ELEN-865 Theory of Linear Control System

Spring 2018

Final Exam

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```
A=[1 1;1 -2]
```

```
A = 2×2
     1     1
     1    -2
```

```
B=[1;2]
```

```
B = 2×1
     1
     2
```

```
C=[1 3]
```

```
C = 1×2
     1     3
```

```
D=[0]
```

```
D = 0
```

```
r=0.1
```

```
r = 0.1000
```

```
Q=[1 0;0 10]
```

```
Q = 2×2
     1     0
     0    10
```

```
ctrl = ctrb(A,B)
```

```
ctrl = 2×2
     1     3
     2    -3
```

```
p = rank(ctrl)
```

```
p = 2
```

```
K=lqr(A,B,Q,r,0)
```

```
K = 1×2
    6.5568    6.8259
```

a)

The LQR control law is u = -KX, where K is given above.

```
sys = ss(A,B,C,D)
```

```
sys =

  A =
         x1  x2
   x1    1   1
   x2    1  -2

  B =
         u1
   x1    1
   x2    2

  C =
         x1  x2
   y1    1   3

  D =
         u1
   y1    0

Continuous-time state-space model.
```

```
isstable(sys)
```

```
ans = logical
   0
```

```
Acl = A-B*K
```

```
Acl = 2×2
   -5.5568   -5.8259
  -12.1136  -15.6518
```

```
Bcl = [0 ; 0]
```

```
Bcl = 2×1
     0
     0
```

```
sys = ss(Acl,Bcl,C,D)
```

```
sys =

  A =
                x1       x2
```

```
  x1  -5.557  -5.826
  x2  -12.11  -15.65

 B =
      u1
  x1   0
  x2   0

 C =
      x1  x2
  y1   1   3

 D =
      u1
  y1   0

Continuous-time state-space model.
```

```
isstable(sys)
```

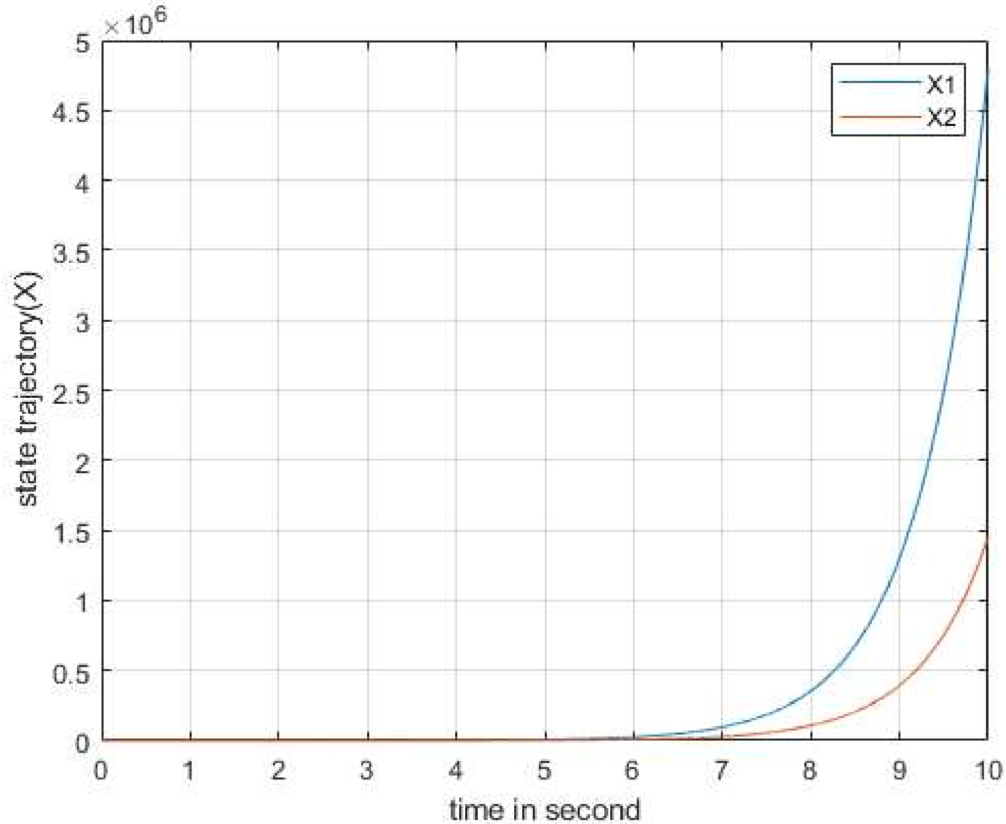
```
ans = logical
   1
```

b)

The open loop system is not stable but the closed loop system is stable.

