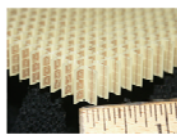
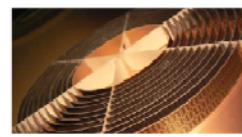


unit cell of a microwave metamaterial consisting of a split-ring resonator and metal wires

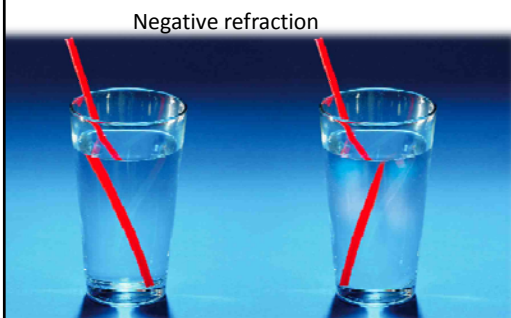


Split-ring resonator array



Metamaterial superlens

Metamaterials



Negative refraction

Solymar & Walsh

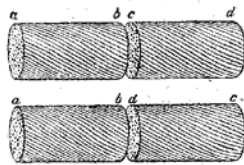
Ch. 15

Handout: #8

Read the [wiki](#) article on **Metamaterial**

Artificial Material

- The idea of artificial material is not new – it has been known for about three scores of years that inclusion of bits of metal in a dielectric will lead to some modest change in the dielectric constant.
- Gabriel Lippman in 1894 started to think about artificial materials – developed a film in the form of a dielectric variation caused by standing waves
- In 1898 **Sir Jagdish Chandra Bose** proposed twisted jute as an artificial material that could rotate the polarization of EM waves – an artificial chiral material



Jute elements.

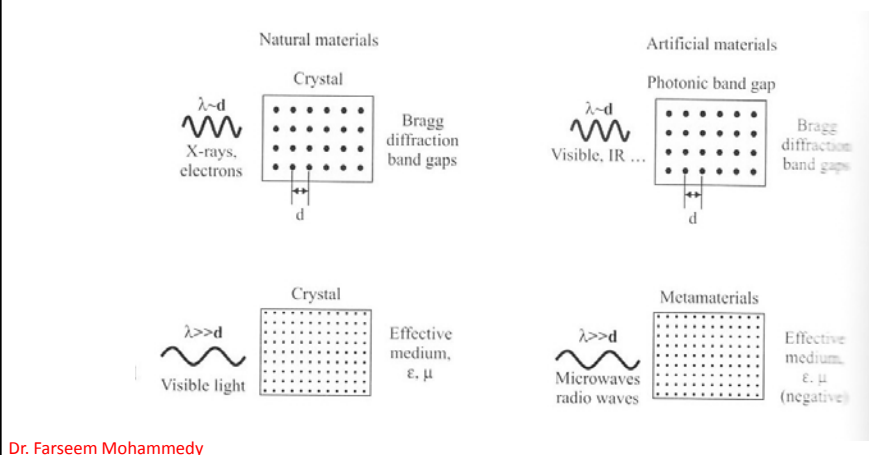


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2

Natural materials vs artificial materials

- Most materials that we discussed were natural – they appeared as they are in nature, or tweaked artificially here and there
- An artificial material is different in every respect from those natural



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3

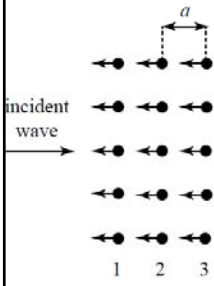
Photonic bandgaps and metamaterials

- We all know what natural materials are made up by lots and lots of small elements like atoms and molecules.
- Some of these materials are amorphous, meaning that all those elements are heaped upon each other in a random manner
- Others are crystalline, which means that they arrange themselves into some regular periodical pattern.
- Recently, however, there have been some new developments - we can now think of **artificial materials** in which atoms and molecules are replaced by macroscopic, man-made, elements.
- They could be roughly divided into **two classes** - in one case the distance between the inclusions is comparable with the wavelength, in the other case it is small relative to the wavelength.
- The first one is known as the subject of **photonic band gaps** and the second one as **metamaterials**.

