Chapter 1

Digital Design and Computer Architecture, 2nd Edition

David Money Harris and Sarah L. Harris



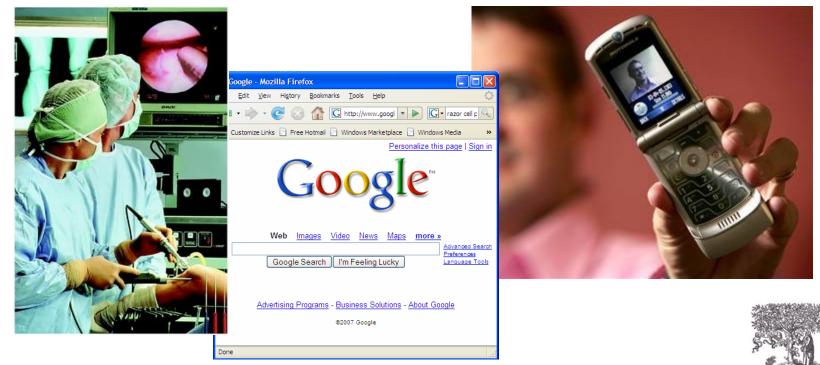
Chapter 1 :: Topics

- Background
- The Game Plan
- The Art of Managing Complexity
- The Digital Abstraction
- Number Systems
- Logic Gates
- Logic Levels
- CMOS Transistors
- Power Consumption



Background

- Microprocessors have revolutionized our world
 - Cell phones, Internet, rapid advances in medicine, etc.
- The semiconductor industry has grown from \$21 billion in 1985 to \$300 billion in 2011



The Game Plan

- Purpose of course:
 - Understand what's under the hood of a computer
 - Learn the principles of digital design
 - Learn to systematically debug increasingly complex designs
 - Design and build a microprocessor



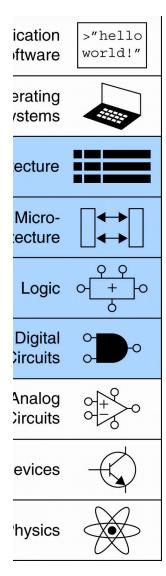
The Art of Managing Complexity

- Abstraction
- Discipline
- The Three -y's
 - Hierarchy
 - Modularity
 - Regularity



Abstraction

• Hiding details when they aren't important





Discipline

- Intentionally restrict design choices
- Example: Digital discipline
 - Discrete voltages instead of continuous
 - Simpler to design than analog circuits can build more sophisticated systems
 - Digital systems replacing analog predecessors:
 - i.e., digital cameras, digital television, cell phones, CDs



The Three -y's

- Hierarchy
- Modularity
- Regularity



The Three -y's

Hierarchy

A system divided into modules and submodules

Modularity

Having well-defined functions and interfaces

Regularity

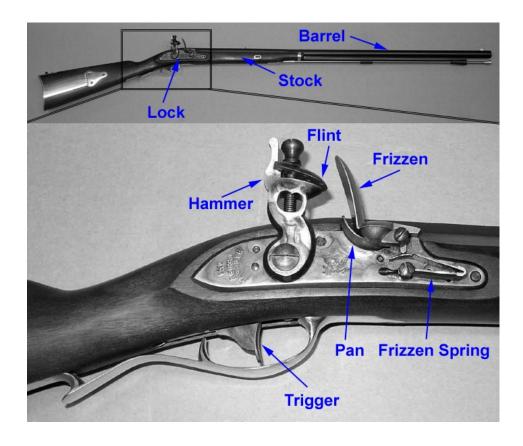
Encouraging uniformity, so modules can be easily reused



Example: The Flintlock Rifle

Hierarchy

- Three main modules: lock, stock, and barrel
- **Submodules of lock:** hammer, flint, frizzen, etc.





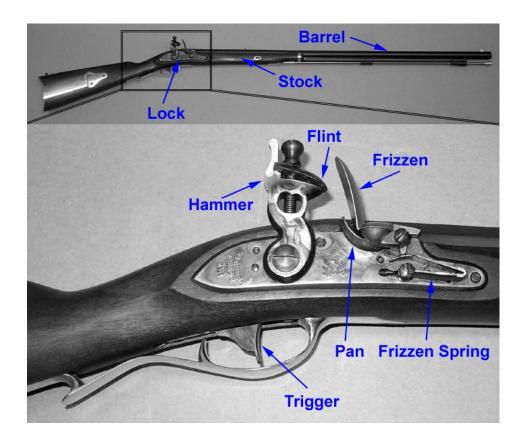
Example: The Flintlock Rifle

Modularity

- Function of stock: mount barrel and lock
- Interface of stock: length and location of mounting pins

Regularity

Interchangeable parts





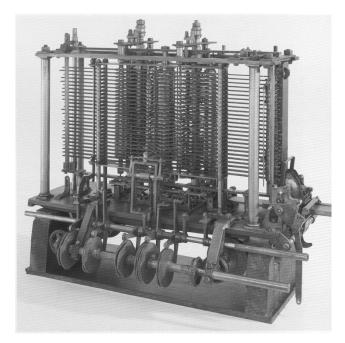
The Digital Abstraction

- Most physical variables are continuous
 - Voltage on a wire
 - Frequency of an oscillation
 - Position of a mass
- Digital abstraction considers discrete subset of values



The Analytical Engine

- Designed by Charles
 Babbage from 1834 1871
- Considered to be the first digital computer
- Built from mechanical gears, where each gear represented a discrete value (0-9)
- Babbage died before it was finished