```
dt=0.001;
tf=10;
t=0:dt:tf;
x0=[10,5]';
x = zeros(length(t), 2);
x(1,:) = x0';
u = zeros(length(t), 1);
for i=2:length(t)

    xx0 = (reshape(x(i-1,:),size(x0)));
    xx = xx0;
    xx_dot = A*xx;
    k1 = dt*xx_dot;

    xx = xx0 + k1./2;
    xx_dot = A*xx + B*(u(i-1,:)');
    k2 = dt*xx_dot;

    xx = xx0 + k2./2;
    xx_dot = A*xx + B*(u(i-1,:)');
    k3 = dt*xx_dot;

    xx = xx0 + k3;
    xx_dot = A*xx + B*(u(i-1,:)');
    k4 = dt*xx_dot;

    xx = xx0 + k1./6 + k2./3 + k3./3 + k4./6;
```

```
    x(i,:) = xx(:);
end
figure
plot(t,x(:,1))
hold on;
plot(t,x(:,2))
grid on
xlabel('time in second')
ylabel('state trajectory(X)')
legend('X1','X2')
```

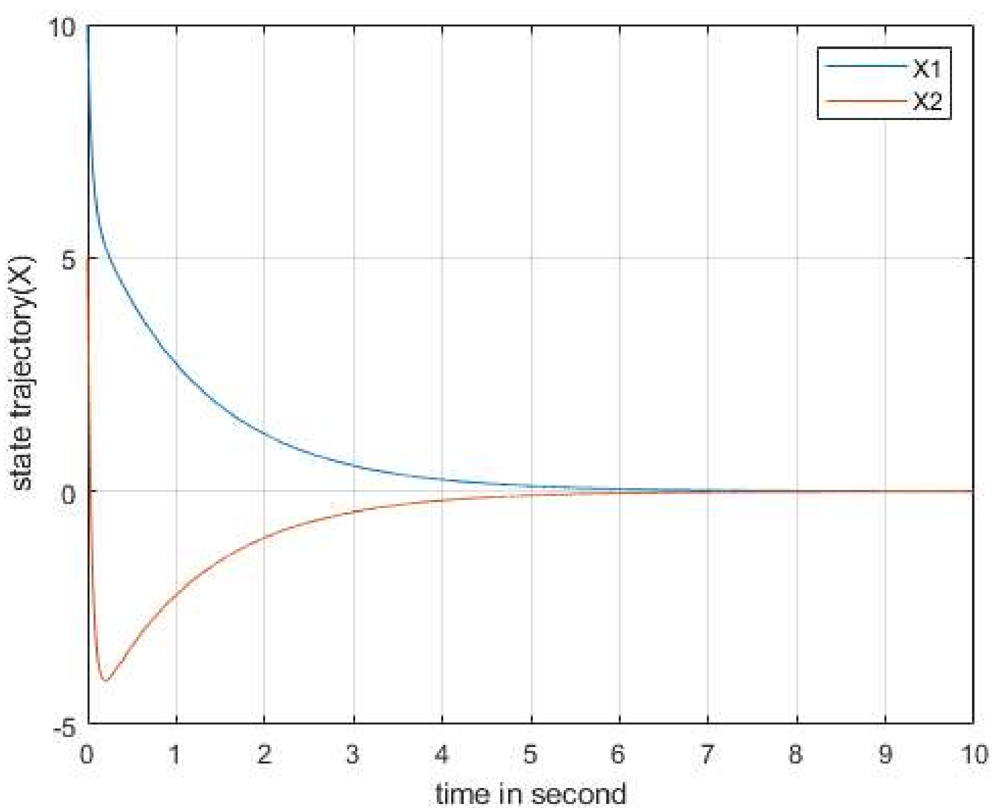


c)

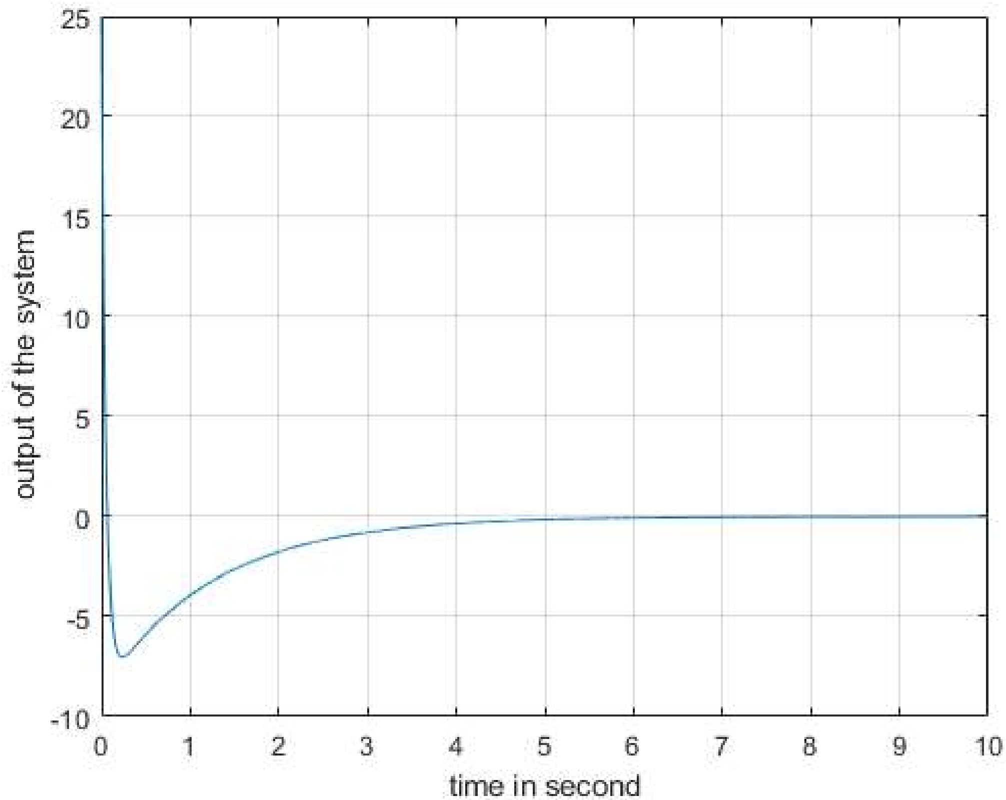
The state trajectories of the open loop system is shown above.