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4

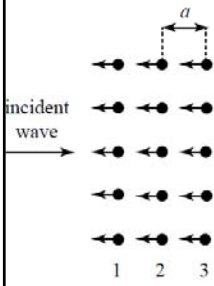
a/λ



- Our main interest is in the interplay of waves and materials restricted to classical physics. The key parameter is a/λ , where a is the distance between elements in the material and λ is the free-space wavelength.
- We may now look at two cases: the wavelength is comparable with a or much larger than a .
- In the first case the **Bragg effect** comes into play.
- An EM wave may be incident perpendicularly at a lattice. The wave propagating then from row 1 to row 2 will cover a path a . The part of the wave that is reflected by row 2 will have covered an additional distance a when arriving back at row 1.
- When a happens to be equal to one-half of a wavelength then the waves reflected by all the rows will have the same phase and will reinforce each other.
- If there are many rows, and there are indeed many of them in a crystal, then most of the incident power may be reflected.
- This effect is at the basis of **X-ray** and **electron diffraction** in crystals.

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a/λ



- When the wavelength \gg lattice period, then no such dramatic effect occurs, but it is nonetheless significant.
- There may not be major reflection or diffraction but the electromagnetic wave is still considerably affected when it enters a material.
- We may then ignore the details and pretend that there is no discrete structure: the material is homogeneous and continuous.
- The aim is then to find some effective parameters like electric permittivity and magnetic permeability.
- This is known as the **effective-medium approximation**.
- **Summarizing**, there is the Bragg effect, when the distance between the elements is comparable with the wavelength, and there is effective-medium response when that distance is much less than the wavelength.

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What is Metamaterials

- We shall give here two definitions –
 - 1. Metamaterials are engineered composites that exhibit superior properties not found in nature and not observed in the constituent materials.
 - 2. A metamaterial is an artificial material in which the electromagnetic properties, as represented by the permittivity and permeability, can be controlled. It is made up of periodic arrays of metallic resonant elements. Both the size of the element and the unit cell are small relative to the wavelength.

Originates from a Greek word **μετα**: "after/beyond"

Example: metaphysics ("beyond nature"):

- a branch of philosophy concerned with giving a general and fundamental account of the way the world is.

Metamaterials are artificially engineered materials possessing properties (e.g., mechanical, optical, electrical) that are **not** encountered in naturally occurring materials

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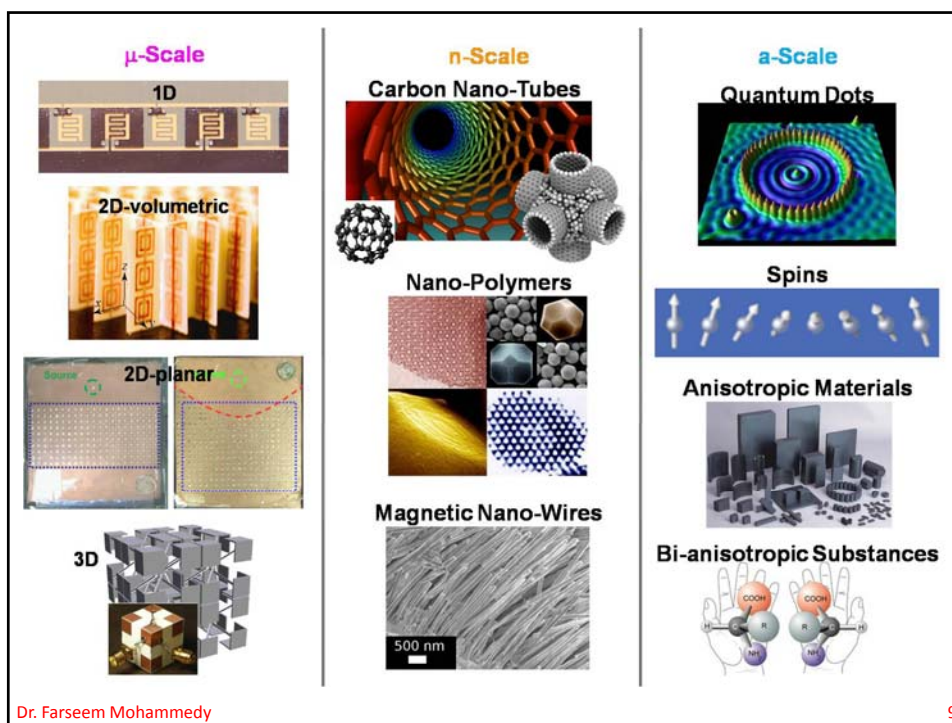
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What is Metamaterials

