Design of a State-Feedback Controller, an Integral Controller and an Observer for a LTI System

Prepared By,
Mrinmoy Sarkar
Presented To,
Dr. Ali Karimoddini

Overview

- State space representation of the system
- Feedforward-state-feedback controller design
- Integral controller design
- Observer design
- Conclusion

State space representation of the system

There are six states in the system, two inputs and one output. So, it is a MISO system.

$$\dot{\mathbf{x}} = A\mathbf{x} + BU$$
$$y = C\mathbf{x} + DU$$

where,

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0.6103 & 6.1025 & 0 & -1.2545 & -3.1869 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -6.1025 & 0 \\ 0 & -1.2545 & -3.1869 & 0 & 0.6103 & 6.1025 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 \\ -3.0513 & -3.1869 \\ 0 & 0 \\ 0 & 0 \\ -3.1869 & -3.0513 \\ 0 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}, \quad D = \begin{pmatrix} 0 & 0 \end{pmatrix}$$

Feedforward-state-feedback controller design

- Step1: The controllability of the system is checked. But the system is not controllable.
- Step2: The system is decomposed to controllable and uncontrollable part. The reduced system matrix is given below:

Ac = *4x4 double*

0.0000	0.0000	0.7071	-0.7071
-0.0000	0.0000	-4.3151	-4.3151
0.0000	0.0000	1.8647	0.0000
0.0000	0.0000	0	-0.6442

Bc = *4x2 double*

-0.0000	0.0000
0.0000	-0.0000
-0.0959	0.0959
-4.4111	-4.4111

Cc = *1x4 double*

-1.0000	1.0000	0.0000	1.4142

- Step3: The desired poles are chosen using ITAE prototype.

```
poles = [-0.424+1.263i -0.424-1.263i -0.6260+0.4141i -0.6260-0.4141i]
```

- Step4: The feedback gain is found using the Algorithm 1. The feedback gain is given below:

K = *2x4 double*

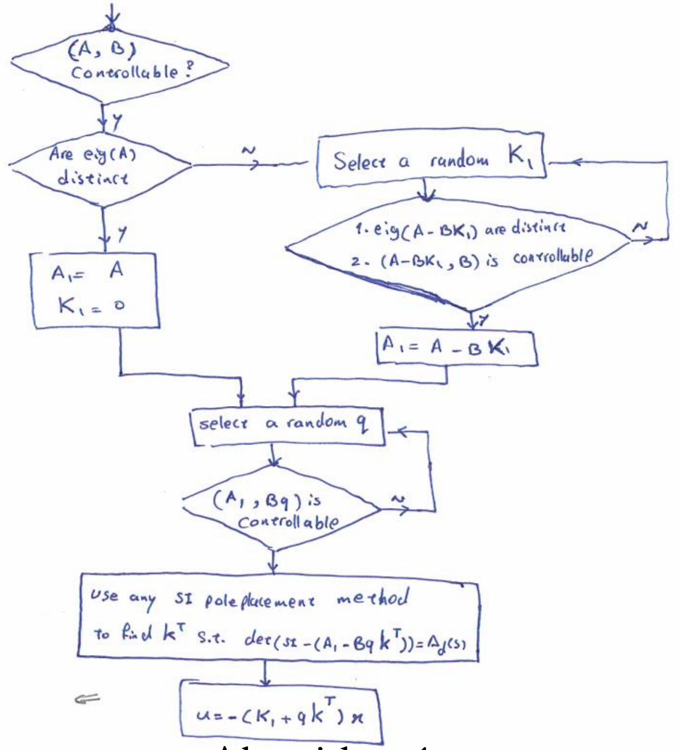
3.4224	-0.1936	38.8003	-0.3764
2.4224	-0.1936	38.8003	-0.3764

- Step5: The feedforward gain is found using the formula given below and the feedforward gain is given below:

For asymptotic tracking, $C(-A+BK^T)^{-1}BF = 1$

where, $F = \begin{pmatrix} F_1 \\ F_2 \end{pmatrix}$

$$F = \begin{pmatrix} 1 \\ 0.6303 \end{pmatrix}$$



Algorithm 1

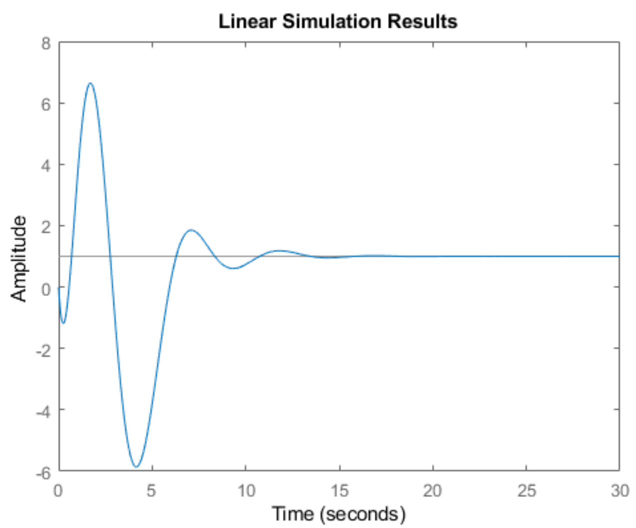
Response of the system without disturbance and with disturbance

The state space representation of the closed-loop system:

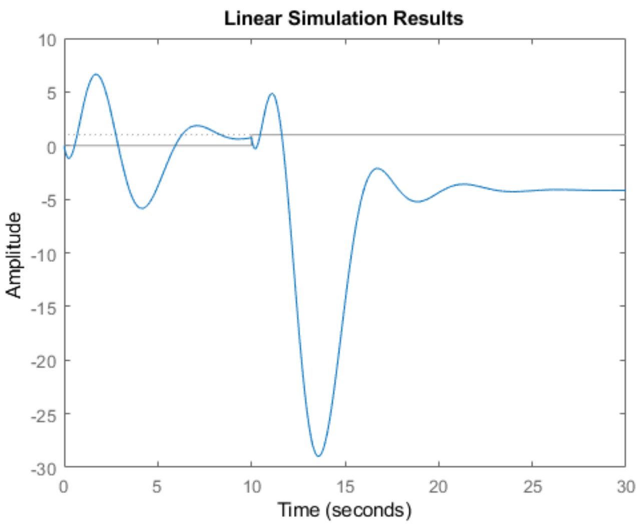
$$\dot{\mathbf{x}} = \begin{pmatrix} 0 & 0 & 0.7071 & -0.7071 \\ 0 & 0 & -4.3151 & -4.3151 \\ 0.0959 & 0 & 1.8647 & 0 \\ 25.7821 & -1.7079 & 342.3021 & -3.9647 \end{pmatrix}\mathbf{x} + \begin{pmatrix} 0 \\ 0 \\ -0.0355 \\ -7.1915 \end{pmatrix} r$$

$$y = (-1 \quad 1 \quad 0 \quad 1.4142)\mathbf{x}$$

When, w = 0



When, w = 1



Integral controller design

- For Integral controller one new pole is added in the system and **“place”** command is used to find out the gain of the system. The new system is given below: `poles = [-0.975 -0.424+1.263i -0.424-1.263i -0.6260+0.4141i -0.6260-0.4141i]`

$$\dot{\bar{X}} = \begin{pmatrix} A_c - BK^T & BK_I \\ -C_c & 0 \end{pmatrix}\bar{X} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} r$$

$$y = \begin{pmatrix} C_c & 0 \end{pmatrix}\bar{X}$$

where,

$$K^T = \begin{pmatrix} -3.7621 & 2.5288 & -18.5399 & 0.6886 \\ 3.8663 & -2.5277 & 18.5363 & -0.8560 \end{pmatrix}$$

