BART Webinar #2: Reproducing CG-SENSE

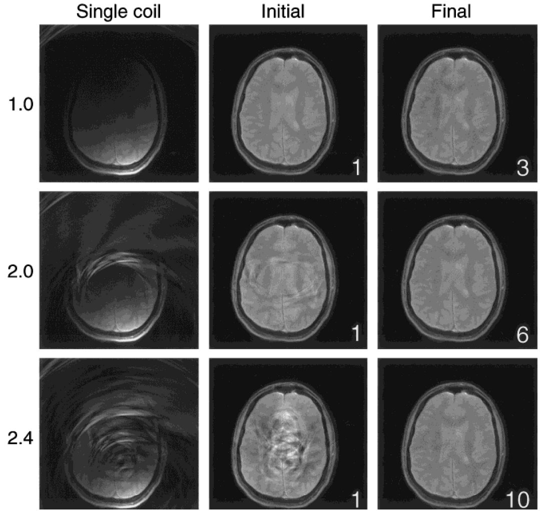
2020-12-02

Intro

- Aim: Reproducing Pruessmann et al. "Advances in Sensitivity Encoding With Arbitrary *k*-Space Trajectories", MRM 2001.
- Reproducing this paper was a challenge of the ISMRM Reproducible Research Study Group in 2019, leading to Maier et al. "CG-SENSE revisited: Results from the first ISMRM reproducibility challenge", MRM 2020.
- This challenge produced many different attempted replications
- We will focus on a reference solution by Oliver Maier

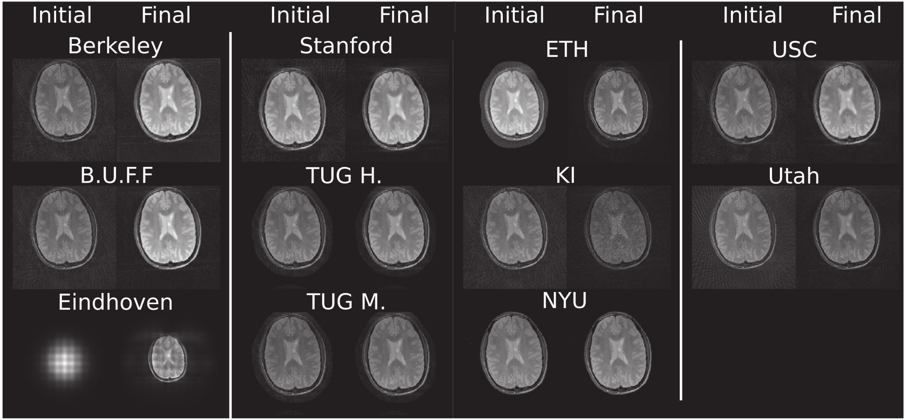
Intro

Pruessmann et al. 2001:



BART

Different replications (Maier et al. MRM 2020):



The Challenge

- Pruessmann et al. describes an iterative reconstruction, using the Conjugate Gradient method, solving the SENSE problem (so parallel imaging in image space) for non-Cartesian data
- This can be written as an optimization problem

$$\mathbf{v}^* = \arg\min_{\mathbf{v}} \|\mathbf{E}\mathbf{v} - \mathbf{m}\|_2^2 + \frac{\lambda}{2}\|\mathbf{v}\|_2^2$$

with image **v**, encoding matrix (including coil sensitivity profiles) **E**, measured multi-channel data **m** and Tikhonov regularization factor λ.

