Hypothesis Testing
Course: CSEG 2036P | *School of Computing Sciences, UPES*
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1 Generalized Scheme for Hypothesis Testing

1.1 Step I: Formulating Hypotheses

1. Establish null hypothesis H_0 and alternative hypothesis H_1.

2. For testing a claimed value θ_0 of parameter θ, consider hypotheses like:

   2.a. $H_0 : \theta = \theta_0$ and $H_1 : \theta \neq \theta_0$ for two-tailed test.
   2.b. $H_0 : \theta \leq \theta_0$ and $H_1 : \theta > \theta_0$ for one-tailed test

For comparing parameters of two populations, e.g., θ_1 and θ_2:

   2.c $H_0 : \theta_1 = \theta_2$ and $H_1 : \theta_1 \neq \theta_2$ for the two-tailed test
   2.d. $H_0 : \theta_1 \leq \theta_2$ and $H_1 : \theta_1 > \theta_2$ for one-tailed test.

1.2 Step II: Set Level of Significance

3. Decide the level of significance (α) for testing the hypothesis, typically 5% or 1%.

1.3 Step III: Choose Test Statistic Under H_0

4. Define a test statistic based on the value of the parameter under H_0.

5. Specify the sampling distribution (e.g., z, χ^2, t, F).

1.4 Step IV: Calculate Test Statistic

Calculate the test statistic using observed sample observations.

1.5 Step V: Obtain Critical Values

Determine critical values from the sampling distribution, constructing a rejection region based on α. Use tables (z-table, χ^2-table, t-table, etc.) for various significance levels.

1.6 Step VI: Compare Calculated vs. Critical Values

Compare the calculated test statistic from Step IV with critical values from Step V. Identify whether it lies in the rejection or non-rejection region.

1.7 Step VII: Reach a Conclusion

If the test statistic falls in the rejection region at α significance level, reject the null hypothesis, otherwise not

2 Hypothesis Testing of Population Mean for Large Samples

When we consider large samples, the sampling distribution of a statistic is approximately normal, irrespective of whether the distribution of the population is normal or not. The distribution of sample means is a normal distribution $\overline{X} \sim \mathcal{N}(\mu, \frac{\sigma^2}{n})$. When we undertake hypothesis testing of population mean for large samples, we establish the following hypothesis (for the two-tailed test):

H_0: $\overline{X} = \mu$ - the population mean is equal to the sample mean
H_1: $\overline{X} \neq \mu$ - the population mean is not equal to the sample mean

We can also similarly define the one-tailed test hypothesis. Once the hypothesis is established, we can define the test statistic z as

$$z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}}$$

1. For the left one-tailed test, if $z < z(df, \alpha)$ where $df = n - 1$, we reject the null-hypothesis.
2. For the right one-tailed test, if $z > z(df, \alpha)$ where $df = n - 1$, we reject the null-hypothesis.
3. For the two-tailed test, if $|z| > |z(df, \frac{\alpha}{2})|$ where $df = n - 1$, we reject the null-hypothesis.

When we have $\overline{X}_1$ and $\overline{X}_2$ as means of two large samples with sizes n_1 and n_2 drawn from general populations with the same mean μ and variances σ_1^2 and σ_2^2 respectively, then the hypothesis (for the two-tailed test) is given by

H_0: $\overline{X}_1 = \overline{X}_2$ The two sample means are equal
H_1: $\overline{X}_1 \neq \overline{X}_2$ The two sample means are equal

We can also similarly define the one-tailed test hypothesis. Once the hypothesis is established, we can define the test statistic z as

$$z = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{\sigma_1^2}{n_2} + \frac{\sigma_1^2}{n_2}}}$$

1. For the left one-tailed test, if $z < z(df, \alpha)$ where $df = n_1 + n_2 - 2$, we reject the null-hypothesis.
2. For the right one-tailed test, if $z > z(df, \alpha)$ where $df = n_1 + n_2 - 2$, we reject the null-hypothesis.
3. For the two-tailed test, if $|z| > |z(df, \frac{\alpha}{2})|$ where $df = n_1 + n_2 - 2$, we reject the null-hypothesis.

Revise *Hypothesis Test of Population Mean: Dependent Samples (Paired Samples)* from Lecture 12, Slide 25.