Chapter 1 < 13>

Digital Discipline: Binary Values

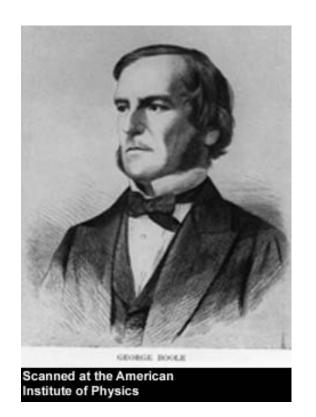
Two discrete values:

- 1's and 0's
- 1, TRUE, HIGH
- -0, FALSE, LOW
- 1 and 0: voltage levels, rotating gears, fluid levels, etc.
- Digital circuits use voltage levels to represent 1 and 0
- Bit: Binary digit



George Boole, 1815-1864

- Born to working class parents
- Taught himself mathematics and joined the faculty of Queen's College in Ireland
- Wrote An Investigation of the Laws of Thought (1854)
- Introduced binary variables
- Introduced the three fundamental logic operations: AND, OR, and NOT





Number Systems

Decimal numbers

Binary numbers



Number Systems

Decimal numbers

$$5374_{10} = 5 \times 10^3 + 3 \times 10^2 + 7 \times 10^1 + 4 \times 10^0$$
five three seven four thousands hundreds tens ones

Binary numbers

$$1101_{2} = 1 \times 2^{3} + 1 \times 2^{2} + 0 \times 2^{1} + 1 \times 2^{0} = 13_{10}$$
one
eight
one
four
two
one



ONE ZER **FROM**

Powers of Two

•
$$2^0 =$$

•
$$2^1 =$$

•
$$2^2 =$$

•
$$2^3 =$$

•
$$2^4 =$$

•
$$2^5 =$$

•
$$2^6 =$$

•
$$2^7 =$$

•
$$2^8 =$$

•
$$2^9 =$$

•
$$2^{10} =$$

•
$$2^{11} =$$

•
$$2^{12} =$$

•
$$2^{13} =$$

•
$$2^{14} =$$

•
$$2^{15} =$$



ONE 0 ZER DO FR

Powers of Two

•
$$2^0 = 1$$

•
$$2^1 = 2$$

•
$$2^2 = 4$$

•
$$2^3 = 8$$

•
$$2^4 = 16$$

•
$$2^5 = 32$$

•
$$2^6 = 64$$

•
$$2^7 = 128$$

•
$$2^8 = 256$$

•
$$2^9 = 512$$

•
$$2^{10} = 1024$$

•
$$2^{11} = 2048$$

•
$$2^{12} = 4096$$

•
$$2^{13} = 8192$$

•
$$2^{14} = 16384$$

•
$$2^{15} = 32768$$

• Handy to memorize up to 29



Number Conversion

- Decimal to binary conversion:
 - Convert 10011₂ to decimal

- Decimal to binary conversion:
 - Convert 47₁₀ to binary



Number Conversion

- Decimal to binary conversion:
 - Convert 10011₂ to decimal
 - $-16\times1+8\times0+4\times0+2\times1+1\times1=19_{10}$

- Decimal to binary conversion:
 - Convert 47₁₀ to binary
 - $-32\times1+16\times0+8\times1+4\times1+2\times1+1\times1=101111_{2}$



Binary Values and Range

- N-digit decimal number
 - How many values?
 - Range?
 - Example: 3-digit decimal number:

- N-bit binary number
 - How many values?
 - Range:
 - Example: 3-digit binary number:



Binary Values and Range

- N-digit decimal number
 - How many values? 10^N
 - Range? $[0, 10^{N} 1]$
 - Example: 3-digit decimal number:
 - 10³ = 1000 possible values
 - Range: [0, 999]
- N-bit binary number
 - How many values? 2N
 - Range: [0, 2^N 1]
 - Example: 3-digit binary number:
 - 2³ = 8 possible values
 - Range: $[0, 7] = [000_2 \text{ to } 111_2]$



Hexadecimal Numbers

Hex Digit	Decimal Equivalent	Binary Equivalent
0	0	
1	1	
2	2	
3	3	
4	4	
5	5	
6	6	
7	7	
8	8	
9	9	
A	10	
В	11	
С	12	
D	13	
Е	14	
F	15	



Hexadecimal Numbers

Hex Digit	Decimal Equivalent	Binary Equivalent
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
В	11	1011
С	12	1100
D	13	1101
Е	14	1110
F	15	1111



Hexadecimal Numbers

- Base 16
- Shorthand for binary



Hexadecimal to Binary Conversion

- Hexadecimal to binary conversion:
 - Convert 4AF₁₆ (also written 0x4AF) to binary

- Hexadecimal to decimal conversion:
 - Convert 0x4AF to decimal



Hexadecimal to Binary Conversion

- Hexadecimal to binary conversion:
 - Convert 4AF₁₆ (also written 0x4AF) to binary
 - **-** 0100 1010 1111₂

- Hexadecimal to decimal conversion:
 - Convert 4AF₁₆ to decimal
 - $16^2 \times 4 + 16^1 \times 10 + 16^0 \times 15 = 1199_{10}$



Bits, Bytes, Nibbles...

Bits

10010110
most least significant bit bit

Bytes & Nibbles

10010110 nibble

Bytes

CEBF9AD7

most least significant byte byte



Large Powers of Two

- $2^{10} = 1 \text{ kilo} \approx 1000 (1024)$
- $2^{20} = 1 \text{ mega } \approx 1 \text{ million } (1,048,576)$
- $2^{30} = 1$ giga ≈ 1 billion (1,073,741,824)



Estimating Powers of Two

• What is the value of 2^{24} ?

