MRI-Reconstruction in a Nutshell

PhD-Training of DS-ISMRM

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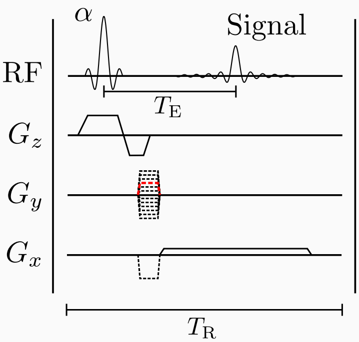
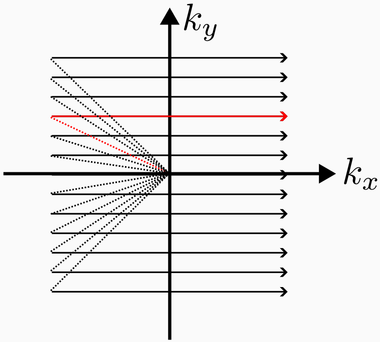
Basic MRI Acquisition and Reconstruction



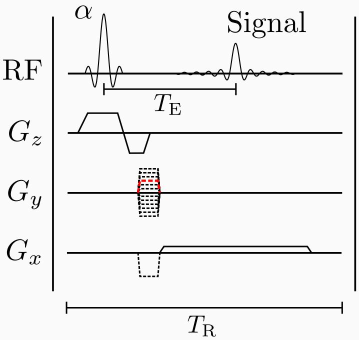
Sequence



k-Space

Signal Equation: $s(t) \propto \int_V \mathrm{d}\boldsymbol{r}\ \rho(\boldsymbol{r}) e^{-i2\pi \boldsymbol{k}(t)\cdot \boldsymbol{r}}$ with $\boldsymbol{k}(t) = \int_0^t \mathrm{d}\tau\ \frac{\gamma}{2\pi} \boldsymbol{G}(\tau)$

Basic MRI Acquisition and Reconstruction



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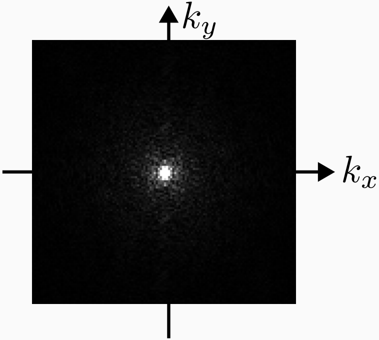

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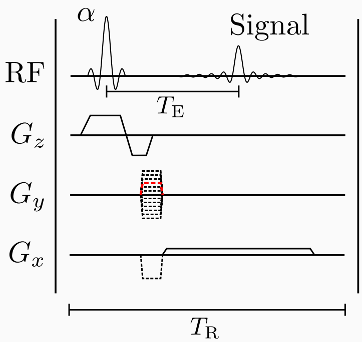

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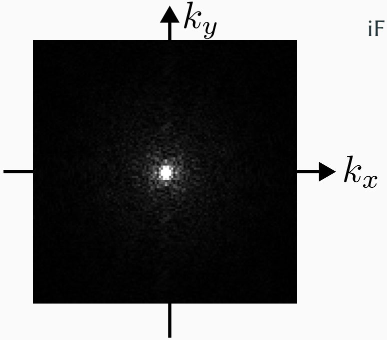
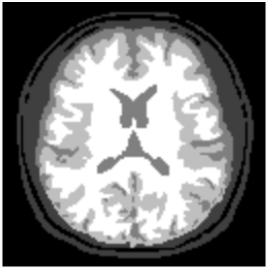


k-Space

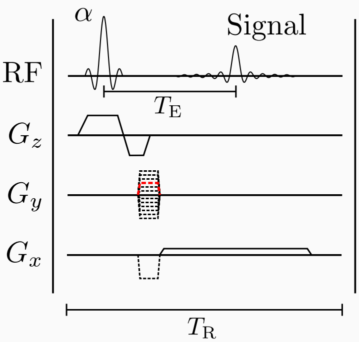
iFFT



Reconstruction

Signal Equation: $s(t) \propto \int_V \mathrm{d}\boldsymbol{r}\ \rho(\boldsymbol{r}) e^{-i2\pi \boldsymbol{k}(t)\cdot\boldsymbol{r}}$ with $\boldsymbol{k}(t) = \int_0^t \mathrm{d}\tau\ \frac{\gamma}{2\pi}\boldsymbol{G}(\tau)$

