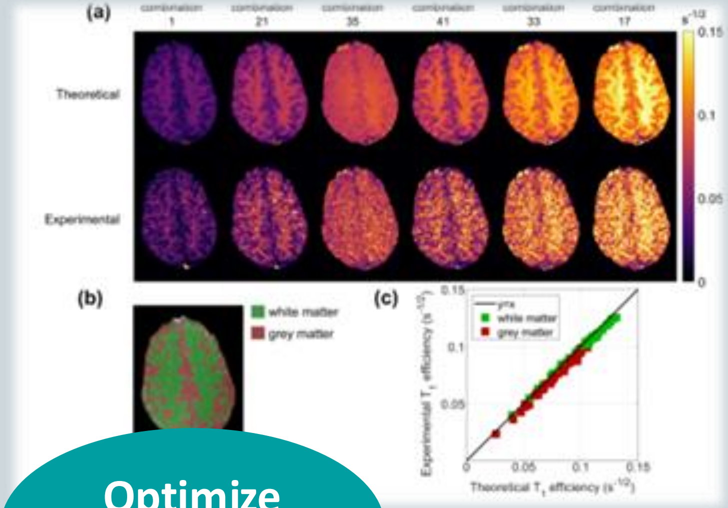
EPGs, sequence simulations, and optimization

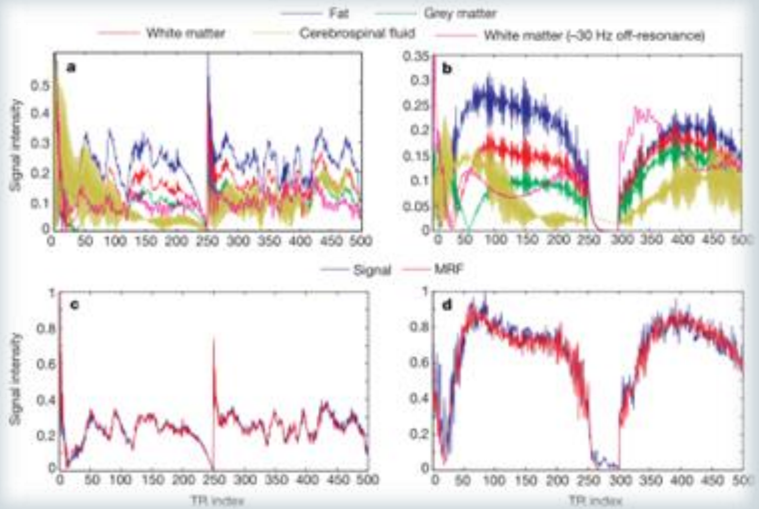
Dr Shaihan Malik

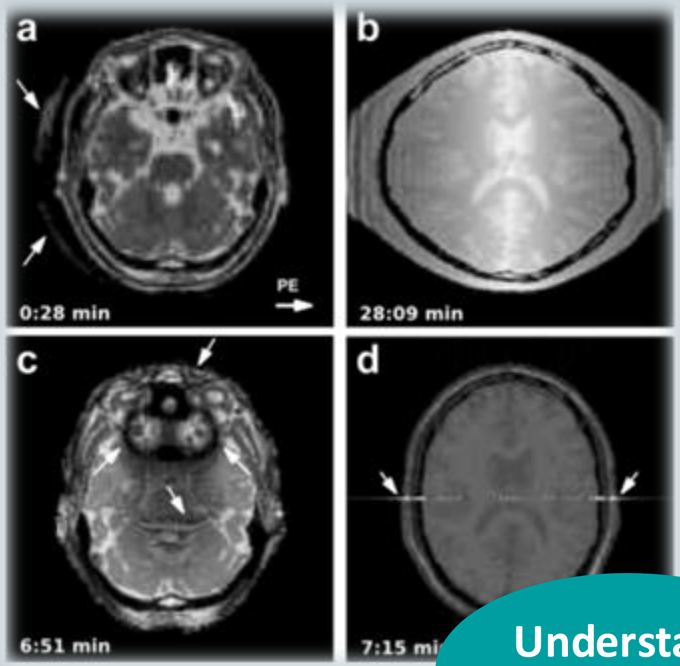
MR simulations – why do we need them?



Optimize Sequences

Quantitative Imaging

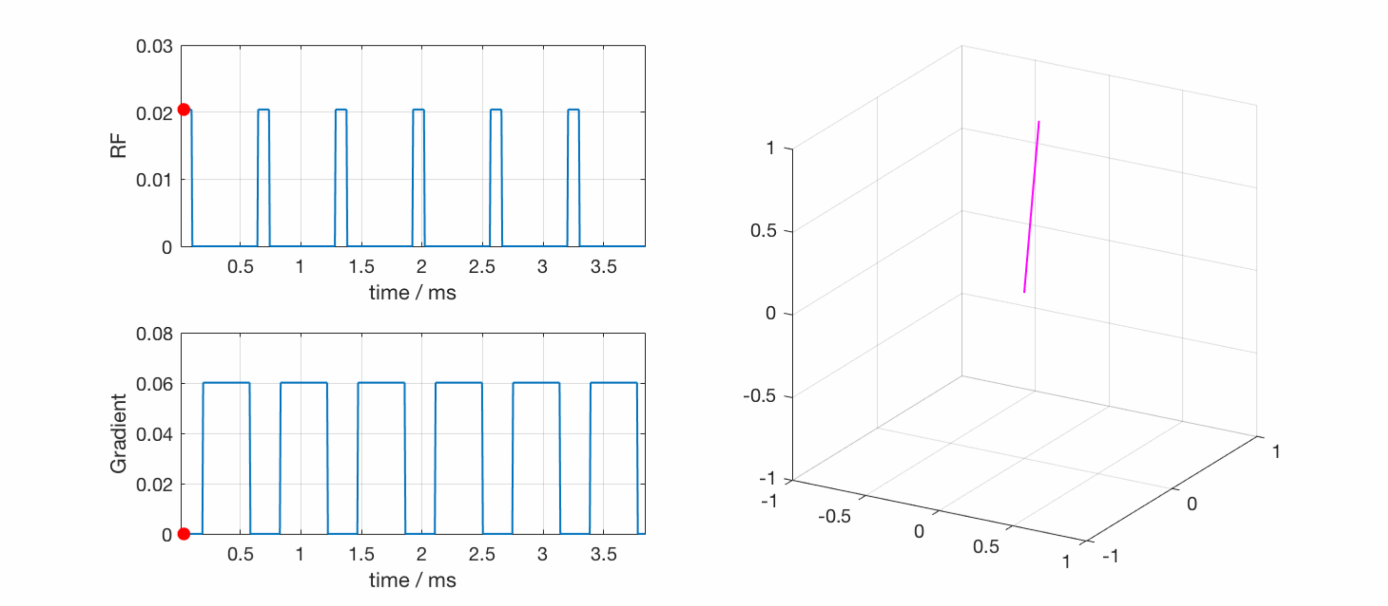




Understand Artefacts

1. *Leitao et al, Phys Med Biol, 2021*
2. *Stöcker et al, Magn Reson Med 2010*
3. *Ma et al, Nature 2013*

Simulation of a single isochromat



From Bloch Equations:

RF pulses rotate M about axis in x-y plane

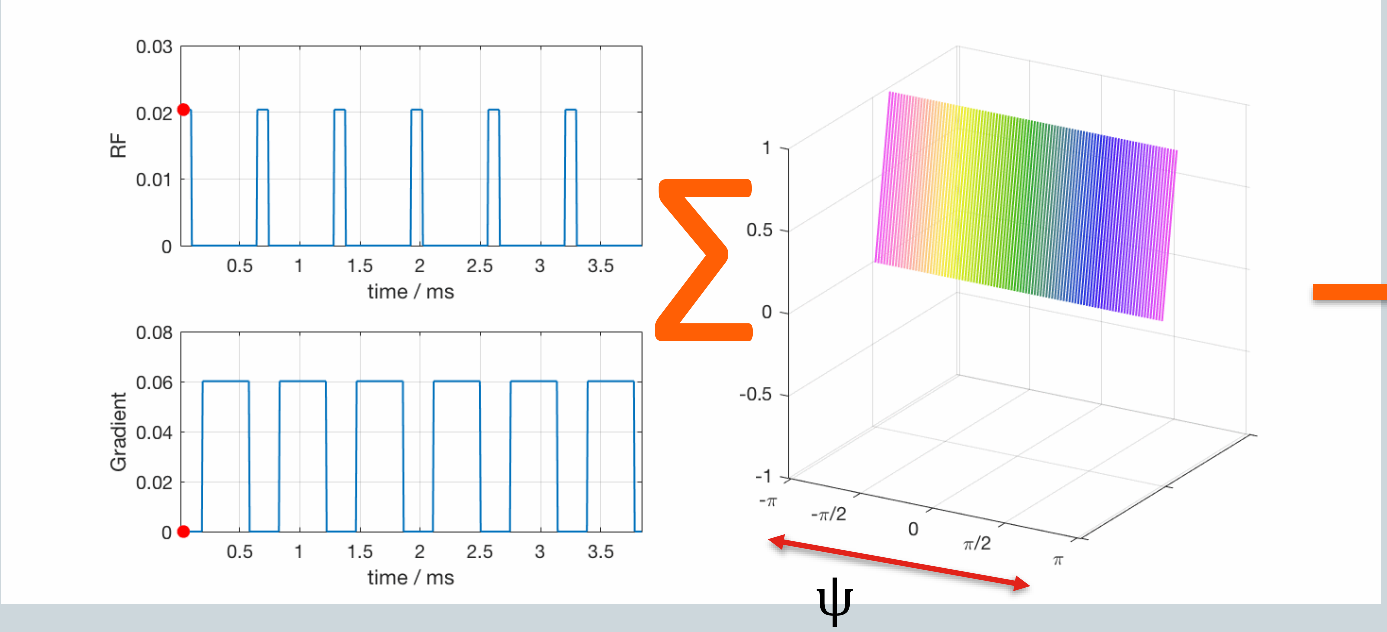
Gradients cause rotation about z depending on position (r): $\psi = -\gamma \int G(t).r\, dt$

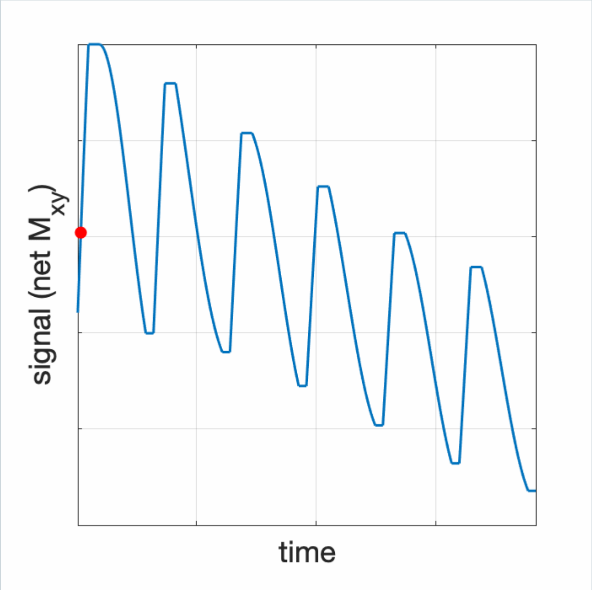
What is the signal we would measure?

We can't tell from this simulation! In reality there is a continuous distribution of magnetization at different spatial locations.

A single spatial location = 'isochromat' (experiences same fields)

‘Ensemble of isochromats’





- We can approximate reality by simulating the behaviour of a large ensemble of isochromats at different spatial positions
- The total signal is then obtained by summing the contributions

Extended Phase Graphs

An alternative picture of the same problem…

To properly simulate sequences with unbalanced gradients we need to look at multiple spatial locations **because the effect of the gradient is inherently space dependent**

In fact it's not *space* that's important, but the rotation angle due to the gradient:

$$M_{x,y,z}(\boldsymbol{r}) \leftrightarrow M_{x,y,z}(\psi) \qquad \psi = -\gamma \int_0^\tau \boldsymbol{G}(t).\boldsymbol{r}\,dt = \boldsymbol{\Delta k}.\boldsymbol{r}$$

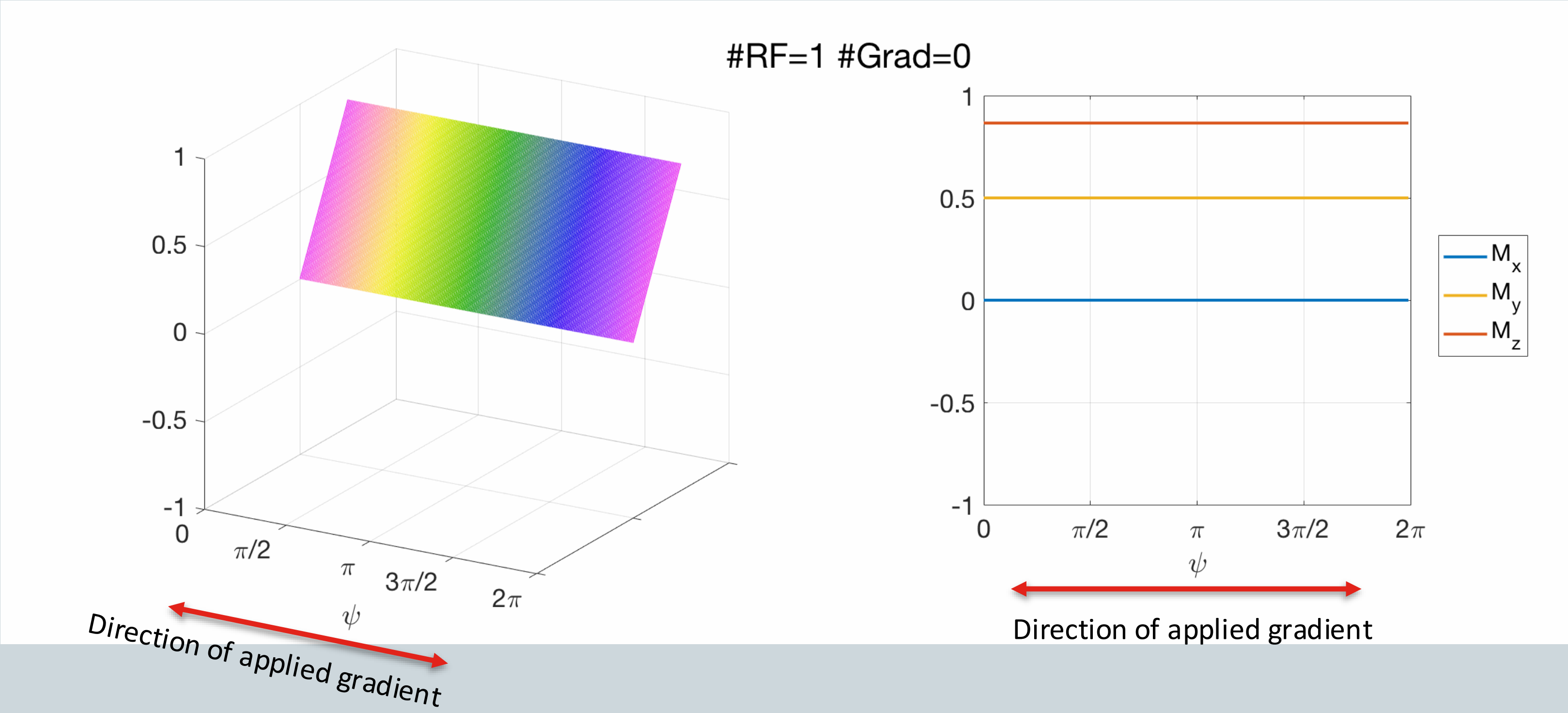
ψ depends on space, but also a *k-space* term

- Isochromat domain: represent M as a function of **r**
- **EPG: represent M as a function of Δk**

Example: start of gradient echo

How does magnetization evolve at the start of a gradient echo scan (no RF spoiling)?

