HYBRID FUZZY LOGIC PID CONTROLLER

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Abstract --This paper investigates the design of a fuzzy logic PID controller that uses a simplified design scheme. Fuzzy logic PD and PI controllers are effective for many control problems but lack the advantages of the fuzzy PID controller. Fuzzy controllers use a rule base to describe relationships between the input variables. Implementation of a detailed rule base, such as a PID controller, increases in complexity as the number of input variables grow and the ranges of operation for the variables become more defined. We propose a hybrid fuzzy PID controller which takes advantage of the properties of the fuzzy PI and PD controllers. The effectiveness of the PID fuzzy controller implementation is illustrated with an example.

I Introduction

Fuzzy logic controllers (FLCs) demonstrate excellent performance in numerous applications such as industrial processes [8] and flexible arm control [5]. Mamdami's [3] work introduced this control technology that Zadeh pioneered with his work in fuzzy sets [9]. Unlike "two valued" logic, fuzzy set theory allows the degree of truth for a variable to exist somewhere in the range [0,1]. For example, if pressure is a linguistic variable that describes an input, then the terms low, medium, high and dangerous describe the fuzzy set for the pressure variable. If the universe of discourse for pressure is [0, 100], then low could be defined as "close to 10", "medium" could be "around 40", and so on. For control applications, linguistic variables describe the control inputs for dynamic plants and rules define the relationships between the inputs. Thus, precise knowledge of a plant's transfer function is not necessary for design and implementation of the controller. The thrust of earlier efforts involved replacing humans in the control loop by describing the operators' actions in terms of linguistic rules.

There are three processes involved in the implementation of an FLC; fuzzification of inputs, a rule base or an inference engine, and defuzzification to obtain a "crisp output." Fuzzification involves dividing each input variables' universe of discourse into ranges called fuzzy sets. A function applied across each range determines the membership of the variable's current value to the fuzzy sets. The value at which the membership is maximum is called the peak value. Width of a fuzzy set is the distance from the peak value to the point where the membership is zero. Linguistic rules express the relationship between input variables. Table I is an example of a matrix of rules that covers all possible combinations of fuzzy sets for two input variables. The rules describe a proportional-plus-derivative FLC (PDFLC). The rule matrix is just a convenience and still represents all the rules in "English" of the form:

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R_N : If error is E_i and change in error is ΔE_j then output is U_{ij}

where $1 \leq i \leq$ number of sets for error, $1 \leq j \leq$ number sets for change in error and $1 \leq N \leq$ (number of sets for error multiplied by the number of sets for change in error). E_i and ΔE_j are fuzzy sets for error and change in error, respectively and U_{ij} are the output fuzzy sets. In this case, each variable has seven fuzzy sets that gives a total of 49 rules. The notation PB means positive big; PM means positive medium; PS means positive small; ZO means zero; NS means negative small; NM means negative medium; and NB means negative big. The defuzzification process determines the "crisp output" by resolving the applicable rules into a single output value.

Table I Rule Matrix for PDFLC

		Error						
		NB	NM	NS	ZO	PS	PM	PB
Change in Error	NB	NB	NB	NB	NB	NM	NS	ZO
	NM	NB	NB	NB	NM	NS	ZO	PS
	NS	NB	NB	NM	NS	ZO	PS	PM
	ZO	NB	NM	NS	ZO	PS	PM	PB
	PS	NM	NS	ZO	PS	PM	PB	PB
	PM	NS	ZO	PS	PM	PB	PB	PB
	PB	ZO	PS	PM	PB	PB	PB	PB

Recent research into fuzzy control has applied classical techniques to stability analysis [1] and design [6,10]. The operation of a fuzzy controller behaves similar to a classical proportional-plus-derivative (PD) or proportional-plus-integral (PI) controller [1,6]. For a classical PD controller, the position and derivative gains remain constant for all values of input. However, for an FLC, the gains depend on the range where the control variables exist at any instant. The piecewise linearity of the FLC provides better system response than a classical controller [2,6,7].