Speeding up by Undersampling



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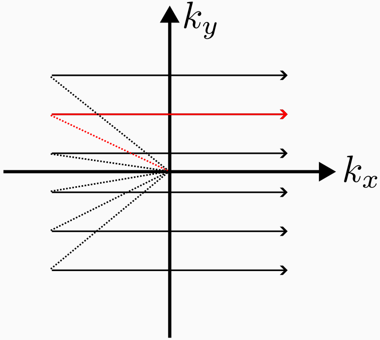

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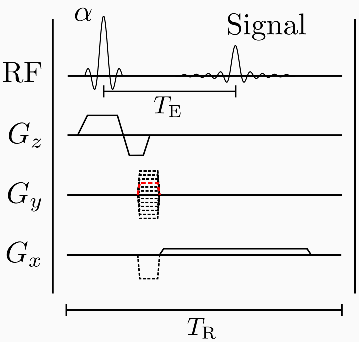

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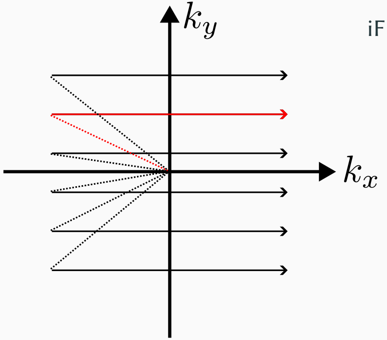

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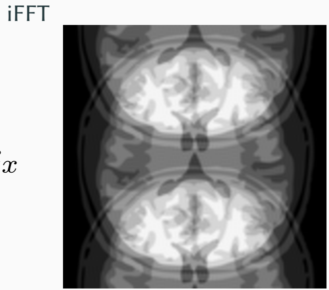

Reconstruction

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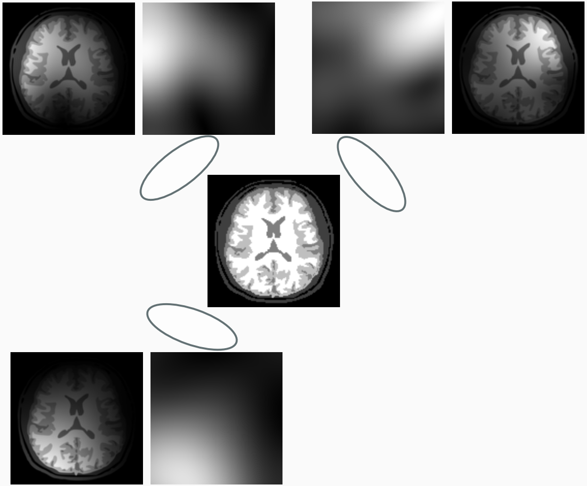
Advanced Reconstruction Techniques[1] - Parallel Imaging with SENSE[2]



[1] As formulated 20-25 years ago

[2] K. P. Pruessmann et al. “SENSE: sensitivity encoding for fast MRI”. Magn. Reson. Med. 1999; 42:952–962

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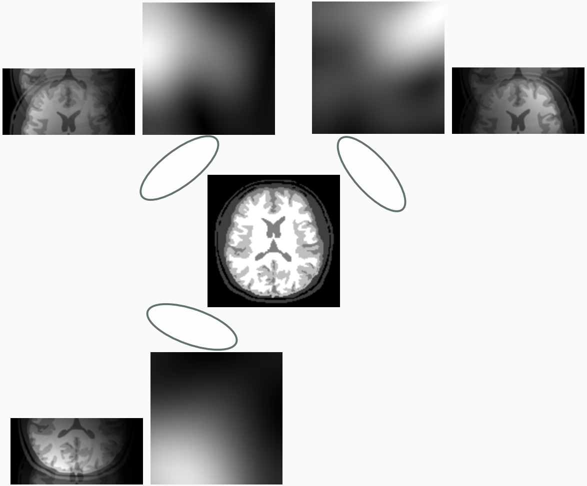