 How many values can a 32-bit variable represent?



Estimating Powers of Two

• What is the value of 2²⁴?

 How many values can a 32-bit variable represent?

$$2^2 \times 2^{30} \approx 4$$
 billion



Addition

Decimal

Binary



Addition

Decimal

Binary



Binary Addition Examples

Add the following
 4-bit binary
 numbers

Add the following
 4-bit binary
 numbers



Binary Addition Examples

Add the following
 4-bit binary
 numbers

Add the following
 4-bit binary
 numbers

Overflow!



Overflow

- Digital systems operate on a fixed number of bits
- Overflow: when result is too big to fit in the available number of bits
- See previous example of 11 + 6



Signed Binary Numbers

- Sign/Magnitude Numbers
- Two's Complement Numbers



Sign/Magnitude Numbers

- 1 sign bit, *N*-1 magnitude bits
- Sign bit is the most significant (left-most) bit
 - Positive number: sign bit = 0 $A: [a_{N-1}, a_{N-2}, \cdots a_2, a_1, a_0]$
 - Negative number: sign bit = 1

$$A = (-1)^{a_{n-1}} \sum_{i=0}^{n-2} a_i 2^i$$

• Example, 4-bit sign/mag representations of \pm 6:

• Range of an *N*-bit sign/magnitude number:



Sign/Magnitude Numbers

- 1 sign bit, *N*-1 magnitude bits
- Sign bit is the most significant (left-most) bit
 - Positive number: sign bit = 0 $A: [a_{N-1}, a_{N-2}, \cdots, a_2, a_1, a_0]$
 - Negative number: sign bit = 1

$$A = (-1)^{a_{n-1}} \sum_{i=0}^{n-2} a_i 2^i$$

• Example, 4-bit sign/mag representations of \pm 6:

$$+6 = 0110$$

$$-6 = 1110$$

• Range of an *N*-bit sign/magnitude number:

$$[-(2^{N-1}-1), 2^{N-1}-1]$$



Sign/Magnitude Numbers

- Problems:
 - Addition doesn't work, for example -6 + 6:

Two representations of $0 (\pm 0)$:

1000

0000



Two's Complement Numbers

- Don't have same problems as sign/magnitude numbers:
 - Addition works
 - Single representation for 0



Two's Complement Numbers

• Msb has value of -2^{N-1}

$$A = a_{n-1}(-2^{n-1}) + \sum_{i=0}^{n-2} a_i 2^i$$

- Most positive 4-bit number:
- Most negative 4-bit number:
- The most significant bit still indicates the sign (1 = negative, 0 = positive)
- Range of an *N*-bit two's comp number:



Two's Complement Numbers

• Msb has value of -2^{N-1}

$$A = a_{n-1}(-2^{n-1}) + \sum_{i=0}^{n-2} a_i 2^i$$

- Most positive 4-bit number: 0111
- Most negative 4-bit number: 1000
- The most significant bit still indicates the sign (1 = negative, 0 = positive)
- Range of an *N*-bit two's comp number:

$$[-(2^{N-1}), 2^{N-1}-1]$$



"Taking the Two's Complement"

- Flip the sign of a two's complement number
- Method:
 - 1. Invert the bits
 - 2. Add 1
- Example: Flip the sign of $3_{10} = 0011_2$



"Taking the Two's Complement"

- Flip the sign of a two's complement number
- Method:
 - 1. Invert the bits
 - 2. Add 1
- Example: Flip the sign of $3_{10} = 0011_2$
 - 1. 1100

$$\frac{2. + 1}{1101} = -3_{10}$$



Two's Complement Examples

• Take the two's complement of $6_{10} = 0110_2$

• What is the decimal value of 1001₂?



Two's Complement Examples

• Take the two's complement of $6_{10} = 0110_2$

```
1. 1001
2. + 1
1010_2 = -6_{10}
```

• What is the decimal value of the two's complement number 1001₂?

```
1. 0110
2. + 1
0111_2 = 7_{10}, so 1001_2 = -7_{10}
```



Two's Complement Addition

• Add 6 + (-6) using two's complement numbers

• Add -2 + 3 using two's complement numbers



Two's Complement Addition

Add 6 + (-6) using two's complement numbers
 111
 0110
 + 1010

• Add -2 + 3 using two's complement numbers 111



Increasing Bit Width

- Extend number from N to M bits (M > N):
 - Sign-extension
 - Zero-extension



Sign-Extension

- Sign bit copied to msb's
- Number value is same

• Example 1:

- 4-bit representation of 3 = 0011
- 8-bit sign-extended value: 00000011

Example 2:

- 4-bit representation of -5 = 1011
- 8-bit sign-extended value: 11111011



Zero-Extension

- Zeros copied to msb's
- Value changes for negative numbers

Example 1:

– 4-bit value =

$$0011_2 = 3_{10}$$

- 8-bit zero-extended value: $00000011 = 3_{10}$

Example 2:

- 4-bit value =

$$1011 = -5_{10}$$

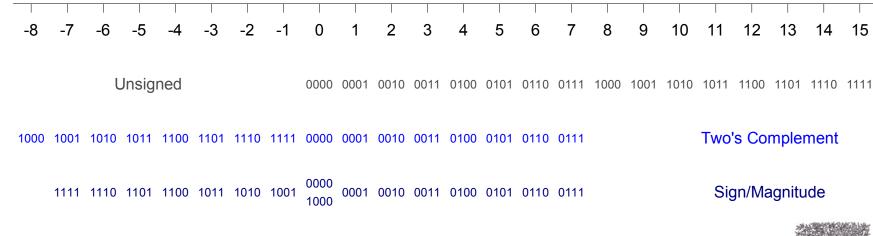
- 8-bit zero-extended value: $00001011 = 11_{10}$



