Think about Motion: Motion $=$ Time

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Translational cardiovascular MR Imaging - PhD Training Ulm

March 18, 2025

Programe

- MR Image Reconstruction in a Nutshell
  Moritz Blumenthal
  Hands-on: How to use BART

- **Think about Motion: Motion = Time**
  Daniel Mackner
  Hands-on: Using BART for CMR Reconstruction

- Real-time MRI
  Philip Schaten
  Hands-on: Real-time reconstruction with BART

Differences to brain MRI - hurdles in CMR

Rosenzweig *et al.*, IEEE TMI, 2020.

Outline

- Gating strategies in cardiac MRI
- Why should we use radial sampling?
- What is the advantage of (tiny) Golden Angle?
- How can we reconstruct Non-Cartesian data?
- Temporal regularization?

Outline

- **Gating strategies in cardiac MRI**

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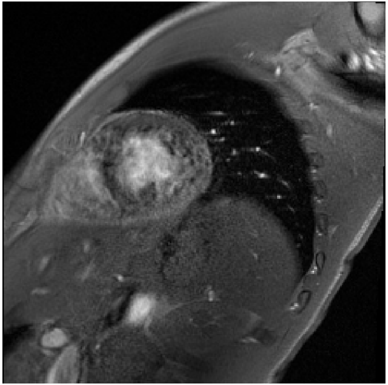
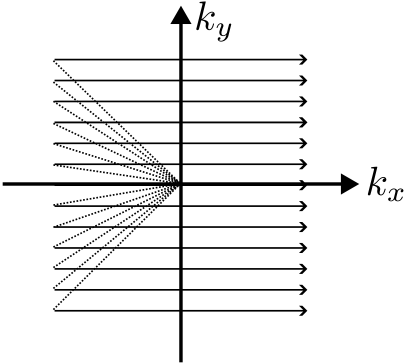
Naive Cartesian cardiac MRI

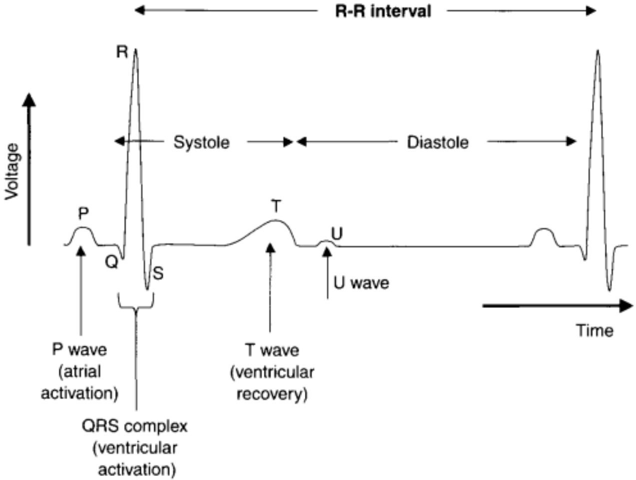




$TR = 5\ ms$, $BR = 256$, **TA=1.3 s** (fully-sampled FLASH)

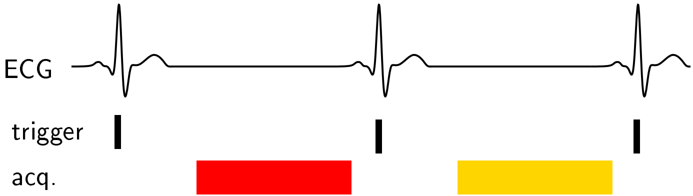
motion artifacts mainly in PE-direction

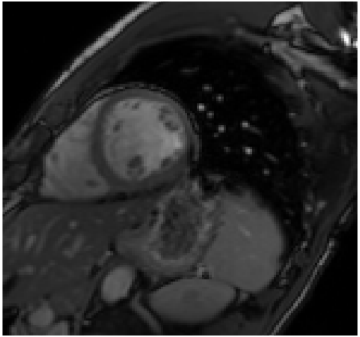
ECG - basics

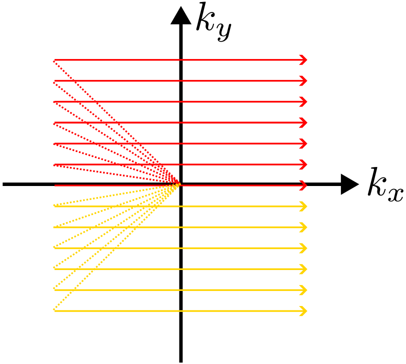


Bernstein *et al.*, Handbook of MRI Pulse Sequences, 2004.