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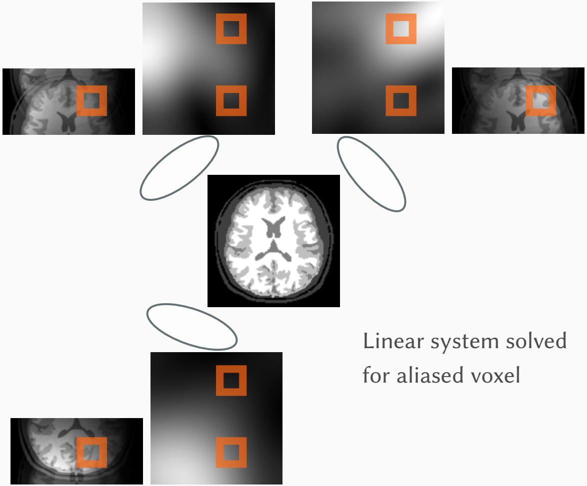
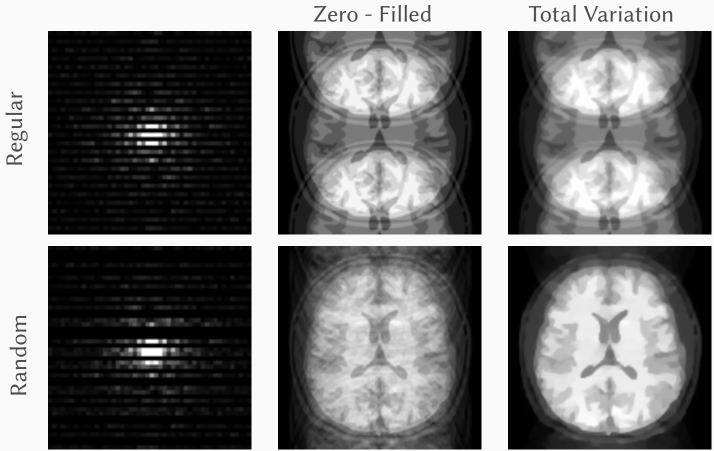


Linear system solved
for aliased voxel

[1] As formulated 20-25 years ago

[2] K. P. Pruessmann et al. "SENSE: sensitivity encoding for fast MRI". Magn. Reson. Med. 1999; 42:952–962

Advanced Reconstruction Techniques[1] - Compressed Sensing[2,3]



1.) Transform sparsity

2.) Incoherence of artifacts

3.) Nonlinear reconstruction
(enforce data consistency
transform sparsity)

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[1] As formulated 20-25 years ago

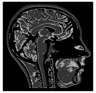
[2] M. Lustig et al. “Sparse MRI: The application of compressed sensing for rapid MR imaging”. Magn. Reson. Med. 2007; 58:1182–1195

[3] K. T. Block et al. “Undersampled radial MRI with multiple coils. Iterative image reconstruction using a total variation constraint”. Magn. Reson. Med. 2007; 57:1086–1098

MRI Reconstruction as Inverse Problem

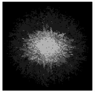
MRI Reconstruction as Inverse Problem - Forward Model

