DEVELOPMENT OF LQ TRACKER FOR INVERTED PENDULUM

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OVERVIEW

Objective

System model and state space representation

LQ tracker formulation

Simulation result

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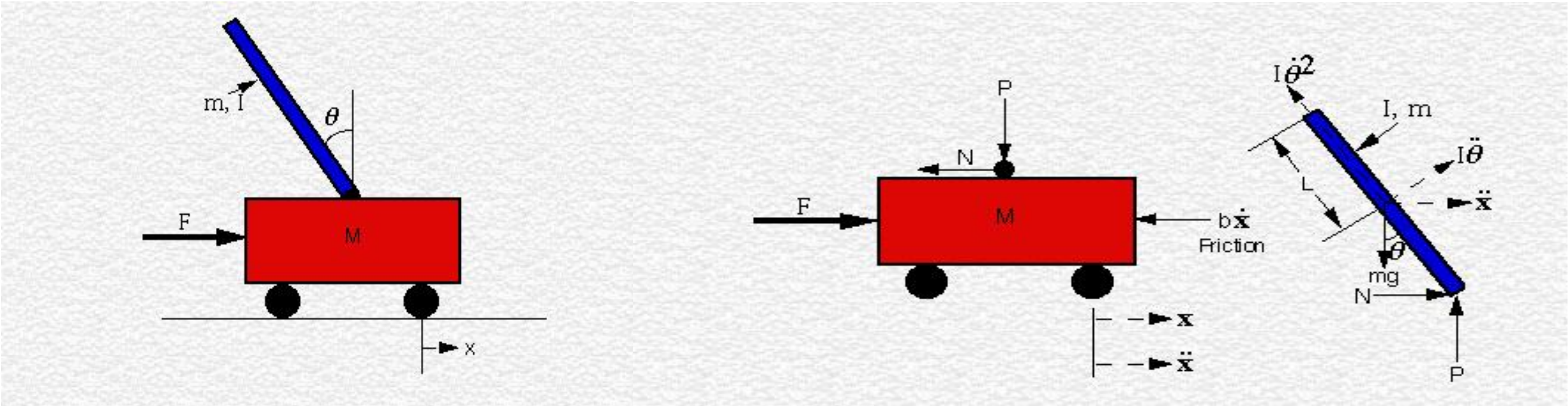
OBJECTIVE

To develop the LQ tracker for an LTI system.

To analyze the performance of the tracker.

To have some hands on experience on how to use LQ tracker to solve real life control problems.

SYSTEM MODEL OF INVERTED PENDULUM AND STATE SPACE REPRESENTATION



$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\Phi} \\ \ddot{\Phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-(I+ml^2)b}{I(M+m)+Mml^2} & \frac{m^2gl^2}{I(M+m)+Mml^2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-mlb}{I(M+m)+Mml^2} & \frac{mgl(M+m)}{I(M+m)+Mml^2} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \Phi \\ \dot{\Phi} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{I+ml^2}{I(M+m)+Mml^2} \\ 0 \\ \frac{ml}{I(M+m)+Mml^2} \end{bmatrix} u$$

$$y = \begin{bmatrix} & & & \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \Phi \\ \dot{\Phi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$$

Here, the system is linearized at

$$\theta = \frac{\pi}{2}, and\ \theta = \pi + \varphi, \varphi\ is\ a\ small\ angle$$

LQ TRACKER FORMULATION

System: $\dot{X} = AX + Bu \; and \; y = CX$

Cost function: $J = \frac{1}{2}\left(y(t_f) - r(t_f)\right)' P(t_f)\left(y(t_f) - r(t_f)\right) + \frac{1}{2}\int_{t_0}^{t_f}[(y-r)'Q(y-r) + u'Ru]dt$