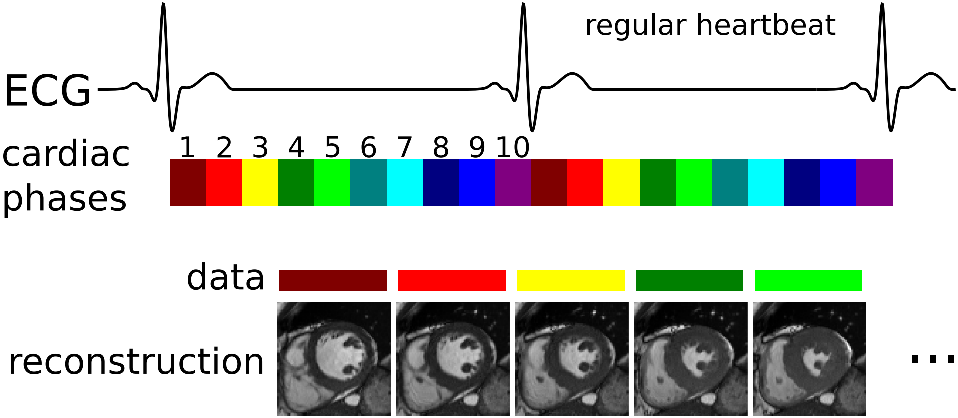
Cartesian cardiac MRI - prospectively triggered







Cardiac MRI - retrospective gating



Schilling, 2024.

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- Gating strategies in cardiac MRI
- **Why should we use radial sampling?**
- What is the advantage of (tiny) Golden Angle?
- How can we reconstruct Non-Cartesian data?
- Temporal regularization?

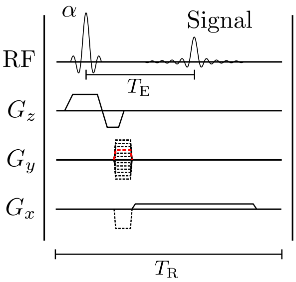
Sampling strategies

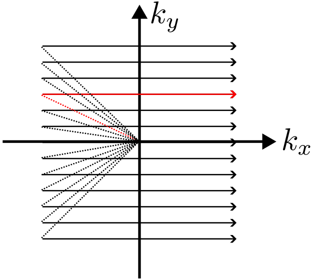
- Signal s is Fourier transform of the spin density ρ with k-space trajectory $\mathrm{k}(t)$

$$\mathrm{k}(t) = \gamma \int_0^t d\tau \mathrm{G}(\tau)$$

$$s(t) = \int_V d\mathrm{x}\rho(\mathrm{x})e^{-i2\pi\mathrm{k}(t)\mathrm{x}}$$

- Cartesian sampling





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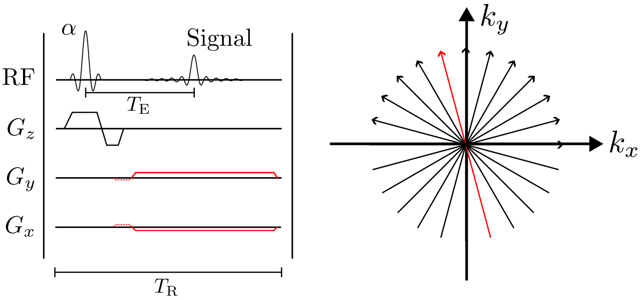
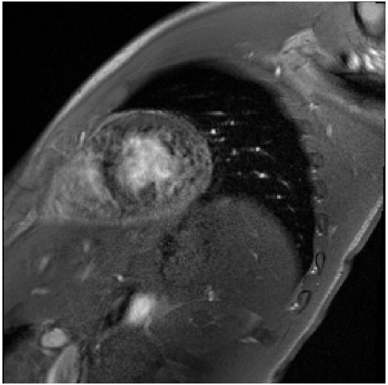
- radial sampling



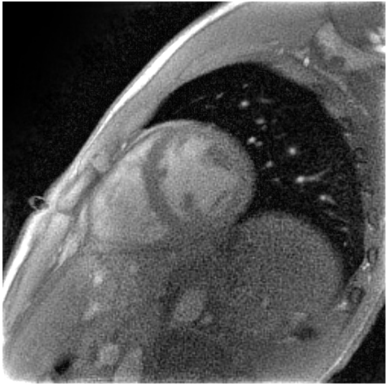
Why should we use radial sampling?