Design concepts for FLCs involve manipulating the fuzzy sets and rules to obtain the desired effective controller gain. Section II discusses steady-state error reduction using a proportional FLC (PFLC). Drawbacks of the PD and PI controllers are discussed in Section III. As the number of control variables increase, implementation of the design of an FLC becomes more complicated. A hybrid proportional-plus-integral-plus-derivative FLC (PIDFLC) that uses a simplified design and implementation is proposed in Section IV. Simulations to demonstrate the effectiveness of the PID fuzzy controllers are given. Some conclusions are drawn in Section V.

II Steady-State Error Reduction

For the classical controller, the steady-state error is dependent on the system type for a specific input and the proportional gain. For example, the steady-state error for a type-0 system with a unit step input is $\frac{1}{1+K_p}$. The larger the proportional gain, the smaller the steady-state error. For a classical controller, the proportional gain is fixed but the effective proportional gain of the PFLC can be increased around zero.

The PFLC is able to minimize the steady-state error by the decreasing the width of the "around zero" fuzzy sets. In terms of linguistic interpretation, this is equivalent to saying "the range of error in which the control action is zero should be smaller". Mathematically, the steady-state error can be approximated by using the rate of change of the control action to calculate the effective gain, $K_{p\text{-eff}}$, and the steady-state error will be $\frac{1}{1+K_p}$. K_p is the effective gain multiplied by the PFLC output gain. The peak values for the error and output fuzzy sets must give at least an effective gain for the desired steady-state.

To obtain the optimum K_p output gain and $K_{p\text{-eff}}$, effective gain of the controller that minimizes steady-state error, a three rule PFLC test needs to be conducted for various values of output gains. This test involves using a PFLC with three input fuzzy sets and three corresponding rules as shown in Table II. PB and NB for the output fuzzy set peak values are the maximum normalized output (i.e. ±1). ZO for the error and the output fuzzy set peak value is zero. PB and NB for the error fuzzy set peak values are ±a which is varied from ±1 to near zero. Figure 1 shows the controller output for PB and NB equal to ±a respectively. Figure 2 shows the corresponding system output for a type-0 system. As "a" becomes closer to 0, the steady-state error is reduced.

Table II Control rule matrix for determining PFLC output.

	NB	ZO	PB
Error	-a	0	a
Output	-1	0	1

Design of a PDFLC is an extension of PFLC design. The PFLC design provides a response that meets the steady-state error specifications. With further manipulation of the input and output fuzzy sets, the overshoot can be minimized. The change in error input provides additional overshoot reduction. By manipulation of the change in error fuzzy sets, the overshoot can be minimized without significantly affecting the rise time.

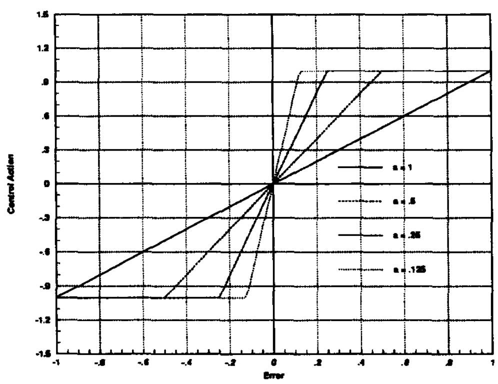

Figure 1 Controller output for PB and NB equal to ±a.

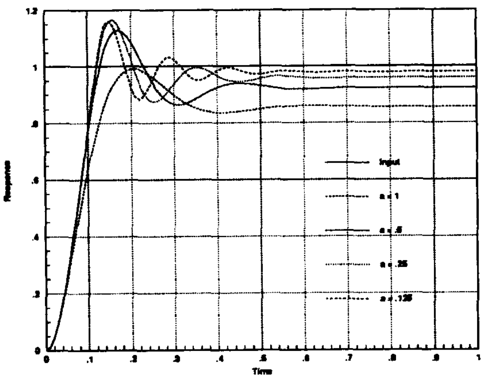

Figure 2 System response for PB and NB equal to ±a.