- Wiki says
- **Metamaterials** are artificial media structured on a size scale smaller than the wavelength of external stimuli. Materials of interest exhibit properties not found in nature, such as [negative index of refraction](#). They are cellular assemblies of multiple elements fashioned from materials including metals and plastics, arranged in periodic patterns. Metamaterials gain their properties not from their constituents, but from their exactly-designed structures. Their precise shape, geometry, size, orientation and arrangement can affect light or sound in a manner that is unachievable with conventional materials.
- Potential applications of metamaterials are diverse and include remote [aerospace](#) applications, [sensor](#) detection and [infrastructure monitoring](#), smart [solar power](#) management, [public safety](#), [radomes](#), [high-frequency battlefield communication](#) and lenses for high-gain [antennas](#), improving [ultrasonic sensors](#), and even [shielding structures from earthquakes](#).
- Metamaterial research is interdisciplinary and involves fields including [electrical engineering](#), electromagnetics, [solid state physics](#), microwave and antennae engineering, [optoelectronics](#), classic [optics](#), [material sciences](#), semiconductor engineering, [nanoscience](#).
- Metamaterials have become a new subdiscipline within [physics](#) and [electromagnetism](#) (especially [optics](#) and [photonics](#)).
- They show promise for [optical](#) and [microwave](#) applications such as new types of beam steerers, [modulators](#), [band-pass filters](#), [lenses](#), [microwave couplers](#), and [antenna systems](#). Furthermore, the lower density of materials means that [components](#), [devices](#), and [systems](#) can be lightweight and small, while at the same time enhancing system and component performance.

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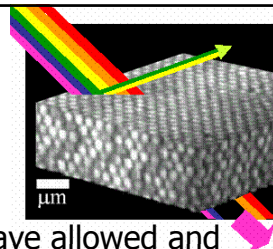
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- The physics of "small-scale" lies at the heart of the metamaterial advantage. The physics at small scale is different than bulk physics and, from a performance standpoint, often significantly better. Quantum confinement, exchange-biased ferromagnetism, and effective media responses are all examples of how the physics at small-scale can result in enhanced electromagnetic properties.

- Gennady Shvets, The University of Texas at Austin

Photonic bandgaps



- As we know, electrons in a semiconductor have allowed and forbidden energies.
- Why do electrons behave that way? It is essentially due to the wave-like nature of the electron. When they see a periodic potential in a periodic medium, they respond.
- Photons can do the same thing if they find themselves in a periodic medium. Put photons into a periodic medium and they will have allowed and forbidden energies which, in this context, means that the propagation of the electromagnetic waves in that medium is allowed or forbidden.
- The modern term for it is **photonic bandgaps**.

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11

Photonic bandgaps

- What are the **applications** of photonic bandgap materials?
- They can be used whenever there is a need for electromagnetic waves propagating in any direction to be reflected.
- They are singularly suitable for constructing resonant cavities – replace a few elements of a photonic bandgap material by one capable of lasing, pump the laser at a wavelength for which the bandgap material is transparent and the whole laser device is ready.
- This is actually the way to produce very small lasers where very small means that its dimensions are submicrometre.
- Another application is for guiding light. If we have a cylindrical photonic bandgap material and we clear the area around the axis, then an optical wave can propagate there without being able to spread outwards in the radial direction. This is because a wave propagating in any but the axial direction will be reflected. These photonic bandgap waveguides promise lower losses for guiding waves to a long distance.

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12

Resonant elements in Metamaterials

- Most metamaterial elements are resonant and then the problem arises how to make them small.
- It is not trivial to satisfy the requirement for the elements to be resonant and at the same time to be small relative to the wavelength.
- We shall concentrate on one resonator, a member of the family of split-ring resonators (SRRs),
- It consists of two concentric split rings with gaps on opposite sides.