'sub-voxel' magnetization distribution



Magnetization appears to evolve as a Fourier series that gets more complex after each gradient and RF pulse

Mathematical basis for this...

First, rewrite magnetization in different basis

$$\begin{bmatrix} M_+ \\ M_- \\ M_z \end{bmatrix} = \begin{bmatrix} 1 & +i & 0 \\ 1 & -i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

M_+ is what we usually use to represent signal in MRI

Now consider effects of gradients...

$$M_+ \rightarrow e^{i\boldsymbol{\Delta k.r}} M_+$$

$$M_- \rightarrow e^{-i\boldsymbol{\Delta k.r}} M_-$$

$$M_z \rightarrow M_z$$

Gradient adds phase to transverse magnetization...

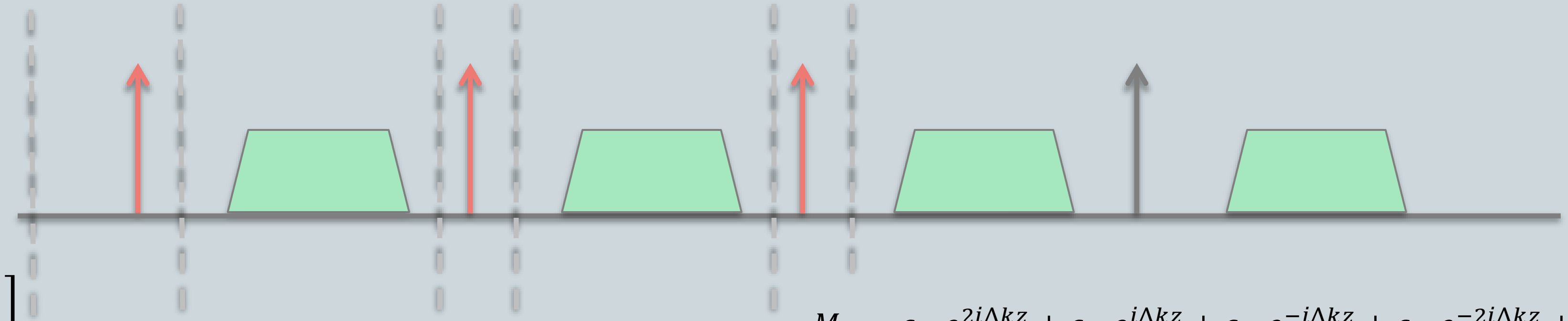
... and RF pulses

$$\widehat{\boldsymbol{M}} \rightarrow \widehat{\boldsymbol{T}}(\theta,\phi)\widehat{\boldsymbol{M}}$$

$$\widehat{\boldsymbol{T}}(\theta,\phi) = \begin{bmatrix} \cos^2\frac{\theta}{2} & e^{2i\phi}\sin^2\frac{\theta}{2} & -ie^{i\phi}\sin\theta \\ e^{-2i\phi}\sin^2\frac{\theta}{2} & \cos^2\frac{\theta}{2} & ie^{-i\phi}\sin\theta \\ -\frac{i}{2}e^{-i\phi}\sin\theta & \frac{i}{2}e^{i\phi}\sin\theta & cos\theta \end{bmatrix}$$

RF pulses mix up the magnetization – i.e. M_+, M_-, and M_z are mixed by this matrix

Simple sequence of RF & gradients



$$\hat{M} = \begin{bmatrix} 0 \\ 0 \\ M_0 \end{bmatrix}$$

$$M_+ = -iM_0 \sin\theta$$
$$M_- = iM_0 \sin\theta$$
$$M_z = M_0 \cos\theta$$

$$M_+ = -iM_0 \sin\theta e^{i\Delta kz}$$
$$M_- = iM_0 \sin\theta e^{-i\Delta kz}$$
$$M_z = M_0 \cos\theta$$

(gradient doesn't affect M_z)

$$M_+ = c_1 e^{i\Delta kz} + c_2 e^{-i\Delta kz} + c_3$$
$$M_- = c_4 e^{i\Delta kz} + c_5 e^{-i\Delta kz} + c_6$$
$$M_z = c_7 e^{i\Delta kz} + c_8 e^{-i\Delta kz} + c_9$$

RF pulse mixes components (partitioning)

$$M_+ = c_1 e^{2i\Delta kz} + c_2 + c_3 e^{i\Delta kz}$$
$$M_- = c_4 + c_5 e^{-2i\Delta kz} + c_6 e^{-i\Delta kz}$$
$$M_z = c_7 e^{i\Delta kz} + c_8 e^{-i\Delta kz} + c_9$$

Gradient shifts transverse terms

$$M_+ = c_{10} e^{2i\Delta kz} + c_{11} e^{i\Delta kz} + c_{12} e^{-i\Delta kz} + c_{13} e^{-2i\Delta kz} + c_{14}$$
$$M_- = c_{15} e^{2i\Delta kz} + c_{16} e^{i\Delta kz} + c_{17} e^{-i\Delta kz} + c_{18} e^{-2i\Delta kz} + c_{19}$$
$$M_z = c_{20} e^{2i\Delta kz} + c_{21} e^{i\Delta kz} + c_{22} e^{-i\Delta kz} + c_{23} e^{-2i\Delta kz} + c_{24}$$

RF partitioning mixes components, transferring higher order terms in Δk to M_z

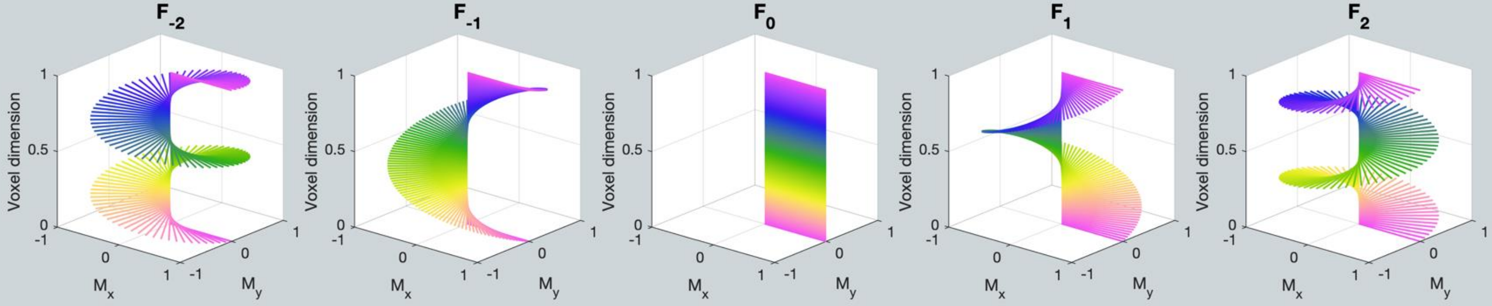
Configuration states

Magnetization naturally forms Fourier series when using constantly repeating sequence ($\mathbf{\Delta k}$)

$$M_+(\boldsymbol{r}) = \sum_{n=-\infty}^{\infty} \tilde{F}_n e^{\mathrm{in}(\boldsymbol{\Delta k}.\boldsymbol{r})} \qquad M_z(\boldsymbol{r}) = Re\left\{\sum_{n=0}^{\infty} \tilde{Z}_n e^{\mathrm{in}(\boldsymbol{\Delta k}.\boldsymbol{r})}\right\}$$

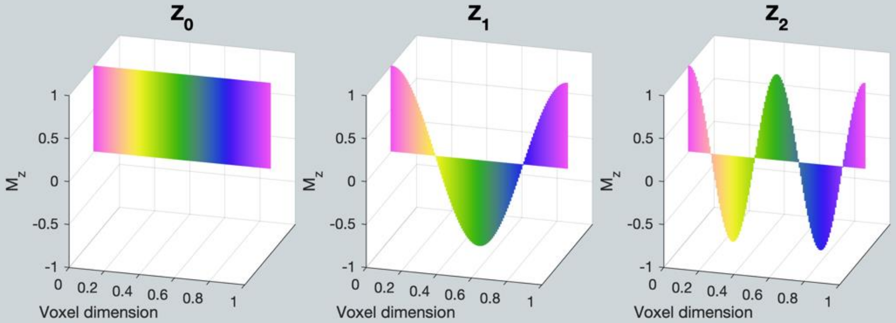
This is a Fourier *series* and not a continuous transform IF the pulse sequence regularly repeats gradients with area $\mathbf{\Delta k}$

Express magnetization as a sum over *configurations* – Fourier expansion in $\boldsymbol{\Delta k}$



Signal = F_0

$$F_0 = \int_{voxel} M_+ dV$$



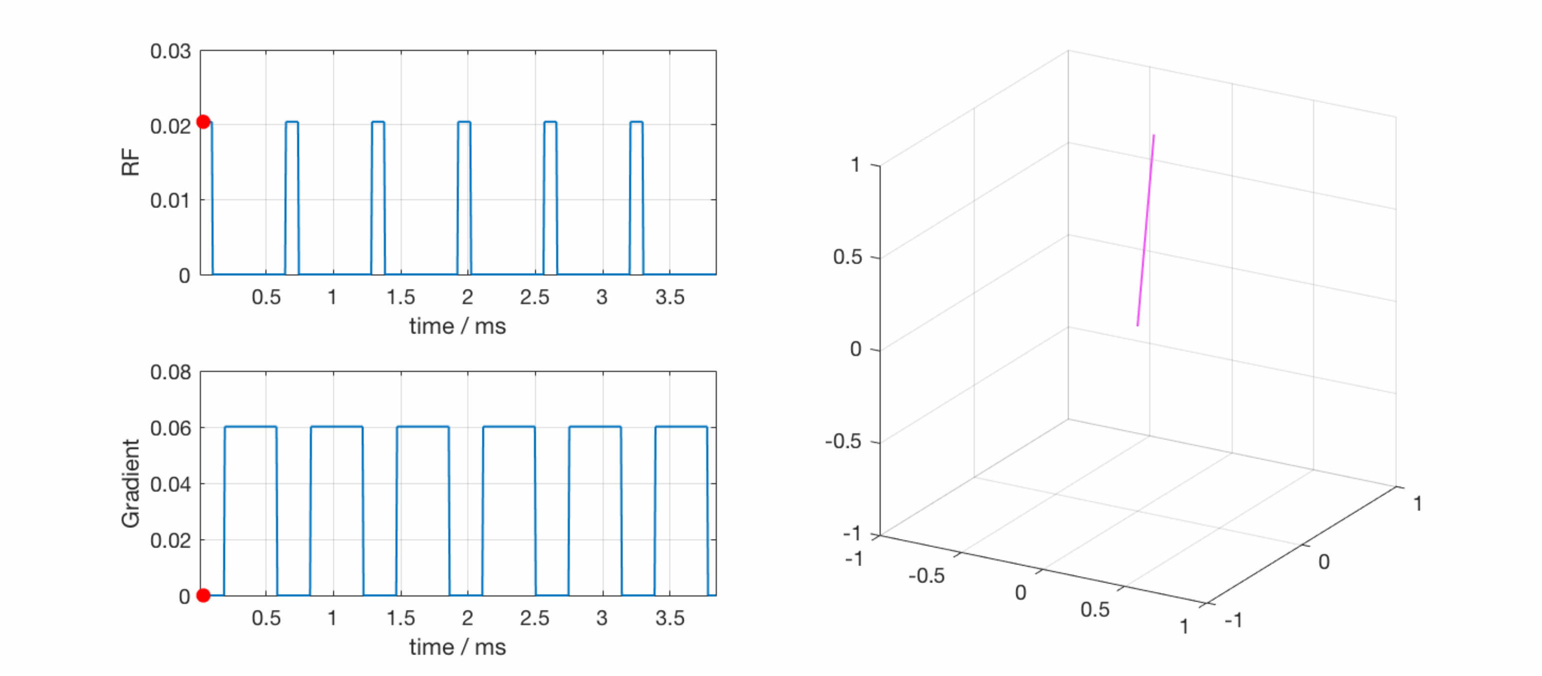
Technically for $\Delta k = 2\pi$ (gradient causes 2π phase over voxel)

F_n are 'helices' where positive and negative n change the sense of rotation

Z_n are cosine modulated longitudinal magnetization

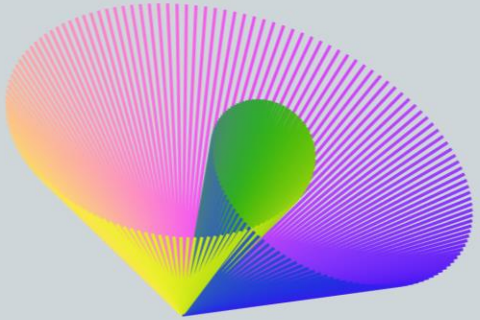
Configurations – for visualisation

Recall the isochromat distribution after a sequence of pulses and gradients….



