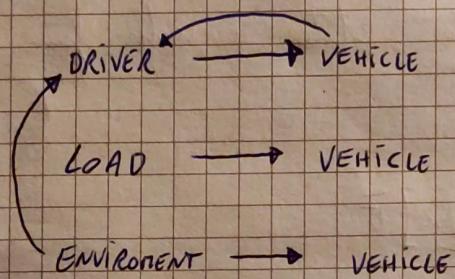


VEHICLE DYNAMICS

ISO 3833 gives us the terminology of the different type of vehicles

- Vehicle dynamics deals with the interaction between:
 - driver
 - environment
 - vehicle
 - loads

type of interaction

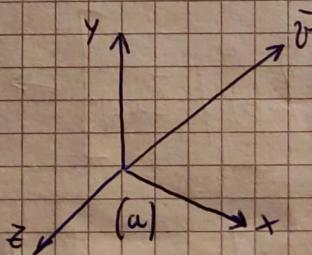


NOTATION

SCALAR, s

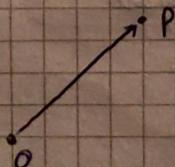
VECTOR, $\bar{v} \in \mathbb{R}^3$

VECTOR in
BASE $\bar{v}(a)$



$$\bar{v} = \{v_x, v_y, v_z\}$$

VECTOR FROM O TO P $\bar{OP}(a) = P(a)$



(NB) The same vector can be measured also using another basis

- MATRICES

$$\begin{bmatrix} \pi \end{bmatrix} \in \mathbb{R}^{n \times m}$$

n = rows

m = columns

property in the slides

ROTATION MATRIX

Very important for us, one 3×3 matrix

$$R^{3 \times 3} \rightarrow ex \quad R = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$$

using this we are able to rotate a vector from one line to another

$$\bar{v}' = [R] \bar{v}$$

$$\bar{v}(b) = \begin{bmatrix} R_{\alpha(b)} \end{bmatrix} \bar{v}(a)$$

$$\begin{bmatrix} R_{a,b} \end{bmatrix} = \begin{bmatrix} X_{a,b} & | & Y_{a,b} & | & Z_{a,b} \end{bmatrix} \rightarrow \text{Composed by 9 numbers!}$$

become the vector repeat the new line

(NB)

Is an orthogonal matrix \Rightarrow inverse = transpose

(2)

PARAMETRIZATION

$$\left[R(\alpha_x) \right] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha_x & -\sin \alpha_x \\ 0 & \sin \alpha_x & \cos \alpha_x \end{bmatrix} \quad \left[R(\alpha_y) \right] = \begin{bmatrix} \cos \alpha_y & 0 & \sin \alpha_y \\ 0 & 1 & 0 \\ -\sin \alpha_y & 0 & \cos \alpha_y \end{bmatrix}$$

$$\left[R(\alpha_z) \right] = \begin{bmatrix} \cos \alpha_z & \sin \alpha_z & 0 \\ -\sin \alpha_z & \cos \alpha_z & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So multiplying we obtain a global rotation that is the sum of the three

→ EXTRINSIC rotation X-Y-Z

→ we have 27 combinations, one with singularities

SAE REGULATION → X → longitudinal
y → side
z → downward!

SINGULARITIES happen when we are not able to go back from rotation matrix to the angle.

→ a type of parametrization that don't suffer the problem of singularities is the QUATERNION

↓

are an extension of the complex numbers

$$q = l_0 + i l_1 + j l_2 + k l_3$$

How we use the quaternions?

