Tutorial Sheet 3: Sets and Probability
Course: CSEG 2036P | *School of Computing Sciences, UPES*
Faculty: Dr. Mrittunjoy Guha Majumdar

1. The joint probability mass function of (X, Y) is given by $p(x, y) = k(2x + 3y), x = 0, 1, 2$ and $y = 1, 2, 3$. Find all the marginal and conditional probability distributions. Also find the probability distribution of $X + Y$.

2. Find the mean, median, quartile deviation, mean deviation and standard deviation of the following probability distribution.

$X = x_i$	0	1	2	3	4	5	6	7	8
p_i	0.004	0.036	0.1	0.232	0.280	0.204	0.112	0.028	0.004

3. If the density function of a random variable is given by $f(x) = kx(1-x), 0 \le z \le 1$, find the median, mode, mean deviation and standard deviation of this distribution.

4. If X is a continuous random variable, prove that

$$E(X) = \int_0^\infty [1 - F(x)]dx - \int_{-\infty}^0 [1 - F(x)]dx$$

where $F(x)$ is the cumulative distribution function.

5. If the joint probability distribution function of (X, Y) is given by

$$f(x, y) = \frac{9(1 + x + y)}{2(1 + x)^4(1 + y)^4}, x > 0, y > 0$$

what are the marginal densities of X and Y? Are the random variables X and Y statistically independent?

6. Prove that $E(E(g(X, Y)|X)) = E(g(X, Y))$.

7. If (X, Y) follows a bivariate normal distribution with the form

$$f(x, y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1 - r^2}}\exp\left\{-\frac{1}{2(1 - r^2)}\left(\frac{x^2}{\sigma_x^2} - \frac{2rxy}{\sigma_x\sigma_y} + \frac{y^2}{\sigma_y^2}\right)\right\}$$

find the expression for $E(Y|X)$.

8. Calculate the correlation coefficient for the following ages of spouses X and Y, using the standard deviation of X and Y:

X	23	27	28	28	29	30	31	33	35	36
Y	18	20	22	27	21	29	27	29	28	29

9. If X, Y and Z are uncorrelated random variables with zero means and standard deviations 5, 12 and 9 respectively, and if $U = X + Y$ and $V = Y + Z$, find the correlation coefficient between U and V.