Nonlinear Regression

Course: CSEG 2036P | *School of Computing Sciences, UPES*

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1 Curvilinear Regression

1.1 Fitting of a Second-Degree Polynomial

We are interested in finding the least-square parabola that fits a set of sample points, with the associated functional form

$$y = a + bx + cx^2$$

where the constants a, b and c are determined by solving the normal equations

$$\sum y = na + b\sum x + c\sum x^2$$

$$\sum xy = a\sum x + b\sum x^2 + x\sum x^3$$

$$\sum x^2 y = a\sum x^2 + b\sum x^3 + x\sum x^4$$

1.2 Geometric Curve or Power Curve

Non-linear curves of the form $y = ax^b$ can be transformed to a linear curve straight line by taking logarithms:

$$\log_{10}(y) = \log_{10} a + b\log_{10} x$$

We can consider $Y = \log_{10}(y)$, $A = \log_{10} a$, $X = \log_{10} x$ and $B = b$, and then we have the equation of the form

$$Y = A + BX$$

We can then apply linear regression and the normal equations

$$\sum Y = nA + B\sum X$$

$$\sum XY = A\sum X + B\sum X^2$$

We could also have done this problem by taking the natural logarithm (ln) instead of $\log_{10}$.

1.3 Exponential Curve

If we are to fit an exponential curve of the form $y = ae^{bx}$, we can begin by taking logarithms on both sides

$$\log_{10}(y) = \log_{10} a + bx\log_{10} e$$

We can consider $Y = \log_{10}(y)$, $A = \log_{10} a$, $X = x\log_{10} e$ and $B = b$, and then we have the equation of the form

$$Y = A + BX$$

We can then apply linear regression and the normal equations

$$\sum Y = nA + B\sum X$$

$$\sum XY = A\sum X + B\sum X^2$$

We could also have done this problem by taking the natural logarithm (ln) instead of $\log_{10}$.

1.4 Hyperbola or Reciprocal Function

We can have to fit a function of the form

$$y = \frac{1}{a_0 + a_1 x}$$

where a_0 and a_1 are constants. We can consider the following equation

$$\frac{1}{y} = a_0 + a_1 x$$

Considering $Y = \frac{1}{y}$ and $X = x$, we have

$$Y = a_0 + a_1 X$$

Then we have the normal equations

$$\sum Y = na_0 + a_1\sum X$$

$$\sum XY = a_0\sum X + a_1\sum X^2$$

A variant of this problem is when we have the function of the form

$$y = \frac{b}{x(x-a)}$$

where a and b are constants. We can consider $Y = \frac{1}{y}$, $X = x$, $A = -\frac{a}{b}$ and $B = \frac{1}{b}$ then we have the equation of the form

$$Y = AX + BX^2$$

Then the normal equations are given by

$$\sum Y = A\sum X + B\sum X^2$$

$$\sum XY = A\sum X^2 + B\sum X^3$$

We can solve these two equations to find the relevant parameters A and B, and from this the original parameters a and b.

2 Multiple Regression

If there is a linear relationship between the dependent variable y and the two independent variables x_1 and x_2, then we can be asked to fit a trend-line of the form

$$y = a_0 + a_1 x_1 + a_2 x_2$$

The constants a_0, a_1 and a_2 can be determined by the normal equations

$$\sum y = na_0 + a_1 \sum x_1 + a_2 \sum x_2$$

$$\sum x_1 y = a_0 \sum x_1 + a_1 \sum x_1^2 + a_2 \sum x_1 x_2$$

$$\sum x_2 y = a_0 \sum x_2 + a_1 \sum x_1 x_2 + a_2 \sum x_2^2$$