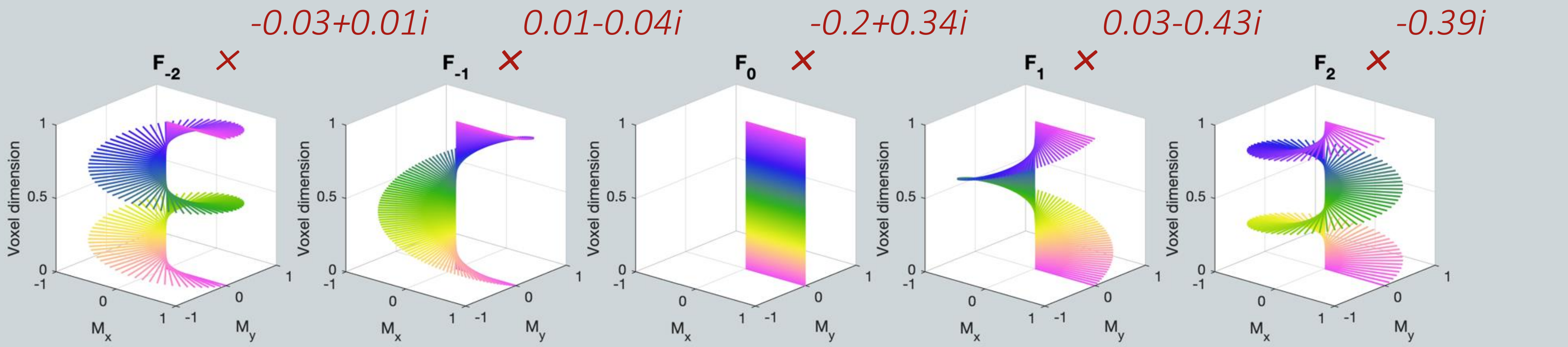
Configurations – for visualisation

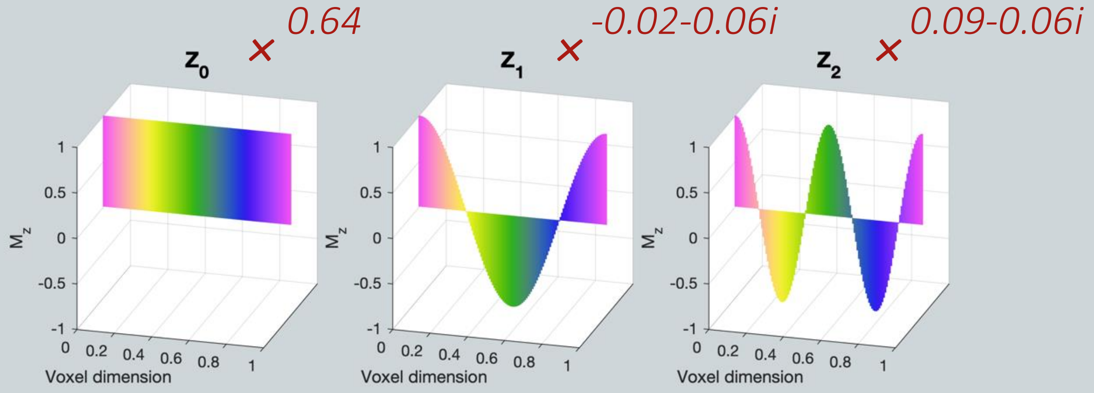
Recall the isochromat distribution after a sequence of pulses and gradients….



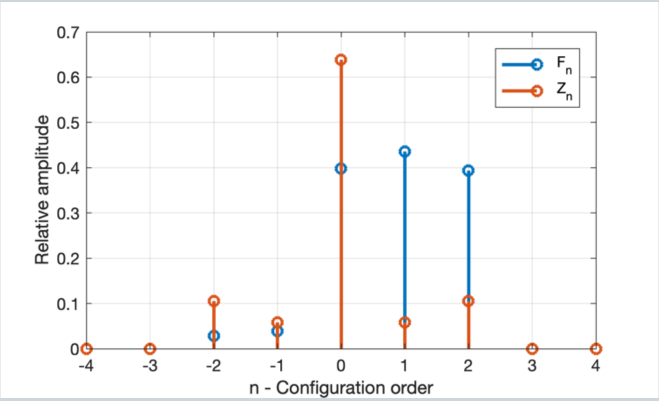
After 3 RF excitations…

=



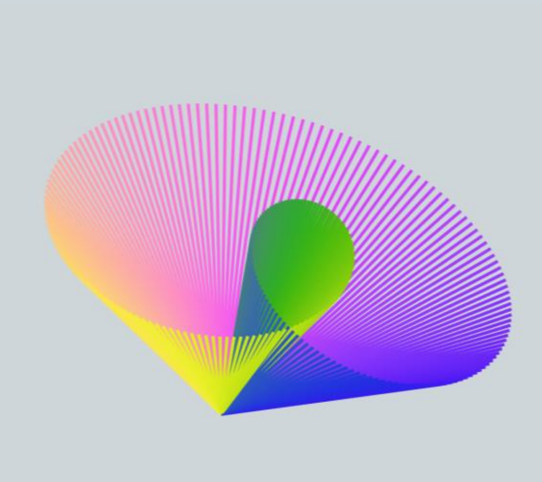


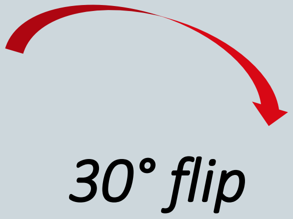
Isochromat distribution can be viewed as a sum over magnetization in different configurations, as illustrated by this example

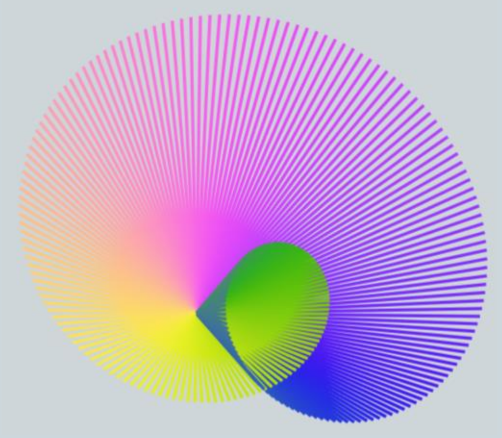


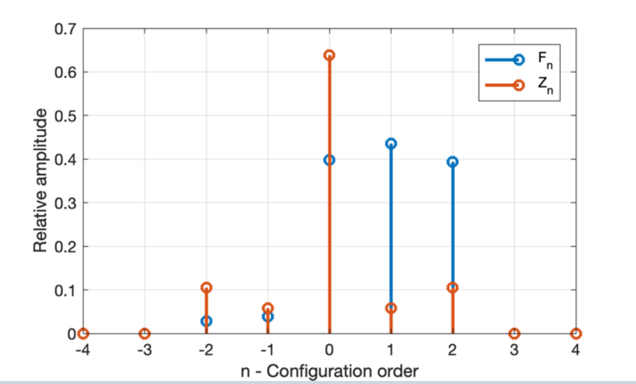
Configurations – effect of RF pulses

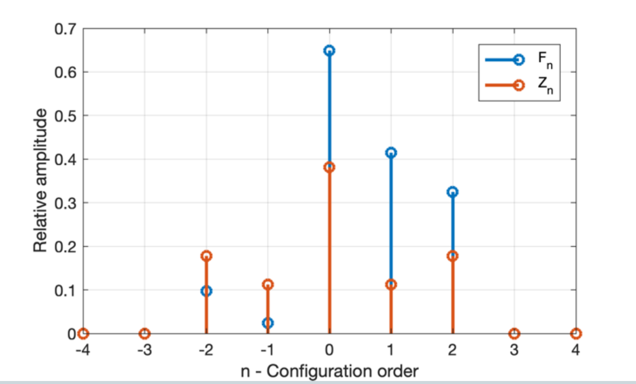
RF pulses rotate all isochromats by the same angle…









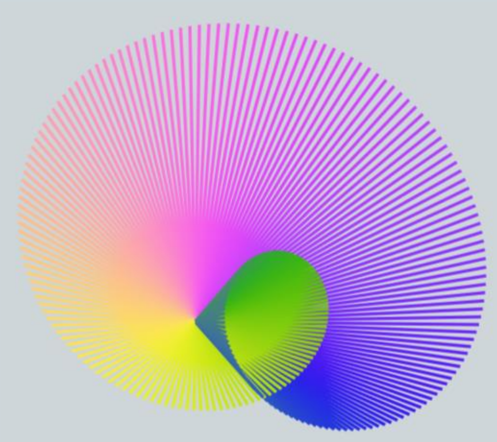


- The effect of the rotation is to alter signal distribution over configurations
- **RF pulses only mix up configurations of same order**

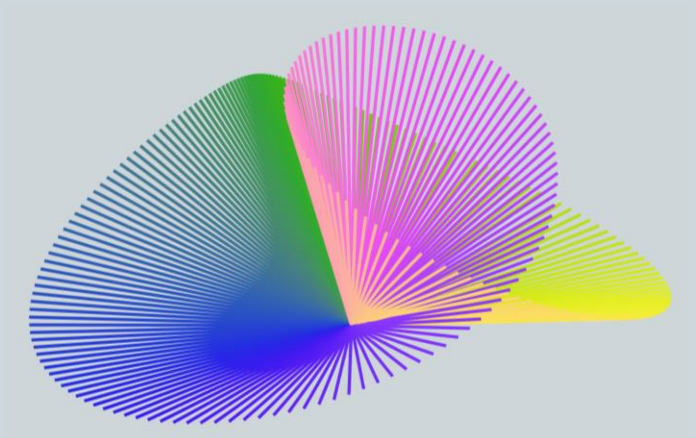
  i.e. F_{-2}, F_2, Z_2 mix etc
- This is a consequence of the distribution only rotating but not changing

Configurations – effect of gradients

Gradients rotate each isochromat around z by different amounts



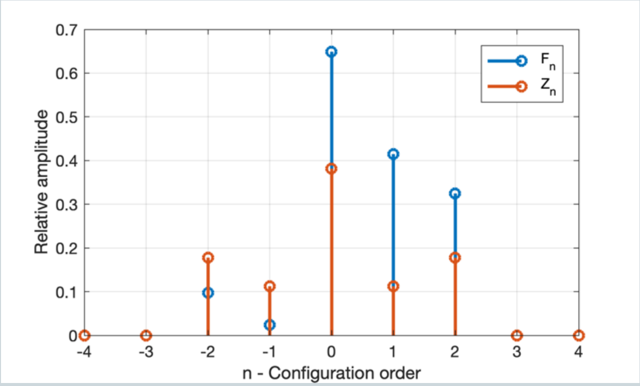
Rotate about z by 0° to 360°

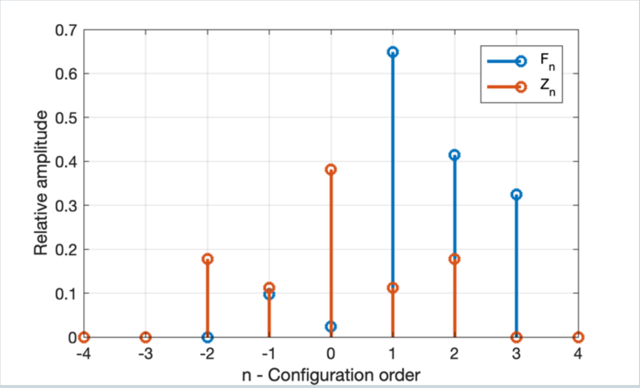


- This **does** change the 'shape' of the distribution
- Transverse configurations **shift to higher orders**

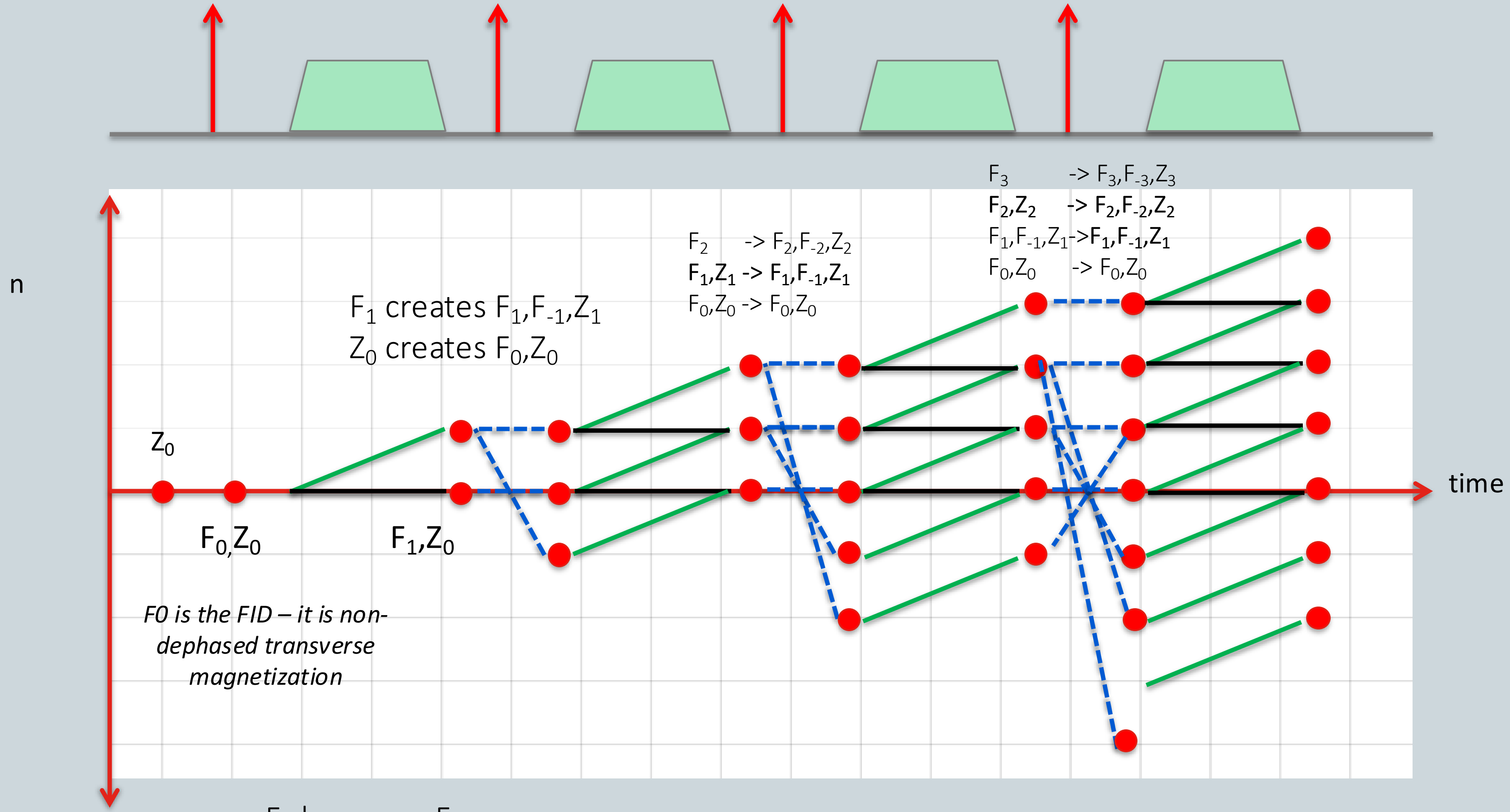
$$F_n \rightarrow F_{n+1}$$

$$Z_n \rightarrow Z_n$$





Phase Diagram

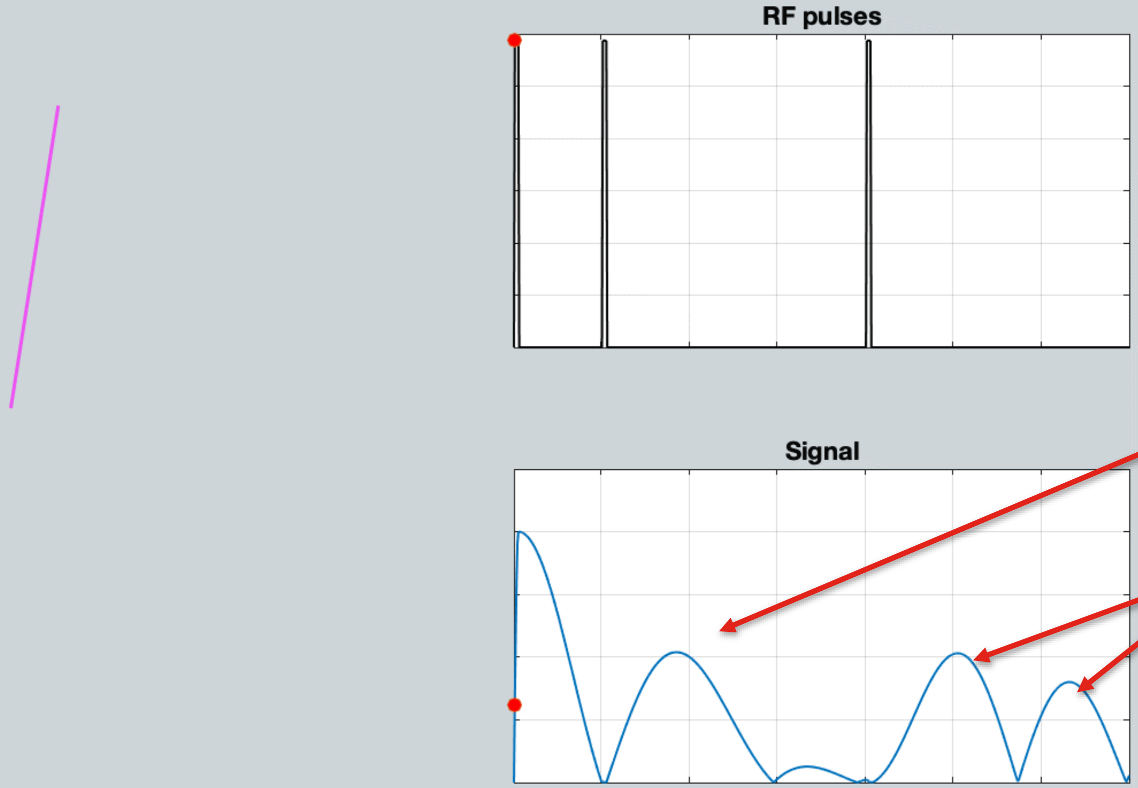


F_0 becomes F_1
Z_0 unaffected

Gradients: All F-states increment by +1
All Z states unaffected

Phase diagrams: intuitive analysis of sequences

E.g. Which echoes do we expect from this 3 pulse sequence (90°-90°-90°)?



