Homework 03

Math 315, Fall 2018

Due: Sept. 28 by 4 p.m.

Instructions: Complete the following problems and submit them by 4 p.m. on the due date. Please make sure that your solution is neatly written, clearly organized, and stapled (if there are multiple pages).

Problem 1. Suppose that a trucking company owns a large fleet of well-maintained trucks and assume that breakdowns appear to occur at random times. The president of the company is interested in learning about the daily rate λ at which breakdowns occur. (Realistically, each truck would have a breakdown rate that depends possibly on its type, age, condition, driver, usage, etc. The breakdown rate for the whole company can be viewed as the sum of the breakdown rates of the individual trucks.) For a given value of the rate parameter λ , it is known that the number of breakdowns y on a particular day has a Poisson distribution with mean λ.

(a) Suppose that one observes the number of truck breakdowns for n consecutive days—denote these numbers by $y_1, \ldots, y_n$. If one assumes that these are exchangeable measurements (conditionally independent given λ), find the find the joint probability distribution of $y_1, \ldots, y_n$ (i.e. the likelihood).

(b) Derive the Jeffreys' prior that goes with the Poisson likelihood.

(c) Write out the mathematical form of the unnormalized posterior density. Identify its parametric family and parameters.

Problem 2: 4H1

- The textbook is ambiguous about whether it wants predictions for the average height at each weight, or predictions for a single individual. Assume that you're making predictions about the average.
- Use the `adults` subset again.

Problem 3: 4H2

- Part a: be sure that your answer discusses why you chose the hyperparameter values that you did.
- Part b: Only superimpose the HPDI for the mean.

Problem 4: 4H3

- Part a: if you haven't taken Math 245, don't panic about the interpretation. Just think about the scale of measurement and carefully try to interpret what's going on. If it doesn't seem to make sense, then tell me that.

- Part b: here's the `ggplot2` starter code (`rangi2` is a color defined in the `rethinking` package):

```
ggplot(data = Howell1) +
  geom_point(mapping = aes(x = weight, y = height), color = rangi2, alpha = 0.4)
```

When you are plotting the predicted mean function and a 97% HPDI for the mean remember that you are plotting them on the *original scale* and that the model refers to the *log scale*.

Only superimpose the HPDI for the mean.