Number System Comparison

Number System	Range
Unsigned	$[0, 2^N-1]$
Sign/Magnitude	$[-(2^{N-1}-1), 2^{N-1}-1]$
Two's Complement	$[-2^{N-1}, 2^{N-1}-1]$

For example, 4-bit representation:





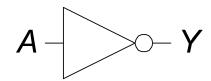
Logic Gates

- Perform logic functions:
 - inversion (NOT), AND, OR, NAND, NOR, etc.
- Single-input:
 - NOT gate, buffer
- Two-input:
 - AND, OR, XOR, NAND, NOR, XNOR
- Multiple-input

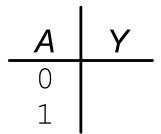


Single-Input Logic Gates

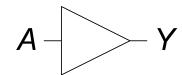
NOT



$$Y = \overline{A}$$



BUF



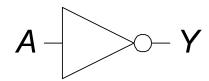
$$Y = A$$

Α	Y
0	
1	

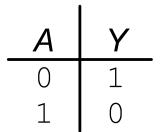


Single-Input Logic Gates

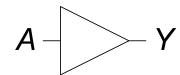
NOT



$$Y = \overline{A}$$



BUF



$$Y = A$$

Α	Y
0	0
1	1



Two-Input Logic Gates

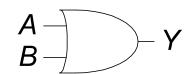
AND



$$Y = AB$$

Α	В	Y
0	0	
0	1	
1	0	
1	1	

OR



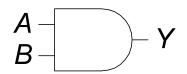
$$Y = A + B$$

Α	В	Y
0	0	
0	1	
1	0	
1	1	



Two-Input Logic Gates

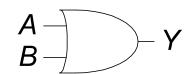
AND



$$Y = AB$$

A	В	Y
0	0	0
0	1	0
1	0	0
1	1	1

OR



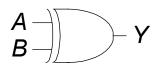
$$Y = A + B$$

A	В	Y
0	0	0
0	1	1
1	0	1
1	1	1

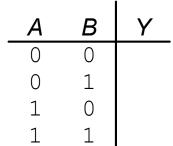


More Two-Input Logic Gates

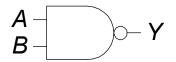
XOR



$$Y = A \oplus B$$



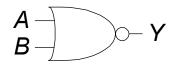
NAND



$$Y = \overline{AB}$$

Α	В	Y
0	0	
0	1	
1	0	
1	1	

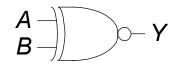
NOR



$$Y = \overline{A + B}$$

Α	В	Y
0	0	
0	1	
1	0	
1	1	

XNOR



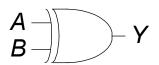
$$Y = \overline{A + B}$$

Α	В	Y
0	0	
0	1	
1	0	
1	1	

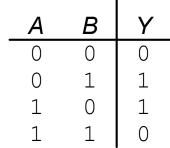


More Two-Input Logic Gates

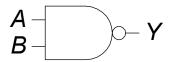
XOR



$$Y = A \oplus B$$



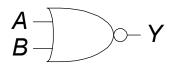
NAND



$$Y = \overline{AB}$$

Α	В	Y
0	0	1
0	1	1
1	0	1
1	1	0

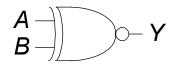
NOR



$$Y = \overline{A + B}$$

A	В	Y
0	0	1
0	1	0
1	0	0
1	1	0

XNOR



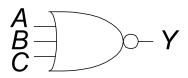
$$Y = \overline{A + B}$$

Α	В	Y
0	0	1
0	1	0
1	0	0
1	1	1



Multiple-Input Logic Gates

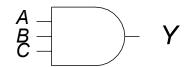
NOR3



$$Y = \overline{A + B + C}$$

A	В	С	Y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

AND3



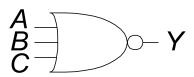
$$Y = ABC$$

A	В	С	Y
0	0	0	
0	0	1	
0	1	0	
Ο	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	



Multiple-Input Logic Gates

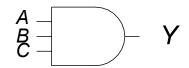
NOR3



$$Y = \overline{A + B + C}$$

Α	В	С	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

AND3



$$Y = ABC$$

A	В	С	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

Multi-input XOR: Odd parity



Logic Levels

- Discrete voltages represent 1 and 0
- For example:
 - -0 = ground (GND) or 0 volts
 - $-1 = V_{DD}$ or 5 volts
- What about 4.99 volts? Is that a 0 or a 1?
- What about 3.2 volts?



Logic Levels

- Range of voltages for 1 and 0
- Different ranges for inputs and outputs to allow for noise

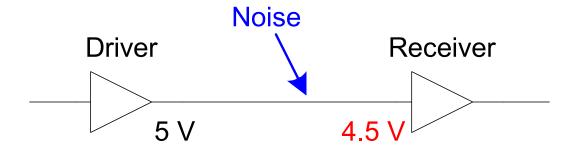


What is Noise?



What is Noise?