ρ

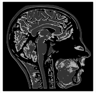


$$\overset{\mathcal{F}}{\Longrightarrow}$$

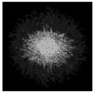
$y = \mathcal{F}\rho$



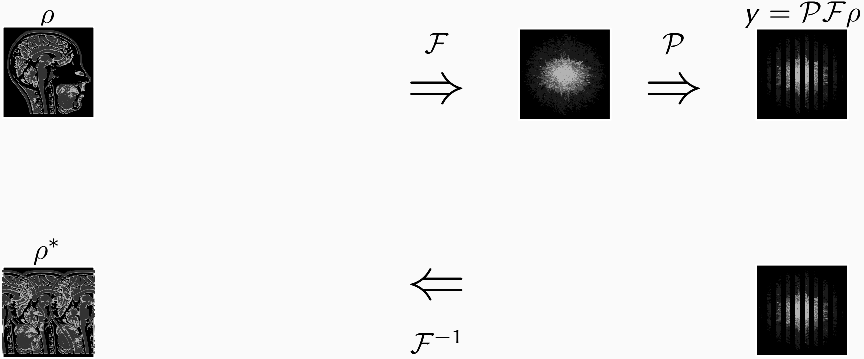
ρ^*



$$\underset{\mathcal{F}^{-1}}{\Longleftarrow}$$

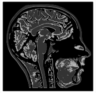


MRI Reconstruction as Inverse Problem - Forward Model

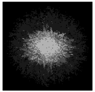


MRI Reconstruction as Inverse Problem - Forward Model

ρ



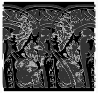
$\overset{\mathcal{F}}{\Longrightarrow}$



$\overset{\mathcal{P}}{\Longrightarrow}$

$\boldsymbol{y} = \mathcal{P}\mathcal{F}\rho$

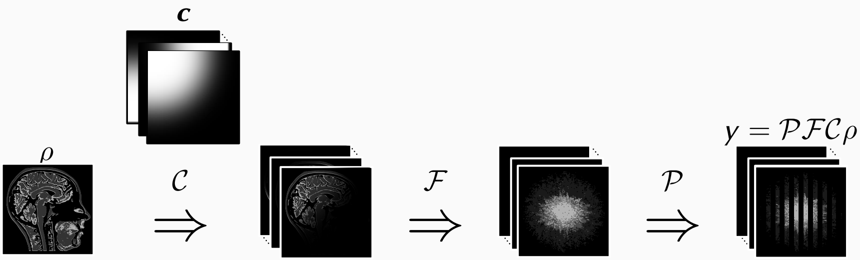
ρ^*

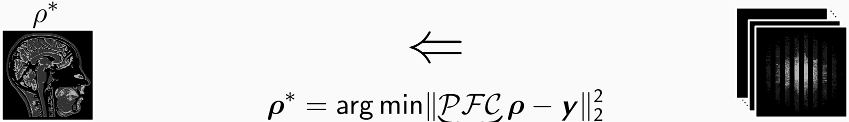


$\Longleftarrow$

$$\boldsymbol{\rho}^* = \arg\min_{\boldsymbol{\rho}} \| \underbrace{\mathcal{P}\mathcal{F}}_{A} \boldsymbol{\rho} - \boldsymbol{y} \|_2^2$$

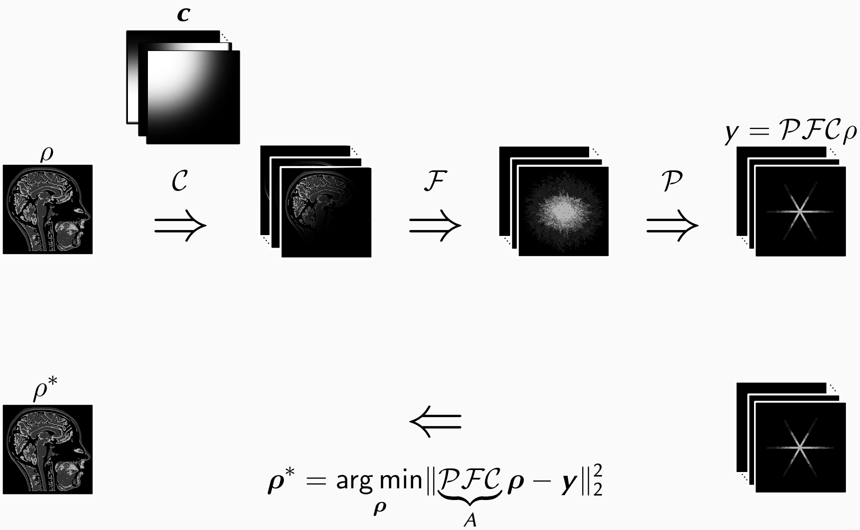
MRI Reconstruction as Inverse Problem - Forward Model





$$\boldsymbol{\rho}^* = \arg\min_{\boldsymbol{\rho}} \|\underbrace{\mathcal{P}\mathcal{F}\mathcal{C}}_{A}\boldsymbol{\rho} - \boldsymbol{y}\|_2^2$$