- unit quaternions represent rotation in 3D about generic axis \vec{u} :

If I have the vector \vec{u} and ϕ I can compute

$$q_0 = \cos\left(\frac{\phi}{2}\right) \quad q_1 = u_x \sin\left(\frac{\phi}{2}\right) \quad q_2 = u_y \sin\left(\frac{\phi}{2}\right) \quad q_3 = u_z \sin\left(\frac{\phi}{2}\right)$$

→ rotation of point \vec{v} as a double quaternion product:

$$\begin{pmatrix} 0, \vec{v}' \\ \vec{v} \end{pmatrix} = q \begin{pmatrix} 0, \vec{v} \\ \vec{v} \end{pmatrix} q^* \quad \longleftrightarrow \quad \vec{v}' = [R] \vec{v}$$

conjugate

There is a formula to compute the rotation matrix given a quaternion

SINGULARITY FREE \rightarrow we are always able to find the rotation matrix given the quaternion

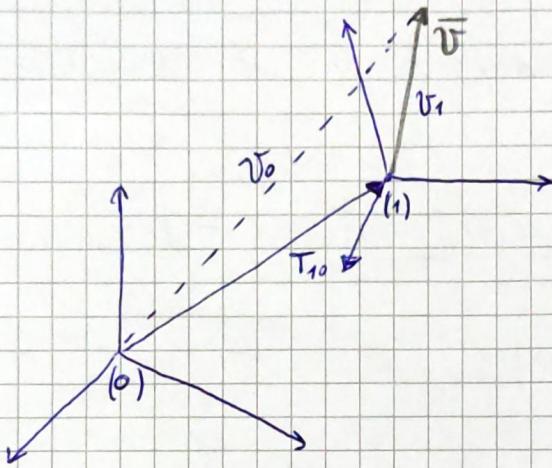
COORDINATE TRANSFORMATION

Consider the problem to TRANSFORM the position of a point moving with a rigid reference

- a reference 1 moves away from a reference 0
- ROTATION $[R_{1,0}]$ of 1 respect to 0
- TRANSLATION $T_{1,0}(a) = \vec{v}_1(a)$

$$* \quad \vec{v}_0(0) = \vec{v}_{1,0}(a) + [R_{1,0}] \vec{v}_1(1) = \vec{v}_1(a)$$

(3)



Something start from o and travel to 1 , and we want to know v respect to the point o and expressed in o reference!

↓ set of three number (the axis)

without rotation, what happen?

$$\bar{v}_o(o) = \bar{T}_{10} + \bar{v}_1(o)$$

We have a sum if all the vector we measured in the same basis

* we can obtain the inverse

$$v_1(1) = [r_{1,0}]^T (\bar{v}_o(o) - \bar{T}_{10}(o))$$

19 operation for doing this

EXAMPLE in the notebook

SPEED

to compute the speed is necessary to do the derivative!

$$\dot{\tilde{v}}_o(0) = \dot{T}_{10}(0) + [R_{1,0}] [\tilde{\omega}_{(1)}] v_{1(1)} + [R_{1,0}] \dot{v}_{1(1)}$$

Velocità di TRASLAMENTO

RELATIVE VELOCITY

What is the meaning of $[\tilde{\omega}]$?

is the rotation matrix multiplied by the angular velocity vector

$$[\tilde{\omega}_{(1)}] = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

- $\omega(1)$ INTRINSIC angular velocity
- $\omega(0)$ EXTRINSIC angular velocity

ACCELERATION

Computing another acceleration, we find the acceleration of a ROTOTRASLATING object

$$\ddot{\tilde{v}}_o(0) = \ddot{T}_{10}(0) + [R_{1,0}] [\tilde{\omega}_{(1)}] [\tilde{\omega}_{(1)}] v_{1(1)} + [R_{1,0}] [\tilde{\omega}_{(1)}] v_{1(1)} + 2[R_{1,0}] [\tilde{\omega}_{(1)}] v_{1(1)}^2$$

CENTRIPETAL ACCELERATION

$$+ [R_{1,0}] \ddot{v}_{1(1)}$$

this is the coriolis acceleration!

example 2 in the notebook

DYNAMICS

there are two classes for equation of motion

$\begin{matrix} \nearrow \text{ODE} \\ \searrow \text{DAE} \end{matrix}$

ODE ordinary differential equations

$$\frac{dx}{dt} = f(x, t)$$

- as many equation as degrees of freedom
- in dynamics : index 2 $M \frac{dv}{dt} = f(v, x, t)$

- index-n can be always converted to index 1

e.g.