3 Hypothesis Testing of Population Mean for Small Samples

If $\overline{X}$ is the mean of a sample of size n and sample standard deviation s, we establish the following hypothesis (for the two-tailed test):

H_0: $\overline{X} = \mu$ - the population mean is equal to the sample mean
H_1: $\overline{X} \neq \mu$ - the population mean is not equal to the sample mean

We can also similarly define the one-tailed test hypothesis. Once the hypothesis is established, we can define the test statistic z as

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n-1}}$$

1. For the left one-tailed test, if $t < t(df, \alpha)$ where $df = n - 1$, we reject the null-hypothesis.
2. For the right one-tailed test, if $t > t(df, \alpha)$ where $df = n - 1$, we reject the null-hypothesis.
3. For the two-tailed test, if $|t| > |t(df, \frac{\alpha}{2})|$ where $df = n - 1$, we reject the null-hypothesis.

4 Hypothesis Test of Proportions

We can compare sample proportions $\hat{p}$ to standard reference value(s) p_0, with the hypothesis for the two-tailed test

H_0: $\hat{p} = p_0$
H_1: $\hat{p} \neq p_0$

We can similarly establish the hypothesis for the one-tailed tests. The test statistic z for this test is

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}}$$

1. For the left one-tailed test, if $z < z(df, \alpha)$ where $df = n - 1$, we reject the null-hypothesis.
2. For the right one-tailed test, if $z > z(df, \alpha)$ where $df = n - 1$, we reject the null-hypothesis.
3. For the two-tailed test, if $|z| > |z(df, \frac{\alpha}{2})|$ where $df = n - 1$, we reject the null-hypothesis.

If we have two large samples of sizes n_1 and n_2 with p_1 and p_2 being the proportions of successes respectively, we can define the hypothesis for the two-tailed test

$H_0 : p_1 = p_2$
$H_1 : p_1 \neq p_2$

We can define the hypothesis for the one-tailed test similarly. We can define the relevant test statistic as

$$z = \frac{p_1 - p_2}{\sqrt{P(1 - P)(\frac{1}{n_1} + \frac{1}{n_2})}}$$

where probability of success for each trial is a constant P.

1. For the left one-tailed test, if $z < z(df, \alpha)$ where $df = n_1 + n_2 - 2$, we reject the null-hypothesis.
2. For the right one-tailed test, if $z > z(df, \alpha)$ where $df = n_1 + n_2 - 2$, we reject the null-hypothesis.
3. For the two-tailed test, if $|z| > |z(df, \frac{\alpha}{2})|$ where $df = n_1+n_2-2$, we reject the null-hypothesis.

5 Hypothesis Testing of Population Standard Deviations

Let s be the standard deviation of a large sample of size n drawn from a normal population with standard deviation σ. Then it is known that s follows a normal distribution $\mathcal{N}(\sigma, \frac{\sigma}{\sqrt{2n}})$ approximately. We can establish the hypothesis (for the two-tailed test),

H_0: $s = \sigma$
H_1: $s \neq \sigma$

We can similarly establish the hypothesis of the one-tailed test. Once the hypothesis is formulated, we can write the test statistic z for this as

$$z = \frac{s - \sigma}{\sigma/\sqrt{2n}}$$

1. For the left one-tailed test, if $z < z(df, \alpha)$ where $df = n - 1$, we reject the null-hypothesis.
2. For the right one-tailed test, if $z > z(df, \alpha)$ where $df = n - 1$, we reject the null-hypothesis.
3. For the two-tailed test, if $|z| > |z(df, \frac{\alpha}{2})|$ where $df = n - 1$, we reject the null-hypothesis.

If we have two large samples of sizes n_1 and n_2 with standard deviations s_1 and s_2 respectively, from a population with population standard deviation, we can compare the sample standard deviations with the hypothesis (for the two-tailed test),

$H_0 : s_1 = s_2$
$H_1 : s_1 \neq s_2$

We can similarly establish the hypothesis for the one-tailed tests. Once the hypothesis is established, we can write the test statistic z,

$$z = \frac{s_1 - s_2}{\sigma\sqrt{\frac{1}{2n_1} + \frac{1}{2n_2}}}$$

1. For the left one-tailed test, if $z < z(df, \alpha)$ where $df = n_1 + n_2 - 2$, we reject the null-hypothesis.
2. For the right one-tailed test, if $z > z(df, \alpha)$ where $df = n_1 + n_2 - 2$, we reject the null-hypothesis.
3. For the two-tailed test, if $|z| > |z(df, \frac{\alpha}{2})|$ where $df = n_1+n_2-2$, we reject the null-hypothesis.