```
u = zeros(length(t), 1);
x = zeros(length(t), 2);
y = zeros(length(t), 1);
x(1,:) = x0';
u(1) = -K*(x(1,:))';
y(1) = C*(x(1,:))';
for i=2:length(t)

    xx0 = (reshape(x(i-1,:),size(x0)));
    xx = xx0;
    xx_dot = A*xx + B*(u(i-1,:)');
```

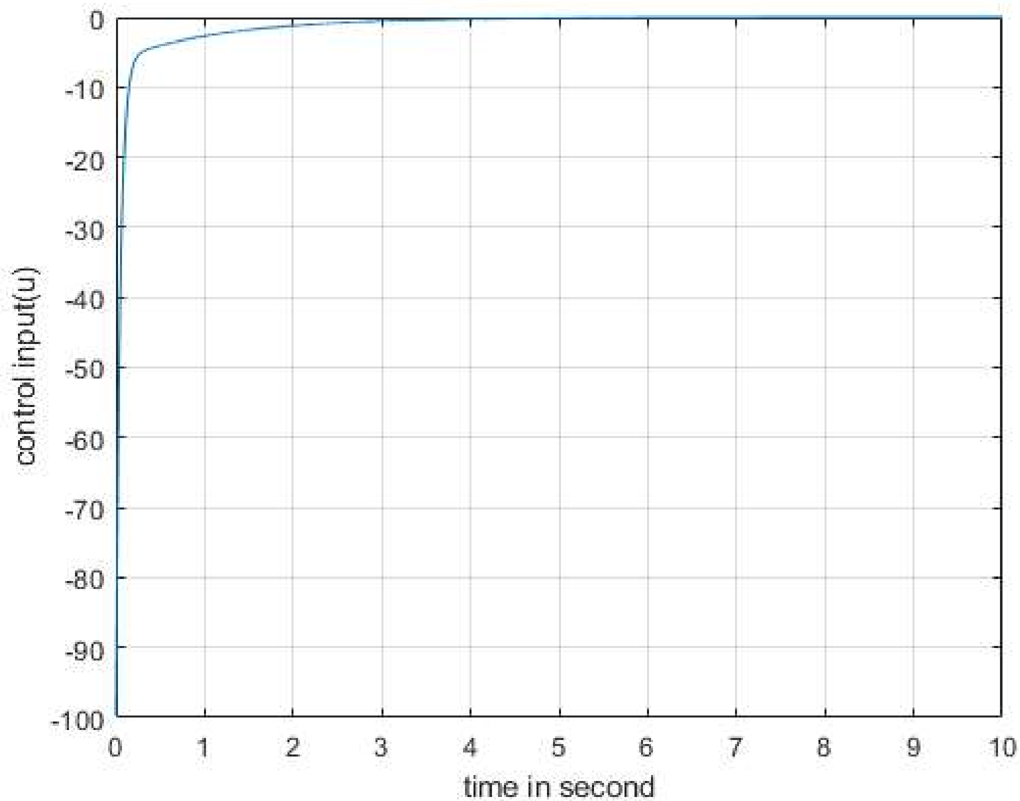
```
    k1 = dt*xx_dot;

    xx = xx0 + k1./2;
    xx_dot = A*xx + B*(u(i-1,:)');
    k2 = dt*xx_dot;

    xx = xx0 + k2./2;
    xx_dot = A*xx + B*(u(i-1,:)');
    k3 = dt*xx_dot;

    xx = xx0 + k3;
    xx_dot = A*xx + B*(u(i-1,:)');
    k4 = dt*xx_dot;

    xx = xx0 + k1./6 + k2./3 + k3./3 + k4./6;

    x(i,:) = xx(:);
    u(i,:) = -K*(x(i,:))';
    y(i) = C*(x(i,:))';
end
figure
plot(t,x(:,1))
hold on;
plot(t,x(:,2))
grid on
xlabel('time in second')
ylabel('state trajectory(X)')
legend('X1','X2')
```



