Project One Template

MAT350: Applied Linear Algebra

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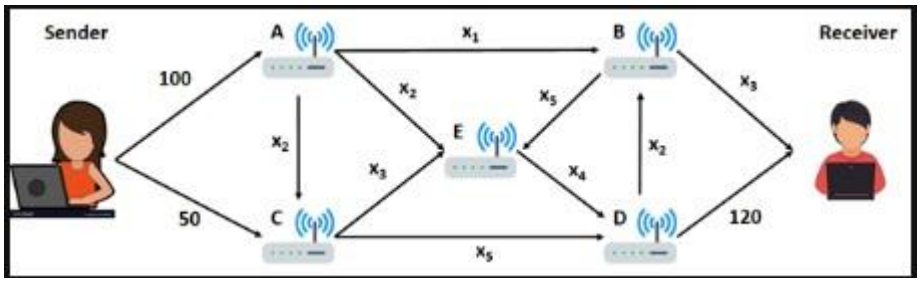
Problem 1

Develop a system of linear equations for the network by writing an equation for each router (A, B, C, D, and E). Make sure to write your final answer as A**x**=**b** where A is the 5x5 coefficient matrix, **x** is the 5x1 vector of unknowns, and **b** is a 5x1 vector of constants.



Solution:

System of linear equations:

The input was set equal to the output and then equations were simplified to put all variables on the left.

Router A recieves 100 and outputs x1 once and x2 twice:

$x1 + 2x2 = 100$

Router B recieves x1 and x2 and outputs x3 once and x5 once:

$x3 + x5 = x1 + x2$

$-x1 - x2 + x3 + x5 = 0$

Router C recieves 100 and x2 while outputing x3 and x5:

$-x2 + x3 + x5 = 50$

Router D recieves x5 and x4 and outputs x2 and 120:

$x4 + x5 = 120 + x2$

$-x2 + x4 + x5 = 120$

Router E recieves x2, x3, and x5 and outputs x4:

$x4 = x2 + x3 + x5$

$x2 + x3 - x4 + x5 = 0$

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ -1 & -1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 & 1 \\ 0 & -1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \\ x4 \\ x5 \end{bmatrix} \begin{bmatrix} 100 \\ 0 \\ 50 \\ 120 \\ 0 \end{bmatrix}$$

Problem 2

Use MATLAB to construct the augmented matrix [A **b**] and then perform row reduction using the rref() function. Write out your **reduced matrix and identify the free and basic variables of the system**.

Solution:

```
%code
A = [1 2 0 0 0; -1 -1 1 0 1; 0 -1 1 0 1; 0 -1 0 1 1; 0 1 1 -1 1]
```

```
A = 5×5
     1     2     0     0     0
    -1    -1     1     0     1
     0    -1     1     0     1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
b = [100; 0; 50; 120; 0]
```

```
b = 5×1
   100
     0
    50
   120
     0
```

```
Ab = [A, b]
```

```
Ab = 5×6
     1     2     0     0     0   100
    -1    -1     1     0     1     0
     0    -1     1     0     1    50
     0    -1     0     1     1   120
     0     1     1    -1     1     0
```

```
[rowreducedAb] = rref(Ab)
```

```
rowreducedAb = 5×6
     1     0     0     0     0    50
     0     1     0     0     0    25
     0     0     1     0     0    30
     0     0     0     1     0   100
     0     0     0     0     1    45
```

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 50 \\ 25 \\ 30 \\ 100 \\ 45 \end{bmatrix}$$

A basic variable correspnds to the variable in one of the matrix's pivot columns and free variables cannot exist within a pivot column. Every column is a pivot column, therefore, there are no free variables and the columns *x1*, *x2*, *x3*, *x4*, and *x5* are basic variables.

Problem 3

Use MATLAB to **compute the LU decomposition of A**, i.e., find A = LU. For this decomposition, find the transformed set of equations L**y** = **b**, where **y** = U**x**. Solve the system of equations L**y** = **b** for the unknown vector **y**.

Solution:

```
%code
[L, U] = lu(A)
```

```
L = 5×5
    1.0000         0         0         0         0
   -1.0000    1.0000         0         0         0
         0   -1.0000    1.0000         0         0
         0   -1.0000    0.5000    1.0000         0
         0    1.0000         0   -1.0000    1.0000
U = 5×5
     1     2     0     0     0
     0     1     1     0     1
     0     0     2     0     2
     0     0     0     1     1
     0     0     0     0     1
```

```
y = L\b
```

```
y = 5×1
   100
   100
   150
   145
    45
```

Problem 4

Use MATLAB to **compute the inverse** of U using the inv() function.

Solution:

```
%code
inv(U)
```

```
ans = 5×5
    1.0000   -2.0000    1.0000         0         0
         0    1.0000   -0.5000         0         0
         0         0    0.5000         0   -1.0000
         0         0         0    1.0000   -1.0000
         0         0         0         0    1.0000
```

Problem 5

Compute the solution to the original system of equations by transforming **y** into **x**, i.e., compute **x** = inv(U)**y**.

Solution:

```
%code
x = inv(U) * y
```

```
x = 5×1
    50
    25
    30
   100
    45
```

Problem 6

Check your answer for x_1 using Cramer's Rule. Use MATLAB to compute the required determinants using the det() function.

Solution:

```
% Initially set all submatrix A1 to equal A
A1 = A
```

```
A1 = 5×5
     1     2     0     0     0
    -1    -1     1     0     1
     0    -1     1     0     1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
% Replace the appropriate variable
A1(:,1) = b
```

```
A1 = 5×5
   100     2     0     0     0
     0    -1     1     0     1
    50    -1     1     0     1
   120    -1     0     1     1
     0     1     1    -1     1
```

```
% Calculate
x1 = det(A1)/det(A)
```

```
x1 = 50
```

Problem 7

The Project One Table Template, provided in the Project One Supporting Materials section in Brightspace, shows the recommended throughput capacity of each link in the network. Put your solution for the system of equations in the third column so it can be easily compared to the maximum capacity in the second column. In the fourth column of the table, provide recommendations for how the network should be modified based on your network throughput analysis findings. The modification options can be No Change, Remove Link, or Upgrade Link. In the final column, explain how you arrived at your recommendation.

Solution:

*Fill out the table in the original project document and export your table as an image. Then, use the **Insert** tab in the MATLAB editor to insert your table as an image.*

I had to make my own table in a libre file document since SNHU's student license doesn't extend to editing word documents on a MacOS.

Network Link	Recommended Capacity (Mbps)	Solution	Recommendation	Explanation
x_1	60	50	No Change	Network throughput for x_1 is 10 Mbps b recommended capacity and does not ne altered.
x_2	50	25	No Change	Network throughput for x_2 is 25 Mbps b recommended capacity and does not ne altered.
x_3	100	30	No Change	Network throughput for x_3 is 70 Mbps b recommended capacity and does not ne altered.
x_4	100	100	Upgrade Link	Network throughput for x_4 is operating recommended capacity and needs to be handle a higher throughput.
x_5	50	45	No Change	Network throughput for x_5 is 5 Mbps bel recommended capacity and does not ne altered.