- Anything that degrades the signal
 - E.g., resistance, power supply noise, coupling to neighboring wires, etc.
- Example: a gate (driver) outputs 5 V but, because of resistance in a long wire, receiver gets 4.5 V





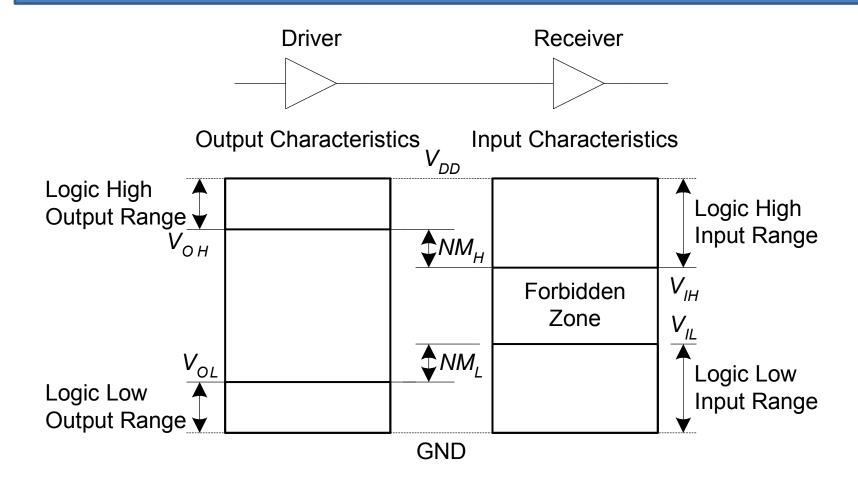
The Static Discipline

 With logically valid inputs, every circuit element must produce logically valid outputs

 Use limited ranges of voltages to represent discrete values

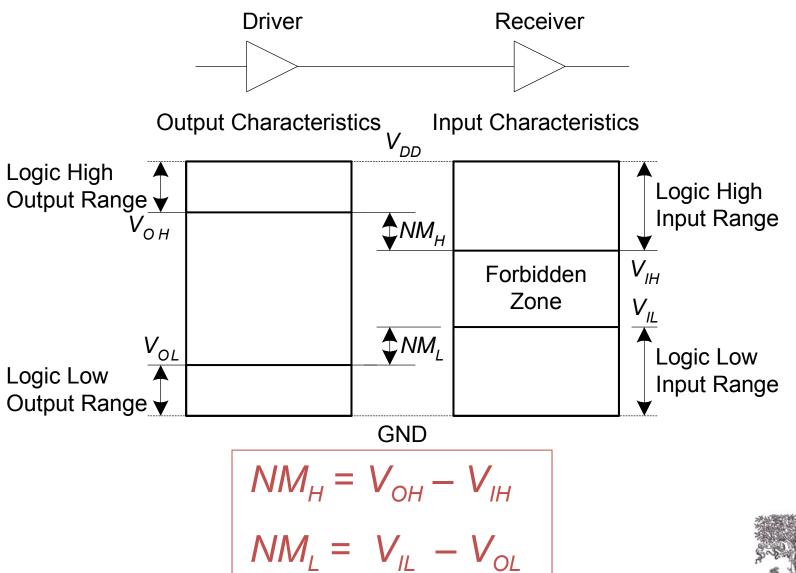


Logic Levels





Noise Margins

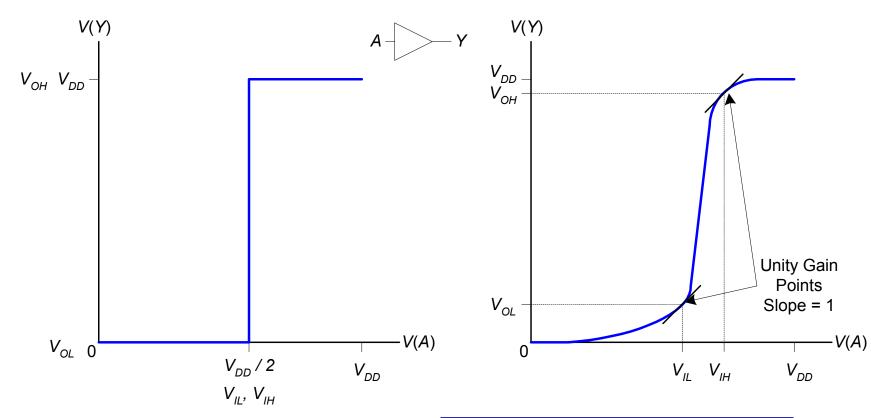




DC Transfer Characteristics

Ideal Buffer:

Real Buffer:

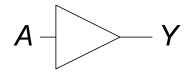


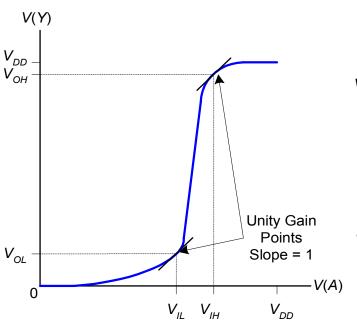
$$NM_H = NM_L = V_{DD}/2$$

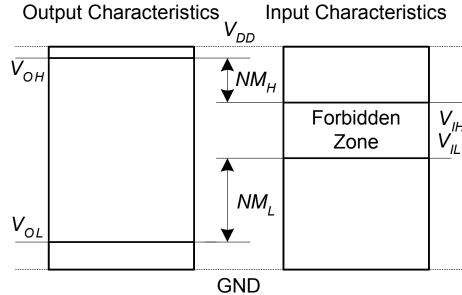
 NM_H , $NM_L < V_{DD}/2$



DC Transfer Characteristics









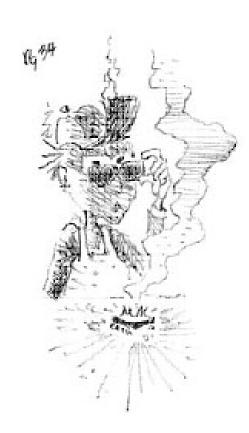
V_{DD} Scaling

- In 1970's and 1980's, $V_{DD} = 5 \text{ V}$
- V_{DD} has dropped
 - Avoid frying tiny transistors
 - Save power
- 3.3 V, 2.5 V, 1.8 V, 1.5 V, 1.2 V, 1.0 V, ...
- Be careful connecting chips with different supply voltages

