Identifying Minimal Set of FDS or

Standard Form of FDs or

Irreducible set of FDs or

Canonical Cover

Standard Form of FDs or Canonical Cover

- A canonical cover of F is a **minimal set of functional dependencies** equivalent to F, having **no redundant dependencies or redundant parts of dependencies**.
- It is denoted by F_c
- A set of FDs F is minimal if it satisfy the following conditions:
  - **F** and F_c must be **equivalent (i.e., F logically implies** all dependencies in F_c and vice-versa)
  - Every **FD of F_c is simple** (i.e., in the RHS there will be single attribute in every FD.

Union Rule

$F = \{A \rightarrow BC\}$

$F_c = \{A \rightarrow B, A \rightarrow C\}$

Decomposition Rule

  - Fc is **left reduced** (set of FDs, where **no extraneous left** attribute present).
  - Fc is **non-redundant.**

NOTE: Let F be set of FDs over relation R and let $A \rightarrow B$, a FD in F, is said to be redundant, if and only if it will be equivalent to $F = F - \{A \rightarrow B\}$

Step-I: Decompose all the FDs such that R.H.S contain only single attribute.

$F = \{A \rightarrow BC\}$

$F_c = \{A \rightarrow B, A \rightarrow C\}$

Important: Decomposition is always done at right side of FD. Never try to decompose at left side. It makes FDs invalid.

$AB \rightarrow CD$	$AB \rightarrow CD$
We can decompose	We can't decompose
$AB \rightarrow C$	$A \rightarrow CD$
$AB \rightarrow D$ ✔	$B \rightarrow CD$ ✘

Step-II: Find the redundant FDs and delete them from FD set.

$F = \{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$

A^+ (take A^+ without considering $A \rightarrow B$) = $\{A, C\}$,

It means $A \rightarrow B$ is must, we can't get B from any other FD.

$B^+ = \{B\}$, it means $B \rightarrow C$ is also must.

$A^+ = \{A, B, C\}$, thus, $A \rightarrow C$ is redundant.

$F = \{A \rightarrow B, B \rightarrow C\}$

Note: Once the non essential FD is identified, then do not include that non essential FD while computing the closure of attributes further. Exclude that FD immediately.

NOTE: You and directly remove all the Trivial FDs, if exists.

Step-III: Find redundant/extraneous attribute on L.H.S and delete them.

- If we are able to remove an attribute from a functional dependency without changing the closure of the set of functional dependencies, that attribute is called as Extraneous Attribute. It is possible when there are more than one attribute on left hand side.

$$F = \{AB \rightarrow C\}$$

This FD has more than one attribute on the Left side, it may contain extra attribute at left side.

Q.1 F = {$A \rightarrow C$, $AC \rightarrow D$, $E \rightarrow AD$, $E \rightarrow H$}

Find the canonical cover for the given set of FDs over F.

Step-I : Decompose all the FDs to get single attribute on RHS.

$A \rightarrow C$
$AC \rightarrow D$
$E \rightarrow A$
$E \rightarrow D$
$E \rightarrow H$

Step-II : Remove redundant FDs by checking the effect of their non-existence one by one.

~~$A \rightarrow C$~~	$A^+ = \{A\}$, thus, $A \rightarrow C$ is must.
~~$AC \rightarrow D$~~	$AC^+ = \{A, C\}$, thus, $AC \rightarrow D$ is must
~~$E \rightarrow A$~~	$E^+ = \{E, D, H\}$, thus $E \rightarrow A$ is must.
~~$E \rightarrow D$~~	$E^+ = \{E, A, H, C, D\}$, thus $E \rightarrow D$ is redundant.
~~$E \rightarrow H$~~	$E^+ = \{E, A, C, D\}$, thus $E \rightarrow H$ is must.

$A \rightarrow C$
$AC \rightarrow D$
$E \rightarrow A$
$E \rightarrow H$

Step-III: Remove extraneous attribute in FDs having more than one attribute on LHS.