```
figure
plot(t,y)
grid on
xlabel('time in second')
ylabel('output of the system')
```




```
figure
plot(t,u)
grid on;
xlabel('time in second')
ylabel('control input(u)')
```



d)

Plot of state trajectory and control signal of the cloosed loop system is shown above.

```
o = obsv(A,C)
```

```
o = 2×2
     1     3
     4    -5
```

```
p = rank(o)
```

```
p = 2
```

```
Q=[5 0;0 50]
```

```
Q = 2×2
     5     0
     0    50
```

```
L = lqr(A',C',Q,r,0)
```

```
L = 1×2
   18.5268   16.2731
```

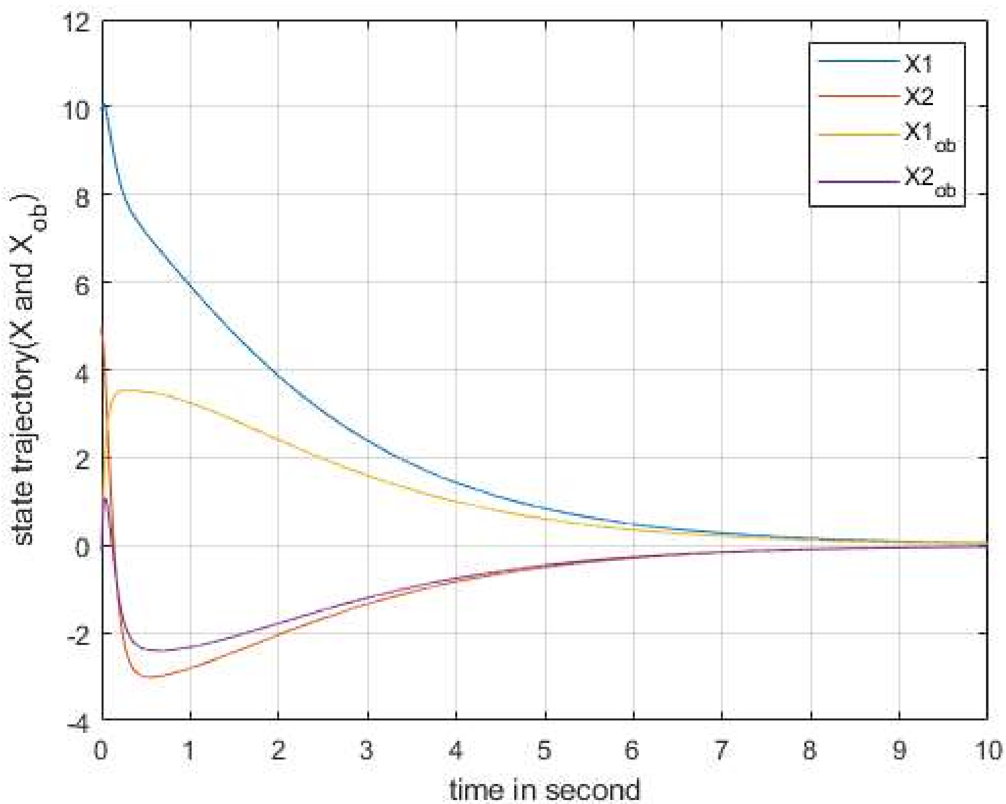
e)

The observer is : X_dot = AX_ + Bu + L(y-y_) and y_ = CX_

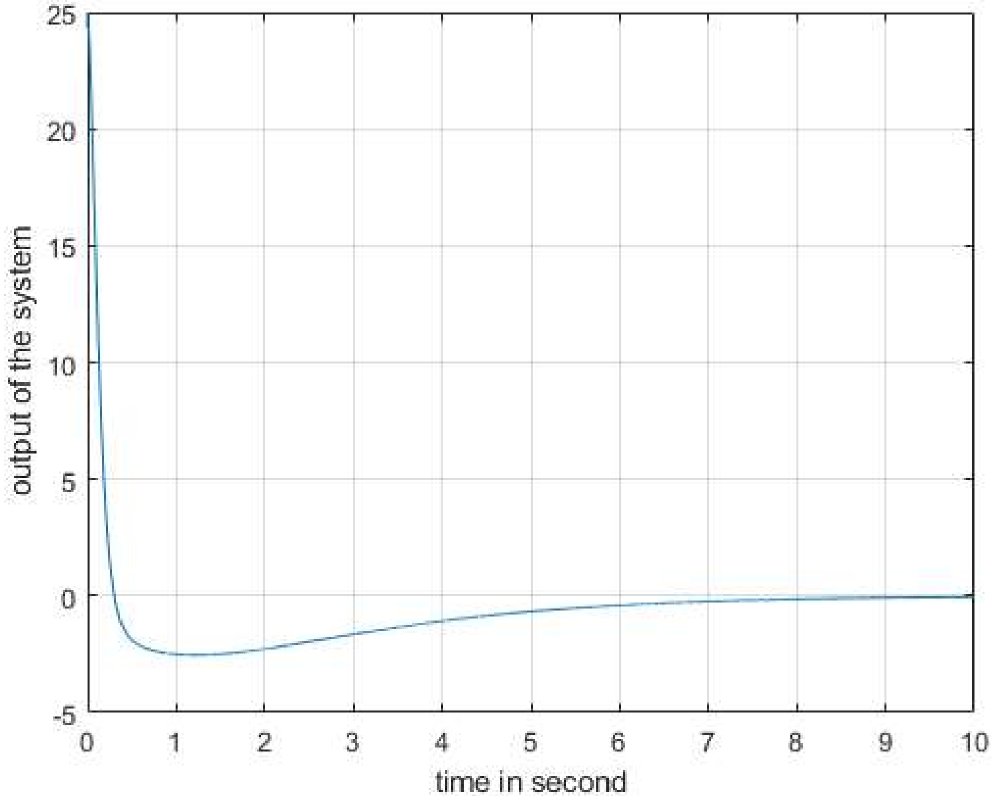
The observer gain L is given above.

