FitzHugh-Nagumo-ing the Hodgkin-Huxley model with RNNs

Reproducing with a lower dimensional model

CAMP 2022

Team (still deciding)

The Hodgkin-Huxley model, and friends

$$I = C_m \frac{\mathrm{d}V_m}{\mathrm{d}t} + \bar{g}_\mathrm{K} n^4 (V_m - V_K) + \bar{g}_\mathrm{Na} m^3 h (V_m - V_{Na}) + \bar{g}_l (V_m - V_l),$$

$$\frac{dn}{dt} = \alpha_n(V_m)(1-n) - \beta_n(V_m)n$$

$$\frac{dm}{dt} = \alpha_m(V_m)(1-m) - \beta_m(V_m)m$$

$$\frac{dh}{dt} = \alpha_h(V_m)(1-h) - \beta_h(V_m)h$$

Hodgkin and Huxley, J Physiol. 1952 Aug 28; 117(4): 500–544

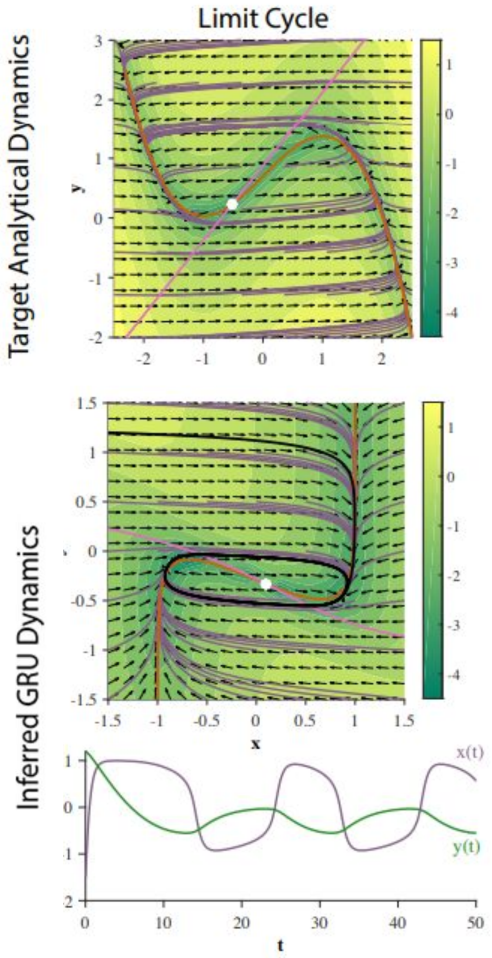
Motivation

- Simplification of HH model from 4D

- Exploring the power of RNNs to analyse dynamical systems

- This article:

  Jordan et al., **Gated recurrent units viewed through the lens of continuous time dynamical systems.** Front. Comput. Neurosci, 2021.



Bonhoeffer-van der Pol (BVP) model

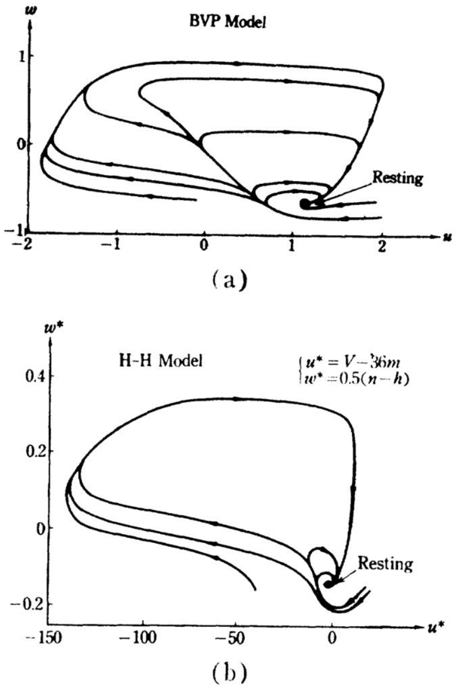
FitzHugh, J. Gen. Physiol, 1960; Thresholds and Plateaus in the Hodgkin-Huxley Nerve Equations.

$$
\begin{cases}
J = \dfrac{1}{c}\dfrac{du}{dt} - w - \left(u - \dfrac{u^3}{3}\right), \\
c\dfrac{dw}{dt} + bw = a - u,
\end{cases}
$$

where $1 > b > 0$, $c^2 > b$, $1 > a > 1 - ⅔ * b$

Correspondence of BVP and and HH model parameters:

u $\rightarrow$ *V, m;* w $\rightarrow$ *h, n;* J $\rightarrow$ *I*

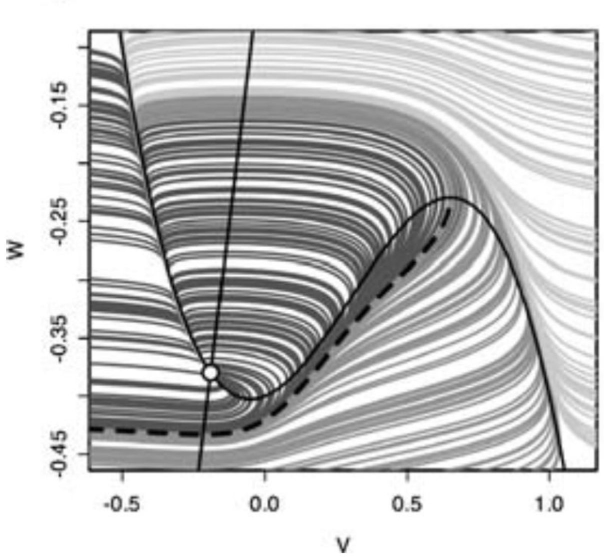


J. Nagumo, S. Arimoto, S. Yoshizawa (1962)

FitzHugh-Nagumo model

$$\psi\, dV/dt = V(1 - V)(a + V) - W + c + I,$$

$$dW/dt = V - bW,$$



a = 0.1, b = 0.5, c = - 0.4, Ψ = 0.015, I = 0

Figure from S. Hong, B. Agüera y Arcas, and A. Fairhall (2007)

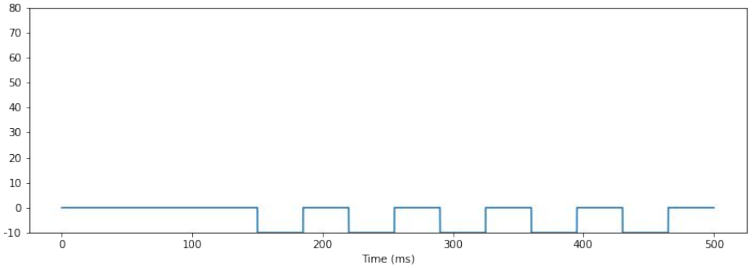
FitzHugh, 1961; Nagumo, Arimoto, & Yoshikawa, 1962

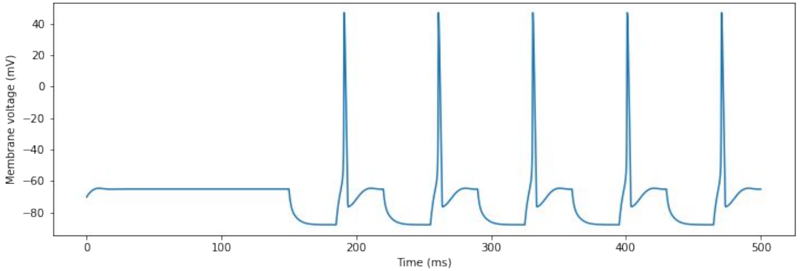
What solutions did we expect?

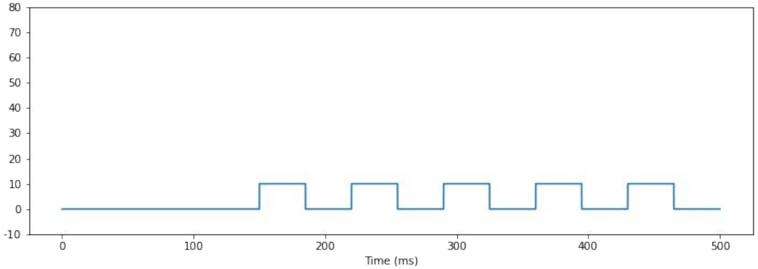
- 2-neuron RNN which can accurately predict HH model from current values
- Try to replicate spikes, rebound spikes, f-I curves resembling the HH model
- Phase portrait depicting dynamics of the HH system
- Comparison of observed spikes using latency, spike width, f-I curve