MRI Reconstruction as Inverse Problem - Forward Model



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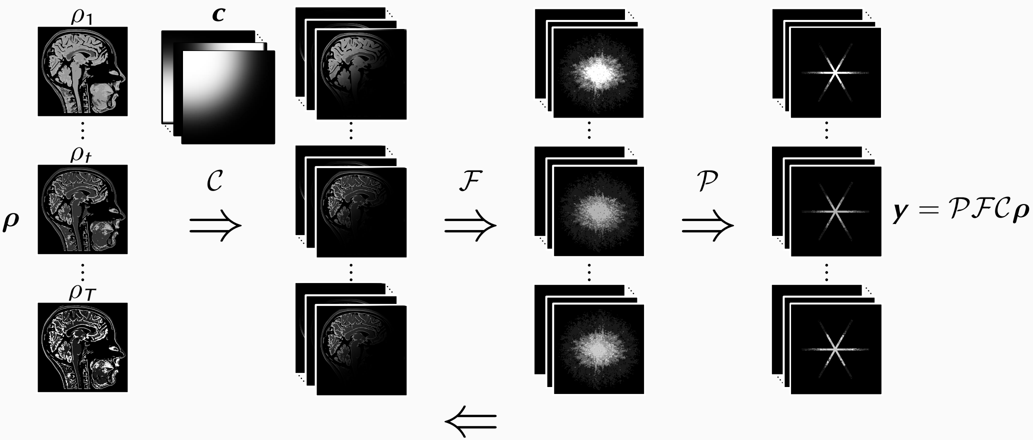
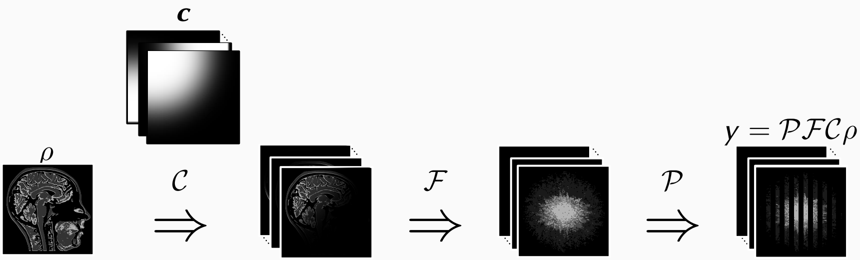
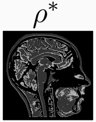


$$\boldsymbol{\rho}^* = \arg\min_{\boldsymbol{\rho}} \|\underbrace{\mathcal{PFC}}_{A}\boldsymbol{\rho} - \boldsymbol{y}\|_2^2$$





$$\Longleftarrow$$

$$\boldsymbol{\rho}^* = \arg\min_{\boldsymbol{\rho}} \|\underbrace{\mathcal{P}\mathcal{F}\mathcal{C}}_{A}\boldsymbol{\rho} - \boldsymbol{y}\|_2^2 + \lambda R(\boldsymbol{\rho})$$

MRI Reconstruction as Inverse Problem - Regularization

$$\rho^* = \arg\min_{\rho} \|A\rho - y\|_2^2 + \lambda R(\rho)$$

Spatial Regularization

- Total Variation[1]: $R(\rho) = \|\nabla\rho\|_1$
- Wavelet sparsity[2]: $R(\rho) = \|W\rho\|_1$
- Total Generalized Variation[3]

Temporal Regularization

- Total Variation: $R(\rho) = \sum_{t=1}^{T} \|\rho_t - \rho_{t-1}\|_1$
- Causal difference: $R(\rho_t) = \left\|\rho_t - \rho_{t-1}^*\right\|_2^2$

Spatio-Temporal Regularization

- Low-rank[4,5]: $R(\rho) = \|\rho\|_*$
- Low-rank + Sparse[6]: $R(L + S) = \|L\|_* + \|TS\|_1$
- Subspace[7]