III Controller Limitations

Like the classical PD controller, its fuzzy counterpart can not eliminate steady-state error. The fuzzy control action drives the plant output to the ranges for the zero set for both the error and change in error (the center rule of the rule matrix). However, since the zero set is defined as a range, it is possible to have a small error when the controller action is near zero. As described in the previous section, to reduce the steady-state error, ZO for the error term can be made smaller so that there is more contribution from the control action of adjacent rules of the matrix. Narrowing the ZO range corresponds to increasing the gain of the controller around the zero error point. If the change in error is zero, and ZO for the error term decreases to a width of zero, the control action is discontinuous.

Figure 3 shows the controller output of a PDFLC for a step input to a closed loop system with a plant that has the transfer function $G_p(s)=\frac{45}{s^2+18s+45}$. All fuzzy sets are equally spaced except for PS and NS which are near zero. The large effective gain of the controller causes the system response to continually oscillate as shown in Figure 4. Therefore, while the steady-state error can be reduced by manipulating the effective gains, it can not be eliminated.

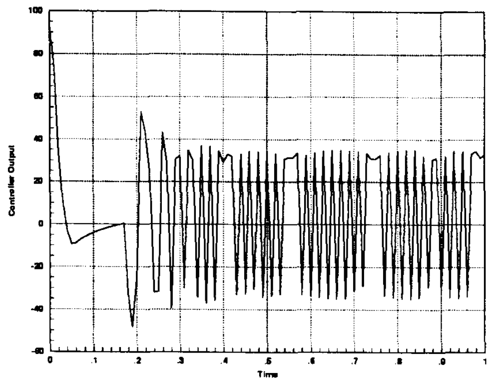

Figure 3 Controller output of example system.

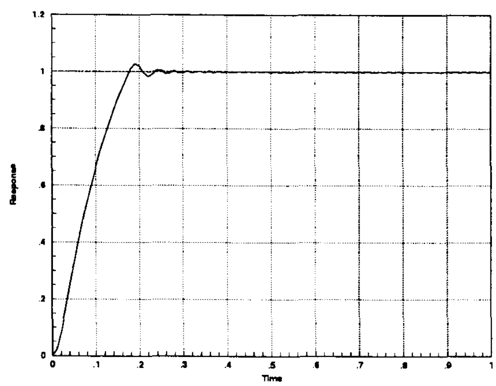

Figure 4 Response of example system

The PI fuzzy controller has a slower response due to the integral error control variable. The controller response maps to a path through the matrix of control rules. Movement from the currently applied rule to the next rule is dependent on the current and next set of control variable. Movement from set to set based on the integral error control variable is slow because the integral error variable changes slowly, thus slowing down the fuzzy controller response.

In terms of classical control, the PI controller has the effect of adding a pole at the origin. The PI controller acts as a low pass filter and reduces the bandwidth of the frequency response. A lower bandwidth corresponds to a slower rise time. However, the main advantage of the PI controller is that it increases the type of the system which is used to reduce or eliminate steady-state error.

IV Hybrid PIDFLC Design

Since neither the PD nor PI controllers are capable of individually meeting design criteria of steady-state error, overshoot and rise time, the PID controller is used. For PIDFLC design, there are three inputs which makes the design and implementation more complex.

The number of rules to cover all possible input variations for a PDFLC is the number of sets for the error multiplied by the number of sets for the change in error. The rule matrix for a controller that has seven sets for error and seven sets for change in error has 49 rules. Similarly, the PI control rule matrix has 49 rules. The addition of another control variable significantly increases the number of rules. For example, if the integral error control variable with seven sets is added to the PD controller, there would be 343 control rules. Design of such a large rule base would be a tedious task.

The following section describes a method of PIDFLC design and implementation. The goal is to obtain a PID like response with a FLC without significantly increasing design complexity.