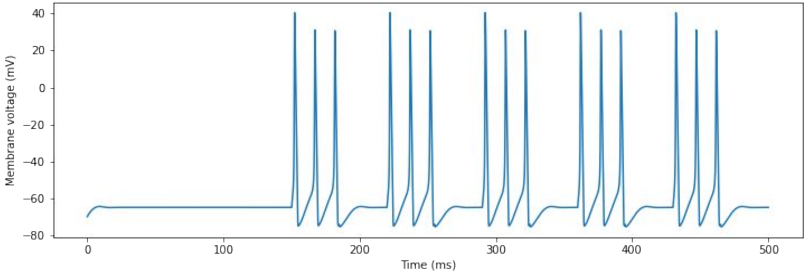
Protocol

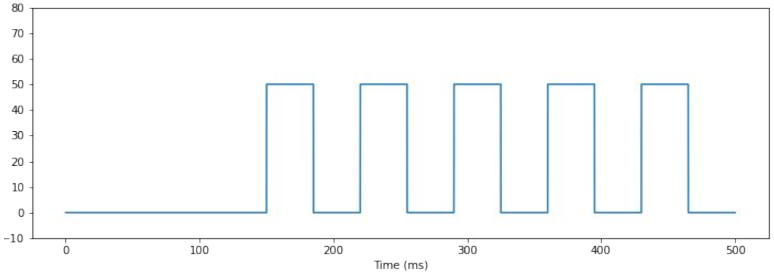
Injected current

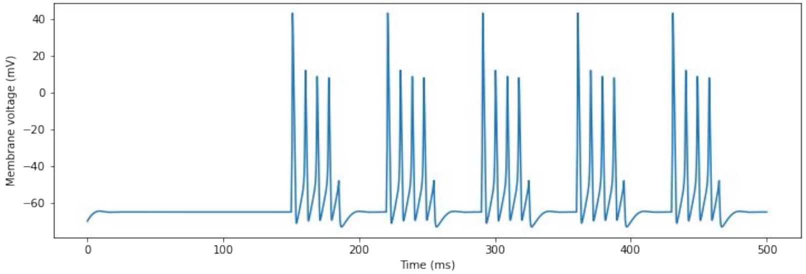












Implementation and Architecture

Hidden state dynamics of the RNN:

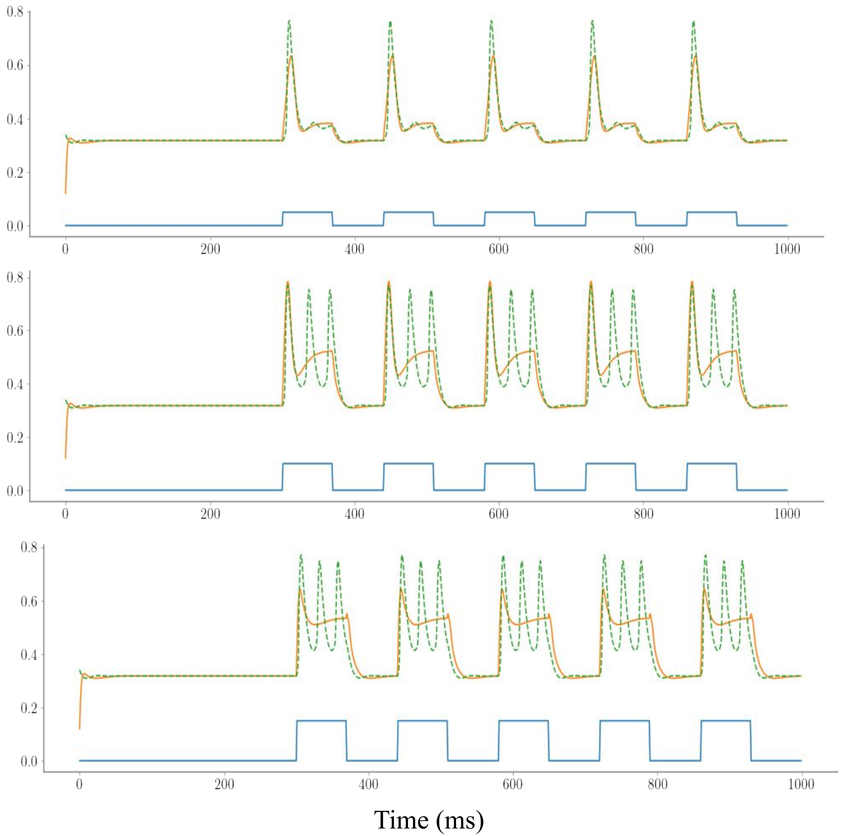
dh/dt = *f*(h(t), x(t))