- motion robust by varying PE and pass of k-space center
  - $\rightarrow$ no ghosting artifacts
- inherent incoherent undersampling property
  - $\rightarrow$ aliasing in Cartesian vs. streakings in radial
- undersampling-induced artifacts look like noise (Compressed Sensing)

Cartesian vs. radial sampling



Cart. FLASH: $TR = 5\ ms$, $BR = 256$



Rad. FLASH: $TR = 3.5\ ms$, 256 spokes

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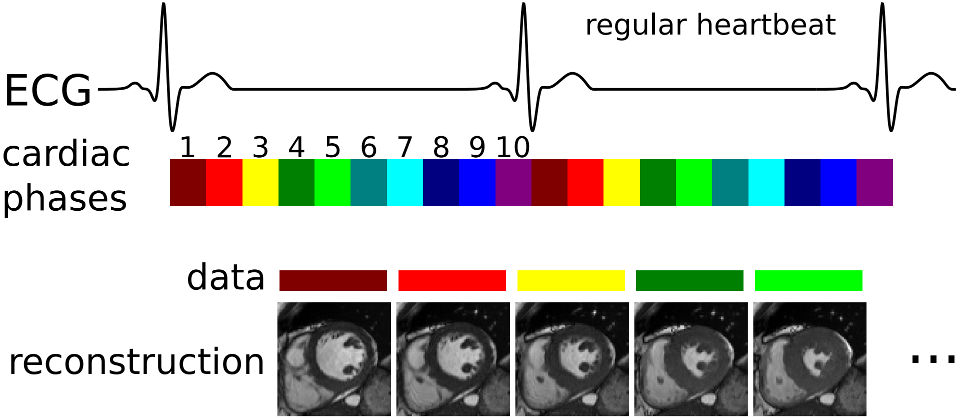
  (Compressed Sensing)

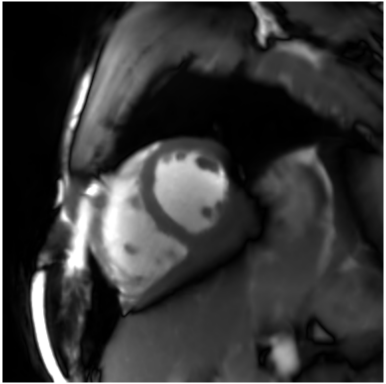
- What about gating/triggering?

Cardiac MRI - retrospective gating

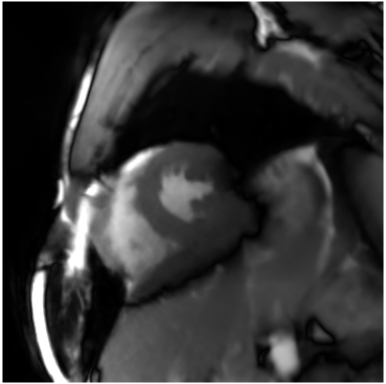


Schilling, 2024.

Self-calibration



diastolic phase



systolic phase

- radial lines through the center of k-space
- assumption of mean signal difference originating from the heart
- can eliminate the need to use navigator signals or external device

Feng *et al.*, MRM, 2015. Rosenzweig *et al.*, IEEE TMI, 2020.