Reduced Rule PID Controller

Logically, it should be possible to divide the action of the PID controller into two separate control actions: PD controller for fastest response and PI controller for the elimination of the steady-state error. The plant must be capable of being compensated by a PI controller. Figure 5 is the implementation of the proposed hybrid fuzzy PID control scheme. Similar to separate control rule tables for "coarse" and "fine" control [2], a PD controller provides the "coarse" control and the PI controller gives the "fine" control. Logic is required to determine the switching between the two control actions.

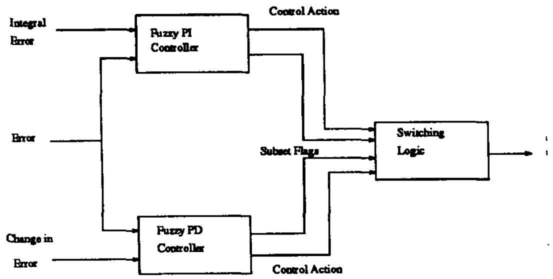

Figure 5 Proposed hybrid PIDFLC design.

The PDFLC is active when the error is large and should be reduced fast. On the other hand, the PI portion activates only when the PD portion reduces the error and change in error to where both are in the ZO fuzzy set range. Therefore, at any instant, calculation of the control action involves only four control rules whereas an FLC with three input variables (i.e. PID) requires eight.

For a PIDFLC with three control variables of seven sets each, 21 sets would have to be checked to determine applicable rules. For the hybrid controller, only a maximum of sixteen sets would be checked. The rule search first checks the two ZO sets for the PD portion and then checks at most all fourteen of the PI portion sets. If the check on the two ZO sets is negative then the remaining 12 sets would be checked.

For the hybrid fuzzy PID controller, the PD and PI portions are designed separately and logic controls when to switch between the two controllers. The logic switches to the PI portion when both change in error and error are in the ZO range. The PD portion must not be re-enabled until the error variable moves out of the ZO range, regardless of the change in error variable. The PI portion in the process of reaching steady-state obviously creates a change in error that might be out of the PD's ZO range and should not reactivate the PD portion.

Numerical Example

The hybrid fuzzy PID controller was used to control the plant that has the transfer function $G_p(s)=\frac{45}{s^2+18s+45}$. The controller gains were $K_p = 19$ and $K_d = .5$. The peak values of the fuzzy sets is given in Table III. The PI portion was designed with K_i/K_p set to .1, K_p is 100, and seven fuzzy set with all peak values equally spaced. Figure 6 shows that after a slight overshoot, the ramp response settles to the desired output.

Table III

	NB	NM	NS	ZO	PS	PM	PB
Error	-1	-.35	.03	0	.03	.35	1
ΔError	-1	-.35	-.10	0	.10	.35	1
Output	-1	-.75	-.3	0	.3	.75	1

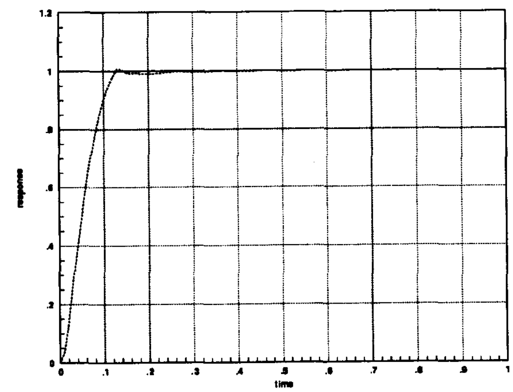

Figure 6 Response for example system controlled by hybrid PIDFLC

IV Conclusions

PD and PI fuzzy controllers have the same design disadvantages as their classical counterparts. Therefore, in some cases a fuzzy

PID controller maybe required. The fuzzy PID controller entails a large rule base which presents design and implementation problems. A reduced rule fuzzy PID scheme was implemented to take advantage of both PD and PI control actions. Some further research is required for the process of switching between the control actions. Results from the simulation demonstrates the effectiveness of the PID controllers.

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