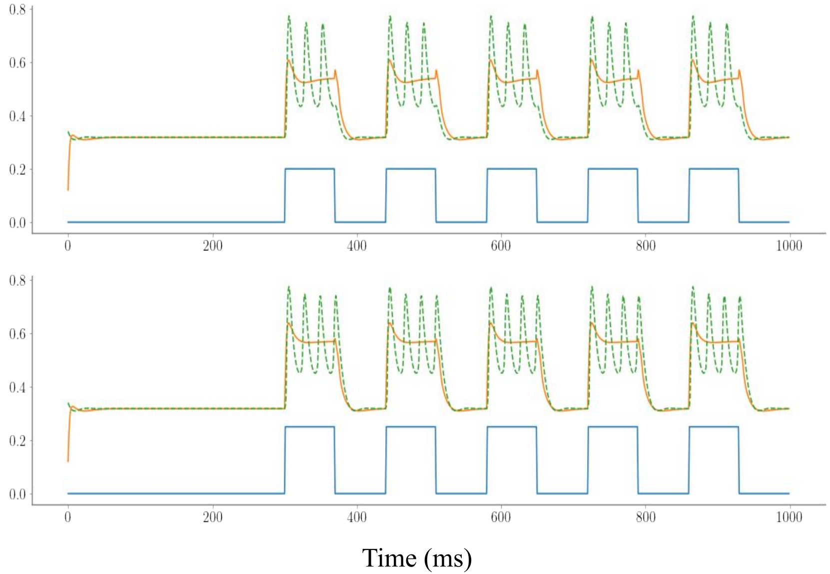
where, h = hidden state

x(t) = time dependent input

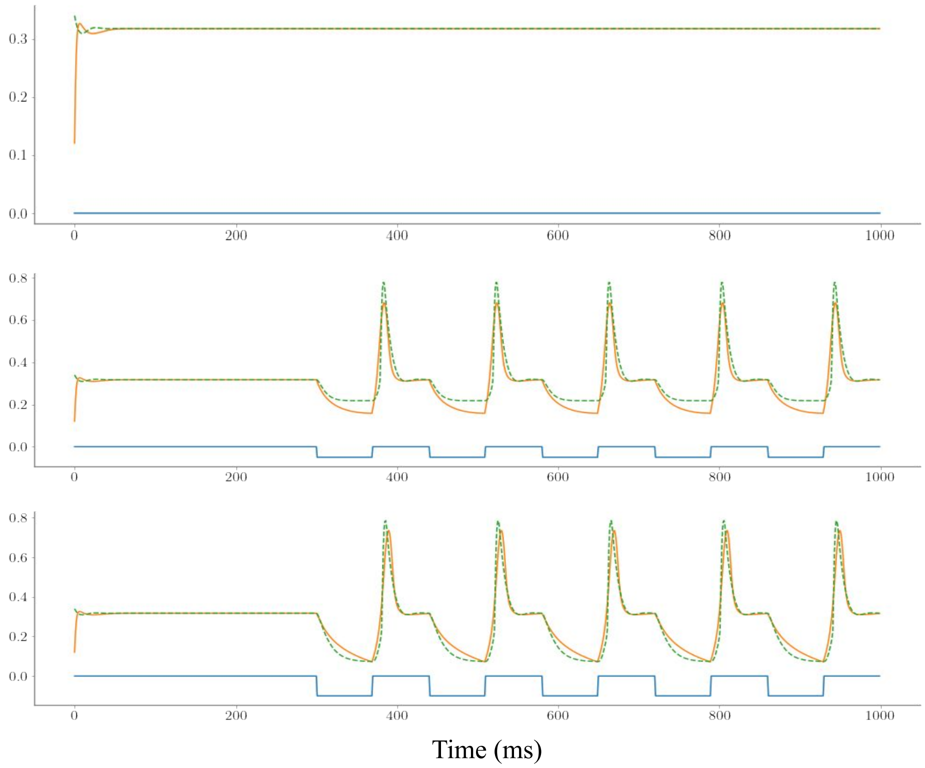
$$\mathbf{z}(t) = \sigma(\mathbf{U}_z\mathbf{h}(t) + \mathbf{b}_z)$$
$$\mathbf{r}(t) = \sigma(\mathbf{U}_r\mathbf{h}(t) + \mathbf{b}_r)$$
$$\dot{\mathbf{h}} = (1 - \mathbf{z}(t)) \odot (\tanh(\mathbf{U}_h(\mathbf{r}(t) \odot \mathbf{h}(t)) + \mathbf{b}_h) - \mathbf{h}(t))$$

Results

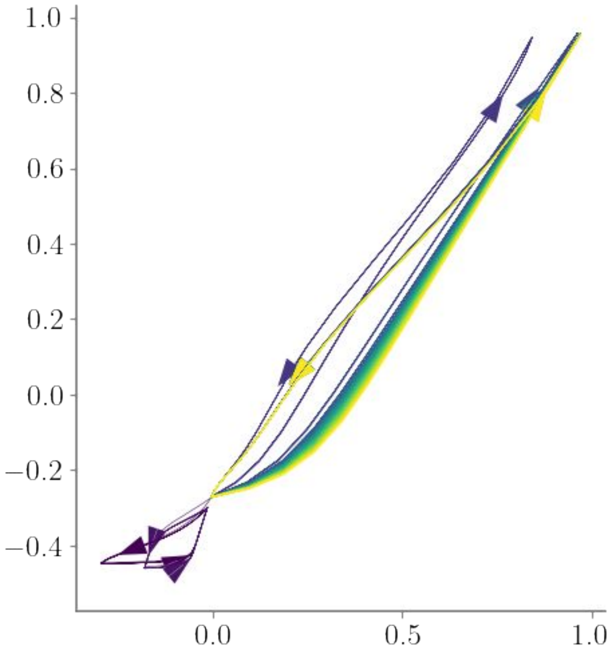




Results



Phase Portrait



Thank you!