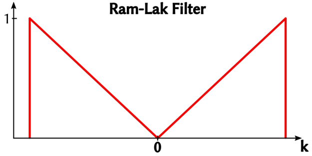
- The Conjugate Gradient method can be applied to the normal equation

$$(\mathbf{E}^H\mathbf{E} + \lambda\mathbf{I})\mathbf{v} = \mathbf{E}^H\mathbf{m}$$

The Challenge

- $$(\mathbf{E}^H\mathbf{E} + \lambda\mathbf{I})\mathbf{v} = \mathbf{E}^H\mathbf{m}$$
- For faster convergence, a density correction $\mathbf{D}$ is introduced
- Weighs each k-space sample with its spatial density $\Rightarrow$ for radial sampling, we use a Ram-Lak Filter
- This leads to the normal equation:

$$\begin{aligned} & (\mathbf{E}^H\mathbf{D}\mathbf{E} + \lambda\mathbf{I})\mathbf{v} = \mathbf{E}^H\mathbf{D}\mathbf{m} \\ \Leftrightarrow \quad & (\mathbf{E}^H\mathbf{D}^{\frac{1}{2}}\mathbf{D}^{\frac{1}{2}}\mathbf{E} + \lambda\mathbf{I})\mathbf{v} = \mathbf{E}^H\mathbf{D}^{\frac{1}{2}}\mathbf{D}^{\frac{1}{2}}\mathbf{m} \\ \Leftrightarrow \quad & (\overline{\mathbf{E}}^H\overline{\mathbf{E}} + \lambda\mathbf{I})\mathbf{v} = \overline{\mathbf{E}}^H\mathbf{D}^{\frac{1}{2}}\mathbf{m}, \text{ with } \overline{\mathbf{E}} = \mathbf{D}^{\frac{1}{2}}\mathbf{E} \end{aligned}$$



- (Pruessmann's paper also includes an intensity correction and a k-space filter, which we will exclude here)

The Challenge

- In the original paper, both radial and spiral sampling are used
- Only radial sampling for the challenge
- Datasets were provided (similar to the original data)
- With that, implement an algorithm and reproduce the main results of the paper

The Challenge

- So the following is our task:
  1. Convert data to BART's cfl format and to BART's conventions
  2. Undersample to the desired degree (1, 2, 3 and 4)
  3. Calculate the density correction (Ram-Lak filter)
  4. Reconstruct with CG and optional regularization

  (5. Apply intensity correction)

  (6. Create plots)

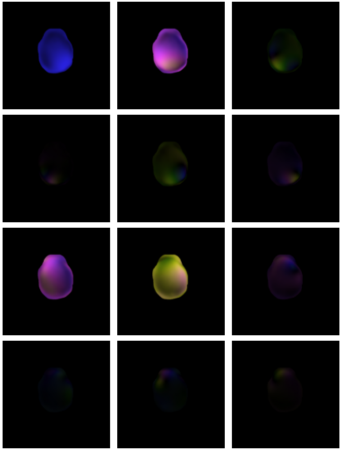
Data Wrangling

- Raw data can be found on Zenodo `https://zenodo.org/record/3975887`

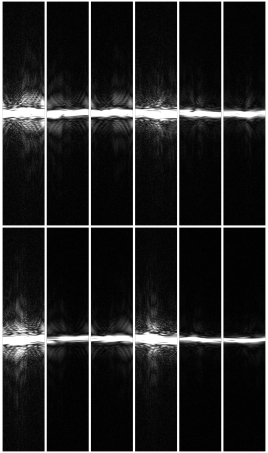
rawdata_brain_radial_96proj_12ch.h5 md5:6d52f9fd01c0441f3e680811a6839824	53.8 MB	Download
rawdata_heart_radial_55proj_34ch.h5 md5:0cfcb7a366b7c8cb52cc9c63f88cf34e	150.4 MB	Download

- HDF5 Format containing raw k-space data, trajectory, coil sensitivities, intensity correction maps and a mask
- BART does not (yet) read HDF5
- Fortunately, it can be done in a few lines of python (which will be shown later)

