Modified Denavit-Hartenberg (DH) parameters

The DH parameters were originally proposed by Denavit and Hartenberg (1955) and widely used to define links' configuration of a robotic manipulator consisting of one degree-of-freedom (DOF) joints, i.e., revolute or prismatic. Later, Khalil and Kleinfinger (1986) showed that the DH parameters are powerful tool for serial robots, but, lead to ambiguities in the case of closed and tree structured robots. They presented the modified DH parameter from its original definition. Craig (1991) also used modified DH notation for serial robots. The modified DH parameters are used in Recursive Dynamics Simulator (ReDySim) and illustrated next.

In order to define configuration of a link relative to its neighbours, coordinate frames are attached to the links. The frames $\mathcal{F}_{k-1}$ (O_{k-1}-$X_{k-1}Y_{k-1}Z_{k-1}$) and $\mathcal{F}_k$ (O_k-$X_kY_kZ_k$) are rigidly attached to links (k-1) and k, respectively. The axis Z_{k-1} is along the $(k-1)^{st}$ joint, whereas the origin O_{k-1} is located on a point where common normal to Z_{k-1} and Z_k intersect Z_{k-1}. The axis X_{k-1} is along common normal to Z_{k-1} and Z_k. Moreover, axis Y_{k-1} is chosen to complete a right handed frame. It may be noted that

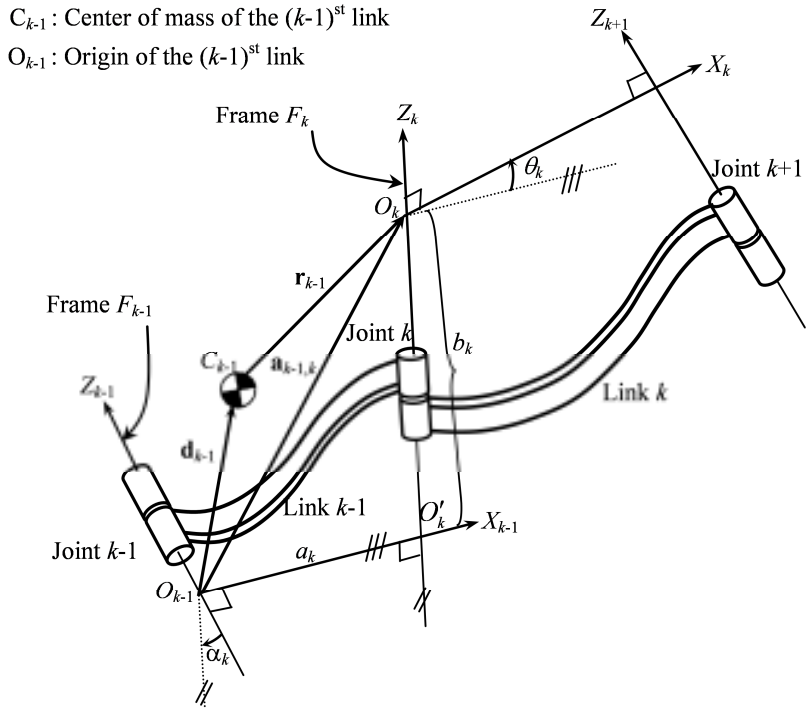

Fig. 1 Frame convention for modified DH parameters

frame $\mathcal{F}_0$, O_0-$X_0Y_0Z_0$, may be arbitrarily chosen, and hence, Z_0 is chosen coincident with Z_1 for the sake of simplification.

Once the link frames have been established, the position and the orientation of the frame k with respect k-1 are specified by the four parameters known as DH parameter. Out of these four parameters only one parameter is variable whereas others are constant. These link parameters in terms of the link frames can be obtained as follows:

Twist angle (α_k) = the angle from Z_{k-1} to Z_k aboutX_{k-1}

Link length (a_k) = the distance from Z_{k-1} to Z_k along X_{k-1}

Joint offset (b_k) = the distance from X_{k-1} to X_k along Z_k

Joint angle (θ_k) = the angle from X_{k-1} to X_k about Z_k

θ_k or b_k being the joint variable depending on the type of the joint. If the joint is revolute than θ_k is the joint variable, whereas b_k is the joint variable in the case of prismatic joint. Based on the above DH parameters, rotation matrix between the frames k and k-1 can be obtained as follows

$$\mathbf{Q}_k \equiv \begin{bmatrix} C\theta_k & -S\theta_k & 0 \\ S\theta_k C\alpha_k & C\theta_k C\alpha_k & -S\alpha_k \\ S\theta_k S\alpha_k & C\theta_k S\alpha_k & C\alpha_k \end{bmatrix} \tag{1}$$

The representation of the position vector $\mathbf{a}_{k-1,k}$, measured from the origin of link k-1 to the origin of link k, in frame k-1 is given by

$$\left[\mathbf{a}_{k-1,k}\right]_{k-1} \equiv \begin{bmatrix} a_k \\ -b_k S\alpha_k \\ b_k C\alpha_k \end{bmatrix} \tag{2}$$

It is worth noting that the vector $\mathbf{a}_{k-1,k}$ in Eq. (2) is constant in frame k-1, if $(k\text{-}1)^{st}$ joint is revolute. Similarly, vector $\mathbf{d}_{k-1}$ is also constant in frame k-1.