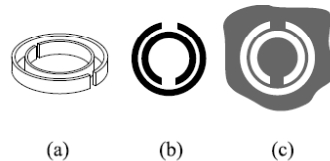


Fig. 2.12 Split-ring resonators (a) as pipes, (b) in printed circuit form, and (c) as a complementary variety

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13

Split ring resonator

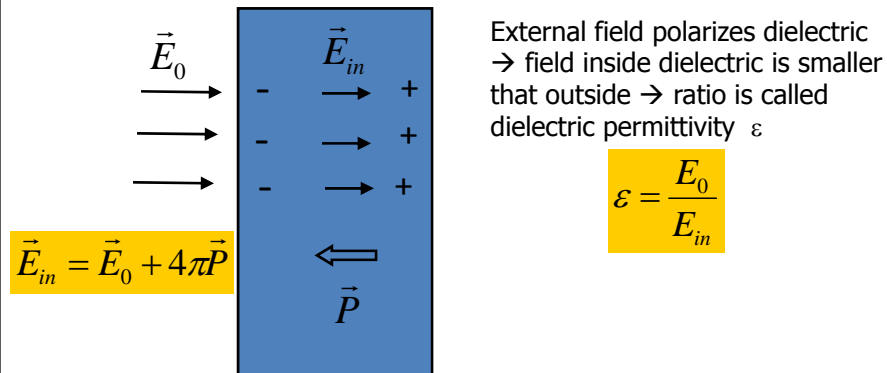
- Each ring has a self-inductance, there is a mutual inductance between them, there is a capacitance between the rings and there are gap capacitances at the splits.
- The exact analysis is difficult, but there is an excellent approximation and that leads to simplified resonant frequency formula – the capacitance of the half-ring is $C_{\text{half-ring}} = \pi \cdot r_0 \cdot C_{\text{pu}}$, where C_{pu} is the inter-ring capacitance per unit length and the resonant frequency is

$$\omega_0 = \sqrt{\frac{\pi r_0 L C_{\text{pu}}}{2}}$$

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14

What is a dielectric permittivity?

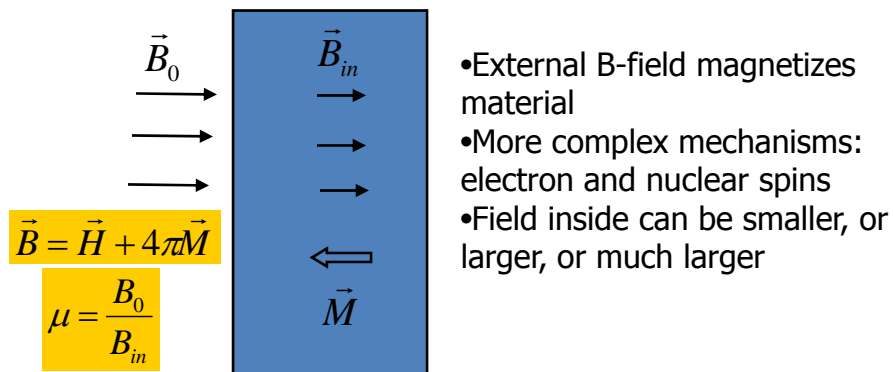


- In most materials $\epsilon > 1$ (e.g., $\epsilon = 12$ for Si, $\epsilon = 2.25$ for glass)
- Permittivity depends on frequency: long lookup tables!
- Not without exceptions: $\epsilon < 0$ in metals (visible, IR,...)

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15

What is a magnetic permeability?



- In most materials $\mu > 0$
- There are exceptions (ferrites), but only at microwave frequencies

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16

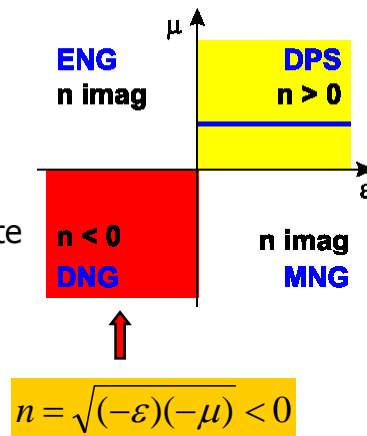
How waves propagate (or not)?