Chips operate because they contain magic smoke

Proof:

 if the magic smoke is let out, the chip stops working





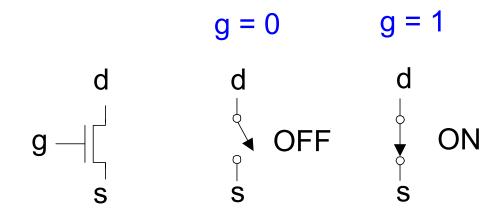
Logic Family Examples

Logic Family	V_{DD}	$V_{I\!L}$	V_{IH}	V_{oL}	V_{OH}
TTL	5 (4.75 - 5.25)	0.8	2.0	0.4	2.4
CMOS	5 (4.5 - 6)	1.35	3.15	0.33	3.84
LVTTL	3.3 (3 - 3.6)	0.8	2.0	0.4	2.4
LVCMOS	3.3 (3 - 3.6)	0.9	1.8	0.36	2.7



Transistors

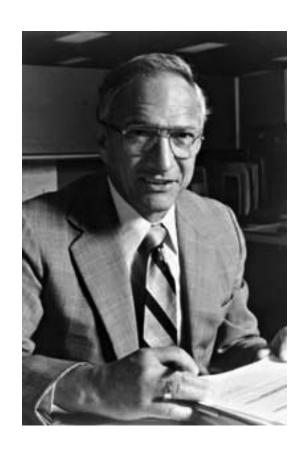
- Logic gates built from transistors
- 3-ported voltage-controlled switch
 - 2 ports connected depending on voltage of 3rd
 - d and s are connected (ON) when g is 1





Robert Noyce, 1927-1990

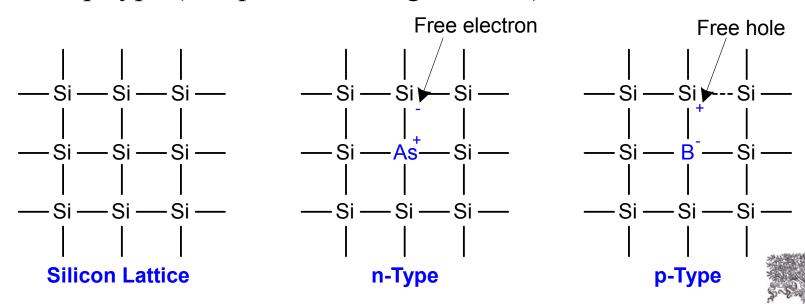
- Nicknamed "Mayor of Silicon Valley"
- Cofounded Fairchild Semiconductor in 1957
- Cofounded Intel in 1968
- Co-invented the integrated circuit





Silicon

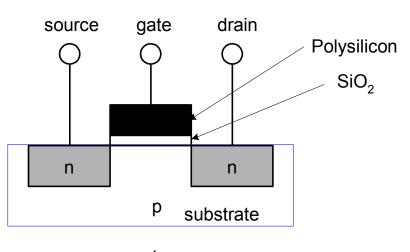
- Transistors built from silicon, a semiconductor
- Pure silicon is a poor conductor (no free charges)
- Doped silicon is a good conductor (free charges)
 - n-type (free negative charges, electrons)
 - p-type (free positive charges, holes)

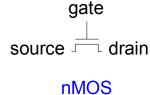


MOS Transistors

Metal oxide silicon (MOS) transistors:

- Polysilicon (used to be **metal**) gate
- Oxide (silicon dioxide) insulator
- Doped silicon







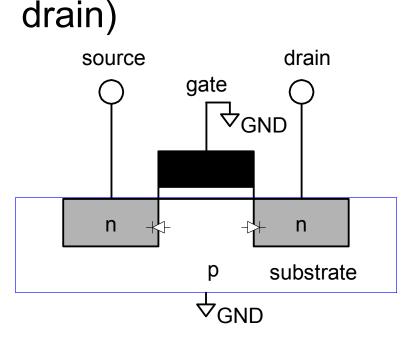
Transistors: nMOS

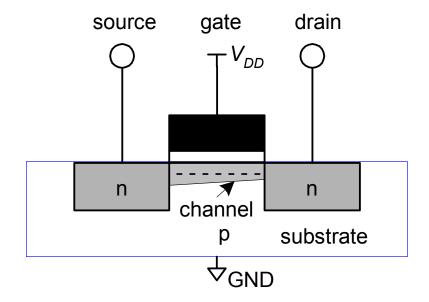
Gate = 0

OFF (no connection between source and

Gate = 1

ON (channel between source and drain)

