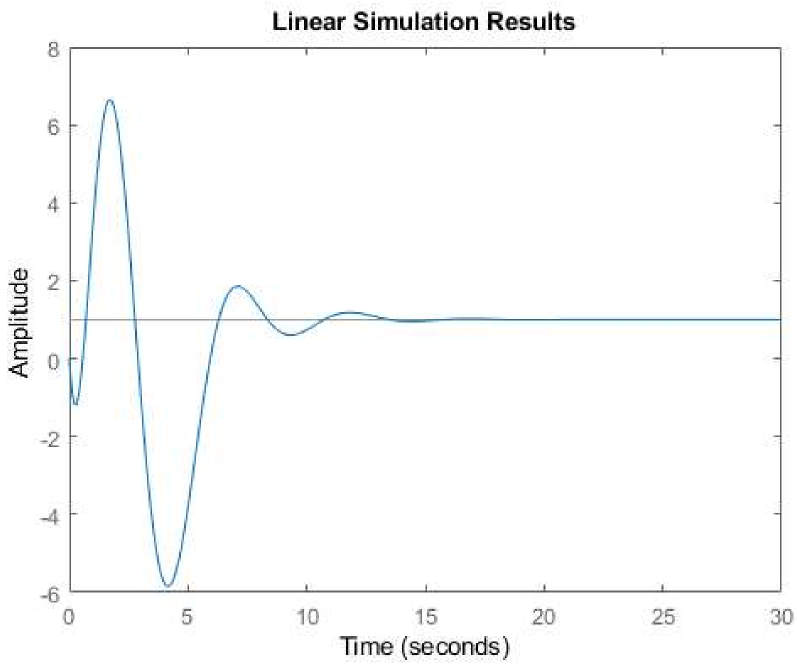
<u># Response of the Augmented Closed-loop System</u>

```
aug_sys = tf(ss(Aco_cL, Bco_cL, Cco_cL, Dco_cL))
```

```
aug_sys =


  -10.17 s^3 + 27.91 s^2

           - 4.429 s + 1.005

  --------------------------

  s^4 + 2.1 s^3 + 3.4 s^2

            + 2.7 s + 0.9999
```

```
Continuous-time transfer function.
```

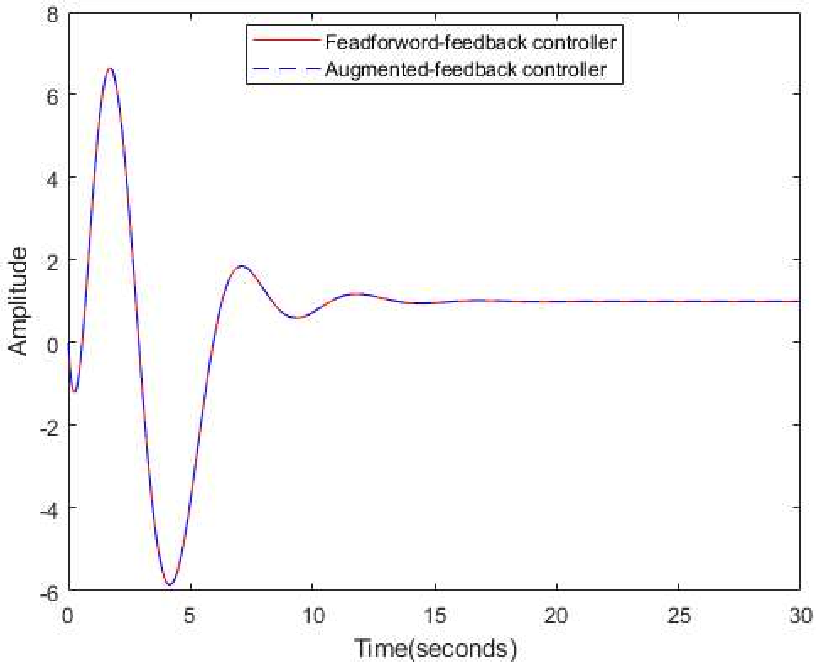
```
y_aug = lsim(aug_sys,r,t);
lsim(aug_sys,r,t);
```



Remark: $y(t)$ **asymptotically tracks the reference signal**

{j}

```
figure
plot(t,y_Ff,'r');
hold on;
plot(t,y_aug,'b--');
xlabel('Time(seconds)')
ylabel('Amplitude')
legend('Feadforword-feedback controller','Augmented-feedback controller','Location','best')
```




```
figure
```

Remark: The Augmented-feedback controller works as good as the Feedforword-feedback controller

END