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  $\rightarrow$ no ghosting artifacts
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  $\rightarrow$ aliasing in Cartesian vs. streakings in radial
- undersampling-induced artifacts look like noise (Compressed Sensing)

- What about gating/triggering?
  - retrospective gating by ECG signal
  - self-gating (ECG-free)

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- Gating strategies in cardiac MRI
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- **What is the advantage of (tiny) Golden Angle?**
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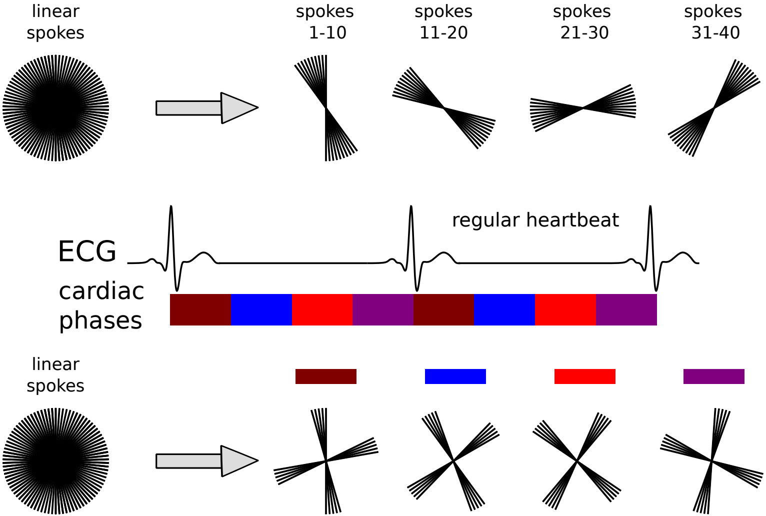
What can be the problem with gating in linear ordering?

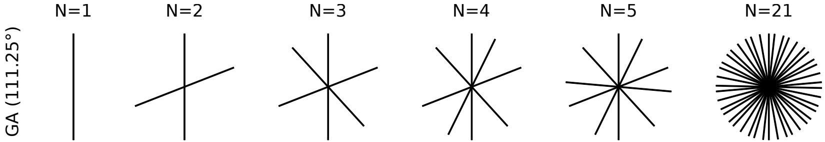


Feng, JMRI, 2022.

What is the Golden Angle?

- solve for Golden Ratio (GR) $\frac{a}{b} = \frac{a+b}{a} \rightarrow GR = \frac{1+\sqrt{5}}{2}$
- for spoke n we have $\theta_n = mod\left(\frac{180^\circ}{GR} \cdot n, 360^\circ\right) \approx 111.25^\circ$

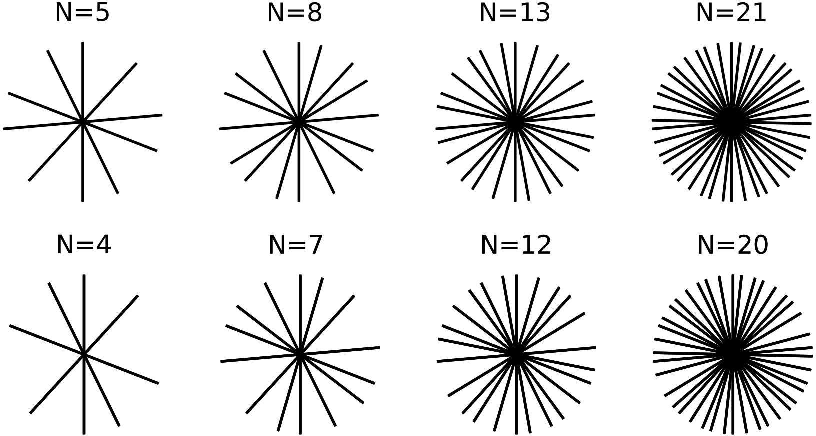
- Golden-Angle based radial spokes never repeat each other
- a rather uniform coverage of k-space with high temporal incoherence



Block *et al.*, MRM, 2007. Winkelmann *et al.*, IEEE TMI, 2006. Feng, JMRI, 2022.