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} f(v, x) \\ v \end{bmatrix}$$

$$\frac{ds}{dt} = f(s, t) \quad s = \begin{bmatrix} v \\ x \end{bmatrix}$$

DAE differential algebraic equations

- more complex than ODE, they contain also constraint

(ex.) $\frac{dx}{dt} = f(x, y, t)$

$$g(x, y, t) = 0$$

configuration \rightarrow expressed by position, rotation

state of a system is equal to ~~position~~ configuration + velocity

LAGRANGE EQUATIONS

variational approach to solve problems with complicated structures and small number of coordinates.

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial R}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i$$

EXAMPLE 3 in the notebook

for a free-rotating bodies in space \rightarrow in the slide the values

$$\left\{ \begin{array}{l} F = m \ddot{x} \\ M = [J_C] \dot{\omega} + \omega \wedge [J_C] \omega \end{array} \right. \begin{array}{l} \text{inertia tensor} \\ \rightarrow \text{angular velocity} \end{array}$$

torque applied to
the body

\downarrow gyroscopic torque
 \circlearrowleft if the angular velocity vector is
aligned to an the three axes

PRECESSION RATE

TIRE MODEL

(5)

Tire forces are the most important loads

F is imposed by (normal, lateral, longitudinal) the tire state

$$F = \{F_x, F_y, F_z\} = f(\text{tire state})$$

- STUCK TO THE ROAD

- SLIDES ACROSS



Tire point can be



can be : - slip angle

- slip ratio

- temperature

TIRE FOOTPRINT FORCES

In real life is a pressure \Rightarrow we need a PATCH but in our case we will consider a single point (average of the points)

SAE TIRE REFERENCE (PAG 63 MIKLEN for an accurate explanation)

6 scalars \Rightarrow three forces : F_x, F_y, F_z



a WRENCH

- LONGITUDINAL
- LATERAL
- VERTICAL (NORMAL)

three torques : M_x, M_y, M_z

- OVERTURNING \rightarrow during an inclination
- ROLLING RESISTANCE
- ALIGNING TORQUE \rightarrow try to align the tire in the direction of rolling

SIMPLIFIED DISK MODEL

Made by ~~so~~ sandwiching a sheet of hard rubber between two metal disks.

- Assumptions:
- NO THICKNESS
 - LOCAL LATERAL ELASTIC DEFORMATION

Look at slide 7!



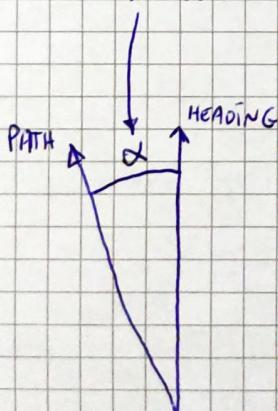
The tire wheel is pushed sideways (small lateral force) the rubber deflects and the wheel moves a small amount laterally

defining so the SLIP ANGLE α = DERIVA

(NB)

with the RUBBER deflection we can ~~roll~~^{ROTATE} and move laterally without slipping

SLIP ANGLE = tire is deflected laterally and then rolled along,
↓
move in a direction at an angle to the wheel plane or heading



RELATIONSHIP BETWEEN LATERAL FORCE and SLIP ANGLE

(6)

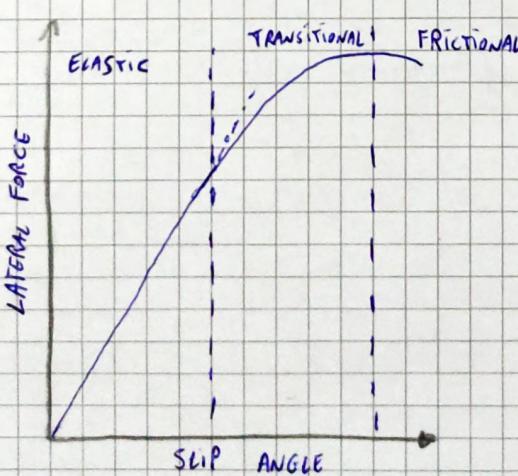
the lateral force may be thought as a result of slip angle, or the slip angle as a result of lateral force.