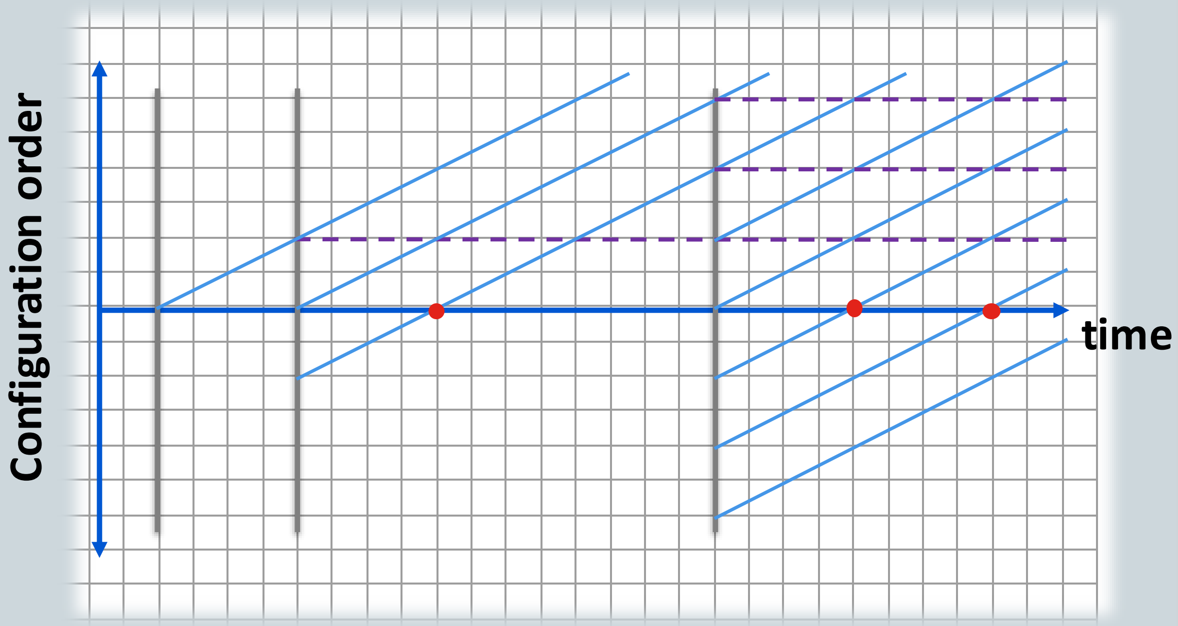
Spin echo from first 2 pulses

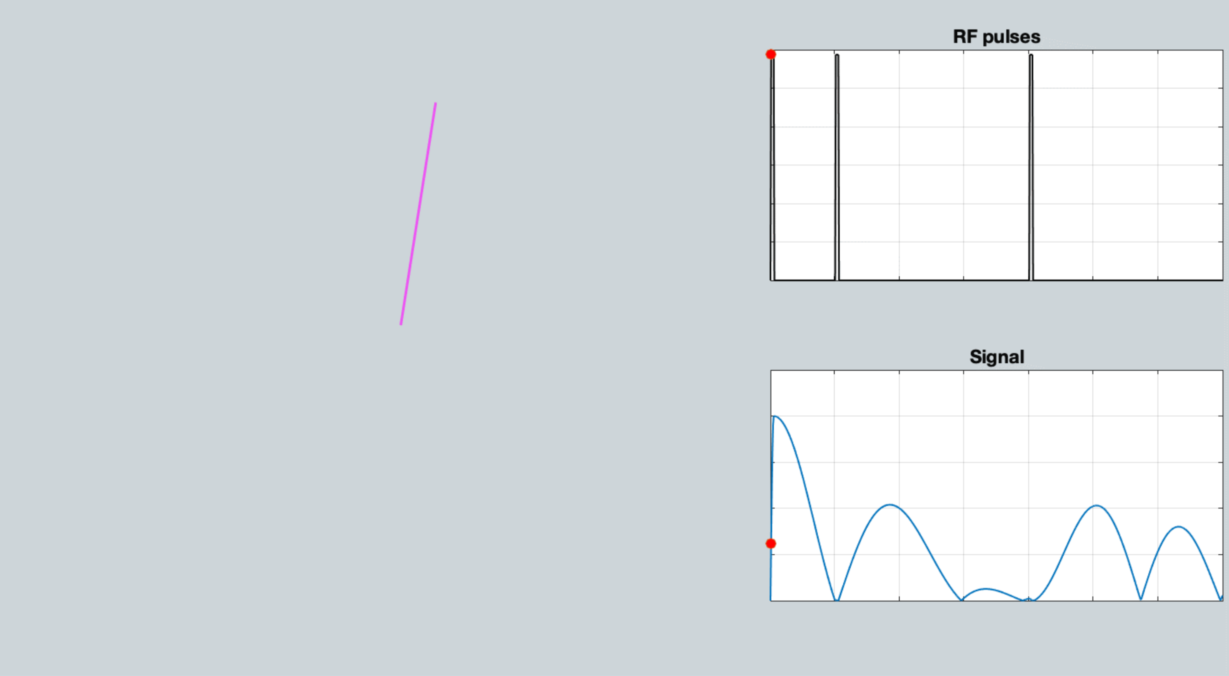
What are these two echoes?

Assume there is a constant background gradient causing spin dephasing

Phase diagrams: intuitive analysis of sequences

E.g. Which echoes do we expect from this 3 pulse sequence (90°-90°-90°)?



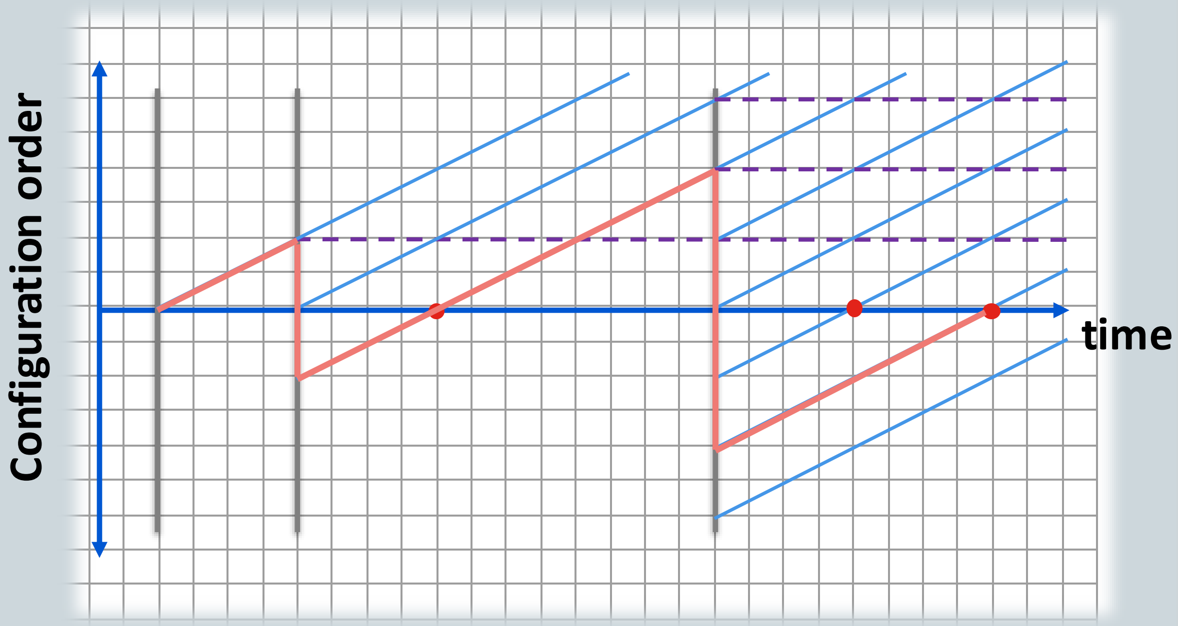


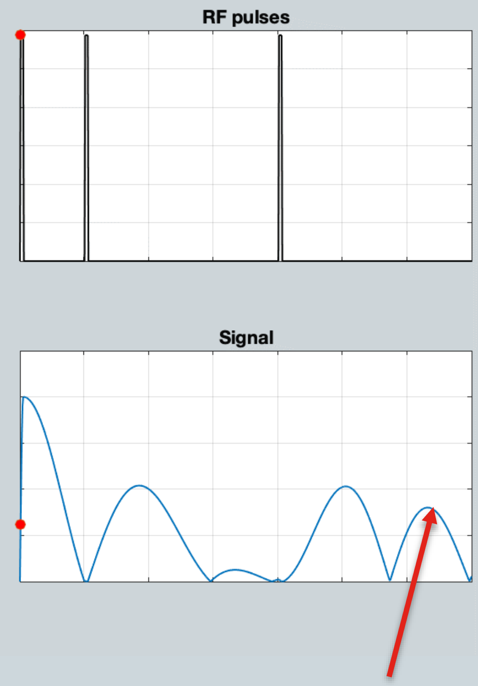
KEY CONCEPT: F_0 corresponds to observed signal

Red dots = echoes, when we have a zero-order configuration (in-phase magnetization)

Phase diagrams: intuitive analysis of sequences

E.g. Which echoes do we expect from this 3 pulse sequence (90°-90°-90°)?





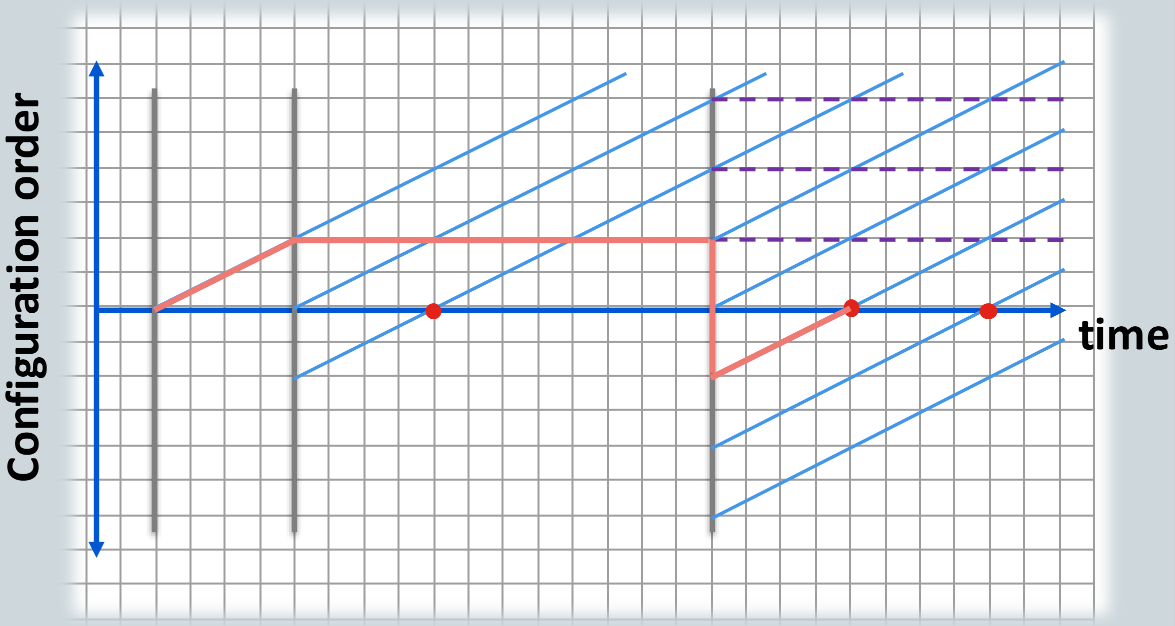
Refocused Spin Echo

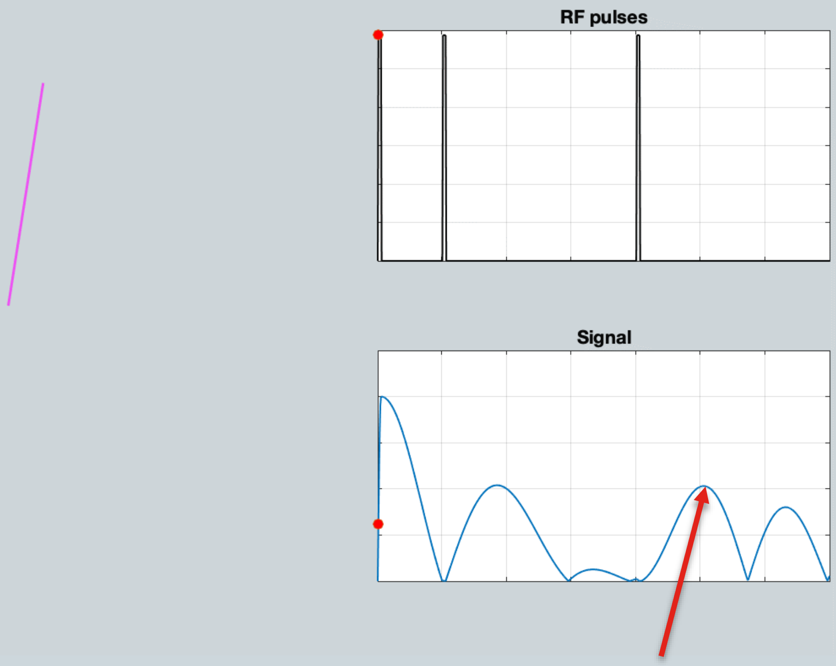
KEY CONCEPT: F_0 corresponds to observed signal

Spin echo from first two pulses is refocused again by the third pulse…

Phase diagrams: intuitive analysis of sequences

E.g. Which echoes do we expect from this 3 pulse sequence (90°-90°-90°)?