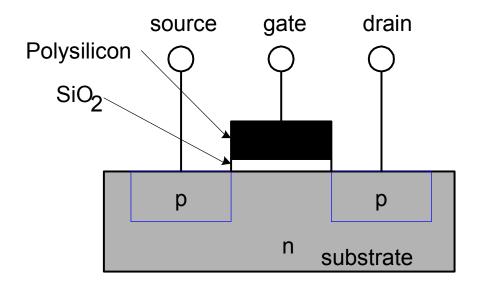


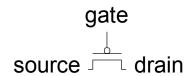




Transistors: pMOS

- pMOS transistor is opposite
 - ON when Gate = 0
 - OFF when Gate = 1



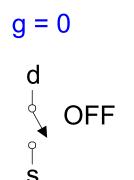


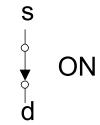


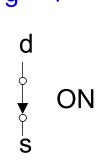
Transistor Function

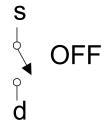
nMOS

pMOS





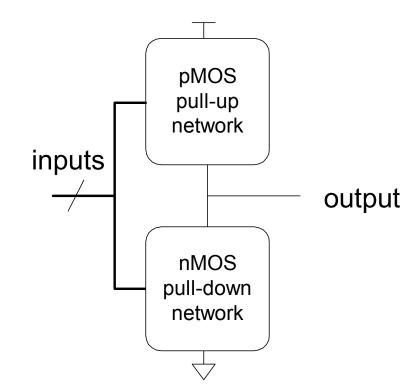






Transistor Function

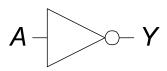
- nMOS: pass good 0's, so connect source to GND
- pMOS: pass good 1's, so connect source to



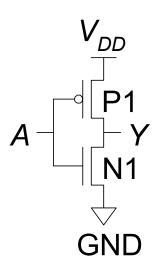


CMOS Gates: NOT Gate

NOT



$$Y = \overline{A}$$

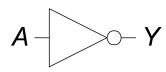


A	P1	N1	Y
0			
1			

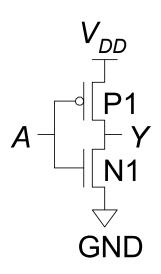


CMOS Gates: NOT Gate

NOT



$$Y = \overline{A}$$

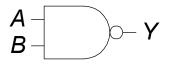


A	P1	N1	Y
0	ON	OFF	1
1	OFF	ON	0



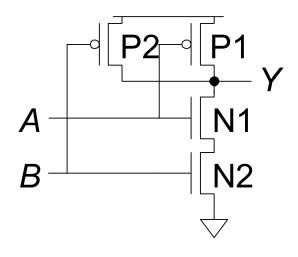
CMOS Gates: NAND Gate

NAND



$$Y = \overline{AB}$$

Α	В	Y
0	0	1
0	1	1
1	0	1
1	1	0

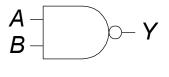


\overline{A}	B	P1	P2	N1	N2	Y
0	0					
0	1					
1	0					
1	1					



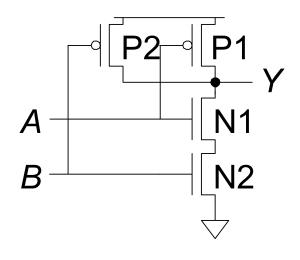
CMOS Gates: NAND Gate

NAND



$$Y = \overline{AB}$$

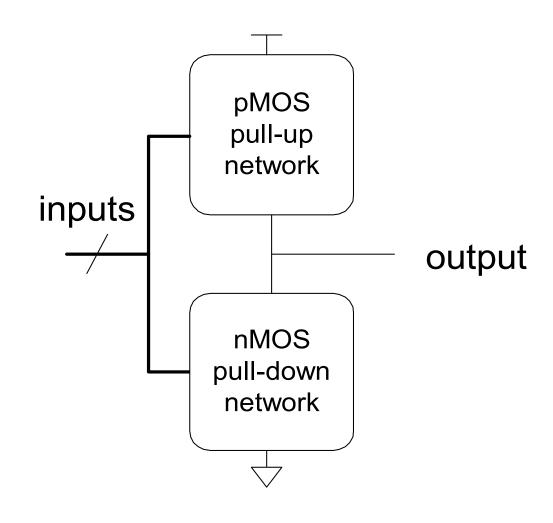
Α	В	Υ
0	0	1
0	1	1
1	0	1
1	1	0



\overline{A}	B	P1	P2	N1	N2	Y
0	0	ON	ON	OFF	OFF	1
0	1	ON	OFF	OFF	ON	1
1	0	OFF	ON	ON	OFF	1
1	1	OFF	OFF	ON	ON	0



CMOS Gate Structure



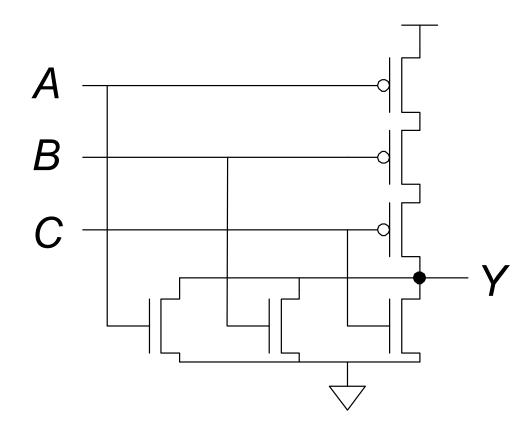


NOR Gate

How do you build a three-input NOR gate?



NOR3 Gate



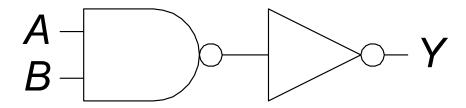


Other CMOS Gates

How do you build a two-input AND gate?