where, $\quad P \geq 0, Q \geq 0, R > 0 \; and \; all \; are \; symmetric$

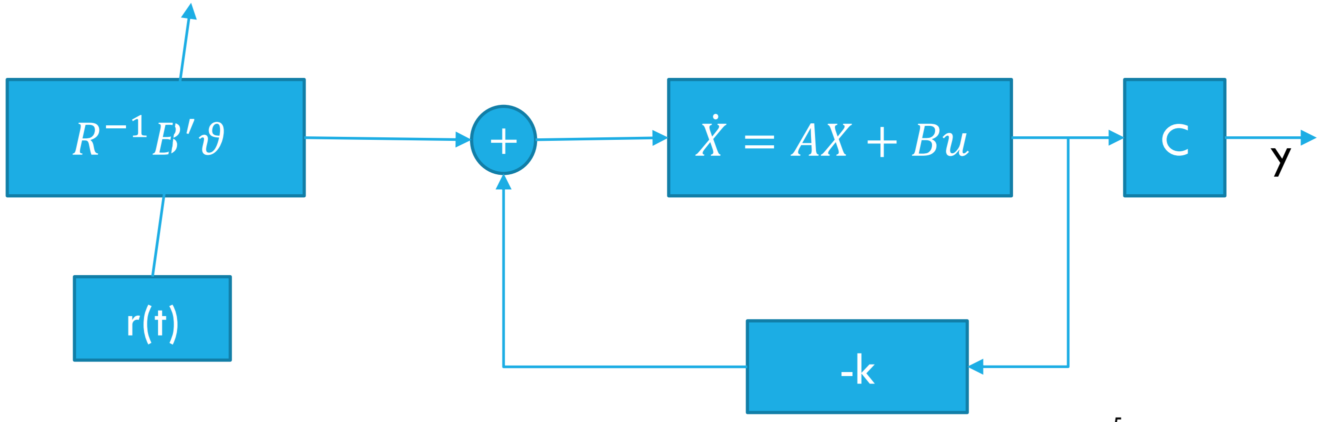
Solution: $u = -kx + R^{-1}B'\vartheta; where, k(t) = R^{-1}B'S(t)$

DREs: $-\dot{S} = A'S + SA - SBR^{-1}B'S + C'QC \; ; S(t_f) = C'P(t_f)C$

$$-\dot{\vartheta} = (A - Bk)'\vartheta + C'Qr \; ; \; \vartheta(t_f) = C'P(t_f)r(t_f)$$

$$r(t) \; is \; given \; over \; [t_0, t_f]$$

Assumptions: X is available.



Block diagram of the system with LQ tracker

SIMULATION

System parameters:

```
M = .5;
m = 0.2;
b = 0.1;
i = 0.006;
g = 9.8;
l = 0.3;
```

```
p = i*(M+m)+M*m*l^2; %denominator for the A and B matricies
A = [0      1              0           0;
     0 -(i+m*l^2)*b/p  (m^2*g*l^2)/p   0;
     0      0              0           1;
     0 -(m*l*b)/p       m*g*l*(M+m)/p  0];
B = [     0;
     (i+m*l^2)/p;
          0;
        m*l/p];
C = [0 0 1 0];
x0 = [0 0 0 0]';
```

```
t0 = 0;
tf = 20;
dt = 0.001;
t = t0:dt:tf;
Q = 2700;
R = 1;
ptf = 0;
F = 1.1;
```

