

# The Yoneda lemma

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- Covariant hom-functor:  $\mathbf{Hom}(a, -)$ , with the action on morphisms given by maps  $\mathbf{Hom}(a, x) \rightarrow \mathbf{Hom}(a, y)$ ,

$$f \mapsto (- \circ f).$$

- Contravariant hom-functor:  $\mathbf{Hom}(-, a)$ , with the action on morphisms given by maps  $\mathbf{Hom}(a, x) \leftarrow \mathbf{Hom}(a, y)$ ,

$$f \mapsto (f \circ -).$$

**Definition.** We say that a functor  $F : C \rightarrow \mathbf{Set}$  is *representable* if there exists a natural transformation such that

$$\alpha : F \simeq \mathbf{Hom}(a, -).$$

Examples:

- Forgetful functors, e.g.  $F : \mathbf{Grp} \rightarrow \mathbf{Set}$ . Notice that  $\mathbf{Hom}(\mathbb{Z}, G)$  is completely determined by where 1 goes, so it is sort of like the set of maps from a one-point test space. So elements of  $\mathbf{Hom}(\mathbb{Z}, G)$  correspond to elements of  $G$ .

All of this is trivial, of course.