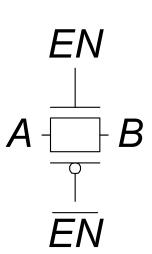
AND2 Gate





Transmission Gates

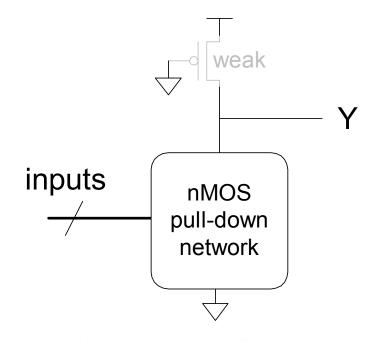
- nMOS pass 1's poorly
- pMOS pass 0's poorly
- Transmission gate is a better switch
 - passes both 0 and 1 well
- When EN = 1, the switch is ON:
 - -EN = 0 and A is connected to B
- When *EN* = 0, the switch is OFF:
 - A is not connected to B





Pseudo-nMOS Gates

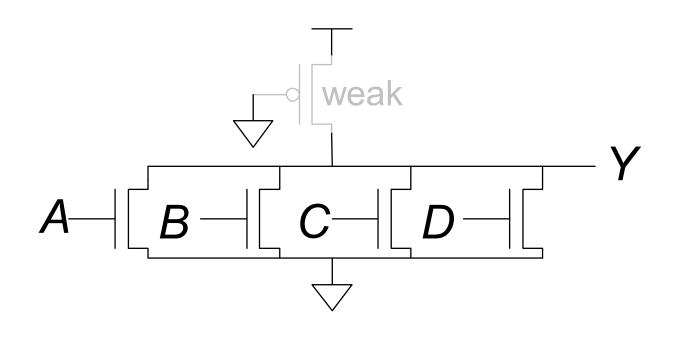
- Replace pull-up network with weak pMOS transistor that is always on
- pMOS transistor: pulls output HIGH only when nMOS network not pulling it LOW





Pseudo-nMOS Example

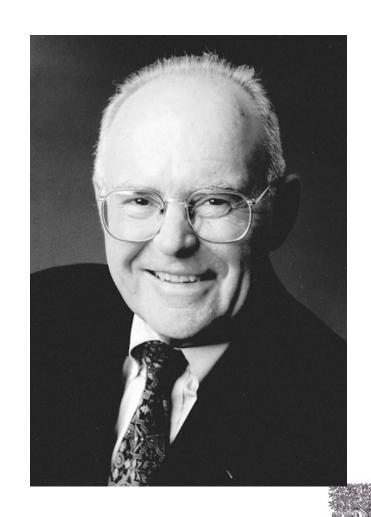
Pseudo-nMOS NOR4



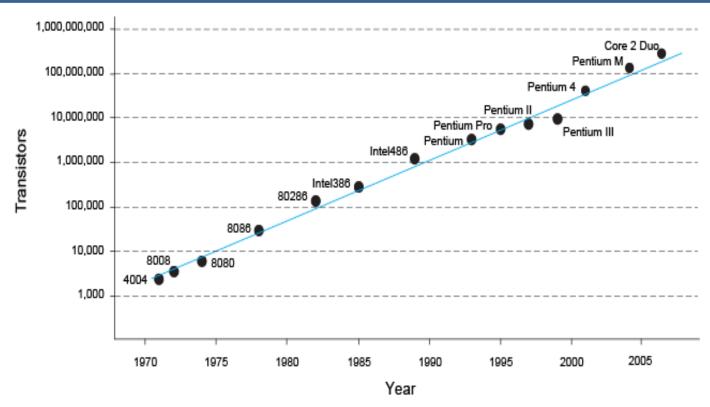


Gordon Moore, 1929-

- Cofounded Intel in 1968 with Robert Noyce.
- Moore's Law: number of transistors on a computer chip doubles every year (observed in 1965)
- Since 1975, transistor counts have doubled every two years.



Moore's Law



- "If the automobile had followed the same development cycle as the computer, a Rolls-Royce would today cost \$100, get one million miles to the gallon, and explode once a year . . ."
 - Robert Cringley



Power Consumption

- Power = Energy consumed per unit time
 - Dynamic power consumption
 - Static power consumption



Dynamic Power Consumption

- Power to charge transistor gate capacitances
 - Energy required to charge a capacitance, C, to V_{DD} is CV_{DD}^2
 - Circuit running at frequency f: transistors switch
 (from 1 to 0 or vice versa) at that frequency
 - Capacitor is charged f/2 times per second (discharging from 1 to 0 is free)
- Dynamic power consumption:

$$P_{dynamic} = \frac{1}{2}CV_{DD}^2f$$



Static Power Consumption

- Power consumed when no gates are switching
- Caused by the quiescent supply current, I_{DD} (also called the leakage current)
- Static power consumption:

$$P_{static} = I_{DD}V_{DD}$$



Power Consumption Example

Estimate the power consumption of a wireless handheld computer

$$-V_{DD} = 1.2 \text{ V}$$

$$-C = 20 \text{ nF}$$

$$-f = 1 \text{ GHz}$$

$$-I_{DD} = 20 \text{ mA}$$



Power Consumption Example

 Estimate the power consumption of a wireless handheld computer

$$-V_{DD} = 1.2 \text{ V}$$

$$-C = 20 \text{ nF}$$

$$-f = 1 \text{ GHz}$$

$$-I_{DD} = 20 \text{ mA}$$

$$P = \frac{1}{2}CV_{DD}^{2}f + I_{DD}V_{DD}$$

$$= \frac{1}{2}(20 \text{ nF})(1.2 \text{ V})^{2}(1 \text{ GHz}) + (20 \text{ mA})(1.2 \text{ V})$$

$$= (14.4 + 0.024) \text{ W} \approx 14.4 \text{ W}$$