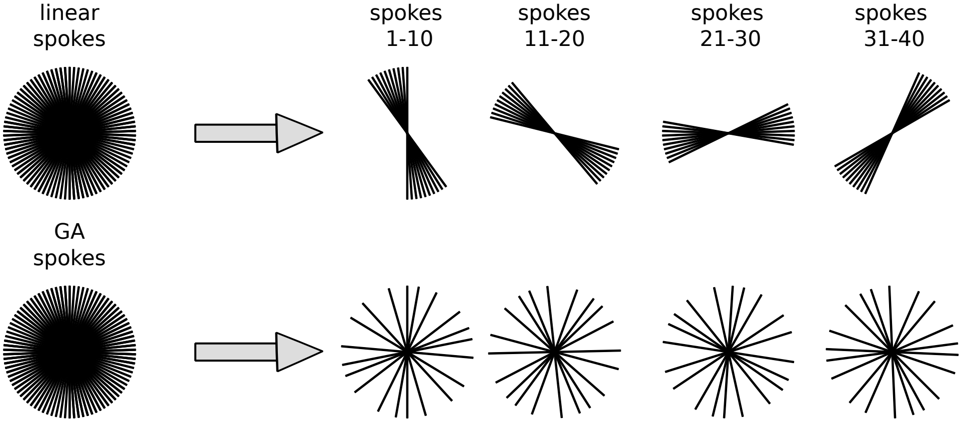
k-space coverage dependent on number of spokes

- Fibonacci series: 1, 2, 3, 5, 8, 13, 21, ...

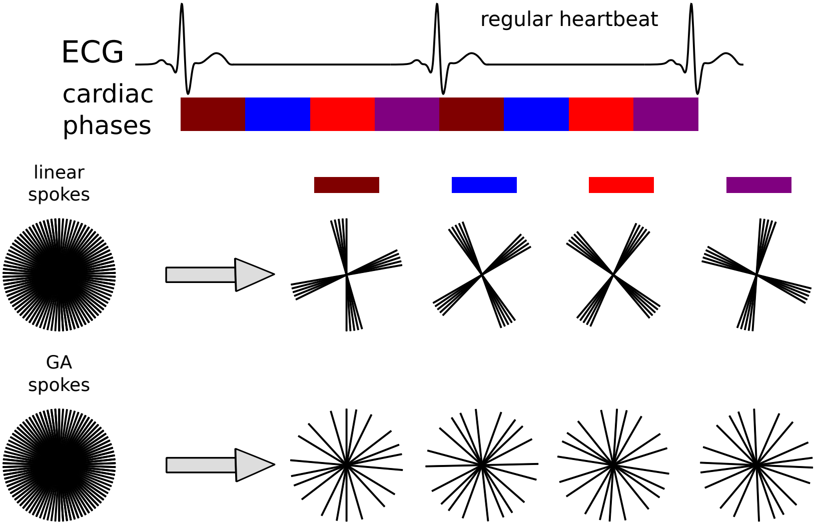


Winkelmann *et al.*, IEEE TMI, 2006.

Linear vs. Golden Angle radial sampling

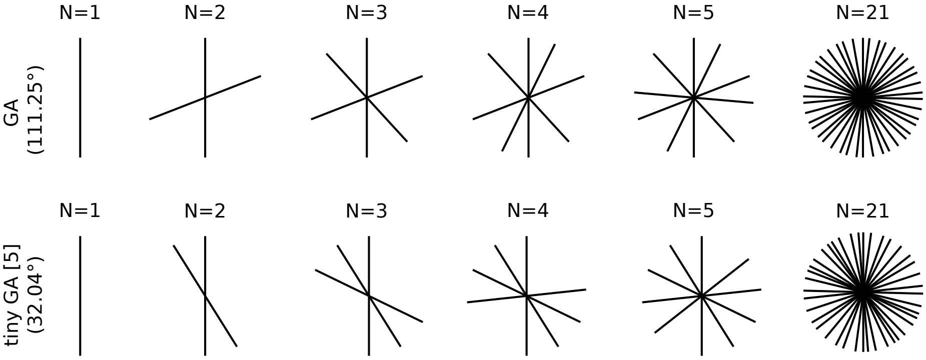


Linear vs. Golden Angle radial sampling



What is the tiny Golden Angle?

- $\theta_n = mod\left(\frac{180^\circ}{GR+N-1} \cdot n, 360^\circ\right)$ with $N > 1$
- reduces sensitivity to eddy currents



Wundrak *et al.*, IEEE TMI, 2015.