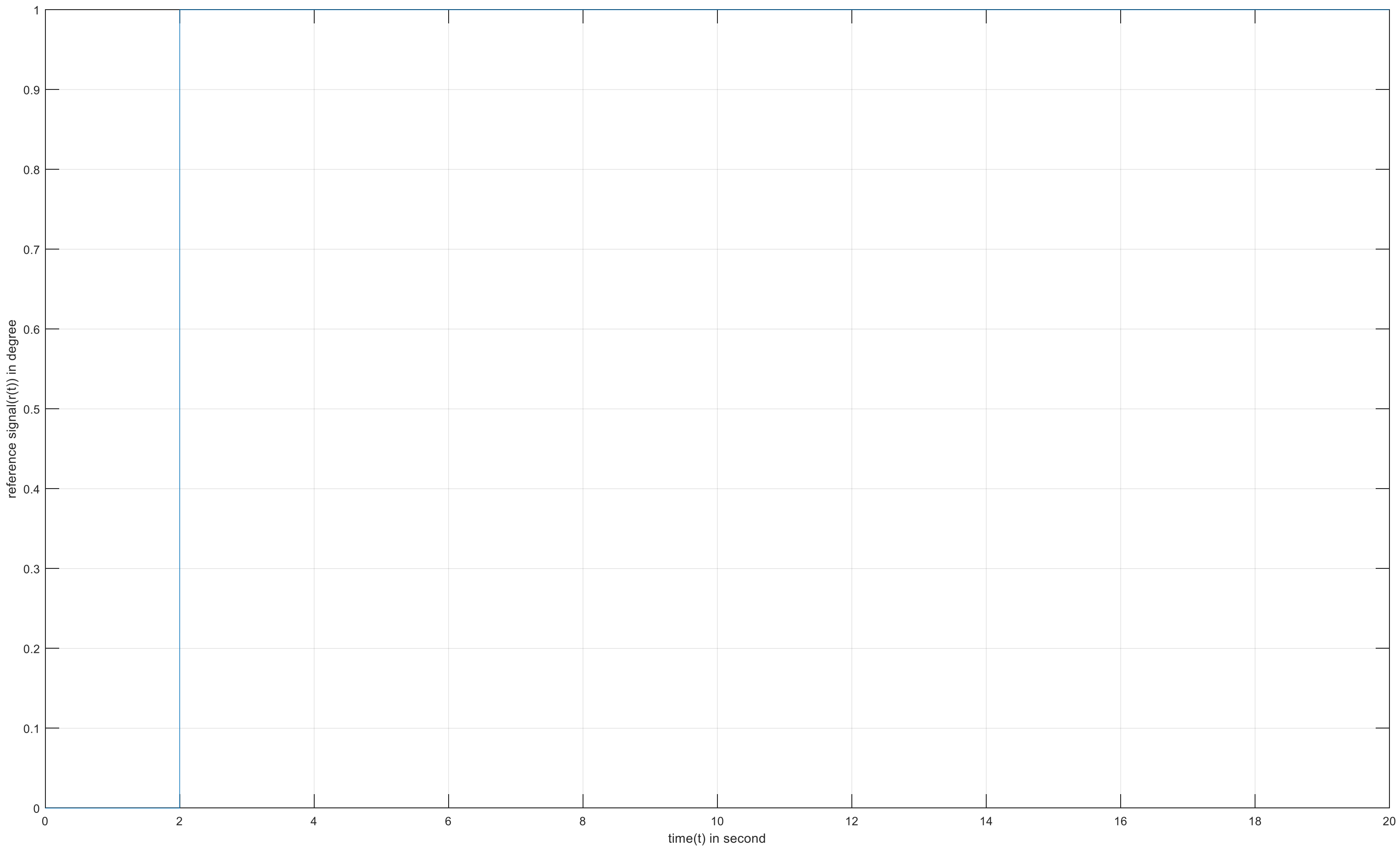
The two DREs are solved simultaneously using Runge–Kutta methods.