(Ex)

- ① if fast wheel are steered a slip angle is created which gives rise to a lateral force. This lateral force then turns the vehicle
- ② on tire test machine it is common to set a series of slip angle and measure the resultant lateral force
- ③ When a lateral wind hit the vehicle, a lateral force must be reacted due to the slip angle change
- ④ When cornering or in a banked road the lateral force is reacted by changes in tire slip angles

TIRE LATERAL FORCE (EFFECT OF SLIP ANGLE α)

SLIDE 9



You can see the input and output as you can like (ex. increase of slip angle cause an increase of lateral force)

We can observe the limit where we reach the maximum lateral force

In slide 10 we saw the effect of the vertical force (normal)

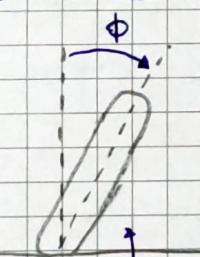
⇒ For basic Coulumb friction all the curves should be the same

⇒ in reality! FOR THE SAME TIRE ~~as~~ a higher normal force leads to
* a lower lateral G forces. (start to be a problem)

so a good solution is to have a lower pressure and so increase
the size of the tire

* this effect is called TIRE LOAD SENSITIVITY

CAMBER EFFECT



view
in positive!

DEF = is defined as the angle between the a tilted wheel plane and the vertical.

a bit of NEGATIVE camber is all the way

• because increase a little bit the lateral force

• NEGATIVE is that we aware the tire (run)

⇒ the camber affect a little bit of adhesion torque!

LONGITUDINAL FORCE

In order to accelerate or brake a vehicle, longitudinal forces must be developed between the tires and the ground.

No change in speed (acceleration) can take place without an applied force acting on a vehicle mass.

⇒ to generate a longitudinal force, ~~must~~ ^{SLIP must} ~~be~~ ^{be} present (both traction or braking)

- Shape of the force is different in traction or braking (look slide 13)

SLIP RATIO

Is defined as the difference between the angular velocity of the driven (or braked) wheel ω and the angular velocity of the free-rolling ~~wheel~~ wheel ω_0 .

$$\text{LONGITUDINAL SLIP VELOCITY } S = \omega - \omega_0$$

SLIP RATIO
FOR SAE

$$SR = \frac{\omega - \omega_0}{\omega}$$

$$SR = \frac{\omega_0}{V} - 1, \alpha=0 \quad \text{with} \quad \omega_0 = \frac{V}{R_e}$$

R_e = radius of the free rolling wheel

- For FREE ROLLING the SR=0

- LOCKED BRAKING SR = -1

slide 14 the graph is interesting!

For traction SR would be as

increasing the slip we increase also the TRACTION FORCE reaching a limit and then there is a decrease

⇒ there are different formulas to obtain the SLIP RATIO
(slide 15)

COMBINING LONGITUDINAL - LATERAL FORCE

Use of SAKAI PLOTS

What ^{we} observe?

- increasing the slip angle we decrease the longitudinal force ("traction force")
- increasing the slip ratio we decrease the lateral force (\approx the ability to steer)

I [slide 18] the continued plot with lateral force and longitudinal force! \Rightarrow there are also skip Anode and skip auto

The NAME is FRiction CIRCLE DIAGRAM

What can we observe?

- \rightarrow high lateral force when no longitudinal force is present and also the opposite
- this graph give us the idea of what to do in different driving condition to obtain the best force from our tyre!

ALIGNING TORQUE | slide 19

- M_z , describe a tire's tendency to steer about a vertical axis through the center of the print
- At low and medium slip angles the tire tends to align its heading with its path

This torque is important because is what we ~~feel~~ feel in our steering wheel!