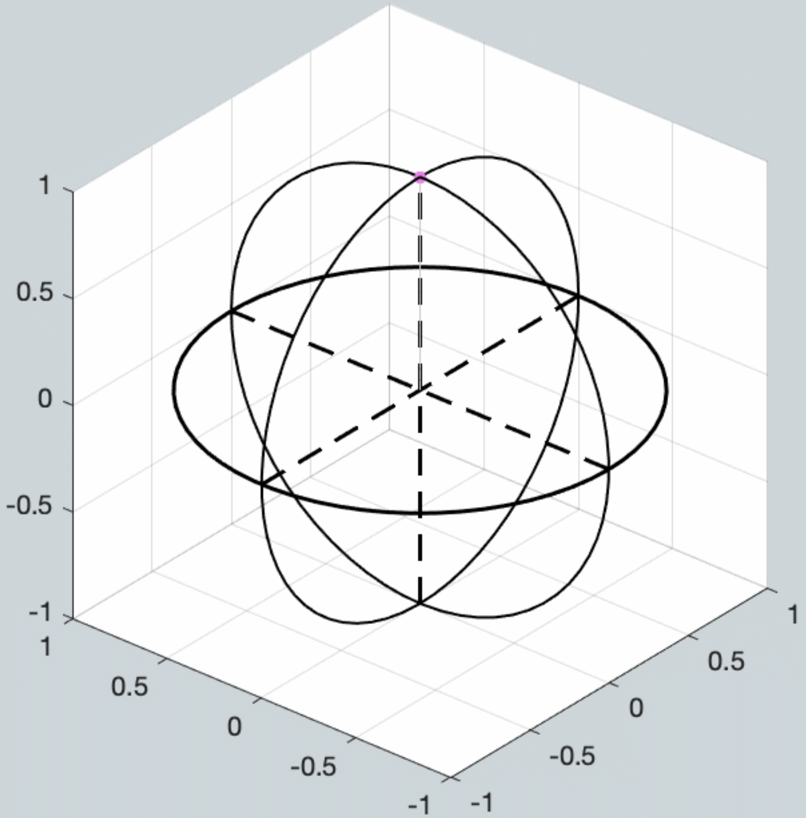


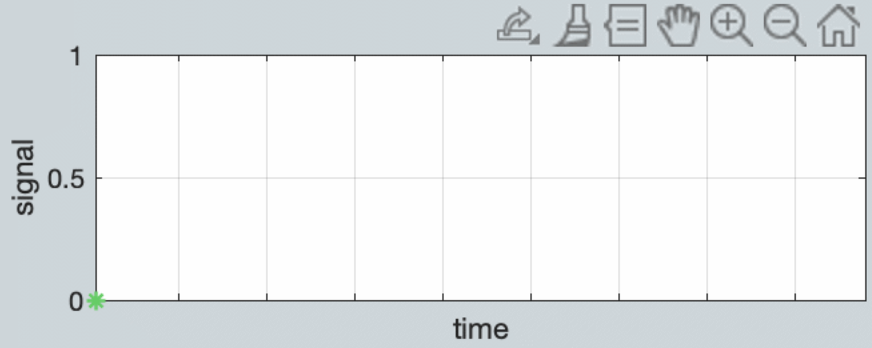
Stimulated echo

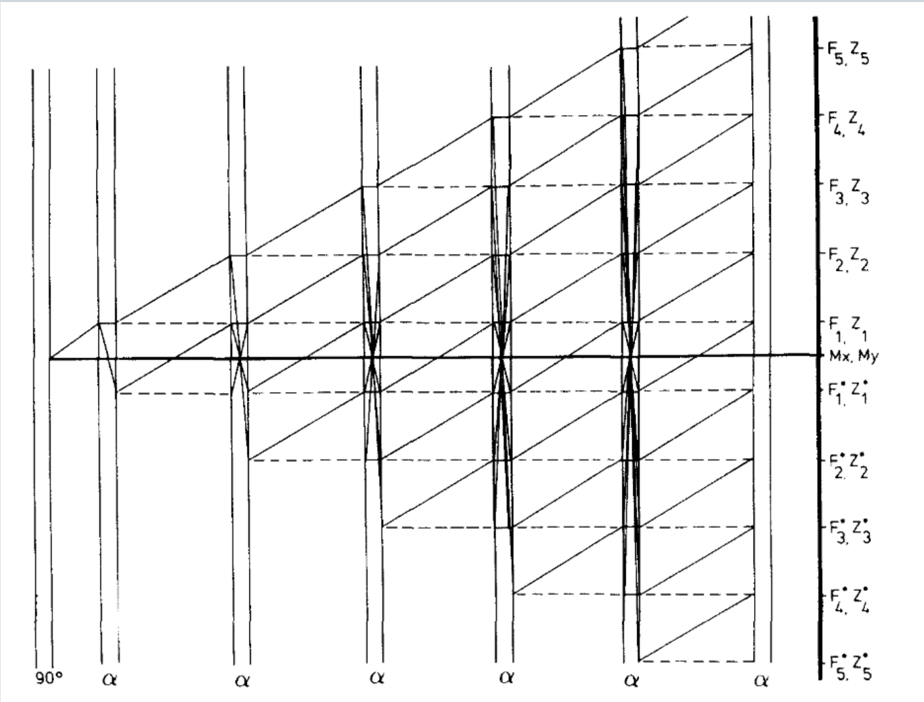
KEY CONCEPT: F_0 corresponds to observed signal

Some magnetization is converted to Mz by second pulse and then refocused by third

Carr Purcell Meiboom Gill (CPMG) sequence…





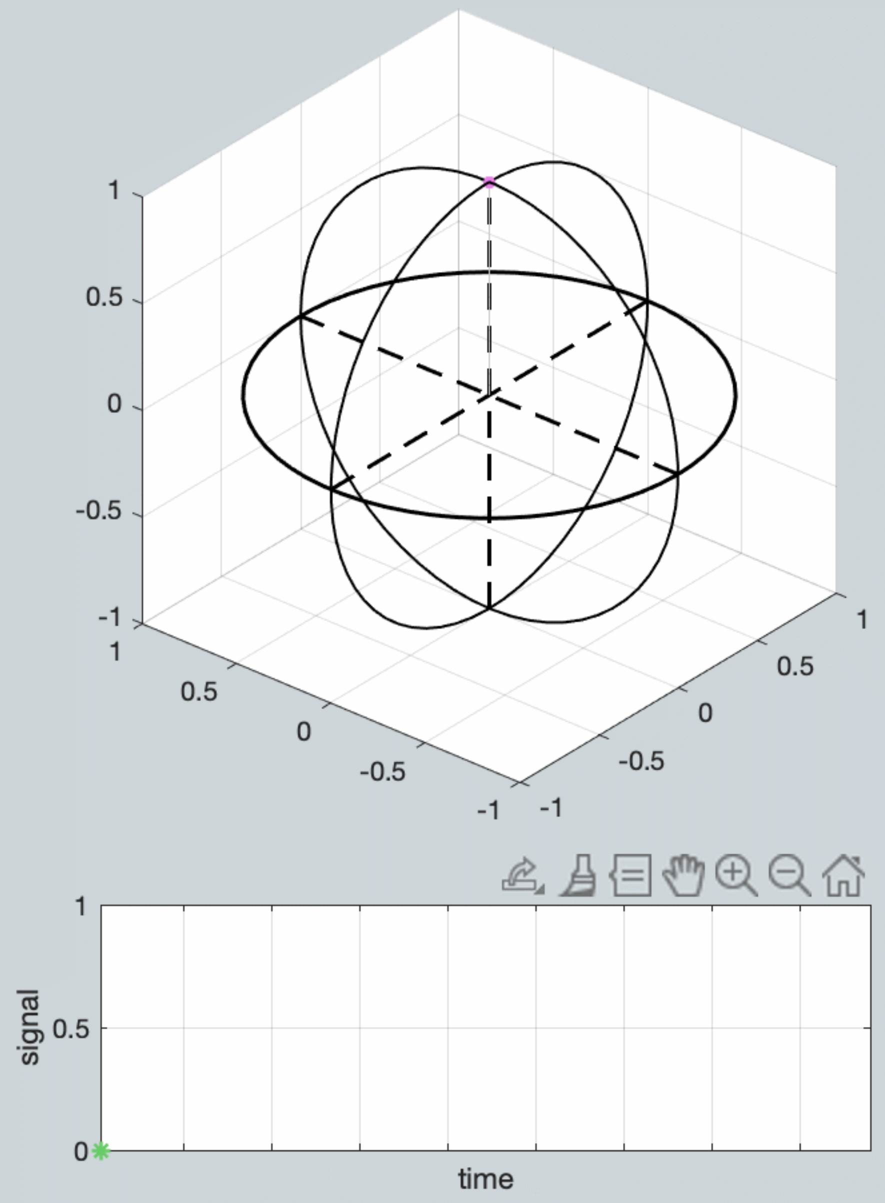


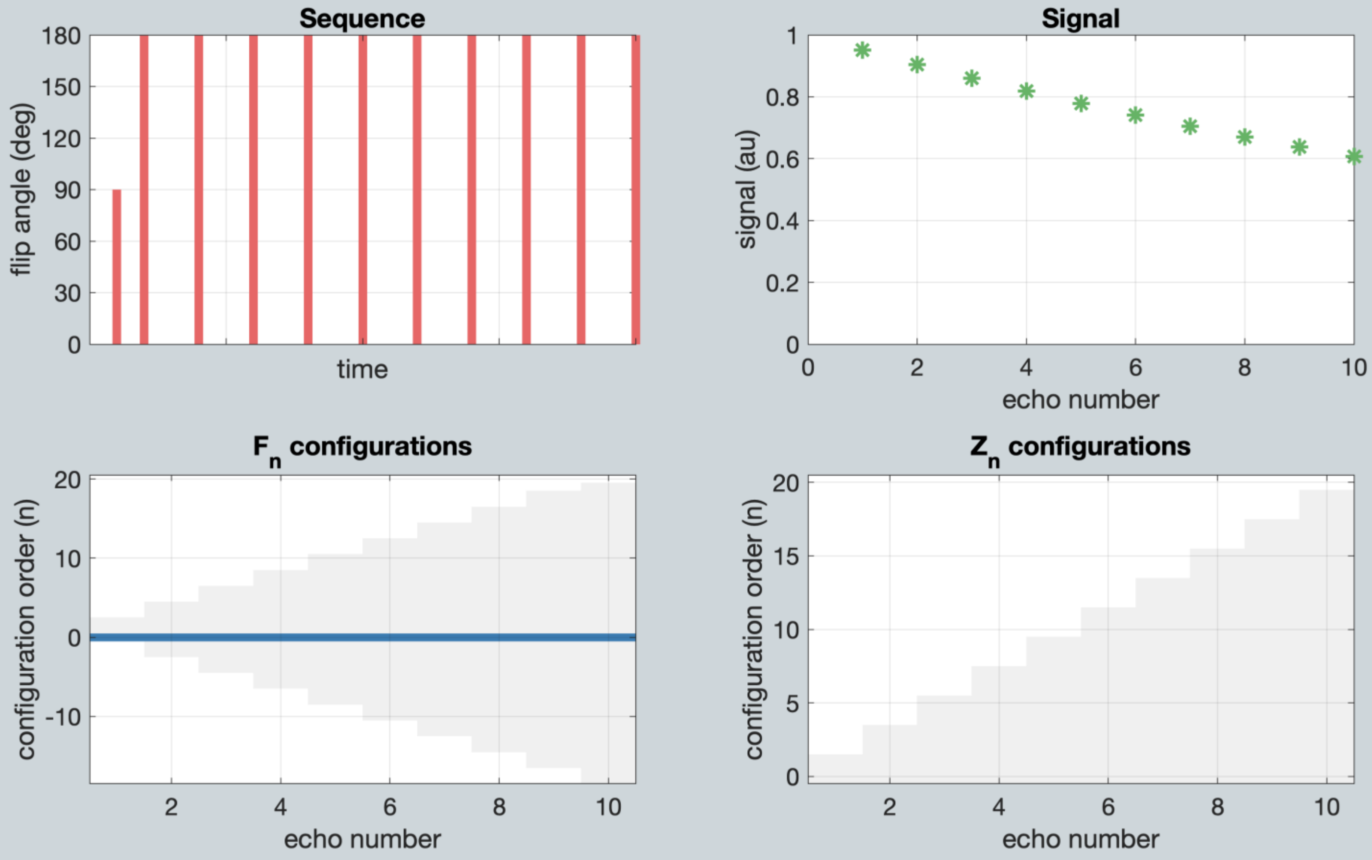
Phase diagram from Hennig 1988

Phase diagram shows all possible configurations

Not all are ‘populated’ - **this depends on the sequence**

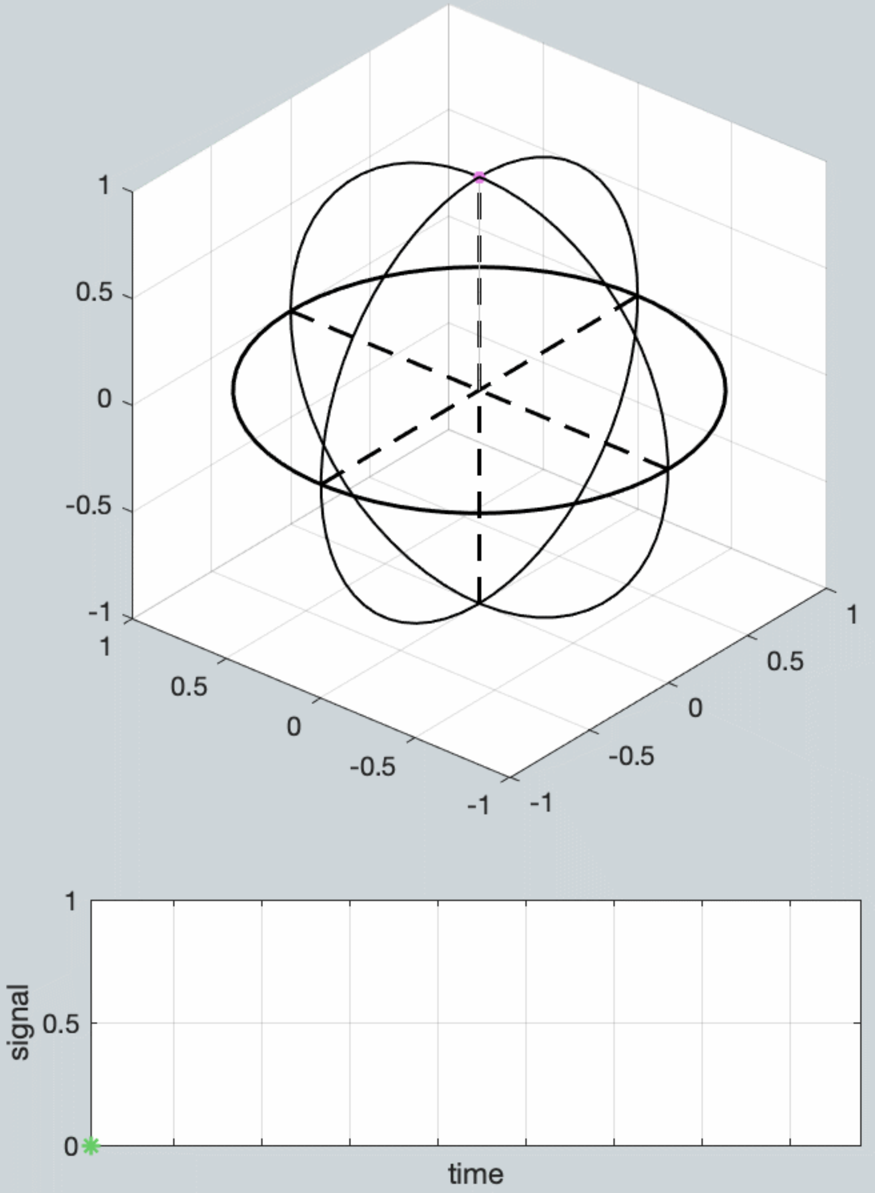
Carr Purcell Meiboom Gill (CPMG) sequence…

