Name:

Enrolment No:

UPES
UNIVERSITY OF TOMORROW

Mid Semester Examination, October 2023

Program Name: B.Tech. CSE

Course Name: Probability and Statistics **Semester:** III

Course Code: CSEG 2036P **Time:** 1.5 hours

No. of Pages: 2 **Max Marks:** 50

Instructions: Attempt all the questions.

Section A
30 Marks

S. No.		Marks	CO
Q 1	Prove that $A-(A-B)=A\cap B$ using the properties of sets. Find $A\cup(B\cup C)\cap(\bar{A}\cap\bar{C})$, if $A=\{3x: 1\leq x\leq 3\}$, $B=\{x^2: 1\leq x\leq 4\}$ and $C=\{11,13\}$ if the universal set is composed of the union of the three sets.	**5**	**CO1**
Q 2	If X denotes the number of heads in a single toss of four fair coins. Determine $P(X>2)+P(X\leq 1)$ and $P(1<X\leq 4)$.	**5**	**CO1**
Q 3	Given the following probability distribution, x: 0, 1, 2, 3, 4, 5, 6, 7 $f(x)$: 0, k, $2k$, $2k$, $3k$, k^2, $2k^2$, $7k^2+k$ What is the value of k as well as the mean and standard deviation of this dataset?	**5**	**CO2**
Q 4	Let X and Y be independent random variables that are uniformly distributed on the interval [0, 1]. Let $Z=X+Y$. Find $E[Z\|X]$.	**5**	**CO2**
Q 5	If the first and second central moments around the point $x=5$ is given as 1 and 5 respectively, for a distribution, find the value of $E((2+X)^2)$ and $Var(4+3X)$	**5**	**CO2**

<table>
<tr><td>Q 6</td><td>What is meant by covariance and correlation coefficient of two random variables? If independent random variables X and Y have variances 24 and 19 respectively, what is the correlation coefficient between $X + Y$ and $X - Y$?</td><td>5</td><td></td></tr>
<tr><td></td><td>Section B
20 Marks</td><td></td><td></td></tr>
<tr><td>Q 7</td><td>(a) A pair of fair dice is tossed. Let X denote the maximum of the number appearing: $X(a,b) = max(a,b)$ and let Y denote the sum of the numbers: $Y(a,b) = a + b$, find the mean and standard deviation of X and Y.
(b) Given the following probability distribution
<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>$P(X = x)$</td><td>0.05</td><td>0.30</td><td>0.25</td><td>0.15</td><td>0.25</td></tr></table>
Find the values of mean μ, standard deviation σ, $E(\frac{2X}{3} - 7)$ as well as the highest probability of finding a value between 0 and 0.4.</td><td>10</td><td>CO1, CO2</td></tr>
<tr><td>Q 8</td><td>In a certain medical test for a rare disease, let D represent the event that a person has the disease, and T represent the event that the test result is positive. The following information is known:
The probability that a person has the disease is $P(D) = 0.02$.
The probability that the test correctly identifies a person with the disease is
$P(T \mid D) = 0.95$
The probability that the test incorrectly identifies a person without the disease as positive is $P(T \mid \bar{D}) = 0.10$.
(a) What is the probability that a randomly selected person tests positive $P(T)$?
(b) What is the probability that a person who tests positive actually has the disease $P(D \mid T)$?
(c) Show that $P(\bar{T}|D) = 1 - P(T|D)$ and find the probability that the test result is a false negative $P(D \mid \bar{T})$ – the person tests negative even though they do have the disease?</td><td>10</td><td>CO1</td></tr>
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