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Design of linear quadratic regulator (LQR) control system for flight stability of LSU-05

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Abstract. Lapan Surveillance UAV-05 (LSU-05) is an unmanned aerial vehicle designed to cruise time in 6 hours and cruise velocity about 30 m/s. Mission of LSU-05 is surveillance for researchs and observations such as traffics and disaster investigations. This paper aims to design a control system on the LSU-05 to fly steadily. The methods used to stabilize LSU-05 is Linear Quadratic Regulator (LQR). Based on LQR controller, there is obtained transient response for longitudinal motion, $t_d = 0.221s, t_r = 0.419s, t_s = 0.719s, t_p = 1.359s$, and $M_p = 0\%$. In other hand, transient response for lateral-directional motion showed that $t_d = 0.186s, t_r = 0.515s, t_s = 0.87s, t_p = 2.02s$, and $M_p = 0\%$. The result of simulation showed a good performance for this method.

1. Introduction

In recent years, the world of flight has suffered rapid development. Many of the developments happened, one of them are made Unmanned Aerial Vehicle (UAV). Making an UAV needed the research of the interdisciplinary, including instruments of supporting. One of them is control system, so UAV can fly independent in run missions who is given.

Use of UAV have been divided into two purposes: UAV for military purposes as the target drone, its spies, patrol in the border areas, and each others. Whereas for the purpose of civil as search and rescue natural disaster victims, monitoring forest, monitoring traffic, shooting from the air on land harvest, mapping and town planning, and each others. UAV can be used for working at high risk, as spying on the enemy in the war, or a rescue mission in the radiation danger, and each others. UAV with small size dont issue the noise like the plane. With color obscured, so UAV can fly to fused with the sky and hard to know by human. Hence, UAV often has used by the department of defense to do controling of the territory[1].

The research and development UAV in Indonesia are done by Aeronautics Technology Center LAPAN. One of a type of UAV LAPAN that used as research in this paper is Lapan Surveillance UAV-05 (LSU-05). According to the specifications, LSU-05 was designed to be flying nonstop 200 km and endurance at least six hours, with an average speed cruising 30 m/s. Mission of LSU-05 is surveillance for researchs and observations such as traffics and disaster investigations[2].

LSU-05 was developed into aircraft with autonomous system. To realize the initiative, so UAV should be equipped with appropriate and reliable control system. Hence, in this paper

design control system was used Linear Quadratic Regulator (LQR). LQR was chosen because LQR able to overcome big disturbance is going on stability the system without reduces working performance and can overcome disturbance that occurred previously[3, 4]. By using LQR controller, characteristic response of LSU-05 appropriate with design and has flight stability.

2. Basic Teory
In this section contains state space of LSU-05 and LQR control system.

2.1. State Space of LSU-05
State space of LSU-05 divided into: state space of longitudinal motion and state space of lateral-directional motion. Based on the result of identification with dummy data, state space of longitudinal motion can be written[5]:

$$\begin{bmatrix} \dot{u} \\ \dot{\alpha} \\ \dot{\theta} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -0.83705 & 1.7696 & -0.35236 & 0 \\ -5.9575 & -21.766 & 0.0056738 & 0.8717 \\ 0 & 0 & 0 & 1 \\ 14.891 & -47.637 & -0.015802 & -7.9269 \end{bmatrix} \begin{bmatrix} u \\ \alpha \\ \theta \\ q \end{bmatrix} + \begin{bmatrix} 0 & 3.9397 \\ -0.91092 & 0 \\ 0 & 0 \\ -30.902 & -6.9048 \end{bmatrix} \begin{bmatrix} \delta_e \\ \delta_T \end{bmatrix}$$

whereas state space of lateral-directional motion can be written[5]:

$$\begin{bmatrix} \dot{\beta} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} -2.3817 & 0 & -1.0019 & 2.1827 \\ -21.063 & -16.055 & 0.87229 & 0 \\ 24.512 & -16.651 & -3.5379 & 0 \\ 0 & 1.0026 & -0.029766 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ p \\ r \\ \phi \end{bmatrix} + \begin{bmatrix} 0 & -0.24719 \\ -36.263 & -688.44 \\ -0.67252 & -67.983 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_A \\ \delta_R \end{bmatrix}$$

2.2. LQR Control System
Problems that are commonplace in the field of control is not only stabilize system, but also how output system can follow reference. If output is desired to follow reference r, then adding an integrator and defining error state (ξ) is output of integrator, with $\dot{\xi}$ is difference between input and output of the system[6, 7, 8]:

$$\dot{x} = Ax + Bu \tag{1}$$
$$y = Cx \tag{2}$$
$$u = -Kx + k_I \xi \tag{3}$$
$$\dot{\xi} = r - y = r - Cx \tag{4}$$

with:

x = state vector
u = Control signal
y = output
r = reference (step function, scalar)
ξ = output of integrator