$$\dot{y} = f(t, y), \quad y(t_0) = y_0.$$

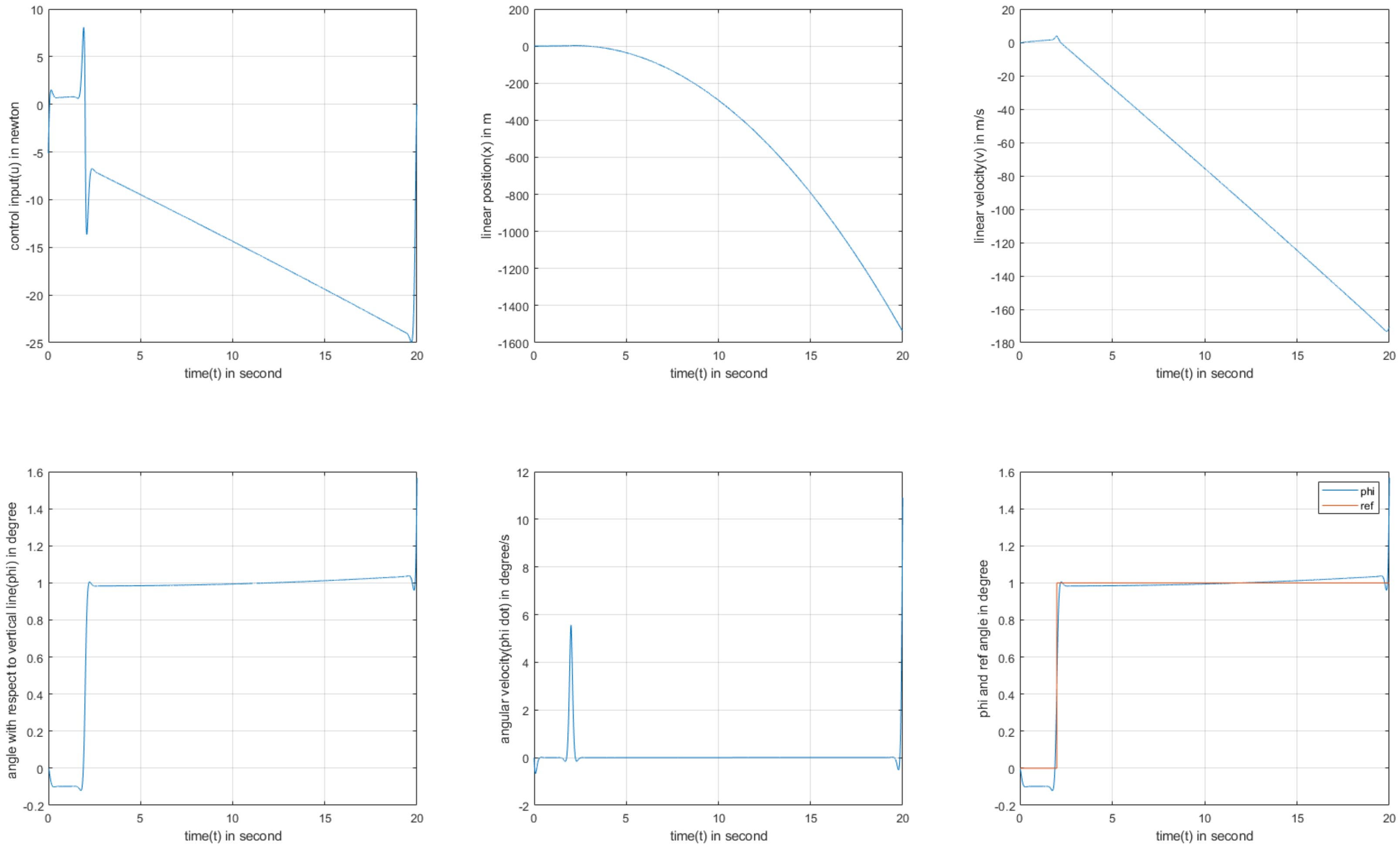
$$y_{n+1} = y_n + \tfrac{h}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right),$$
$$t_{n+1} = t_n + h$$

$$\begin{aligned}
k_1 &= f(t_n, y_n),\\
k_2 &= f\left(t_n + \frac{h}{2}, y_n + h\frac{k_1}{2}\right),\\
k_3 &= f\left(t_n + \frac{h}{2}, y_n + h\frac{k_2}{2}\right),\\
k_4 &= f\left(t_n + h, y_n + hk_3\right).
\end{aligned}$$

