Tutorial Sheet 5: Special Distributions

Course: CSEG 2036P | *School of Computing Sciences, UPES*
Faculty: Dr. Mrittunjoy Guha Majumdar

1. Define a Bernoulli random variable $X \sim \text{Ber}(p)$ and its probability distribution, along with two conditions assumed for Bernoulli Distributions. Derive its mean and standard deviation.

2. What is a Characteristic Function and a Moment-Generating Function? What are these for a Bernoulli distribution?

3. Define a Binomial random variable $X \sim \text{Bin}(n, p)$ and its probability distribution. Highlight the relation of Binomial distributions with Bernoulli trials.

4. Derive the mean and standard deviation of $X \sim \text{Bin}(n, p)$. What is the Characteristic Function and Moment-Generating function for Binomial distributions?

5. What is a Poisson process? What is its probability distribution function? Derive its mean and standard deviation. What are its Moment-Generating function and Characteristic function?

6. Define a normal distribution and a standard normal table. Derive the points of inflection of a normal distribution.

7. Calculate the probability that a randomly selected student from UPES is shorter than 140 cm, given that the heights of a population of students at UPES follow a normal distribution with a mean height of 150 cm and a standard deviation of 7.5 cm.

8. Define a Gamma Function and highlight any two properties of the Gamma Function. Expand on your understanding of the Gamma Distribution $X \sim \text{Gamma}(\alpha, \beta)$, with the expression for its probability distribution. What are the Characteristic function and Moment-Generating function of the Gamma distribution?

9. Derive the mean and standard deviation of the Gamma distribution. What are the values of $\Gamma(3)$, $\Gamma\left(\frac{11}{2}\right)$ and $\Gamma(-6)$?

10. Define the Beta Functions and Beta distribution. Derive the mean and standard deviation of the Beta distribution. What are the Characteristic function and Moment-Generating function of the Beta distribution?

11. Consider a hypothetical biological inheritance on exo-planet (a planet outside our solar systam) MOA 2007 BLG 192Lb: Each offspring possesses the dominant (success), recessive (failure) or a hybrid phenotype, with ratio 6.5:3.5:1. Suppose that we observe $n = 9$ independent trials and let X_1 denote the total number of these offsprings who will possess the dominant phenotype, X_2 denote the total number of these offsprings who will possess the recessive phenotype and X_3 denote the total number of these offsprings who will possess the hybrid phenotype. Find $P(X_1 = 3, X_2 = 4, X_3 = 2)$.

12. * Suppose you have a biased coin that has a probability of success (landing on heads) equal to p. You start flipping the coin until you get heads for the first time. Let X be the number of flips needed. Now, imagine you have two such coins, one with $p = \frac{1}{2\mathcal{T}_n}$ (where $\mathcal{T}_n =^{(n+1)} C_2$) and the other with $p = 0.3$.

13. Prove the Poisson Approximation of Binomial Distribution.

14. If 6% of electric bulbs manufactured by a certain company are defective. Find the probability that in a sample of 300 bulbs, more than 3 bulbs are defective.

15. Suppose the scores on a standardized test follow a normal distribution with a mean of 100 and a standard deviation of 15. What percentage of students scored above 130? If the top 5% of students are eligible for a special award, what is the minimum score a student needs to qualify for the award? If a student scored at the 75th percentile, what is their score?