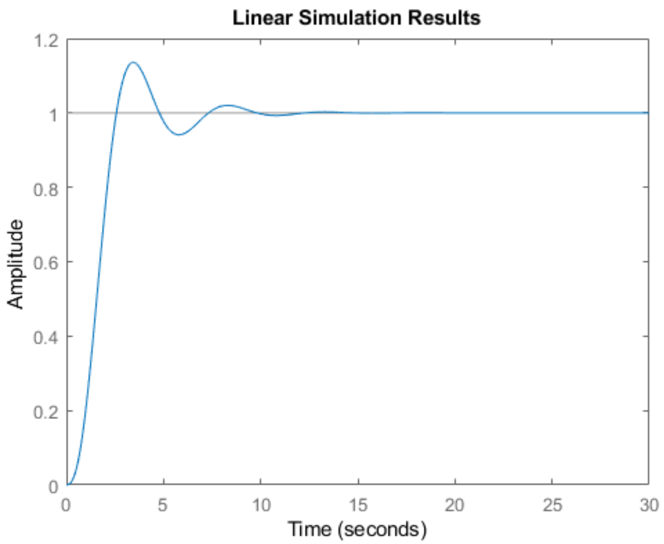
$$K_I = \begin{pmatrix} 2.0719 \\ -2.0957 \end{pmatrix}$$

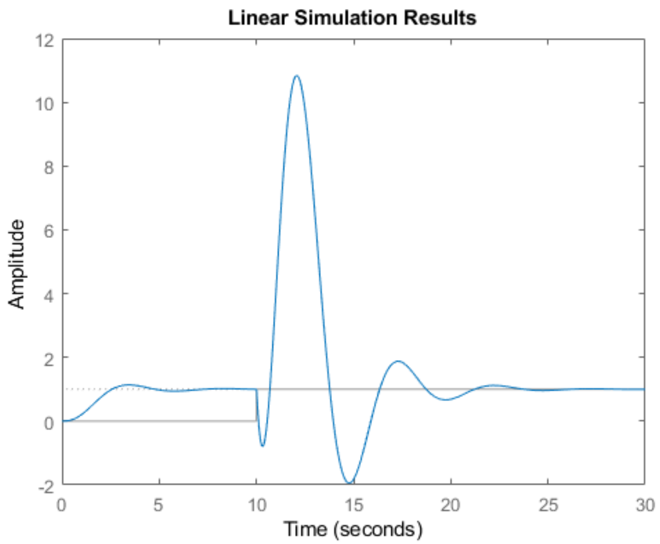
w = 0

$$\dot{\bar{X}} = \begin{pmatrix} A_c - BK^T & BK_I \\ -C_c & 0 \end{pmatrix}\bar{X} + \begin{pmatrix} B \\ 0 \end{pmatrix} w + \begin{pmatrix} 0 \\ 1 \end{pmatrix} r$$

$$y = \begin{pmatrix} C_c & 0 \end{pmatrix}\bar{X}$$

w = 1





Response of the system when, w = 0 & when, w = 1

Observer design

- Step1: The observability of the system is checked. The system is not observable.
- Step2: The system is decomposed to observable and unobservable part.
- Step2: The desired poles are selected as 5 times the real part of the feedback-feedforward gain.

```
multiplier = 5;
poles = [multiplier*-0.7081  multiplier*-0.5210+1.068i multiplier*-0.5210-1.068i]
```

- Step3: Now the observer gain is calculated using the Ackermann's formula:

```
L_T = [zeros(1,size(Ao,2)-1) 1]*pinv(obsv(Ao,Co))*del_d_Ao
```