```
u1 = zeros(length(t), 1);
x1 = zeros(length(t), 2);
y1 = zeros(length(t), 1);
x1(1,:) = x0';
u1(1) = -K*(x1(1,:))';
x_ob = zeros(length(t), 2);
y1(1) = C*(x1(1,:))';
for i=2:length(t)

    xx0 = (reshape(x1(i-1,:),size(x0)));
    xx = xx0;
    xx_dot = A*xx + B*(u1(i-1,:)');
    k1 = dt*xx_dot;

    xx = xx0 + k1./2;
    xx_dot = A*xx + B*(u1(i-1,:)');
    k2 = dt*xx_dot;

    xx = xx0 + k2./2;
    xx_dot = A*xx + B*(u1(i-1,:)');
    k3 = dt*xx_dot;

    xx = xx0 + k3;
    xx_dot = A*xx + B*(u1(i-1,:)');
    k4 = dt*xx_dot;

    xx = xx0 + k1./6 + k2./3 + k3./3 + k4./6;

    x1(i,:) = xx(:);
    %%%%
    xx0 = (reshape(x_ob(i-1,:),size(x0)));
    xx = xx0;
    xx_dot = A*xx + B*(u1(i-1,:)') + L'*(C*x1(i-1,:)' - C*x_ob(i-1,:)');
    k1 = dt*xx_dot;

    xx = xx0 + k1./2;
    xx_dot = A*xx + B*(u1(i-1,:)');
    k2 = dt*xx_dot;

    xx = xx0 + k2./2;
    xx_dot = A*xx + B*(u1(i-1,:)');
    k3 = dt*xx_dot;

    xx = xx0 + k3;
    xx_dot = A*xx + B*(u1(i-1,:)');
    k4 = dt*xx_dot;