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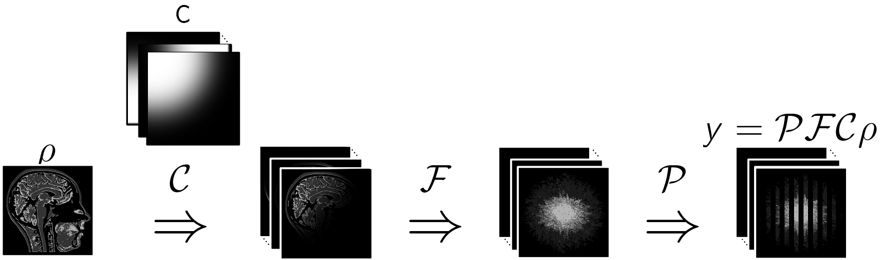
- What is the advantage of (tiny) Golden Angle?

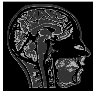
- **How can we reconstruct Non-Cartesian data?**

- Temporal regularization?

Forward Model for Non-Cartesian Sampling



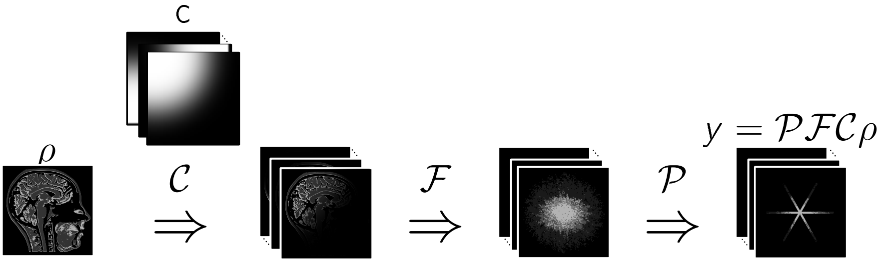
ρ^*

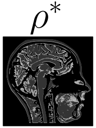


$$\Leftarrow$$

$$\boldsymbol{\rho}^* = \arg\min_{\boldsymbol{\rho}} \|\underbrace{\mathcal{P}\mathcal{F}\mathcal{C}}_{A}\boldsymbol{\rho} - \mathrm{y}\|_2^2 + \lambda R(\boldsymbol{\rho})$$

Forward Model for Non-Cartesian Sampling





$\Leftarrow$

$$\boldsymbol{\rho}^* = \arg\min_{\boldsymbol{\rho}} \| \underbrace{\mathcal{P}\mathcal{F}\mathcal{C}}_{A} \boldsymbol{\rho} - \mathrm{y} \|_2^2 + \lambda R(\boldsymbol{\rho})$$

Non-Cartesian MRI Reconstruction - PICS in BART

$$\rho^* = \arg\min_{\rho} \|\mathcal{P}\mathcal{F}\mathcal{C}\boldsymbol{\rho} - \mathrm{y}\|_2^2 + \lambda R(\boldsymbol{\rho})$$

Cartesian Reconstruction - pattern is provided by mask

```
bart pics -RW:7:0:0.001 -p<pattern> <kspace> <sens> <img>
```

Non-Cartesian Reconstruction - pattern is provided by trajectory

```
bart pics -RW:7:0:0.001 -t<trajectory> <kspace> <sens> <img>
```

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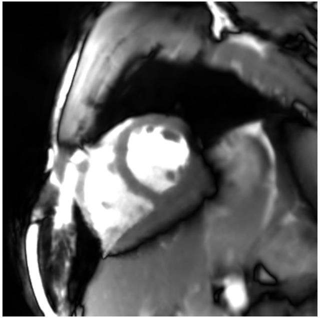
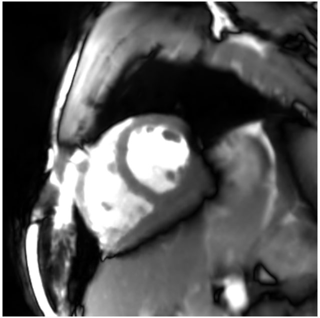
- **Temporal regularization?**

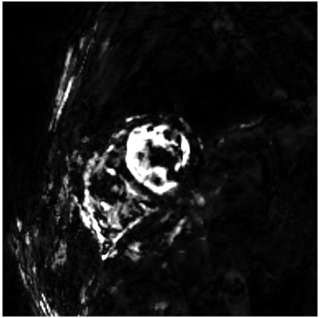
Why is temporal regularization useful?