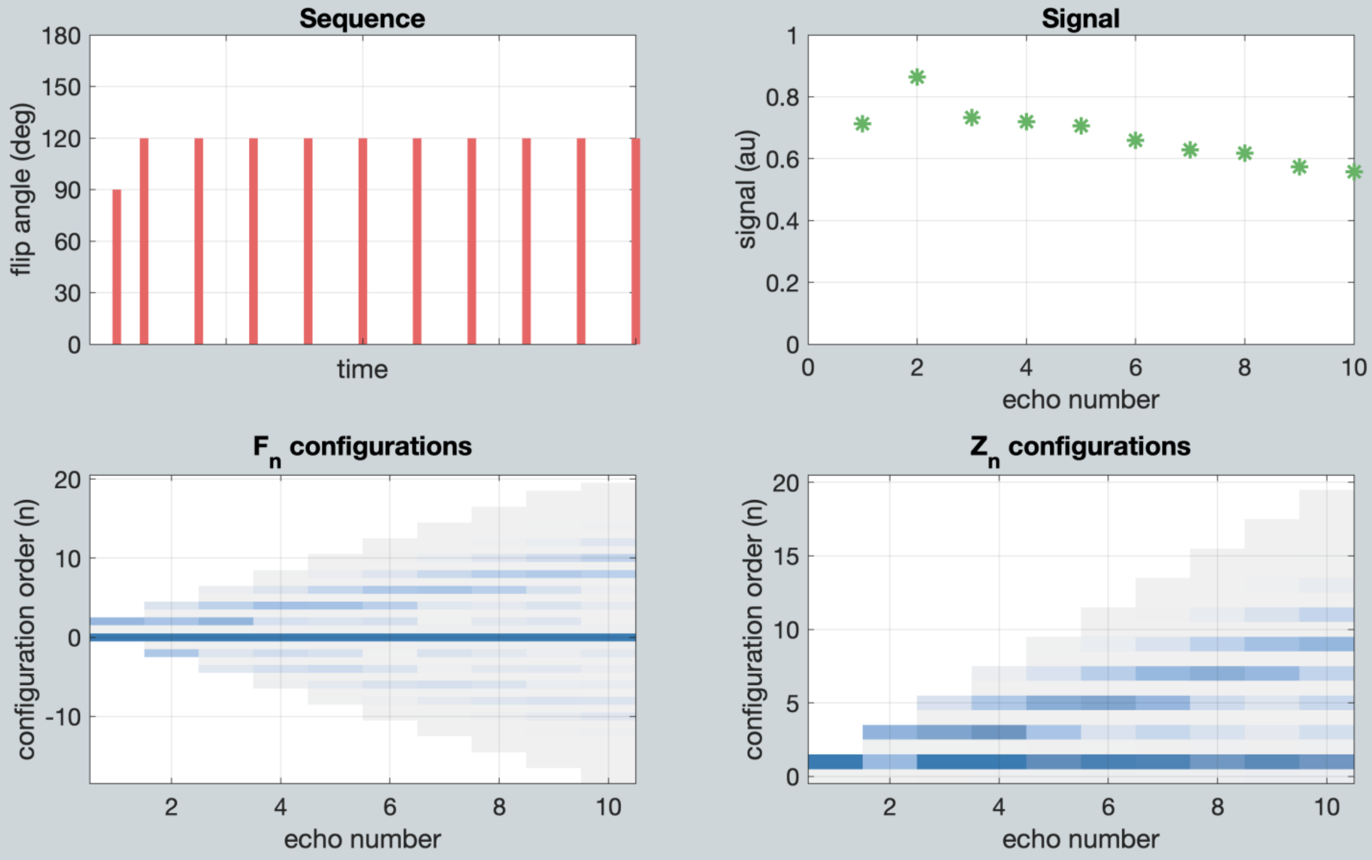




Classic 90-180*n sequence results in fully refocused magnetization – very simple phase graph!

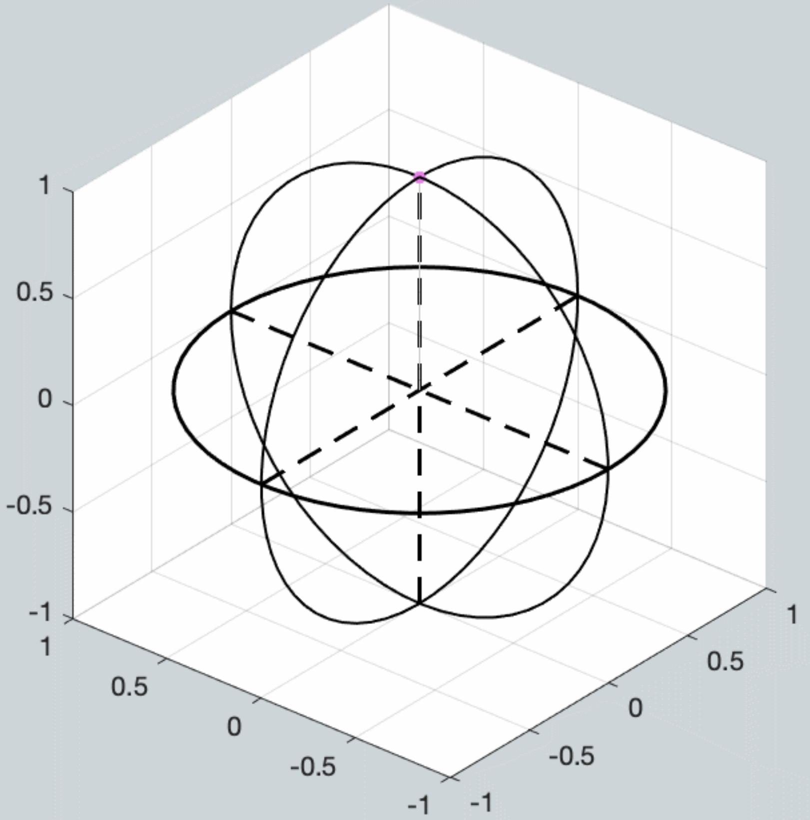
Carr Purcell Meiboom Gill (CPMG) sequence…

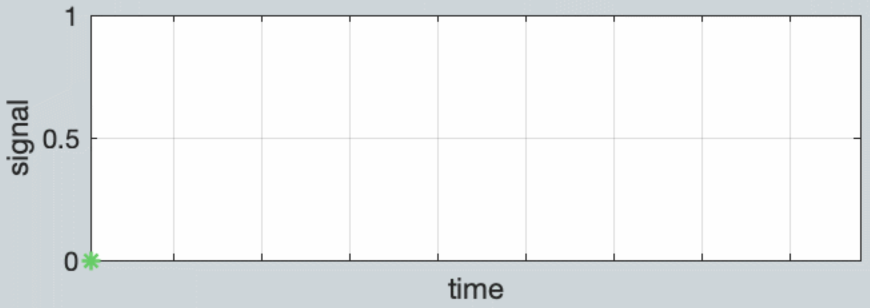


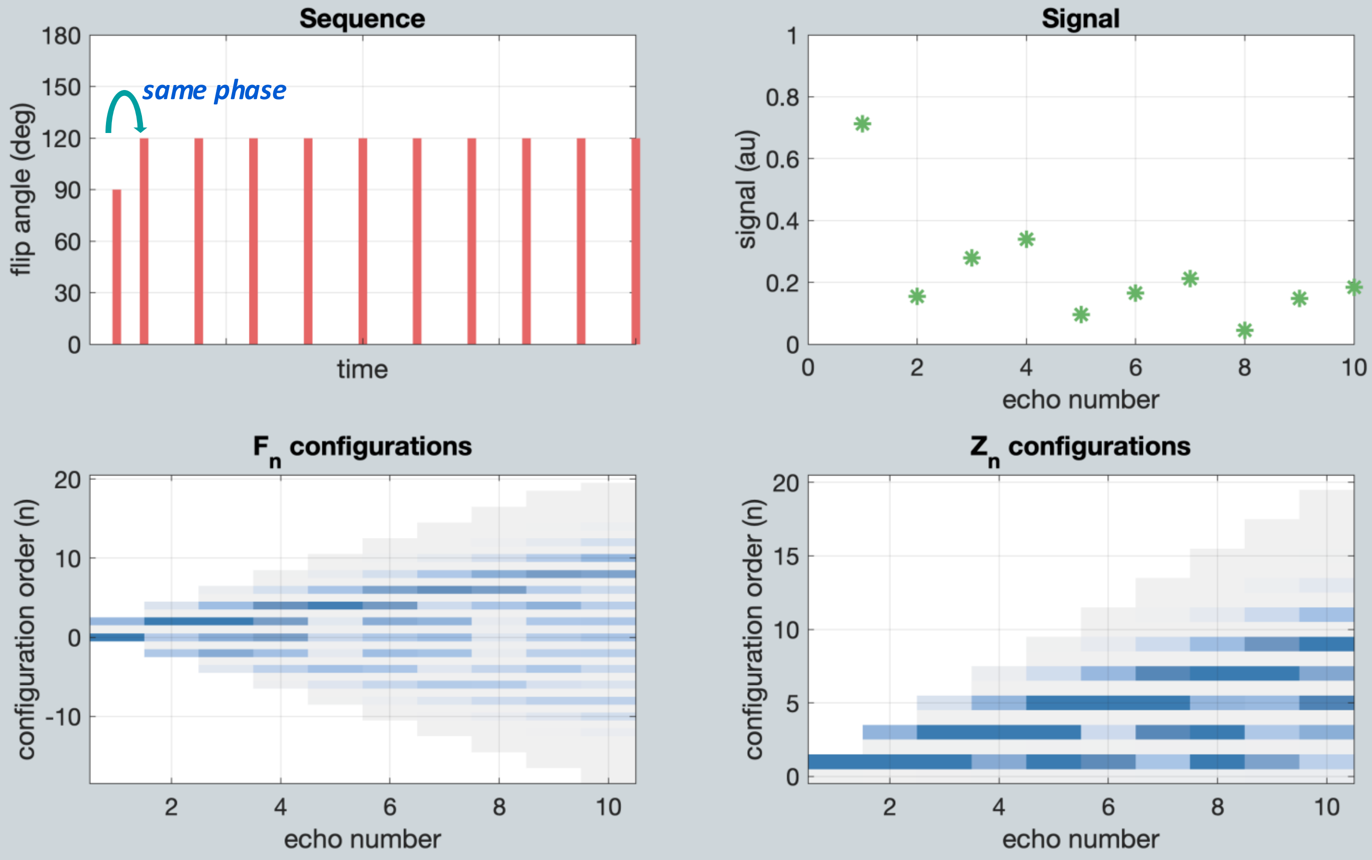


120° refocusing – now the distribution is more complex!