    xx = xx0 + k1./6 + k2./3 + k3./3 + k4./6;
    x_ob(i,:) = xx(:);
    u1(i,:) = -K*(x_ob(i,:))';
    y1(i) = C*(x1(i,:))';
```

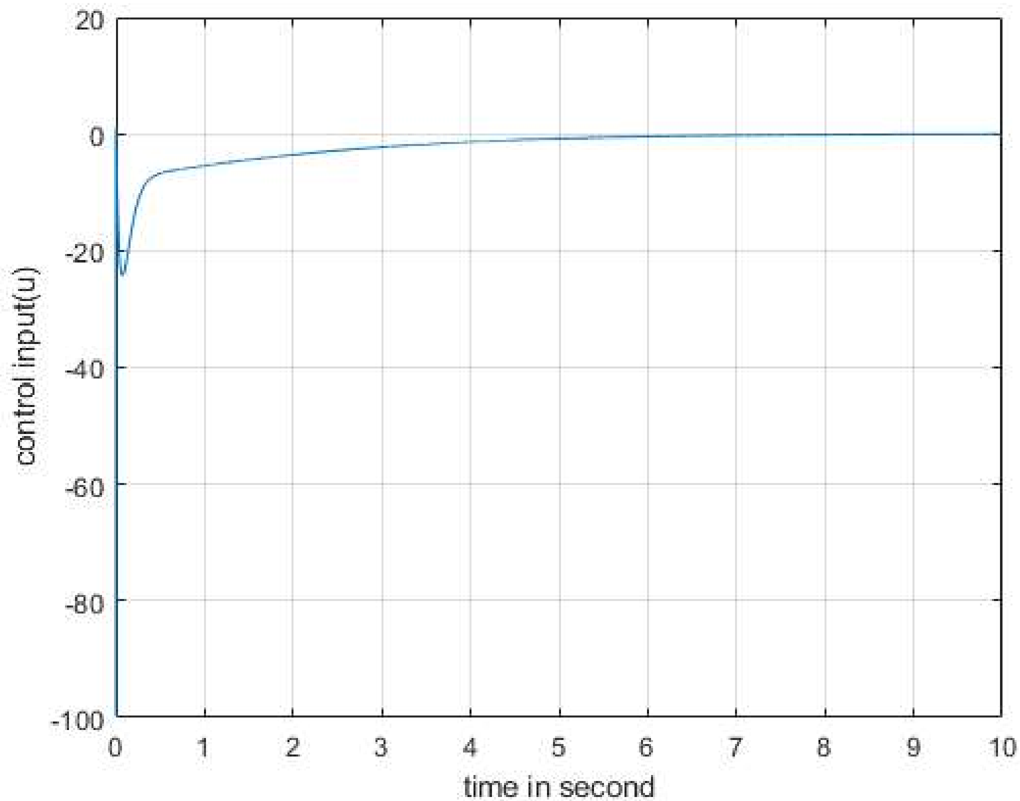
```
end
figure
plot(t,x1(:,1))
hold on;
plot(t,x1(:,2))
plot(t,x_ob(:,1))
hold on;
plot(t,x_ob(:,2))
grid on
xlabel('time in second')
ylabel('state trajectory(X and X_{ob})')
legend('X1','X2','X1_{ob}','X2_{ob}')
```




```
figure
plot(t,y1)
grid on
xlabel('time in second')
ylabel('output of the system')
```



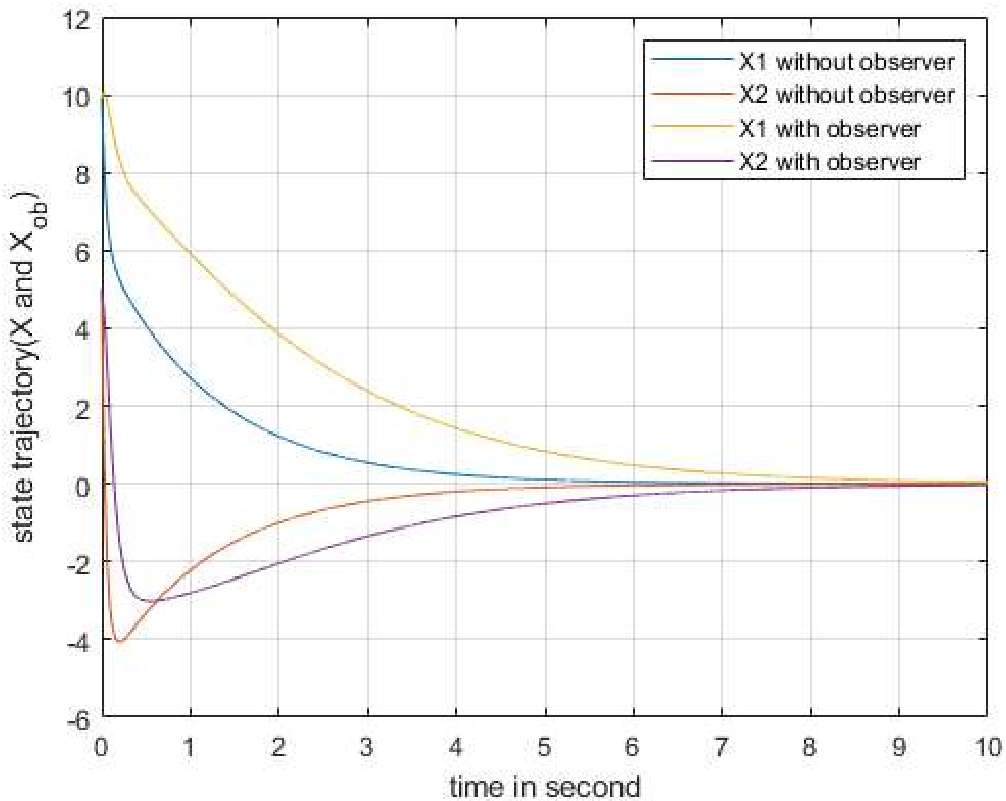