ρ_t



ρ_{t+1}

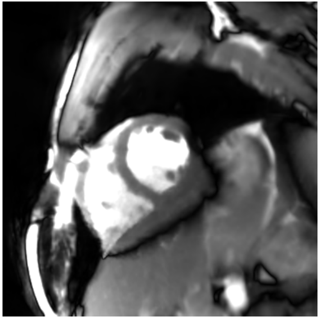


$|\rho_{t+1} - \rho_t|$ (scaling $\times 5$)

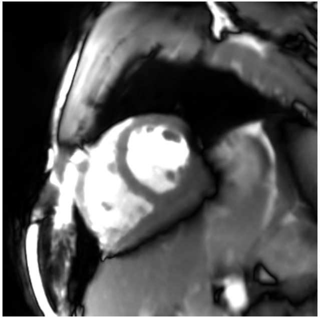
- How can this sparse difference in time be exploited?

Rosenzweig *et al.*, IEEE TMI, 2020.

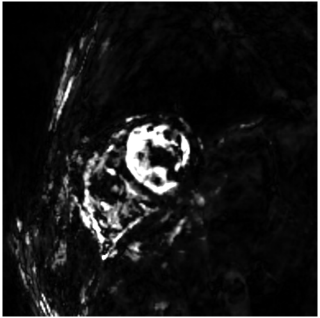
Why is temporal regularization useful?



ρ_t



ρ_{t+1}

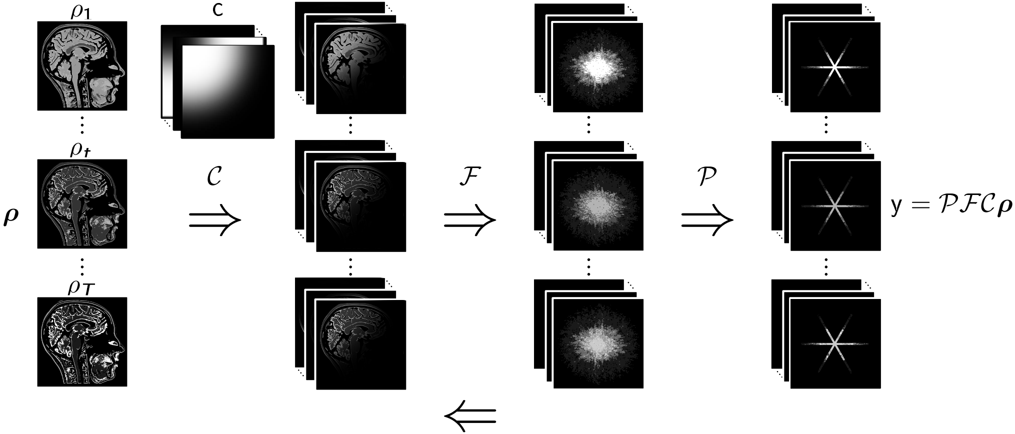


$|\rho_{t+1} - \rho_t|$ (scaling $\times 5$)

- How can this sparse difference in time be exploited?
- Additional regularization term in temporal direction
  $R(\rho_t) = \|\rho_{t+1} - \rho_t\|_1$

Rosenzweig *et al.*, IEEE TMI, 2020.

Temporal regularization



$$\boldsymbol{\rho}^* = \arg\min_{\boldsymbol{\rho}} \sum \|\underbrace{\mathcal{P}\mathcal{F}\mathcal{C}}_{A}\boldsymbol{\rho} - \mathrm{y}\|_2^2 + \lambda\|W\rho_t\|_1 + \mu\|\rho_t - \rho_{t-1}\|_1$$

Non-Cartesian MRI Reconstruction with temporal regularization - PICS in BART

$$\rho^* = \arg\min_{\rho} \|\mathcal{P}\mathcal{F}\mathcal{C}\boldsymbol{\rho} - \mathrm{y}\|_2^2 + \lambda R_W(\boldsymbol{\rho}) + \mu R_{TV}(\boldsymbol{\rho})$$

Regularization on spatial position in image

```
bart pics -RW:7:0:0.001 -t<traj> <ksp> <sens> <img>
```