\Rightarrow after a certain slip angle this torque goes to zero and the feeling is like you loose the steer!

ROLLING RESISTANCE TORQUE

$$M_R$$

is caused by :-
- hysteresis
- dirt road
- brushing effect in tire threads

Sine disruptive effect!

$$M_R = F_z \times R$$

NORMAL FORCE

ROLLING PARAMETER

R is few mm length and tells that the contact point is moved in front of the tire

$$R = f_z R$$

ROLLING COEFFICIENT

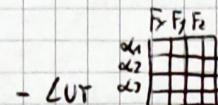
RADIUS OF TIRE

TIRE MODEL

We can't go outside the friction circle diagram (center)

→ the goal is to implement a formula that give 6 numbers with respect to state
 depending on the state (α, SR)
 ex.

3 type of models : - LUT very expensive!



- Semi-empirical ex → (PACEJKA)

able to build more flat starting from limited number of experimental data

- PHYSICAL METHOD widespread based on this!

↓
used by the manufacturer that want to improve the tyres

- high frequency vibration → NOISE!

use FINITE ELEMENT APPROACH

COULOMB MODEL

assume only one μ (friction coefficient)

$$\|F_T\| \leq \mu_s F_N \rightarrow \text{in case of static friction } (\dot{\nu} = 0)$$

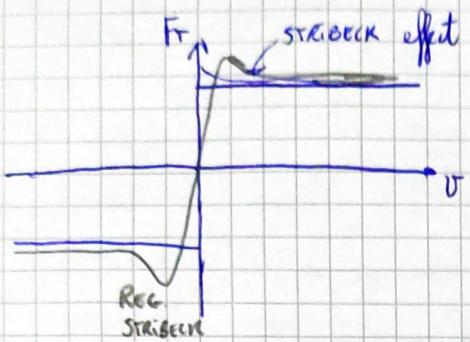
the sum between F_N and F_T must stay inside a cone, obtaining a fulcrum.

$$\|F_T\| = \mu_d F_N \rightarrow \text{in case of dynamic friction } (\dot{\nu} \neq 0)$$

if $\dot{\nu} < 0$ the F_T is opposite of the v



the model is described by μ_s and μ_0 or μ in simplified world



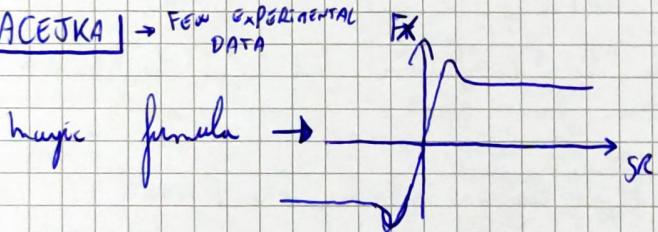
v^2

FTIRE

single formula and need only 9 parameters (easy to find)

it doesn't consider the camber

PACEJKA → few experimental data



(Stony)

create this curve in Matlab

↓

the formula with on the slide give
this shape

need only → plunk, and other

$5H, 5V \rightarrow$ how much is shifted from the origin \Rightarrow for example with one camber

- different version of PACEJKA FORMULA

- use of other formula to mix the longitudinal and lateral force plots

MF-TIRE

based on PACEJKA, introduce also the inflation, the model of the wheel.

MF-SWIFT

- the tyre is modelled by multiple slices and each slice is represented by few DOF (6) that represent a ring able to move up and down respect to the ring thanks to "dampers and springs"

TH layer

allow parking and stand still!

presence of spring and dampers in "horizontal and tangential" direction

FT tire | PHYSICAL model

represent also the deformation of the tyre!

CD tire

→ one of the most advanced model

- good results of simulation when the tyre is in resonance

- simulation of the different layers present in a tire!

- temperature model! → link to the friction coefficient!

\downarrow
useful for racing car tire ...

