Sheafification

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Consider a presheaf $\mathcal{F}$ on a space X. We want to canonically associate a sheaf $\mathcal{F}^+$ to X. To do this, associate to every open set $U \subset X$ the set of "formal sections" of the form

$$f := (f_p \in \mathcal{F}_p)_{p \in U} \tag{1}$$
$$\text{with } f_q = s_q \text{ at all } q \in V, \text{ for } p \in \text{open } U \subset V \tag{2}$$

The idea is to call a set of germs at points in U a "section" if it locally corresponds to functions in the presheaf. Now call the rule that associates

$$U \mapsto \{f_1, f_2, \ldots, \}$$

$\mathcal{F}^+$.

Theorem. $\mathcal{F}^+$ is a sheaf.

Let's verify all the properties. This will necessarily be a dull task.

Theorem. $\mathcal{F}^+$ is a presheaf.

Proof.

1. *The trivial restriction does nothing to sections.* This is obvious, since the open subsets and the sections of $\mathcal{F}$ over the open subsets are the same for U and . . . U.

□