Carr Purcell Meiboom Gill (CPMG) sequence…

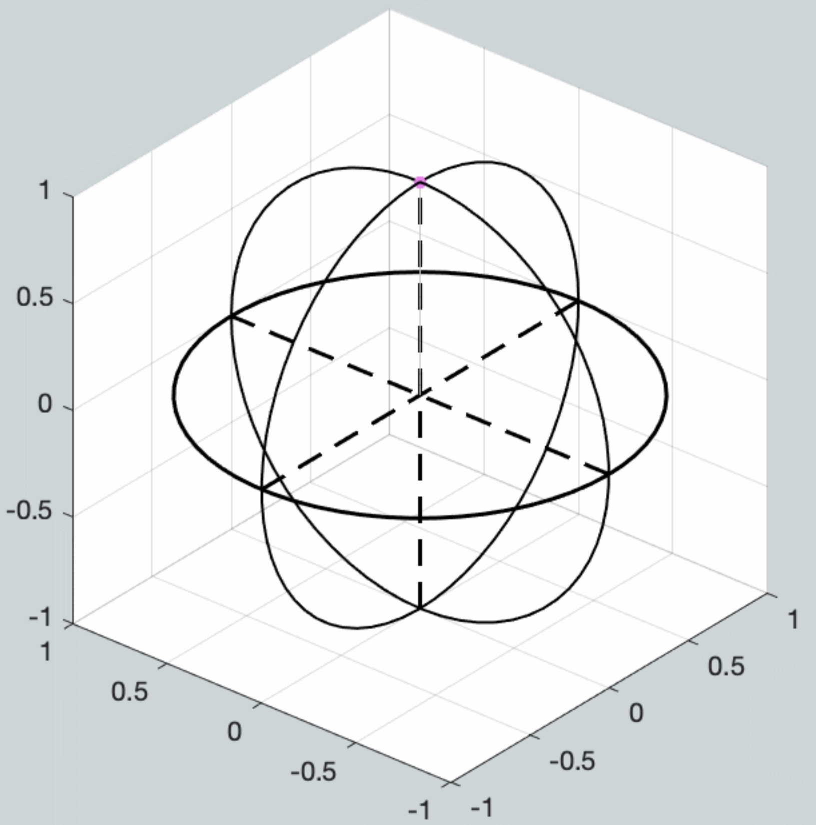


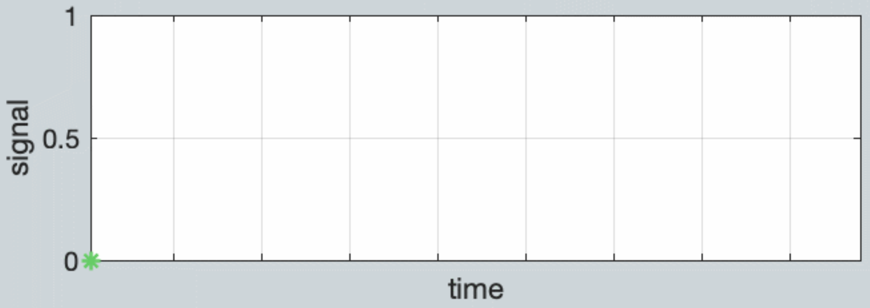


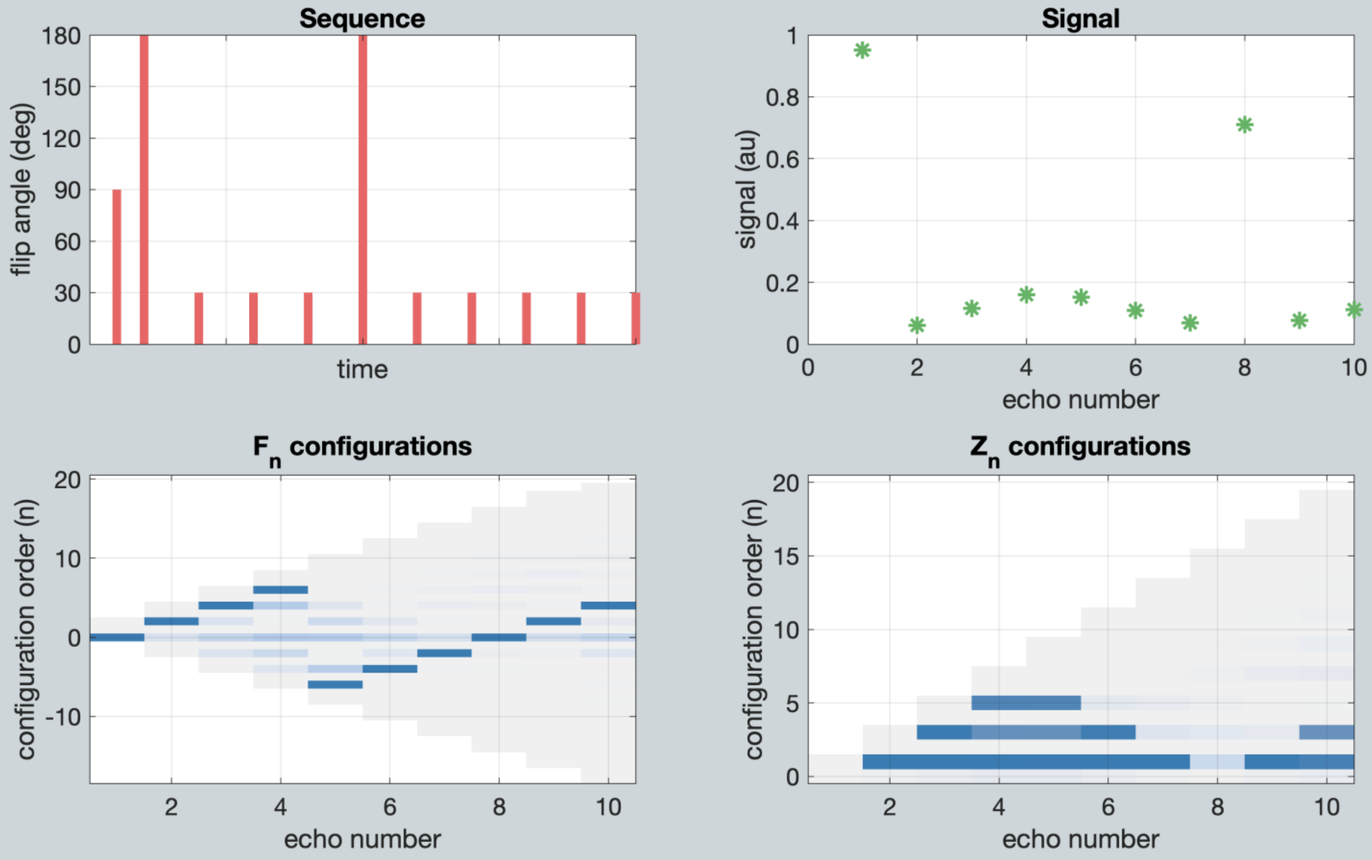


Remove the 90° phase between excitation & refocusing ('non-CPMG')

Carr Purcell Meiboom Gill (CPMG) sequence…

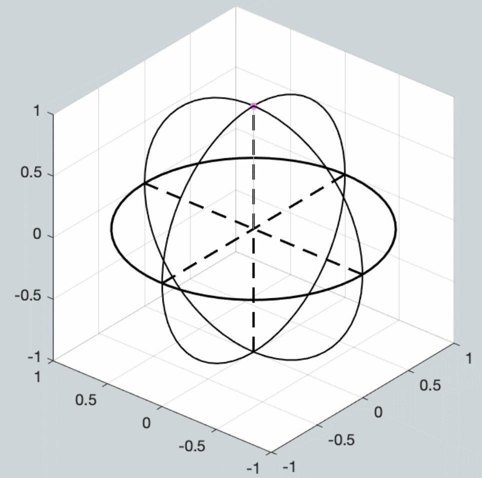


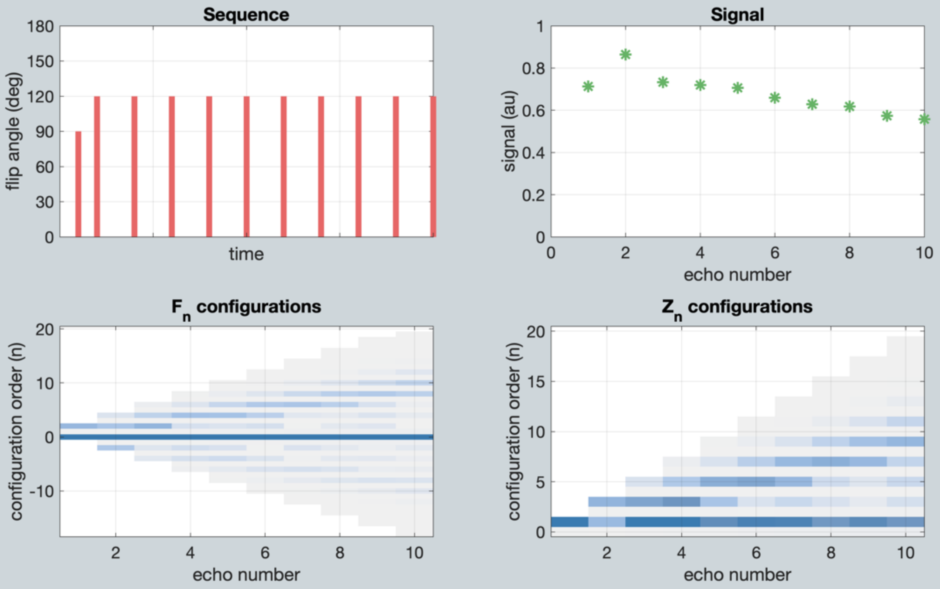




Hyperecho! (Hennig & Scheffler, 2001)

CPMG sequence – take home points



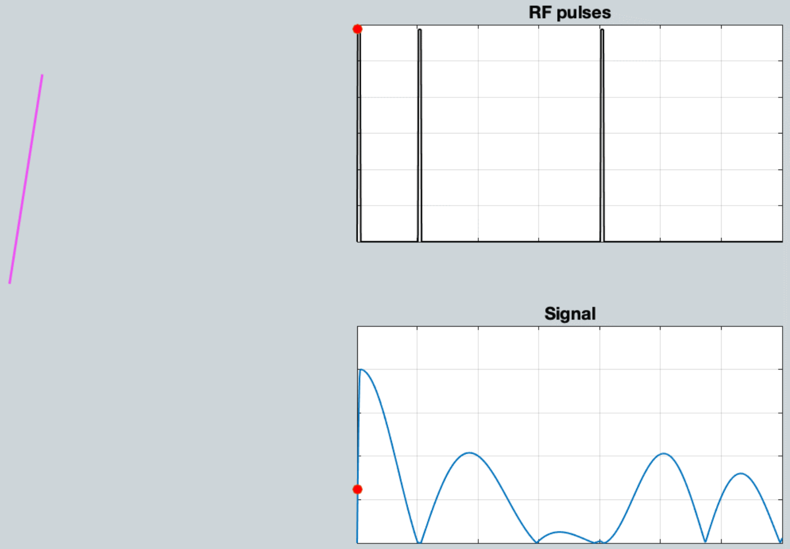


- EPGs are a very good way to understand these sequences
- *Hennig developed the framework for this purpose*
- Amount of magnetization in each configuration tells us more than the very chaotic isochromat distribution

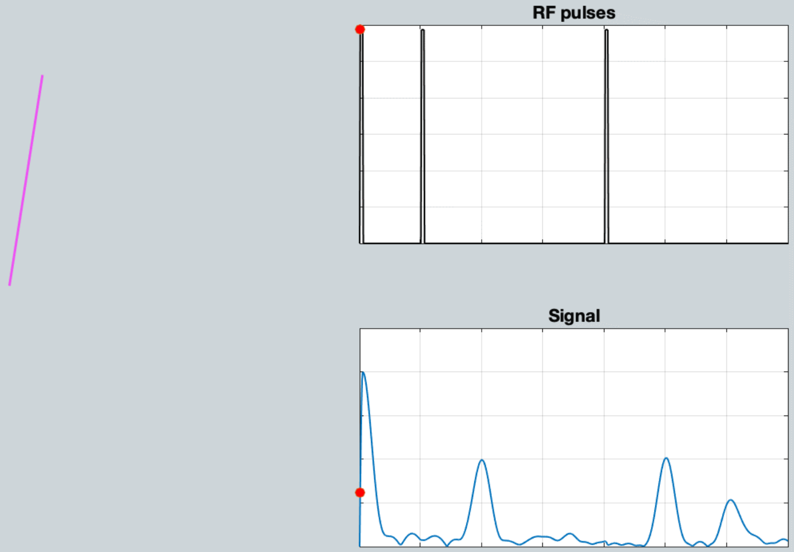
A note on echo shapes…

Uniform frequency distribution => unfamiliar echo shapes…

Uniform frequency distribution (classic EPG)

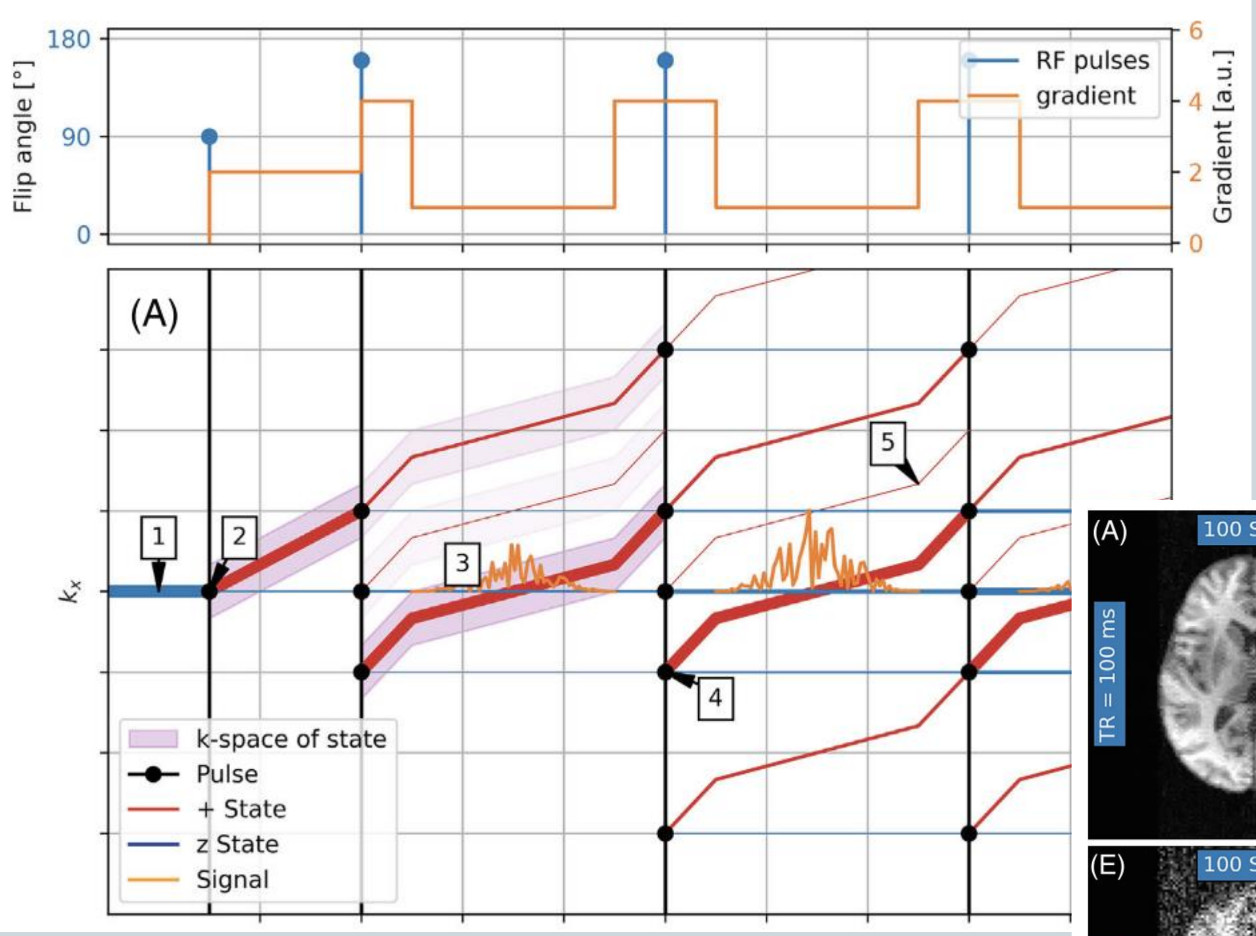


Gaussian frequency distribution



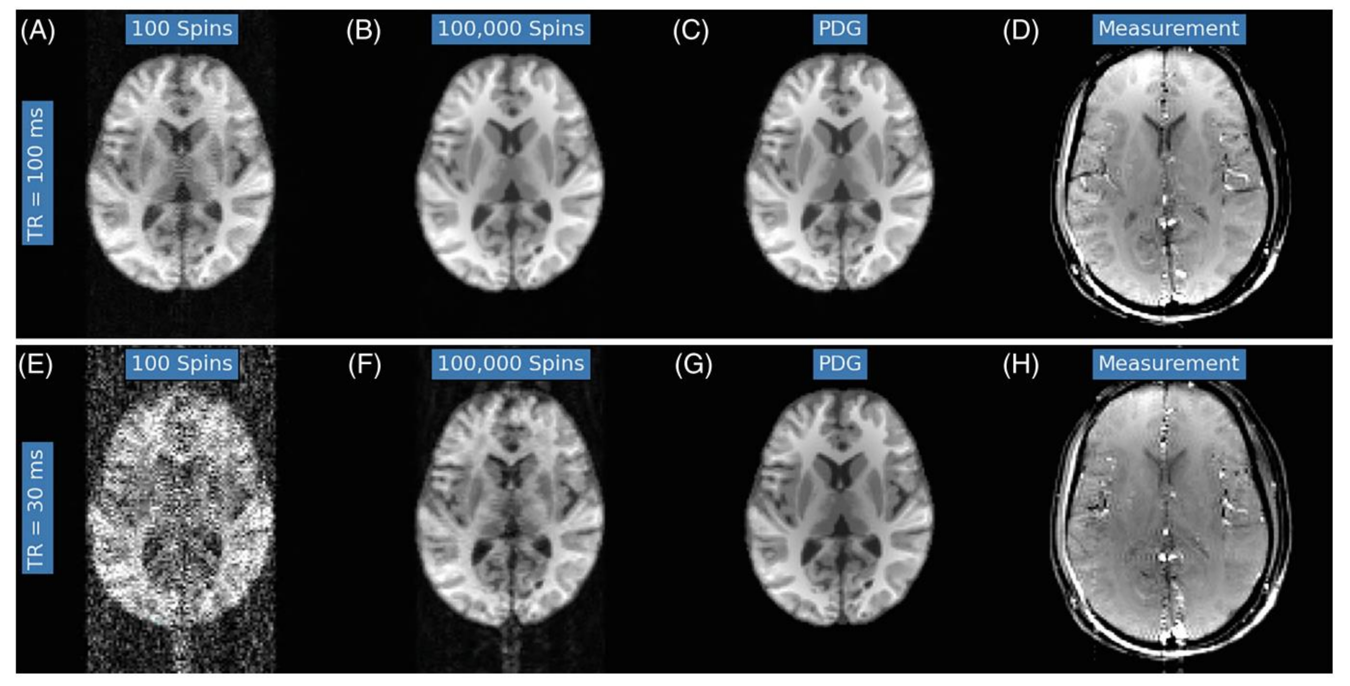
…. Randomizing the isochromat distribution leads to more familiar echoes. This is actually beyond the usual conception of EPG…