- Cavity model → air trapped inside the tyre

AERODYNAMICS

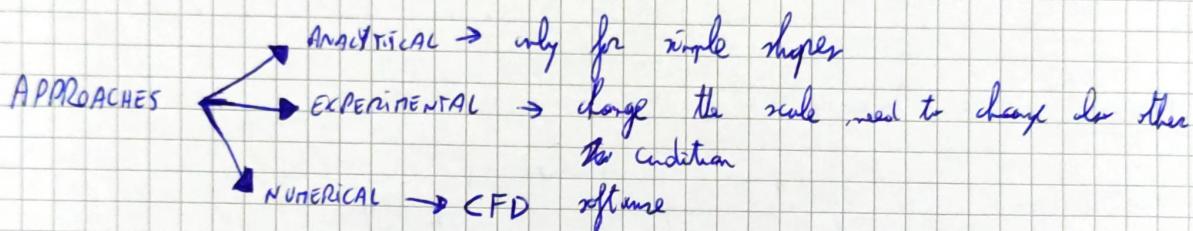
What is high speed?

→ Why want to study it? DRAG → LESS AUTONOMY

NOISE → COMFORT

DOWNGRAVE → higher performance in lateral dynamics.

→ it is a non linear effect, difficult to study



BASIC CONCEPTS

- STATIC PRESSURE p → measured with a U-PIPE
in a wing there is a pressure tap
- DYNAMIC PRESSURE q → also called stagnation pressure

- TOTAL PRESSURE

$$H = p + \underbrace{\frac{1}{2} \rho V^2}_q$$

- PIOT TUBE

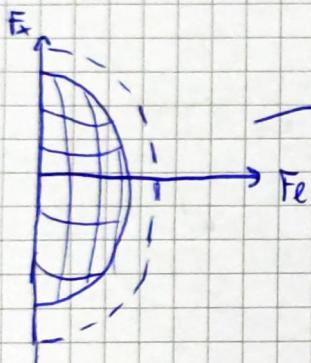
↳ complete airspeed (V) → $\xrightarrow{\text{compute same}} q = H - p$

given q is easy to find V

→ the pressure can be controlled by the shape to generate lift or downforce!

- airplane net results of the pressure ↑
- for a racecar ↓

ex. a car taking a curve



→ for a certain load applied to the tire
increasing the load we change the plot

OTHER EFFECT

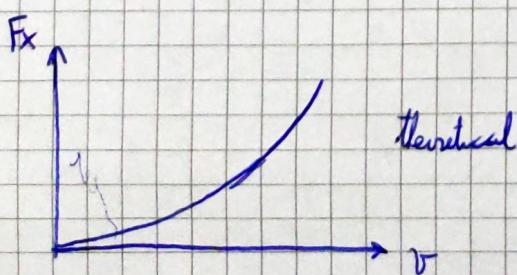
sharp corners cause separation and turbulence!

DRAG COEFFICIENT

measure how well we cut the air! we want to reduce it (always)

$$C_D = \frac{D_{air}}{A q}$$

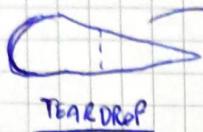
$$F_x = \frac{1}{2} A C_D \rho V^2 \quad \text{"FRONTAL DRAG"}$$



pay attention on both C_D and A

C_D of a F1 car is very bad! $C_D = 1,7 \sim$

- need of very high downforce!



optimal shape! use of long tail

TEARDROP

→ create downforce to ^{cancel free} reduce the contact between the tire and the ground.

with wings / spoilers!

→ to evaluate the vertical force $F_Z = \frac{1}{2} C_L A \rho V^2$

C_L change with $h \rightarrow$ height of spoiler! without spoiler we can have the lift effect ("dangerous")

(NB) in some cases the introduction of a spoiler doesn't ~~affect~~ increase the drag, even improving!

First spoiler used in Al FERRARI DINO, Chapman strange idea to connect the wing to the suspension bracket and not on the frame

REGARDING the FLAP, if the angle of attack is too high we get a separation. for high angle we can attach a slot in the part of the wing and a flap on the rear (with a gap)

→ use of side fins ("end plate") improve the flow of the air when touches the wing.