[1] K. T. Block et al. "Undersampled radial MRI with multiple coils. Iterative image reconstruction using a total variation constraint". Magn. Reson. Med. 2007; 57:1086–1098

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[3] F. Knoll et al. "Second order total generalized variation (TGV) for MRI". Magn. Reson. Med. 2011; 65:480–491

[4] Z.-P. Liang. "Spatiotemporal imaging with partially separable functions". IEEE Int. Symp. Biomed. Imag. 4. 2007:988–991

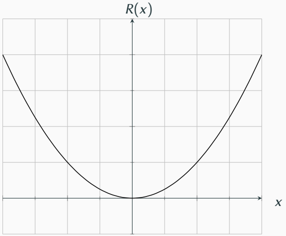
[5] A. G. Christodoulou et al. "High-resolution cardiac MRI using partially separable functions and weighted spatial smoothness regularization". Int. Conf. IEEE Eng. Med. Bio. 32. Buenos Aires, 2010:871–874

[6] R. Otazo et al. "Low-rank plus sparse matrix decomposition for accelerated dynamic MRI with separation of background and dynamic components: L+S Reconstruction". Magn. Reson. Med. 2014; 73:1125–1136

[7] F. H. Petzschner et al. "Fast MR parameter mapping using k-t principal component analysis". Magn. Reson. Med. 2011; 66:706–716

MRI Reconstruction as Inverse Problem - Optimization

$$\boldsymbol{\rho}^* = \arg\min_{\boldsymbol{\rho}} \|A\boldsymbol{\rho} - \boldsymbol{y}\|_2^2 + \lambda R(\boldsymbol{\rho})$$



Iterative Reconstruction

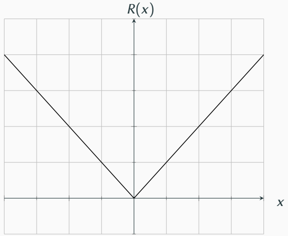
- Optimal algorithm depends on regularization
- CG, FISTA, ADMM, Primal-Dual, etc.

Example: Gradient Descent

$$\boldsymbol{\rho}^{(0)} = \underbrace{\mathcal{C}^H\mathcal{F}^H\mathcal{P}^H}_{A^H}\boldsymbol{y} \qquad \text{(Initialization)}$$

$$\boldsymbol{\rho}^{(k+1)} = \boldsymbol{\rho}^{(k)} - \eta\underbrace{\mathcal{C}^H\mathcal{F}^H\mathcal{P}^H\mathcal{P}\mathcal{F}\mathcal{C}}_{A^HA}\boldsymbol{\rho}^{(k)} + \eta\underbrace{\mathcal{C}^H\mathcal{F}^H\mathcal{P}^H}_{A^H}\boldsymbol{y} - \eta\nabla\lambda R(\boldsymbol{\rho}^{(k)}) \qquad \text{(Iterations)}$$

$$\rho^* = \arg\min_{\rho} \|A\rho - \mathbf{y}\|_2^2 + \lambda R(\rho)$$



Iterative Reconstruction

- Optimal algorithm depends on regularization
- CG, FISTA, ADMM, Primal-Dual, etc.
- Proximal operators prox_R for non-smooth R

Example: ISTA

$$\rho^{(0)} = \underbrace{\mathcal{C}^H \mathcal{F}^H \mathcal{P}^H}_{A^H} \mathbf{y} \qquad \text{(Initialization)}$$

$$\rho^{(k+1)} = \mathrm{prox}_{\eta R}(\rho^{(k)} - \eta \underbrace{\mathcal{C}^H \mathcal{F}^H \mathcal{P}^H \mathcal{P} \mathcal{F} \mathcal{C}}_{A^H A} \rho^{(k)} + \eta \underbrace{\mathcal{C}^H \mathcal{F}^H \mathcal{P}^H}_{A^H} \mathbf{y}) \qquad \text{(Iterations)}$$

MRI Reconstruction as Inverse Problem

$$\boldsymbol{\rho}^* = \arg\min_{\boldsymbol{\rho}} \|A\boldsymbol{\rho} - \boldsymbol{y}\|_2^2 + \lambda R(\boldsymbol{\rho})$$