Dynamic system in Equation (1)-(4) can be written

$$\begin{bmatrix} \dot{x} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x \\ \xi \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ I \end{bmatrix} r \tag{5}$$

Design of tracking has to make system into stabilize, if $x(\infty), \xi(\infty)$ and $u(\infty)$ approach constant value, then $\dot{\xi} = 0$, so $y(\infty) = r$. In steady state Equation (5) into:

$$\begin{bmatrix} \dot{x}(\infty) \\ \dot{\xi}(\infty) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(\infty) \\ \xi(\infty) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(\infty) + \begin{bmatrix} 0 \\ I \end{bmatrix} r(\infty) \tag{6}$$

because $r(t)$ is signal step, then $r(\infty) = r(t) = r$ is constant value, for $t > 0$. subtracting Equation (5) with (6), then we get:

$$\begin{bmatrix} \dot{x}(t) - \dot{x}(\infty) \\ \dot{\xi}(t) - \dot{\xi}(\infty) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(t) - x(\infty) \\ \xi(t) - \xi(\infty) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} [u(t) - u(\infty)]$$

For example:

$$\begin{aligned} x(t) - x(\infty) &= x_e(t) \\ \xi(t) - \xi(\infty) &= \xi_e(t) \\ u(t) - u(\infty) &= u_e(t) \end{aligned}$$

Equation error state can be written:

$$\begin{bmatrix} \dot{x}_e(t) \\ \dot{\xi}_e(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x_e(t) \\ \xi_e(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_e(t) \qquad (7)$$

with:

$$u_e(t) = -K x_e(t) + k_I \xi_e(t) \qquad (8)$$

Vector error size $(n + 1)$ can be defined into:

$$e(t) = \begin{bmatrix} x_e(t) \\ \xi_e(t) \end{bmatrix}$$

then Equation (7) into:

$$\dot{e} = \hat{A}e + \hat{B}u_e \qquad (9)$$

with:

$$\hat{A} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}, \hat{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

and Equation (8) into:

$$u_e = -\hat{K}e$$

with:

$$\hat{K} = \begin{bmatrix} K & -k_I \end{bmatrix}$$

so Equation (9) into:

$$\dot{e} = (\hat{A} - \hat{B}\hat{K})e$$

Value of $\hat{K}$ is found with LQR method and cost function of LQR is defined by:

$$J = \frac{1}{2} \int_0^\infty (e^T Q e + u_e^T R u_e) dt$$

and Riccati's equation is:

$$\hat{A}^T \hat{P} + \hat{P}\hat{A} + \hat{Q} - \hat{P}\hat{B}R^{-1}\hat{B}^T\hat{P} = 0$$

3. Design LQR Control System and Simulation

In this section contains simulation without disturbance and simulation with disturbance.

3.1. Simulation Without Disturbance

Simulation without disturbance represented ideal condition, there isn't influence wind when flight. This simulation was done with input signal step with value 0.2 rad. Weighting matrix Q_{long} and R_{long} in longitudinal motion of LSU-05 as follow:

$$Q_{long} = \begin{bmatrix} 0.01 & 0 & 0 & 0 & 0 \\ 0 & 0.15 & 0 & 0 & 0 \\ 0 & 0 & 50 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 1675 \end{bmatrix}, R_{long} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}$$

Gain control of longitudinal motion was gotten with Matlab:

$$K_{long} = \begin{bmatrix} -0.8354 & 1.4009 & -103.4526 & -7.1251 \\ 1.4098 & -0.0857 & -21.5873 & -1.4830 \end{bmatrix}$$

$$k_{I_{long}} = \begin{bmatrix} 399.9417 \\ 86.8715 \end{bmatrix}$$

whereas weighting matrix Q_{lat} and R_{lat} in lateral-directional motion are

$$Q_{lat} = \begin{bmatrix} 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0.01 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 0 & 1975 \end{bmatrix}, R_{lat} = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix}$$

Gain control for lateral-directional is

$$K_{lat} = \begin{bmatrix} 0.4373 & -0.0826 & 0.5153 & 4.3401 \\ -1.0495 & -0.6014 & 0.1653 & -49.6914 \end{bmatrix}$$

$$k_{I_{lat}} = \begin{bmatrix} -11.5172 \\ 198.4121 \end{bmatrix}$$

The result of pitch angle and roll angle previously and after controlled can be seen in figure 1 and figure 2
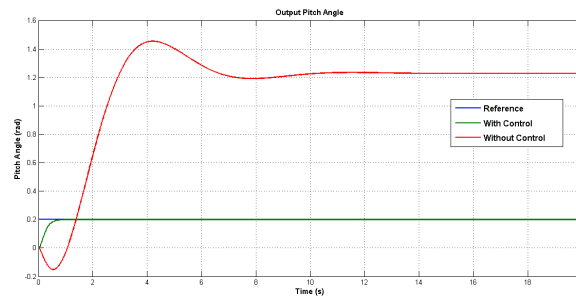

Figure 1. Output Pitch Angle with Input Signal Step.