$A \rightarrow C$
$AC \rightarrow D$
$E \rightarrow A$
$E \rightarrow H$

Check $(AC)^+ = ACD$
Check $A^+ = AC$ (by ignoring AC -> D)

In AC -> D, C is extraneous since $A^+ = AC$

$F_C = \{ A \rightarrow C, A \rightarrow D, E \rightarrow A, E \rightarrow H\}$
OR
$F_C = \{ A \rightarrow CD, E \rightarrow AH\}$

Q.2 $F = \{A \rightarrow B, C \rightarrow B, D \rightarrow ABC, AC \rightarrow D\}$.

Find the canonical cover for the given set of FDs over F.

Step-I: $A \rightarrow B$

$C \rightarrow B$

$D \rightarrow A$

$D \rightarrow B$

$D \rightarrow C$

$AC \rightarrow D$

Step-II: $A \rightarrow B$

$C \rightarrow B$

$D \rightarrow A$

$\cancel{D \rightarrow B}$

$D \rightarrow C$

$AC \rightarrow D$

Step-III: $AC \rightarrow D$

$A^+ = AB$

$C^+ = CB$

Neither A, nor C is extraneous attribute.

F_C: $\{A \rightarrow B, C \rightarrow B, D \rightarrow AC, AC \rightarrow D\}$

Q. 4 Given a relational Schema R(A, B, C, D) and set of Function Dependency FD = { B → A, AD → BC, C → ABD }. Find the canonical cover?

Step-I: Split FDs

1. B → A
2. AD → B (using decomposition inference rule on AD → BC)
3. AD → C (using decomposition inference rule on AD → BC)
4. C → A (using decomposition inference rule on C → ABD)
5. C → B (using decomposition inference rule on C → ABD)
6. C → D (using decomposition inference rule on C → ABD)

Step-II: Remove redundant FDs

B → A, AD → C, C → B, C → D

Step-III: Remove extraneous attributes

B → A, AD → C, C → BD

Q. 4 Given a relational Schema R(W, X, Y, Z) and set of Function Dependency FD = { W → X, Y → X, Z → WXY, WY → Z }. Find the canonical cover?

Step-I: Split FDs

1. W → X
2. Y → X
3. Z → W (using decomposition inference rule on Z → WXY)
4. Z → X (using decomposition inference rule on Z → WXY)
5. Z → Y (using decomposition inference rule on Z → WXY)
6. WY → Z

Step-II: Remove redundant FDs

Hence resultant FD = { W → X, Y → X, Z → W, Z → Y, WY → Z }

Step-III: Remove extraneous attributes

Hence resultant FD = { W → X, Y → X, Z → W, Z → Y, WY → Z } and we can rewrite as:

FD = { W → X, Y → X, Z → WY, WY → Z } is Canonical Cover of FD = { W → X, Y → X, Z → WXY, WY → Z }.

Q. 5 Given a relational Schema R(V, W, X, Y, Z) and set of Function Dependency FD = { V → W, VW → X, Y → VXZ }. Find the canonical cover?

Step-I: Split FDs

1. V → W
2. VW → X
3. Y → V (using decomposition inference rule on Y → VXZ)
4. Y → X (using decomposition inference rule on Y → VXZ)
5. Y → Z (using decomposition inference rule on Y → VXZ)

Step-II: Remove redundant FDs

Hence resultant FD = { V → W, VW → X, Y → V, Y → Z }.

Step-III: Remove extraneous attributes

Hence resultant FD = { V → W, V → X, Y → V, Y → Z } and we can rewrite as

FD = { V → WX, Y → VZ } is Canonical Cover of FD = { V → W, VW → X, Y → VXZ }.

Q. 6 Suppose a relational schema R(w x y z), and set of functional dependency as following

$$F : \{ x \rightarrow w,\ wz \rightarrow xy,\ y \rightarrow wxz \}$$

Find the canonical cover F_c (Minimal set of functional dependency).

Step-I: Decompose FDs

$x \rightarrow w$,

$wz \rightarrow x$,

$wz \rightarrow y$,

$y \rightarrow w$,

$y \rightarrow x$,

$y \rightarrow z$

Step-II: Remove redundant FDs

$x \rightarrow w$

$wz \rightarrow y$

$y \rightarrow x$

$y \rightarrow z$

Step-III: Remove extraneous attributes

$$F_c : \{ x \rightarrow w,\ wz \rightarrow y,\ y \rightarrow xz \}$$