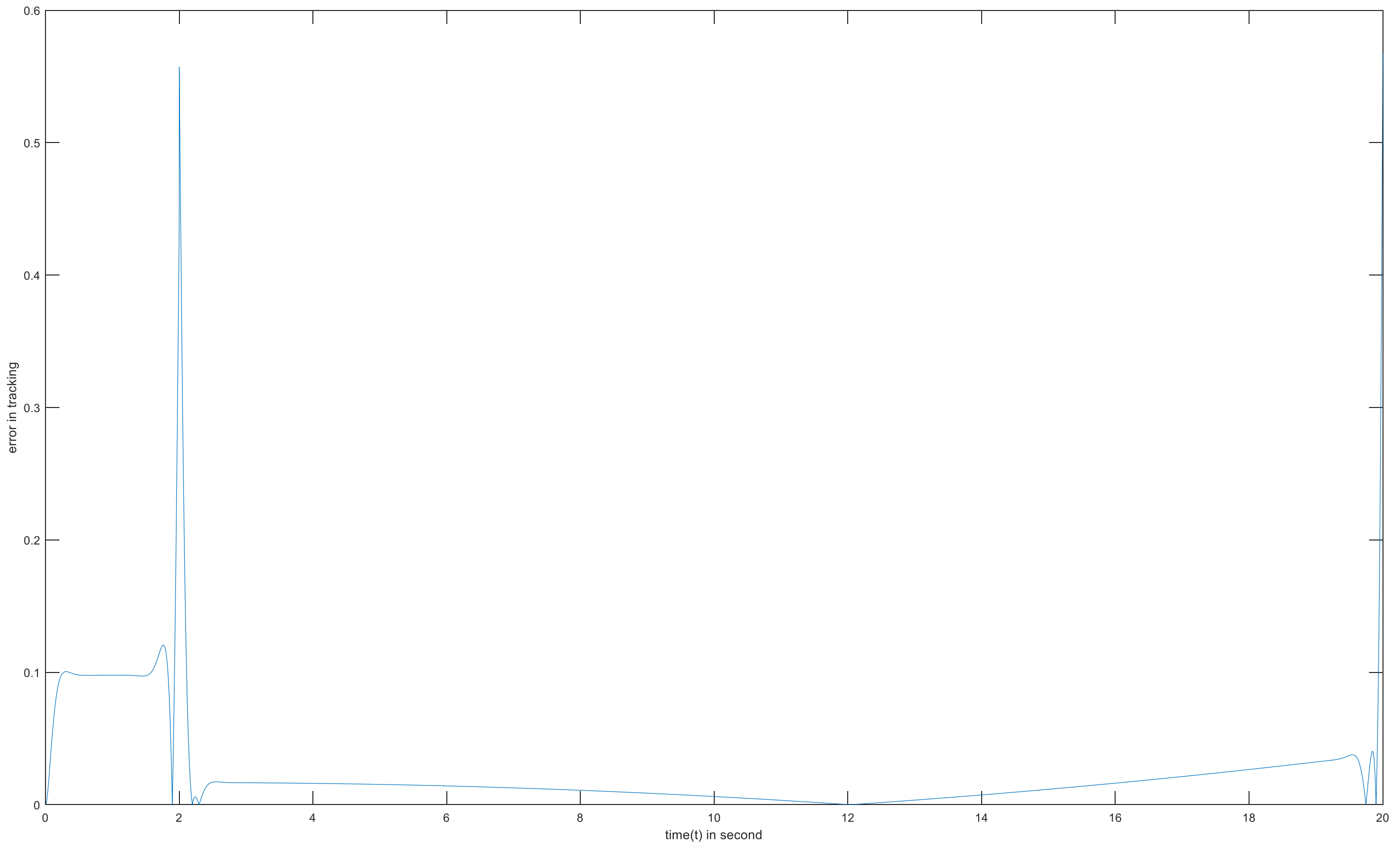
REFERENCE SIGNAL



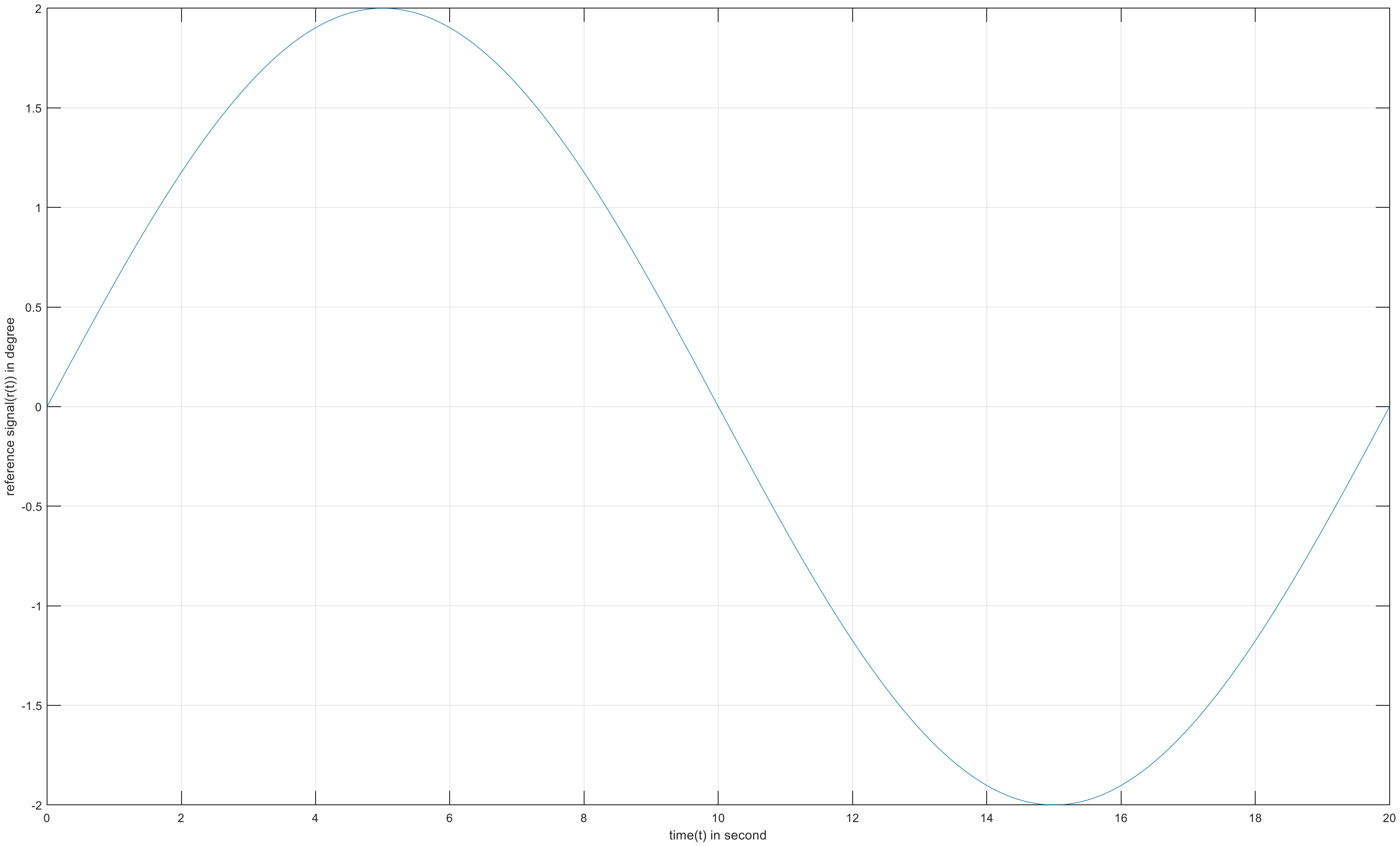