- Step4: The observer gain is given below:

$$\hat{\dot{X}} = (A_o - LC_o)\hat{X} + Bu + Ly$$

$$\hat{y} = C_o\hat{X}$$

where,

$$L = \begin{pmatrix} 7.3609 \\ -26.3331 \\ 8.5076 \end{pmatrix}$$

The Augmented System

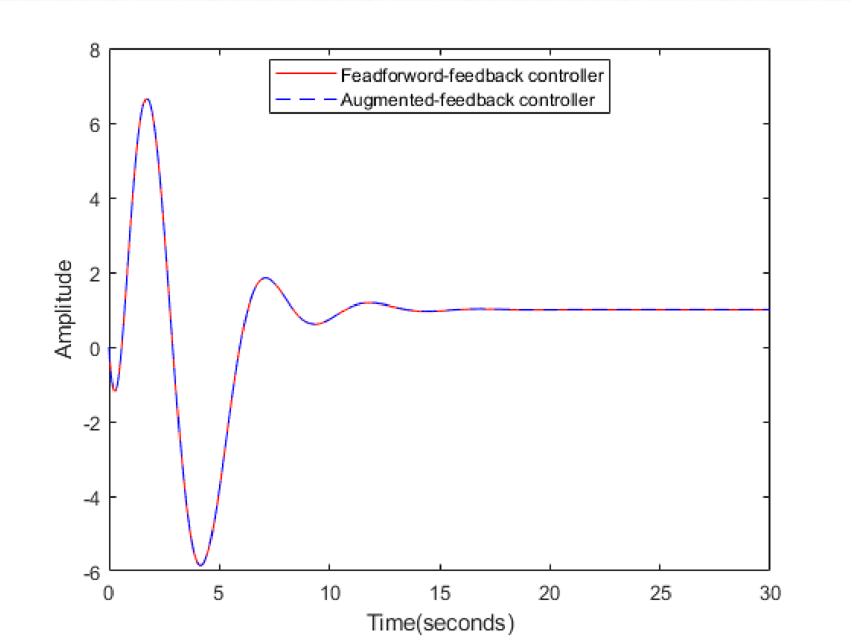
The state space representation of the closed-loop augmented system is given below:

$$\begin{pmatrix} \dot{X} \\ \dot{\bar{X}} \end{pmatrix} = \begin{pmatrix} A - BK^T & BK^T \\ 0 & A - LC \end{pmatrix} \begin{pmatrix} X \\ \bar{X} \end{pmatrix} + \begin{pmatrix} BF \\ 0 \end{pmatrix} r$$

$$y = \begin{pmatrix} c & 0 \end{pmatrix} \begin{pmatrix} x \\ \bar{x} \end{pmatrix}$$

$$\begin{pmatrix} \dot{X} \\ \dot{\bar{X}} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0.7071 & -0.7071 & 0 & 0 & 0 & 0 \\ 0 & 0 & -4.3151 & -4.3151 & 0 & 0 & 0 & 0 \\ 0.0959 & 0 & 1.8647 & 0 & -0.0959 & 0 & 0 & 0 \\ 25.7821 & -1.7079 & 342.3021 & -3.9647 & -25.7821 & 1.7079 & -342.3021 & 3.3205 \\ 0 & 0 & 0 & 0 & 5.9195 & -5.9195 & 0.7071 & -9.0785 \\ 0 & 0 & 0 & 0 & -5.9195 & 5.9195 & -4.3151 & 4.0562 \\ 0 & 0 & 0 & 0 & 18.2658 & -18.2658 & 1.8647 & -25.8317 \\ 0 & 0 & 0 & 0 & 20.4029 & -20.4029 & 0 & -29.4983 \end{pmatrix} \begin{pmatrix} X \\ \bar{X} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -0.0355 \\ -7.1915 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} r$$

$$y = \begin{pmatrix} -1 & 1 & 0 & 1.4142 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ \bar{x} \end{pmatrix}$$



Response of the feedforward-state feedback controller and the augmented system

Conclusion

- In this project we have designed a feedforward-state feedback controller and observed that this controller can track the reference signal when there is no disturbance but can not track the reference signal when there is a constant disturbance.
- Then we have designed an Integral controller which can track the reference signal in the presence of constant disturbance.
- At last we have designed an observer to get feedback from output and estimate the state of the system and combined this with the feedforward-state feedback controller and observed that the response of the system is as good as the feedforward-state feedback controller.

Thank You