Propagation of electromagnetic waves in medium is determined by ϵ and μ of the medium (J. C. Maxwell):

$$k \equiv \frac{n\omega}{c} = \frac{\omega}{c} \sqrt{\epsilon\mu}$$

In most natural materials
 $\mu > 0, \epsilon > 0 \rightarrow$ waves propagate

Sometimes either $\epsilon < 0$, or
 $\mu < 0 \rightarrow$ no propagation



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17

- We know the expression for the refractive index in terms of the relative permittivity and permeability, $n = \sqrt{\epsilon_r \mu_r}$.
- When both μ_r and ϵ_r are positive then there is no problem, everything is familiar
- Some may ask what μ_r is doing in that equation. That objection can be easily overcome by asserting that the relative permeability may indeed be different from unity for metamaterials.
- What happens when only ϵ is negative? That happens in plasmas

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18

Negative index *

- But Veselago, in a paper written in Russian in 1967 and published in English in 1968, asked a more daring question: What happens when both μ and ϵ are negative?
- He discusses the possible responses as quoted below:
- "The situation can be interpreted in various ways. First, we may admit that the properties of a substance are actually not affected by a simultaneous change of the signs of ϵ and μ . Second, it might be that for ϵ and μ to be simultaneously negative contradicts some fundamental law of nature, and therefore no substance with $\epsilon < 0$ and $\mu < 0$ can exist. Finally, it could be admitted that substances with negative ϵ and μ have some properties different from those of substances with positive ϵ and μ ."

* For further details, read Solymar and Shamonina's *Waves in Metamaterials*, section 2.11. Also quoted in *Handout-8*.

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19

Negative index

- Assuming a plane wave propagating in a medium with material constants ϵ and μ in the form $\exp(-j \mathbf{k} \cdot \mathbf{r})$, we can rewrite Maxwell's first two equations as: $\mathbf{k} \times \mathbf{H} = \omega \epsilon \mathbf{E}$ and $\mathbf{k} \times \mathbf{E} = -\omega \mu \mathbf{H}$.
- It may be seen from the above equations that it makes a difference whether the material constants are both positive or both negative. In the former case the vectors \mathbf{E} , \mathbf{H} and \mathbf{k} constitute a right-handed set, whereas for negative ϵ and μ we have a left-handed set.
- The wave vector \mathbf{k} tells us the direction of the phase velocity, the Poynting vector tells us the direction of the group velocity. If the two are in opposite directions we have a backward-wave material with all that implies.
- Thus, negative refraction at the boundary of two materials, one having positive material constants and the other negative ones, follows immediately.
- The most striking example of what we can do with a negative-index material is Veselago's flat lens. For $n = -1$ the angle of refraction is equal to the negative angle of incidence hence all the rays emanating from a line source will be refocused inside the material and brought to another focus outside the material

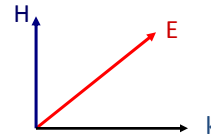
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20

Basic properties of Negative Index Waves

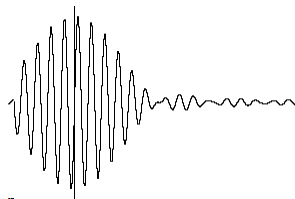
In vacuum "right-hand rule" relates E , H , and k .

Note: normally $\mu > 0$ and $\varepsilon > 0$

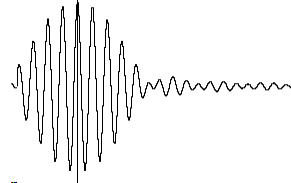


- Consequence: phase velocity (along k) and group velocity (along the Poynting $\underline{E} \times \underline{H}$ vector) are in the same direction

- In NIMs group and phase velocity are in opposite directions



Positive Index Medium

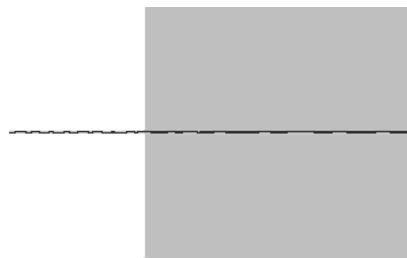


Negative Index Medium

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21

Positive/Negative Index Interface



Positive Index
Medium

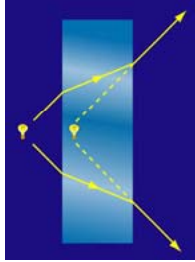
Negative Index
Medium

What happens for the oblique incidence?