Regularization on spatial position in image and Total variation in time

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bart pics -RW:7:0:0.001 -RT:1024:0:0.003 -t<traj>
<ksp> <sens> <img>
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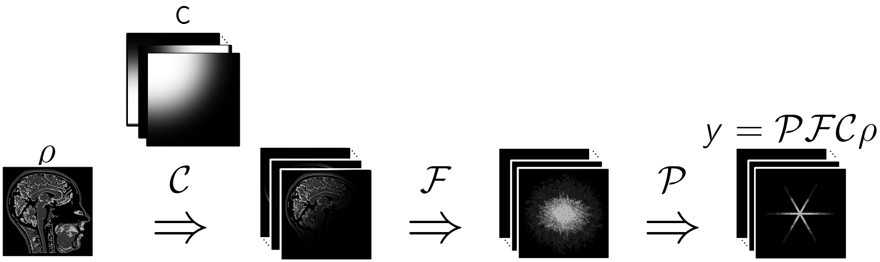
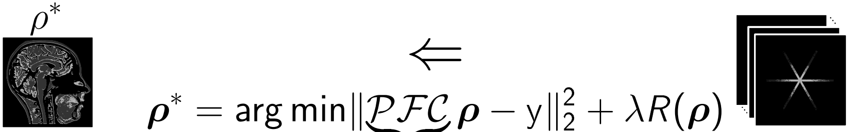
MRI Reconstruction as (Nonlinear) Inverse Problem





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Uecker *et al.*, MRM, 2008 .

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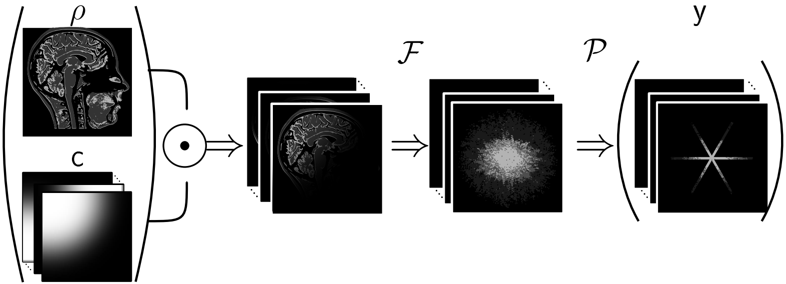


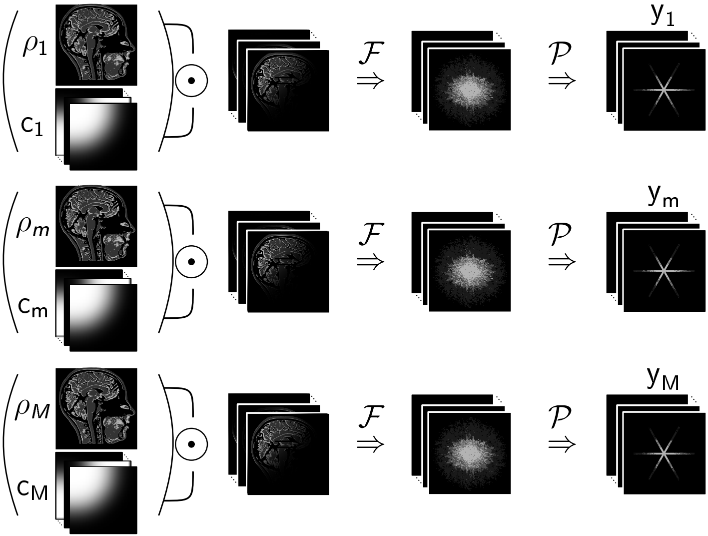
$$F: \mathrm{x} = (x_m, \mathrm{c}) \mapsto \mathcal{P}\,\mathcal{F}\,(\mathrm{c} \odot x_m)$$

$$\hat{\mathrm{x}} = \underset{\mathrm{x}=(\mathrm{x_c},\ \mathrm{c})}{\operatorname{argmin}} ||F(\mathrm{x}) - \mathrm{y}||_2^2 + \alpha\,||\mathrm{x_c}||_2^2 + \alpha\,||W\mathrm{c}||_2^2$$

W .. preconditioning matrix (corresponds to Sobolev norm in the original space)

Uecker *et al.*, MRM, 2008 .

Coils per motion state



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Translational cardiovascular MR Imaging - PhD Training Ulm
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