Use of the ground effect! \rightarrow use of the VENTURI effect

\downarrow under the car the flow is very high, so we have a low pressure
so we obtain a diff. of pressure between the different part (top and bottom)
WITH SKIRTS to exploit this effect!

REAR DIFFUSER \rightarrow create downforce

\downarrow
the diverging effect will create additional downforce!

POWER TRAIN

engine + driveline



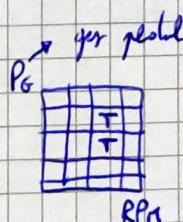
pay attention on the definition!

transmission include :- clutch

- torque converter
- gears

FREE SOFTWARE \rightarrow OPEN MODELICA

ENGINE \rightarrow use of LUT



torque converter \rightarrow packets of oil (need to watch a youtube video)

\hookrightarrow to design it, just 2 numbers K^* and R_0

sequential gearbox \rightarrow you can pass from I to II to III

H-type \rightarrow synchronizer to match the speed of the gear!

$$\tilde{\tau} = \frac{w_o}{w_i}$$

→ output
↓ input

DIFFERENTIAL

to obtain different angular velocity!

$$\tilde{\tau}_o = -1$$

↑
original

$$\omega_p = \frac{\omega_L + \omega_R}{2}$$

enter with speed in a direction and in output we have another direction

in case of - tire is stuck with ice we lose torque hence the engine inverse the speed due to the sum of speed of the wheel!
(DA Riserize)

avoid this with SELF-LOCKING DIFFERENTIAL

LONGITUDINAL DYNAMICS

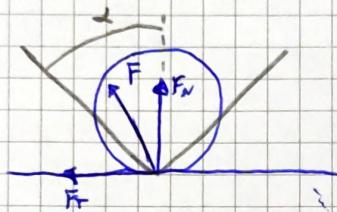
used at the very beginning of the design process!

(13)

Why? predict : acceleration, braking, steady state ~~etc~~

constant speed \rightarrow evolution of different ~~age~~ of a car

use of tire model simplified



F must stay inside the friction cone
(in this case the ~~age~~ friction angle)

How to model?

analytical formulas
numerical methods \rightarrow more complex

DYNAMIC WHEEL LOADS

- we have horizontal, vertical force acting on both tires.

and we write the ~~age~~ equation of motion! $\rightarrow \left\{ \begin{array}{l} \sum H = 0 \\ \sum V = 0 \end{array} \right.$

\downarrow
and NO SPINNING
(so NO BRAKING??)

$$\left\{ \begin{array}{l} \sum H = 0 \\ \sum V = 0 \\ \sum M_o = 0 \end{array} \right.$$

- the vertical loads are governed by 3 geometrical parameters!

$$(a_1 + a_2) = \text{wheel base}$$

- we can notice that increasing the ~~age~~ acceleration we decrease the F_{x1}
(lift of the front wheel)

\hookrightarrow here an input on this!

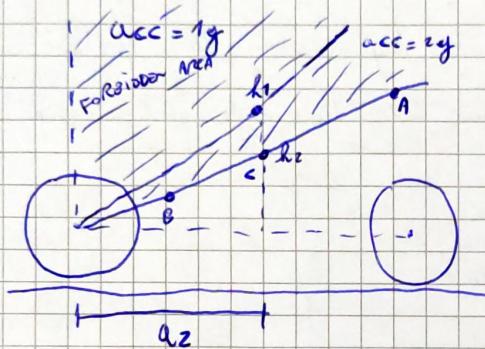
slide 9

In dynamic system \rightarrow multiplied and divided by g

LOAD TRANSFER

When we accelerate a lot! rear tire pressed to the ground, front left!

TILTING LINE CASE \rightarrow To avoid ~~the~~ flip over



the COG can stay in the limit case, the optimal point depend on different situation

B \rightarrow better for braking

A \rightarrow very suitable for braking

C \rightarrow better for steering

at the end probably C is the best option!