RESPONSE OF THE TRACKER



ABSOLUTE ERROR OF THE TRACKER



REFERENCE SIGNAL



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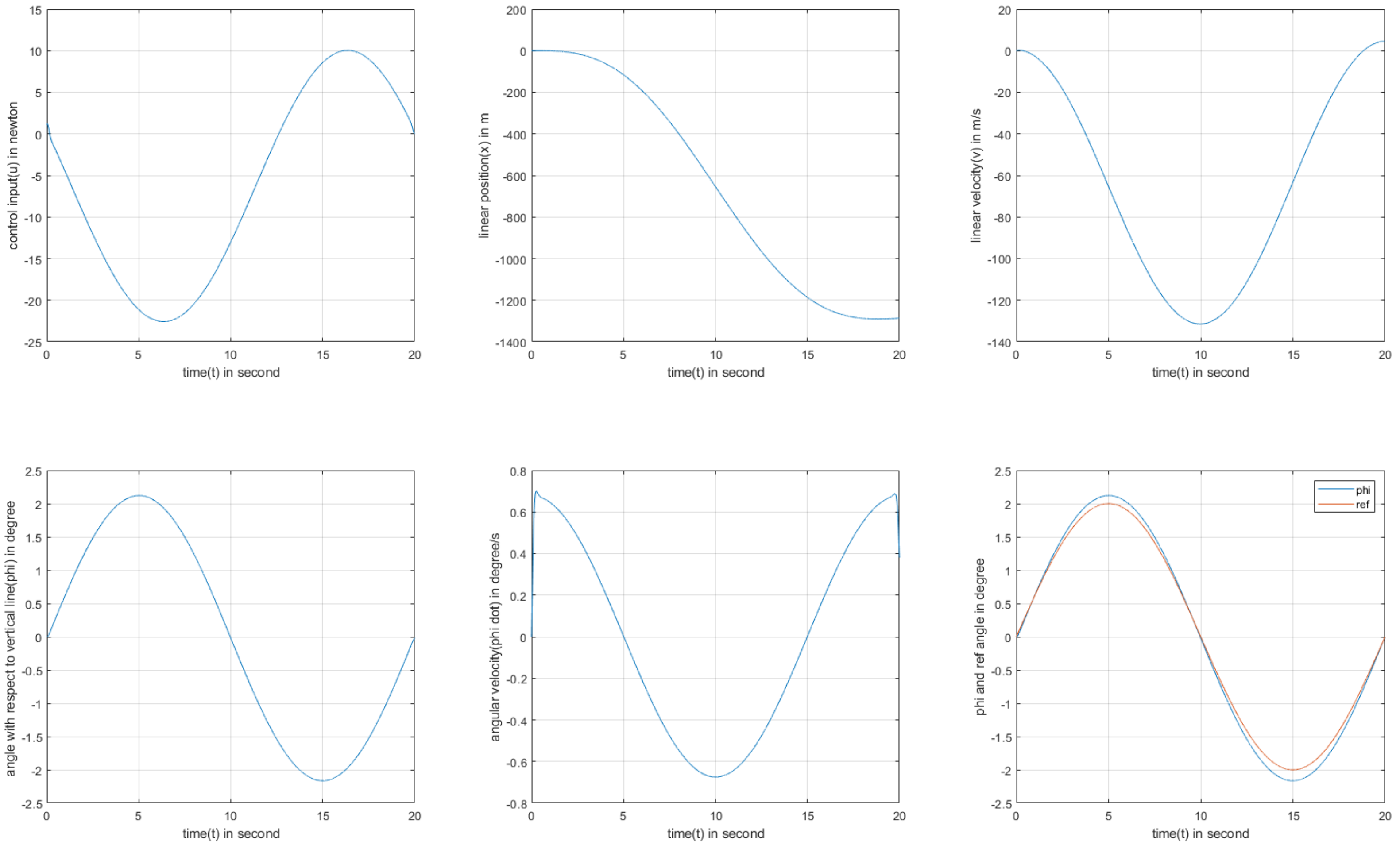