Based on figure 1 dan figure 2 can be seen that output of pitch angle and roll angle previously and after controlled very different. Previously controlled, output of pitch angle and roll angle can't follow reference, nevertheless after controlled, output of pitch angle and roll angle can follow reference. The result of comparison transient response for output of pitch angle and roll angle previously and after controlled can be seen in Table 1.
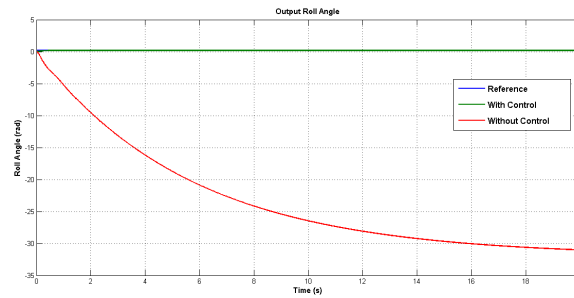
Figure 2. Output Roll Angle with Input Signal Step.

Table 1. Comparison of Pitch Angle and Roll Angle Without Control and With Control.

Spesification	Pitch Angle		Roll Angle	
	Without Control	With Control	Without Control	With Control
Delay time (t_d)	1.949 s	0.221 s	3.919 s	0.186 s
Settling time (t_s)	6.7 s	0.719 s	22.3 s	0.87 s
Rise time (t_r)	1.11 s	0.419 s	12.6 s	0.515 s
Maximum overshoot (M_p)	18.4%	0%	0%	0%
Peak time (t_p)	3.26 s	1.359 s	39.94 s	2.222 s

3.2. Simulation With Disturbance
Robustness simulation of controller is used to know strength of controller, how robustness controller can overcome disturbance that was received by system. This simulaton was done with disturbance that was appeared from external system, in this case, the disturbance is wind. The result of this simulation can be seen in figure 3 and figure 4.
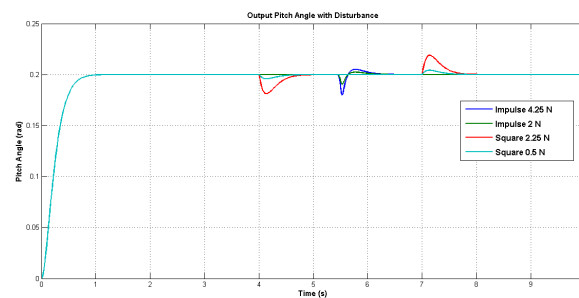

Figure 3. Output Pitch Angle with Disturbance.

Based on figure 3 and figure 4 can be seen when occured disturbance in the form of signal square on second into 4-7, overshoot was produced by output pitch angle consecutive for 9.4% and 2.1% for square disturbance each one of 2.25 N and 0.5 N, whereas output roll angle consecutive for 9.25% and 2.3% for square disturbance each one of 1 N and 0.25 N. Controller can return output in reference value in 0.75 s. Nevertheles, after disturbance was eliminated, output pitch angle and roll angle came back to overcame overshoot and then stable again.
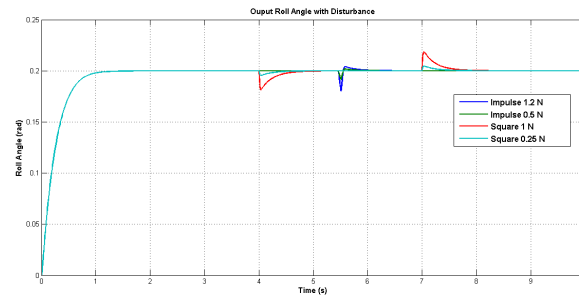
5

Figure 4. Output Roll Angle with Disturbance.

4. Conclusion
Based on the result of simulation in previous section, we can get a conclusion as follow:

(i) The result of LQR controller, there is obtained transient response for longitudinal motion, $t_d = 0.221s, t_r = 0.419s, t_s = 0.719s, t_p = 1.359s$, and $M_p = 0\%$. In other hand, transient response for lateral-directional motion showed that $t_d = 0.186s, t_r = 0.515s, t_s = 0.87s, t_p = 2.02s$, and $M_p = 0\%$.

(ii) In pitch angle Controller able to overcome external disturbance in the form of signal impulse until 4.25 N, signal square until 2.25 N, whereas in roll angle, controller able to overcome disturbance in the form of signal impulse until 1.2 N and signal square until 1 N.

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