```
figure
plot(t,u1)
grid on;
xlabel('time in second')
ylabel('control input(u)')
```



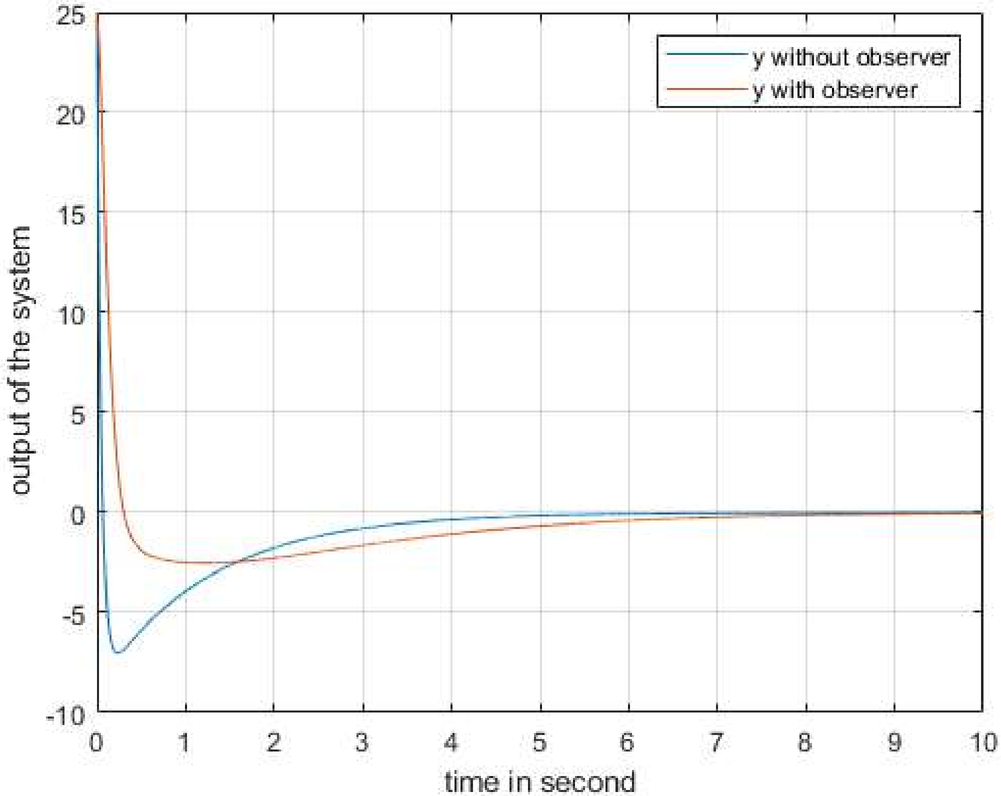
f)

The comparison between part (d) and (e) are given below:

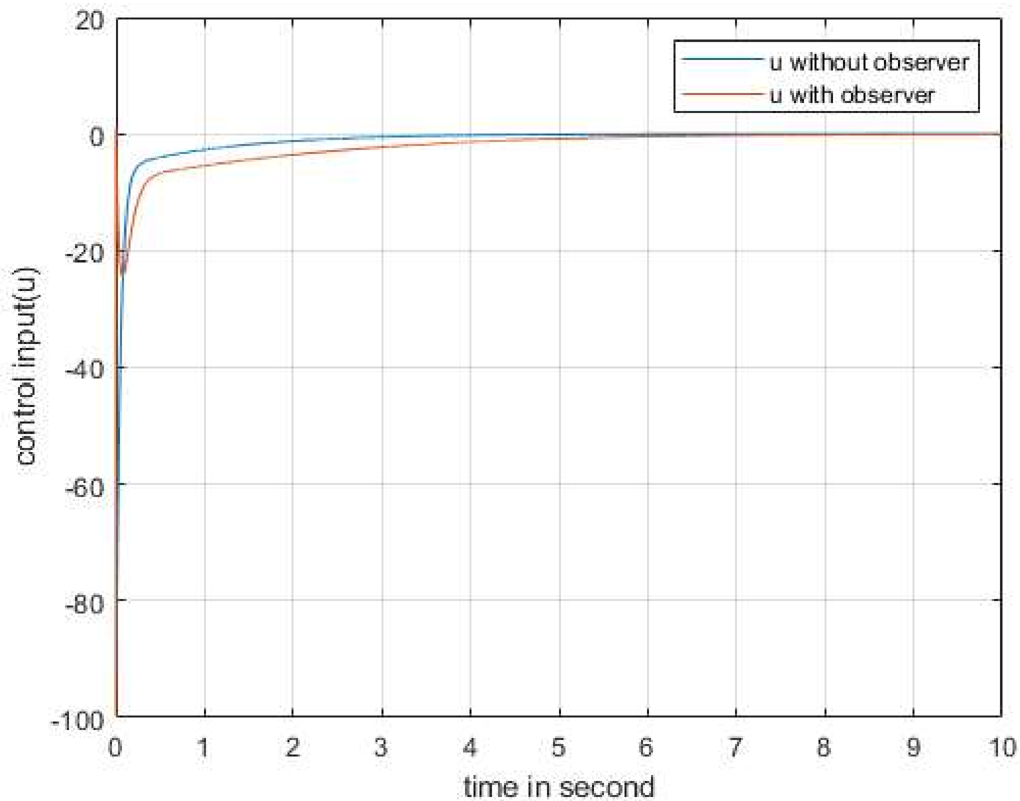
```
figure
plot(t,x(:,1))
hold on;
plot(t,x(:,2))
plot(t,x1(:,1))
hold on;
plot(t,x1(:,2))
grid on
xlabel('time in second')
ylabel('state trajectory(X and X_{ob})')
legend('X1 without observer','X2 without observer','X1 with observer','X2 with observer')
```