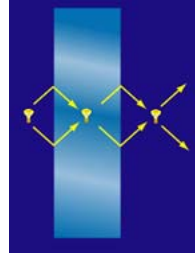
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22

Unusual refractive properties of NIMs



Light enters $n > 0$
material \rightarrow deflection



Light enters $n < 0$ material
 \rightarrow focusing ("Veselago
Lens")

Surface waves make Veselago's lens a super-lens! (Pendry, 2000)

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23

Straw in a negative index water



empty glass



regular water,
 $n = 1.3$



"negative" water,
 $n = -1.3$

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24

How to Make a Negative-Index Material

In microwave range: use "perfectly" conducting components to simulate $\epsilon < 0$ and $\mu < 0$, Smith et.al., (2000)



Metal poles: $\epsilon = 1 - \omega_p^2/\omega^2 < 0$
 Split-ring resonators, Pendry'99:
 "geometric" resonance at ω_M

$$\omega_p = \frac{c}{D} \left(\frac{2\pi}{\log(D/r)} \right)^{1/2}$$

$$\mu = 1 - \frac{F\omega_M^2}{\omega^2 - \omega_M^2} < 0$$

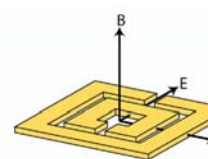
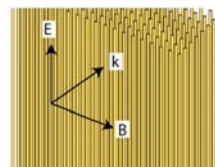
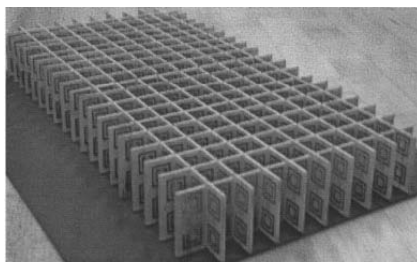
Challenges:

- (a) moving to optical frequencies (infrared, visible, UV)
- (b) simplifying the structure ($\epsilon < 0$ and $\mu < 0$ from same element)

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25

Another Example a microwave NIM



Basic Elements of a NIM:

- (a) Split ring resonator: just a well designed inductor resonating at $\omega \ll c/L \rightarrow$ gives $\mu < 0$
- (b) Metal wires (continuous or cut): $r \ll L$ to ensure that $\epsilon < 0$ for $\omega \ll c/L$

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26

The Perfect Lens

- Does it exist?
- It is an old subject, going back at least for half a century (Torald di Francia, 1953), how one can beat the classical limit of resolution.
- There are essentially two approaches: one uses the far field and relies on changing the field distribution in the aperture of the lens, the second one is based on near fields.
- The first attempt at high-resolution near-field imaging was made by Ash and Nicholls (1972)
- The idea was to make use of the field leaking out of a microwave cavity through a small hole. If an object with a structure somewhat larger than the hole is scanned in front of the hole then the resonant frequency of the cavity depends on the relative position of the object.
- By monitoring the resonant frequency it turned out to be possible to obtain information about the structure with a resolution of $\lambda/60$, close to that of the size of the hole. Their work initiated the whole new field of scanning near-field optical microscopy.
- An entirely new idea of near-field imaging came with a proposal by Pendry (2000). He calculated that the flat lens of Veselago (which required a refractive index of minus unity) will be able to image an object with infinite resolution, provided $\epsilon_r = -1$ and $\mu_r = -1$.

