Tutorial Sheet 1: Sets and Probability
Course: CSEG 2036P | *School of Computing Sciences, UPES*
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1. Use Venn diagrams to verify that

   (a) $\overline{(A \cap B)} = \overline{A} \cup \overline{B}$
   (b) $A \cup (A \cap B) = A$
   (c) $(A \cap B) \cup (A \cap \overline{B}) = A$
   (d) $A \cup B = (A \cap B) \cup (A \cap \overline{B}) \cup (\overline{A} \cap B)$
   (e) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

2. If 3 of 20 tires in a storage facility are defective and 4 of them are randomly chosen for inspection, what is the probability that only one of the defective tires will be included?

3. If the probabilities are 0.87, 0.36, 0.29 that, while under warranty, a new car will require repairs on the engine, drive-train or both, what is the probability that a car will require one or the other or both kinds of repairs under the warranty?

4. When we roll a pair of balanced dice, what are the probabilities of getting

   (a) 2 or 12
   (b) 4, 7 or 11
   (c) a prime number

5. If A and B are mutually exclusive events $P(A) = 0.29$ and $P(B) = 0.43$, find

   (a) $P(\overline{A})$
   (b) $P(A \cup B)$
   (c) $P(A \cap \overline{B})$
   (d) $P(\overline{A} \cap \overline{B})$

6. Write the power-set of $\{x^2 : x \in \mathbb{N}, 1 \leq x \leq 5\}$.

7. If $A \subset B \subset C$, then which of the following is the cardinality of $(A \cup B \cup C) - (A \cap B \cap C)$:

   (a) Cardinality of $C - B - A$
   (b) Cardinality of $C - A$
   (c) Cardinality of $\overline{A \cap B \cap C}$
   (d) Cardinality of $\overline{C}$

8. A person has undertaken a mining job. The probabilities of completion of the job on time with and without rain are 0.42 and 0.90 respectively. If the probability that it will rain is 0.45, then determine the probability that the mining job will be completed on time.

9. Three cities X, Y and Z suffer from high levels of pollution. The probabilities that the pollution level is "severe" are 0.40, 0.30 and 0.90 for cities X, Y and Z respectively. A city is chosen at random, and its pollution level measured. Find the probability that the measured pollution level is "severe".

10. A card is lost from a shuffled deck of 52 cards. From the remaining cards, a card is pulled out at random and is found to be a spade. Find the probability that the lost card was a spade.

11. If the probability that a communication system will have high fidelity is 0.81 and the probability that it will have high fidelity and selectivity is 0.18, what is the probability that a system with high fidelity will also have high selectivity?

12. Prove that $P(A|B) = P(A)$ implies that $P(B|A) = P(B)$ provided that $P(A) \neq 0$ and $P(B) \neq 0$.

13. Given that $P(A) = 0.50$, $P(B) = 0.30$ and $P(A \cap B) = 0.15$, verify that

    (a) $P(A|B) = P(A)$
    (b) $P(A|\overline{B}) = P(A)$
    (c) $P(B|A) = P(B)$
    (d) $P(B|\overline{A}) = P(B)$

14. If $P(A) = 0.80, P(B) = 0.35$ and $P(A \cap C) = 0.28$, are the events A and C independent?

15. In an automatic production line, four technicians regularly make repairs when breakdown occurs. Junaki, who services 20% of the breakdowns, makes an incomplete repair 1 time in 20; Tarun, who services 60% of the breakdowns, makes an incomplete repair 1 time in 10; Gopal, who services 15% of the breakdown, makes an incomplete repair 1 time in 10, and Pramodini, who services 5% of the breakdowns, makes an incomplete repair 1 time in 20. For the next problem with the production line diagnosed as being due to an initial repair that was incomplete, what is the probability that this initial repair was made by Junaki?