Data Wrangling



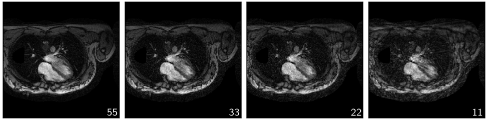
Coil sensitivity profiles

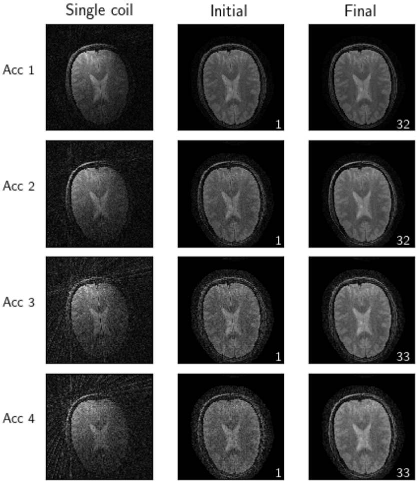


k-space data

The Challenge: Solution

- Oliver Maier's solution looks like this: (`https://github.com/MaierOli2010/ISMRM_RRSG`)





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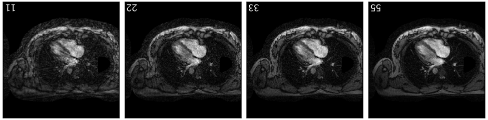

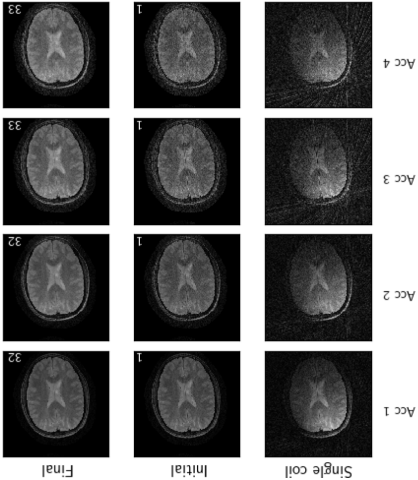

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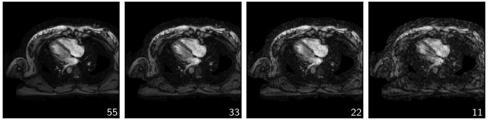

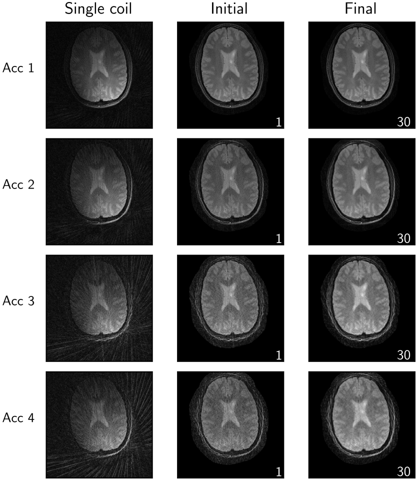

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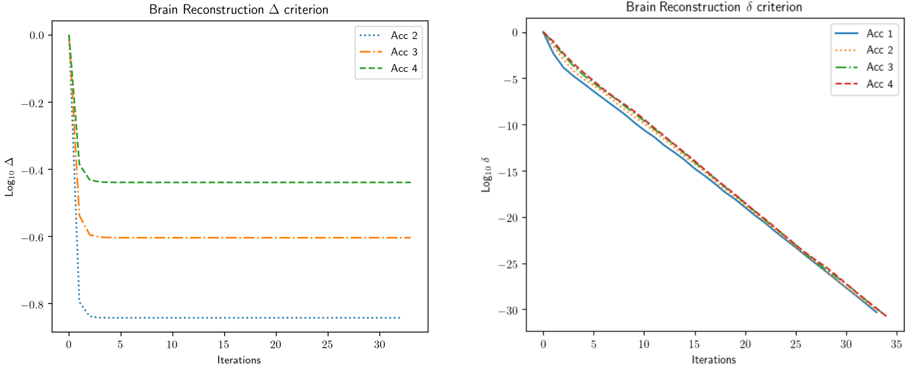

Δ: squared normalized root-mean-square error to a ground truth – here to the reconstruction using undersampling 1.
δ: squared residual of the CG, normalized to the initial residual.