A, B, C \rightarrow good for lateral transfer and obtaining the best acceleration!

GRADE EFFECT

by split in two connects, one is vertical and one in horizontal direction!

\rightarrow the static moment change a little due to the grade

AERODYNAMICS

in vertical direction we introduce the F_D , in horizontal we have the aerodynamic resistance!

F_D and F_{air} depend on speed!

MAX ACCELERATION

work in friction limits

for normal tire $\mu=1$... at a certain point the power doesn't change the acceleration, we reach the limit of the tire!

(NB)

the limit depend on the friction coefficient

\Rightarrow MAX ACCELERATION is limited by tires!

What happen for a (2WD) ? different! slide \rightarrow 17

the results is worse compared to 4WD

reproducing the plot \rightarrow for a FWD better to have the engine in the front
slide 18

For a RWD better in the back
(going attention to the lifting limits)

MAX DECELERATION

better to brake with the front brake

Braking torque higher at the front!

NO SLIPPING ACCELERATION ~~basic exercise like ?!~~

LATERAL DYNAMICS

Why? predict:

- max cornering speed for a given radius
- understeer / oversteer
- stability

How? → refers to the longitudinal dynamics!

Coordinate use the standard from SAE

→ normal velocity not used in lateral dynamics (more for roll and pitch velocity)

we only care about yaw angle!

STANDBY STATE STEERING

cornering at constant speed

the ideal steering \Rightarrow ACKERMAN steering

→ the hubs face is far to the rotation axis \rightarrow that should stay in the line that connect the rear wheels

real life \rightarrow difficult to turn the two wheels in order to have the hubs that pivot the rotation axis

\Rightarrow with Ztl we can achieve an approximation of the ideal steering!

or obtain the PARALLEL or the zero ackerman

long to achieve

- definition of ACKERMAN STEERING ANGLE based on the BICYCLE MODEL

↓

$$\delta = \frac{l}{R}$$

for the same R , if l is higher we get
a higher δ

- SPACE REQUIREMENTS \rightarrow important for BUS, TRUCK

Bicycle car model

+ v your velocity

+ N torque applied to the vehicle

+ β slipping angle

+ δ angle given by the user to the steering wheel

slide 15

$\alpha_r \rightarrow$ slipping angle of the rear wheel

$\alpha_f \rightarrow$ for the front tire

- CONVENTIONAL FORCE applied to the tire is neglected, or the force is perpendicular to the tires (only the lateral force is present)

$$Y_F, Y_R$$

for balance of forces

centrifugal force $\rightarrow C_F = Y_F + Y_R$

changing the position of the CG we can obtain a NEUTRAL STEER, UNDERSTEERING, OVERSTEERING CAR

slide 18 \rightarrow for an understeering car we need to increase the S at the inverse of the centripetal acceleration! (is the opposite for understeering car)

- for a TILTED Road a NEUTRAL STEERING vehicle Keep the trajectory!

STABILITY

Considering a vehicle that goes straight with a constant speed V , there is a value of V that if we have a disturbance in the ride the vehicle start to oscillating (possible flipping)

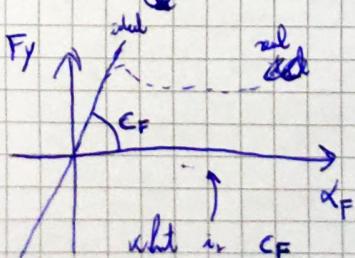
A NON STABLE VEHICLE is the one that receive the disturbance and lose the control

What happen to ζ for the stability?

$N \rightarrow$ yaw torque $I\ddot{\epsilon} \rightarrow$ inertia in z axis

$y \rightarrow$ displacement

for the exam \rightarrow slide 26 \rightarrow under the z ~~ode~~ ODE and the equation of $N\beta$



$N\beta = 0$ for a NEUTRAL STEER CAR
 > 0 understeer
 < 0 oversteer