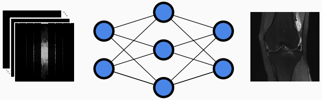
- Modeling the measurement by forward model A
- Include prior knowledge by regularization term R
- Solve optimization problem by iterative algorithms

A Full Reconstruction Pipeline

A non-complete list of steps from raw k-space to DICOM image:

1. Noise prewhitening
2. Gradient delay corrections
3. Coil compression
4. Estimation of coil sensitivities
5. $\boldsymbol{\rho}^* = \arg\min_{\boldsymbol{\rho}} \|A\boldsymbol{\rho} - \boldsymbol{y}\|_2^2 + \lambda R(\boldsymbol{\rho})$
6. Distortion correction
7. Denoising (post-processing)

$\mathrm{Net}(\boldsymbol{y};\boldsymbol{\theta}) = \boldsymbol{x}$



Learn Reconstruction (forget about the model)

- AUTOMAP[1]: $\boldsymbol{\rho}^* = \textsc{Net}(\boldsymbol{y};\theta)$
- Improve zero-filled: $\boldsymbol{\rho}^* = \textsc{Net}(A^H\boldsymbol{y};\theta)$

Keep Model and Learn Regularization

- Directly learn R, ∇R or prox_R independent of model
  - Diffusion models, generative models, plug-and-play priors, etc.
- Unrolled / model-based neural network[2]
  - $\boldsymbol{\rho}^{(k+1)} = \boldsymbol{\rho}^{(k)} - \eta A^H A\boldsymbol{\rho}^{(k)} + \eta A^H\boldsymbol{y} - \underbrace{\textsc{Net}(\boldsymbol{\rho}^{(k)};\theta)}_{\eta\nabla\lambda R(\boldsymbol{\rho}^{(k)})}$

[1] B. Zhu et al. "Image reconstruction by domain-transform manifold learning". Nature. 2018; 555.7697:487–492

[2] K. Hammernik et al. "Learning a Variational Network for Reconstruction of Accelerated MRI Data". Magn. Reson. Med. 2017; 79:3055–3071

BART

Purposes

- Rapid prototyping
- Reproducible research
- Clinical translation

Command line tools for MRI reconstruction

- Estimation of coil sensitivity maps
- Parallel Imaging and Compressed Sensing
- Non-linear (model-based) recos: NLINV, ENLIVE and MOBA

Numeric Backend

- Written in C, few external dependencies
- Operations on multidimensional arrays
- Accelerated by GPU, OpenMP and MPI

A Full Reconstruction Pipeline

A non-complete list of steps from raw k-space to DICOM image:

1. Noise prewhitening - `bart whiten`
2. Gradient delay corrections - `bart estdelay`
3. Coil compression - `bart cc`
4. Estimation of coil sensitivities - `bart ecalib / bart ncalib`
5. $\boldsymbol{\rho}^* = \arg\min_{\boldsymbol{\rho}} \|A\boldsymbol{\rho} - \boldsymbol{y}\|_2^2 + \lambda R(\boldsymbol{\rho})$ - `bart pics`
6. Distortion correction - NA
7. Denoising (post-processing) - `bart nlmeans`

Translate the optimization problem

$$\boldsymbol{\rho}^* = \arg\min_{\boldsymbol{\rho}} \|\mathcal{P}\mathcal{F}\mathcal{C}\boldsymbol{\rho} - \boldsymbol{y}\|_2^2 + \lambda R(\boldsymbol{\rho})$$

to a BART command line call:

```
bart pics -RW:7:0:0.001 -p<pattern> <kspace> <sens> <img>
```

Let’s get started

References i

[1] K. P. Pruessmann et al. “SENSE: sensitivity encoding for fast MRI”. Magn. Reson. Med. 1999; 42:952–962.

[2] M. Lustig, D. Donoho, and J. M. Pauly. “Sparse MRI: The application of compressed sensing for rapid MR imaging”. Magn. Reson. Med. 2007; 58:1182–1195.

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