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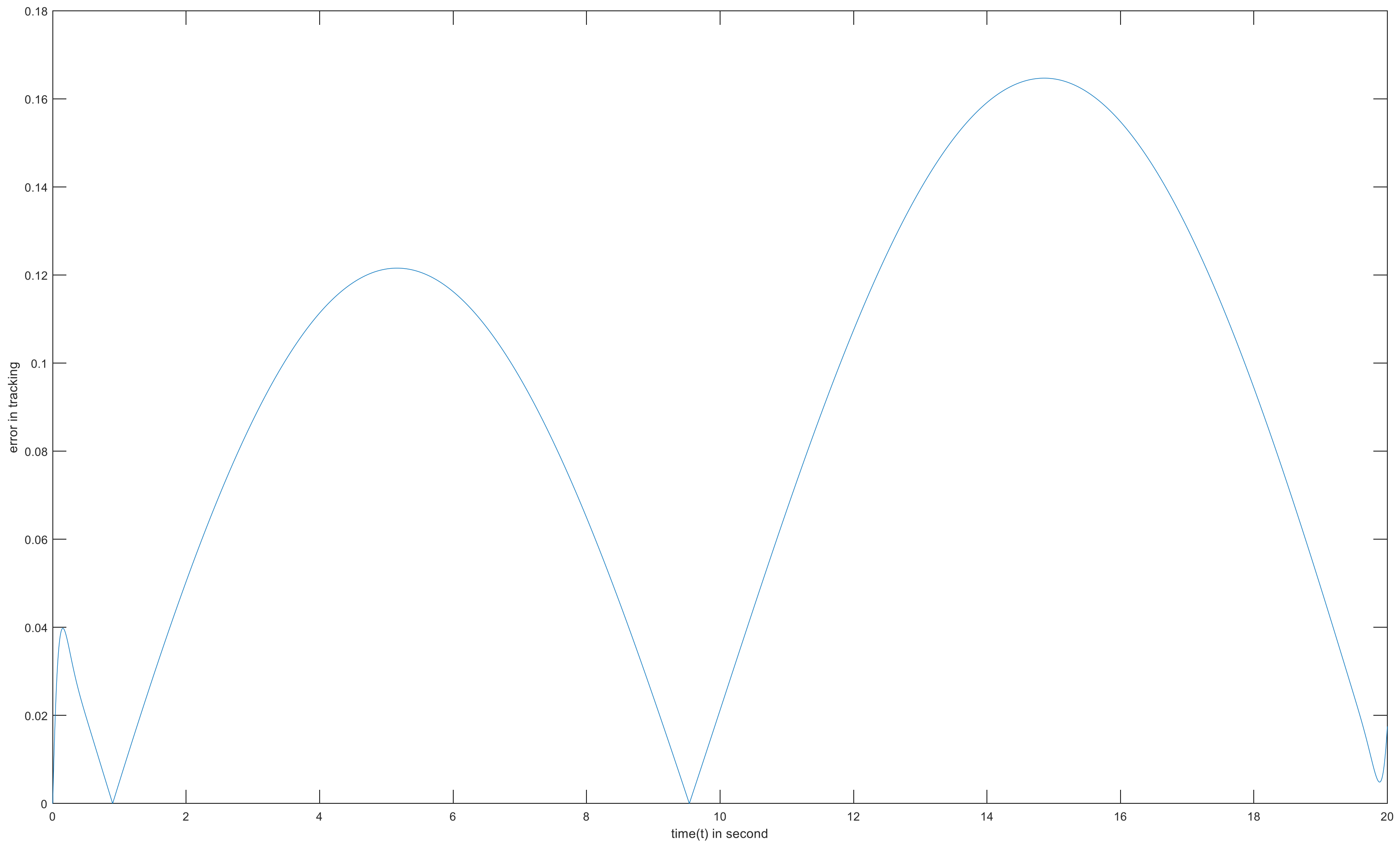

CONCLUSION

In this project LQ tracker is developed and simulated with two reference signal.

The two DRE is solved simultaneously using Runge–Kutta method.

The control input is kept in reasonable value.

The over all absolute error of the tracker is less than 0.1

REFERENCES

[1]https://www.ee.usyd.edu.au/tutorials_online/matlab/examples/pend/invpen.html

[2]https://en.wikipedia.org/wiki/Runge%E2%80%93Kutta_methods

[3]*ECEN865* Lecture Notes

THANK YOU