Phase distribution graphs…



Generalization of EPGs to include:

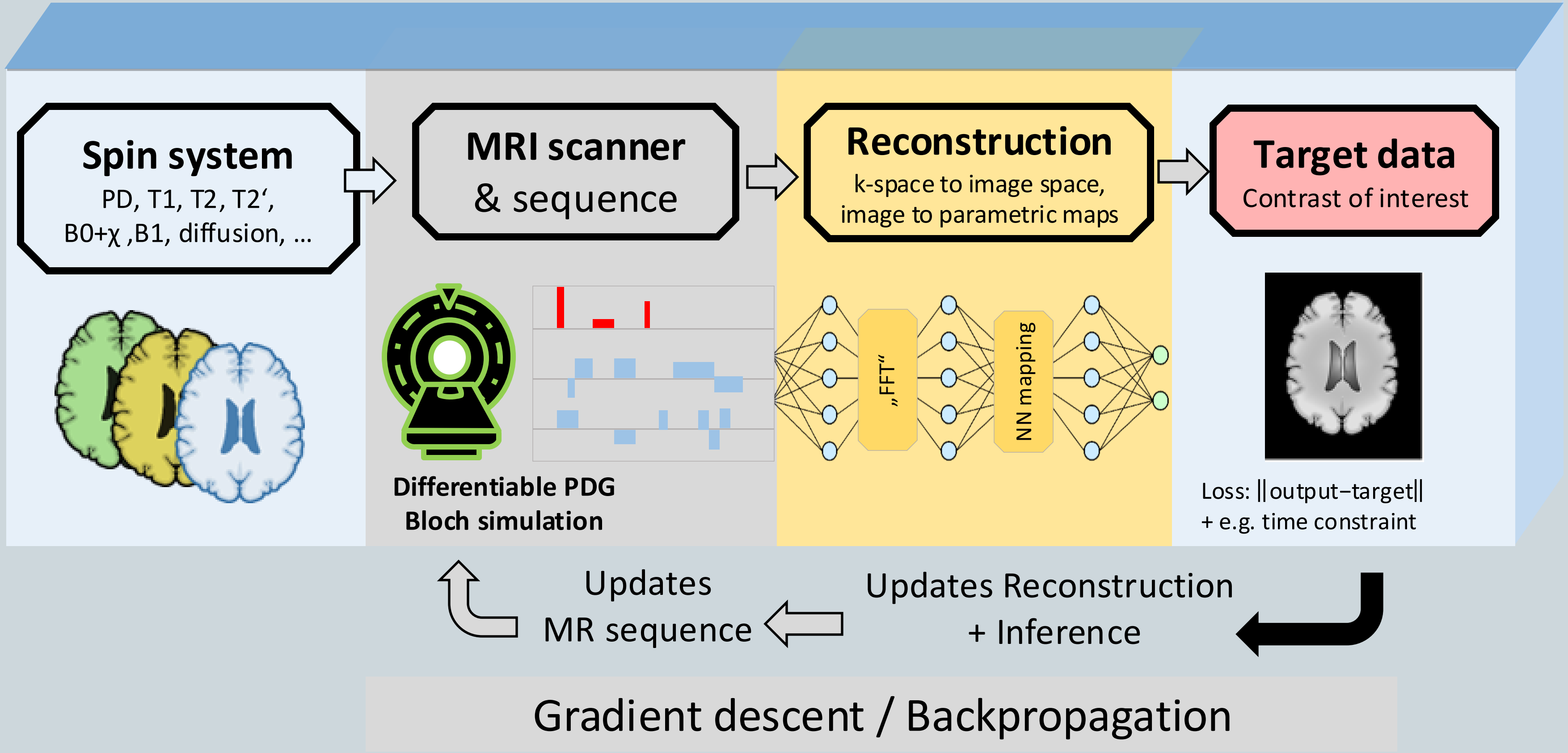
- Spatial signal distribution
- Non-uniform sub-voxel frequency distribution
- Irregular gradients (direction and duration)



Endres et al, MRM 2024

Phase distribution graphs…

- Fast, **differentiable**, **open source** simulations



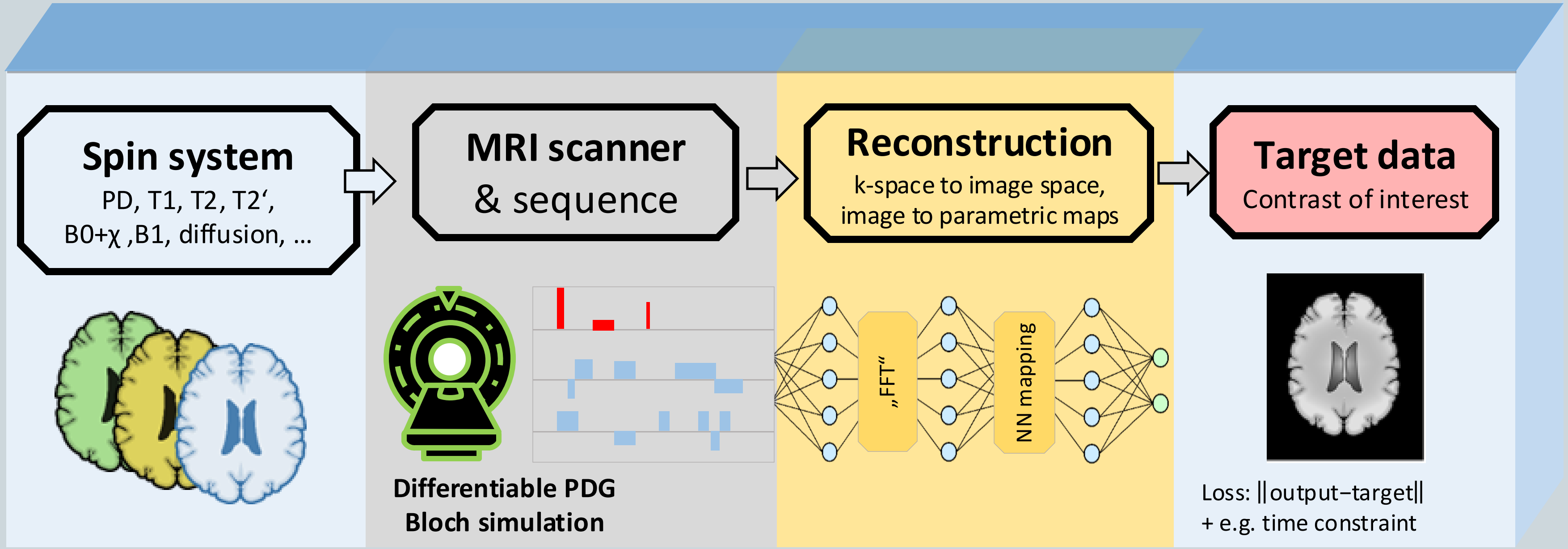
Differentiable Digital Twin of the MRI process

Other work from Endres & co at this ESMRMB:

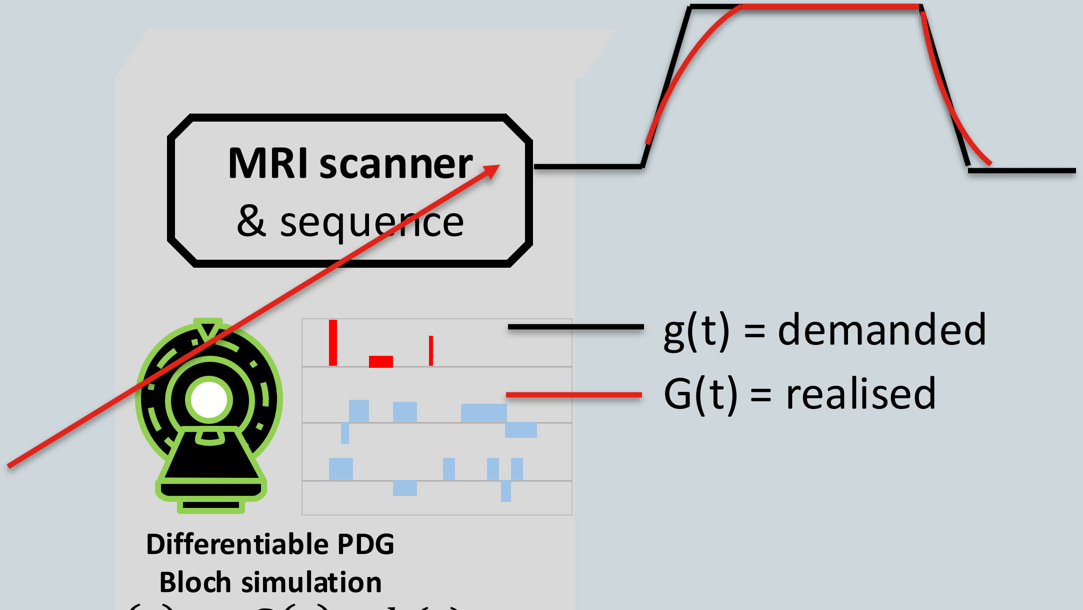
0423, 0421, 0121, 0028

‘MRzero’ Loktyushin et al, MRM 2021

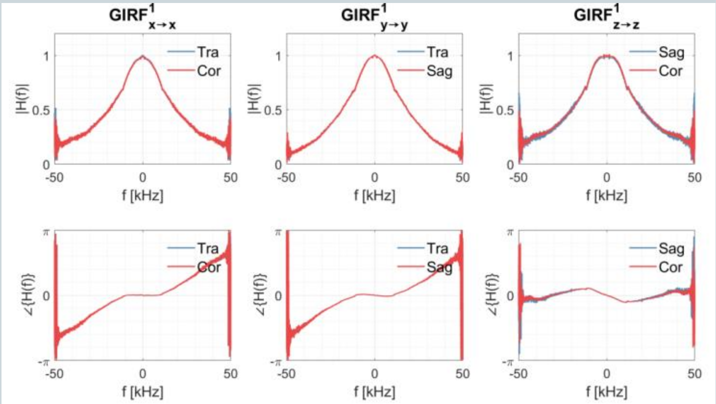
Non-ideal system optimization



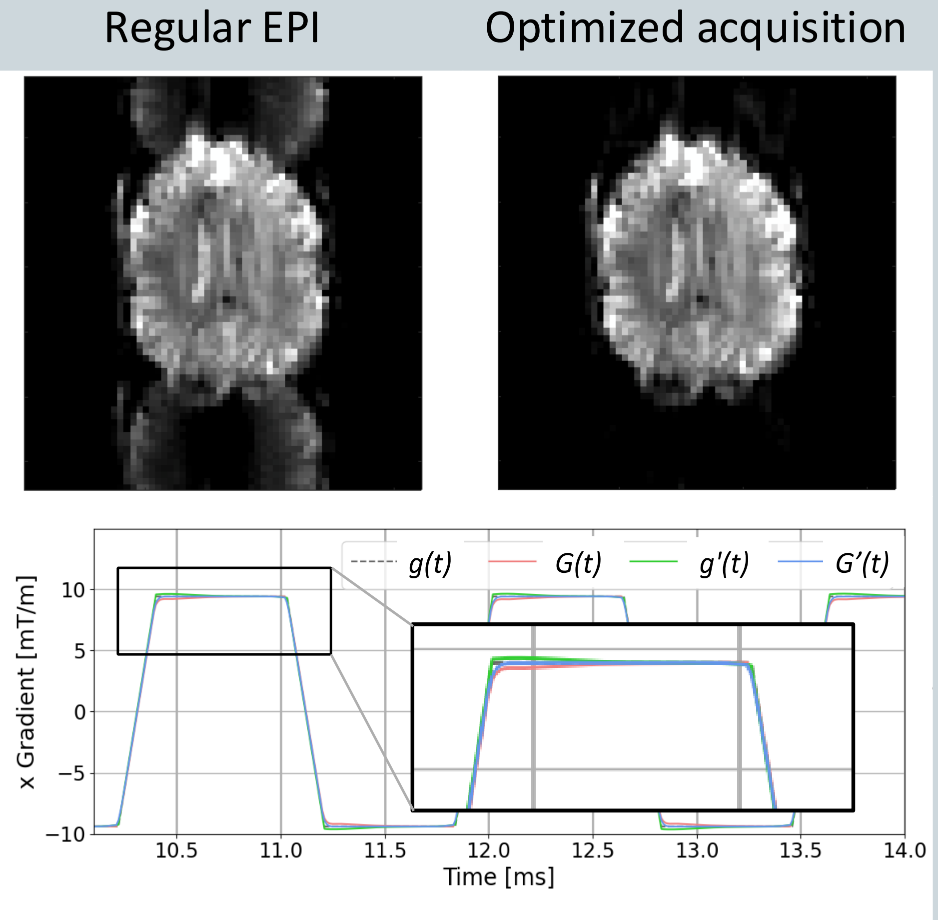
Non-ideal system optimization



$$g(t) = G(t) * h(t)$$



Measured gradient impulse response function = ‘hardware twin’



Optimize sequences using efficient **signal** model (DPG) *and* empirical hardware model (e.g. GIRF)

West et al, 10.48550/arXiv.2403.17575

Summary

Simulations are everywhere in this field!

Thank you!

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Dr Shaihan Malik

Shaihan.malik@kcl.ac.uk