```
figure
plot(t,y)
hold on
plot(t,y1)
grid on
xlabel('time in second')
ylabel('output of the system')
legend('y without observer','y with observer')
```




```
figure
plot(t,u)
hold on
plot(t,u1)
grid on;
xlabel('time in second')
ylabel('control input(u)')
legend('u without observer','u with observer')
```

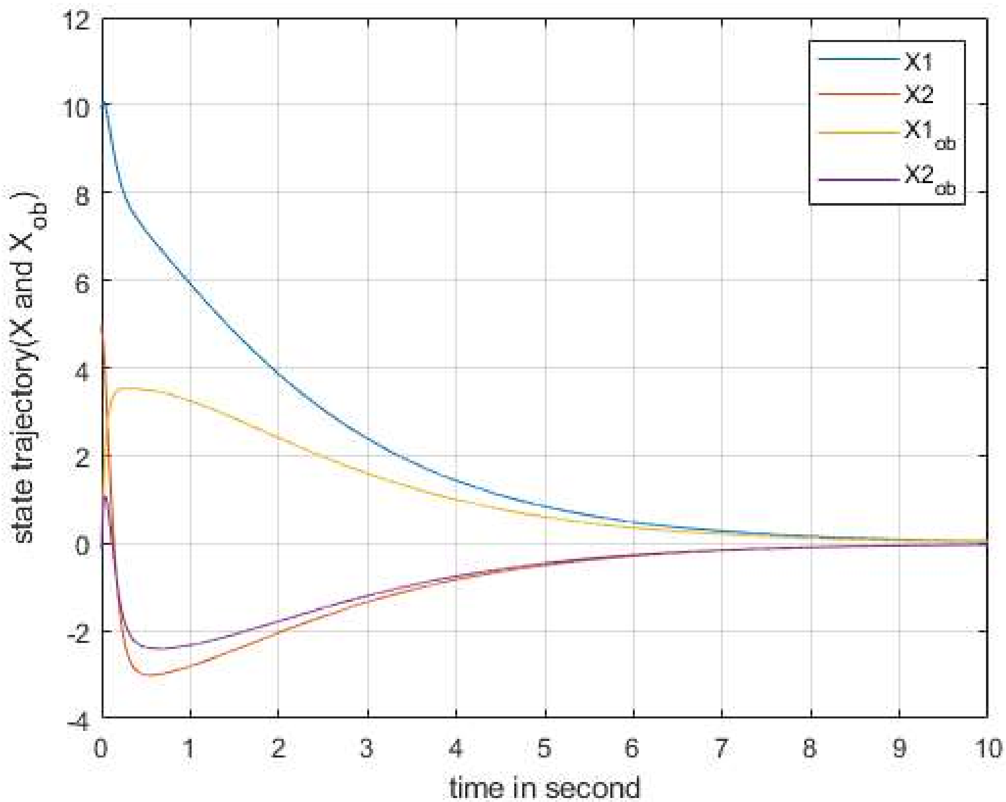


Remark: From the plots we see that the performance degrades when we add observer.

g)

The comparisone between the estimated state variable and their actual value are shown below:

```
figure
plot(t,x1(:,1))
hold on;
plot(t,x1(:,2))
plot(t,x_ob(:,1))
hold on;
plot(t,x_ob(:,2))
grid on
xlabel('time in second')
ylabel('state trajectory(X and X_{ob})')
legend('X1','X2','X1_{ob}','X2_{ob}')
```



Remark: From the plots we see as time goes the estimated state value converges to the real state value.

%%%%%%%%%%%%%%%%%%%The End%%%%%%%%%%%%%%%%%%%%%%%%%