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27

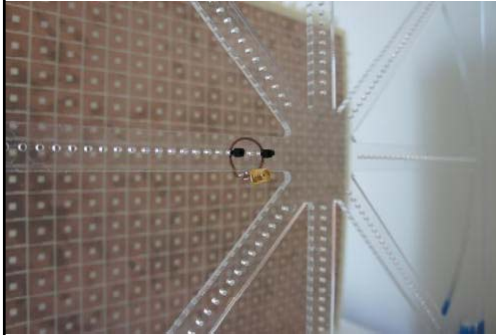
The Perfect Lens

- Infinite resolution means perfect imaging. **Perfect imaging** means that every single detail of the object is reproduced in the image (including both propagating and evanescent components). In terms of a spatial frequency, it means that the spatial frequency spectrum of the image (including both propagating and evanescent components) will be identical with the spatial frequency spectrum of the object.
- In terms of a transfer function it means that the transfer function is flat. Entirely flat. It is the same for every spatial frequency component.
- Is that possible? Not really. A limit will be set, if by nothing else, then by the period of the negative-index material. If we can make metamaterial elements of the size of 100 nm and if the distance between them is also 100 nm then there would be a chance of making a lens with a resolution approaching 100 nm.
- Another chance is obtained with a material in which only the dielectric constant is negative. That will not yield a flat transfer function but it would be flat enough for many purposes, and it would have the great advantage that natural materials (e.g. silver) with that property exist. The period in that material will be of the order of one tenth of a nanometer, thus, at least on that account, the resolution could be extremely high.

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28

Super lens



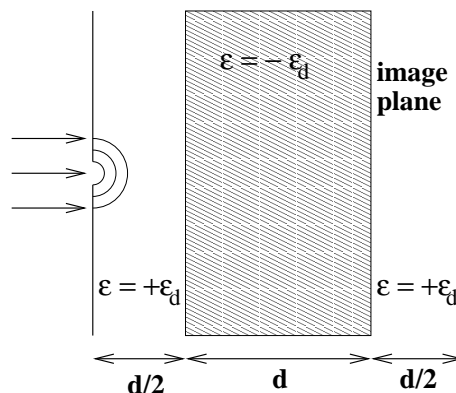
- This small copper coil created the electromagnetic field by running an alternating electric current through it. In the background is the metamaterial "superlens" that focused the electromagnetic field onto another identically sized copper coil on the other side, which greatly increased the wireless transfer's power.
- Courtesy of Guy Lipworth and Joshua Ensworth, graduate student researchers at Duke University

Read more at: <http://phys.org/news/2014-01-superlens-range-wireless-power.html#jCp>

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29

"Poor Man's Super-Lens": $\epsilon < 0$, $\mu > 0$



Inserting a slab of matched material with negative ϵ (and, one day, μ) can prevent image degradation
Super-lensing is a highly resonant phenomenon: frequency-dependent permittivities must match

Recent UV results: Fang et.al, Science '05, Melville and Blaikie, Opt. Expr. '05

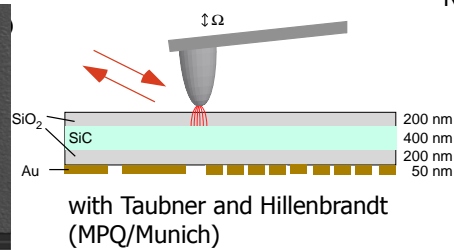
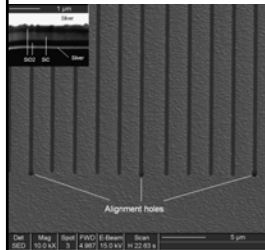
We have demonstrated super-lensing in IR and (a) proved its resonant nature, (b) demonstrated a new application: sub-surface imaging

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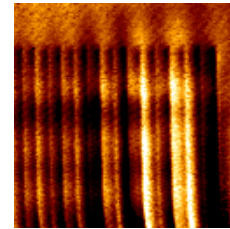
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Superlens in mid-IR: sub-surface imaging

pattern on bottom



NSOM image from top



SiO₂/SiC/SiO₂ superlens with a metallic pattern (0.5 μm slits in Ag film separated by 3 μm on the bottom side) was imaged from the top using NSOM

Sub-surface imaging of sub-λ features at 800 nm depth accomplished at 10.85 μm (CO₂ laser) using a superlens → opens the way to applications of super-lensing to sub-surface imaging of integrated circuits

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31

Summary

Optical meta-materials have been shown to have remarkable applications:

- Can be used to engineer exotic meta-media: Negative Index Materials → plasmonic approach to making a sub-λ NIM
- NIMs and negative ϵ materials can be used to overcome diffraction limit and construct a super-lens
- A super-lens enables ultra-deep sub-surface imaging using NSOM probe

Very new field → lots of work to do (theory and experiments)

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32