SMI College & Career™

Technical Guide

An Assessment of College and Career Math
Readiness Across Grades K–Algebra II

SCHOLASTIC

SMI College & Career™

Technical Guide

ISBN-13: 978-0-545-79640-8
ISBN-10: 0-545-79640-7

Table of Contents

List of Figures and Tables

Figures

Tables

List of Figures and Tables *(continued)*

Introduction

Introduction

Scholastic Math Inventory™ College & Career, developed by Scholastic Inc., is an objective assessment of a student's readiness for mathematics instruction from Kindergarten through Algebra II (or High School Integrated Math III), which is commonly considered an indicator of college and career readiness. SMI College & Career quantifies a student's path to and through high school mathematics and can be administered to students in Grades K–12. A computer-adaptive test, SMI College & Career delivers test items targeted to students' ability level. The measurement system for SMI College & Career is the Quantile® Framework for Mathematics and, while SMI College & Career can be used for several instructional purposes, a completed SMI College & Career administration yields a Quantile® measure for each student. Teachers and administrators can use the students' Quantile measures to:

- **Conduct universal screening:** identify the degree to which students are ready for instruction on certain mathematical concepts and skills
- **Differentiate instruction:** provide targeted support for students at their readiness level
- **Monitor growth:** gauge students' developing understandings of mathematics in relation to the objective measure of algebra readiness and Algebra II completion and to being on track for college and career at the completion of high school

The Quantile Framework for Mathematics, developed by MetaMetrics, Inc., is a scientifically based scale of mathematics skills and concepts. The Quantile Framework helps educators measure student progress as well as forecast student development by providing a common metric for mathematics concepts and skills and for students' abilities. A Quantile measure refers to both the level of difficulty of the mathematical skills and concepts and a student's readiness for instruction.

SMI was developed during 2008–2010 and launched in Summer 2010. Additional items were added in 2011 and again in 2013. Studies of SMI validity began in 2009. For a more robust validity analysis, Phase II of the SMI validation study was conducted during the 2009–2010 school year. Phase III of the validity study was completed in 2012 with data collected from the 2010–2011 school year.

This technical guide is intended to provide users with the broad research foundation essential for deciding how SMI College & Career should be administered and what kinds of inferences can be drawn from the results pertaining to students' mathematical achievement. In addition, this guide describes the development and psychometric characteristics of this assessment and the Quantile Framework.

Features of *Scholastic Math Inventory* College & Career

SMI College & Career is a research-based universal screener and growth monitor designed to measure students' readiness for instruction. The assessment is computer-based and features two components:

- The actual student test, which is computer adaptive, with more than 5,000 items
- The management system, Scholastic Achievement Manager, where enrollments are managed and customized features and data reports can be accessed

SMI College & Career Test

When students are assessed with SMI College & Career, they receive a single measure—a Quantile measure—that indicates their instructional readiness for calibrated content to and through Algebra II/High School Integrated Math III. As a computer-adaptive test, SMI College & Career provides items based on students' previous responses. This format allows students to be tested on a large range of skills and concepts in a shorter amount of time and yields a highly accurate score.

SMI College & Career can be completed in 20–40 minutes and is presented in three parts: Math Fact Screener (for Kindergarten and Grade 1 an Early Numeracy Screener, which tests students on counting and quantity comparison, is used), Practice Test, and Scored Test.

The Math Fact Screener is a simple and fast test focused on basic addition and multiplication skills. Once students demonstrate relative mastery of math facts, they will not experience this part of the assessment again.

The Practice Test is a three- to five-question test calibrated far below the students' current grade level. The purpose of this part is to ensure students can interact with a computer test successfully and to provide them an opportunity to practice with the tools provided in the assessment. Teachers can allow students to skip this part of the test after its first administration.

The Scored Test part of the assessment produces Quantile measures for the students. In this part, students engage with at least 25 and as many as 45 test items that follow a consistent four-option, multiple-choice format. Items are presented in a simple, straightforward format with clear unambiguous language. Because of the depth of the item bank, students do not see the same item twice in the same administration or in the next two administrations. Items at the lowest level are sampled from Kindergarten and Grade 1 mathematical skills and topics; items at the highest level are sampled from Algebra II/High School Integrated Math III topics. Some items may include mathematical representations such as diagrams, graphs, tables, and charts. Optional calculators and formula sheets are included in the program. Calculators will not be visible for problems whose purpose is computational proficiency. Providing students with paper and pencil during their assessment is recommended. All assessments can be saved and accessed at another time for completion. This feature is important for students with extended time accommodations.

Scholastic Achievement Manager

Scholastic Achievement Manager (SAM) is the data backbone for all Scholastic technology programs, including SMI College & Career. In SAM, educators can manage enrollment, create classes, and assign usernames and passwords.

SAM also provides teachers and administrators with nine template reports that support this formative assessment with actionable data for teachers, students, parents, and administrators. These reports feature a growth report that provides a quantifiable trajectory to and through Algebra II/High School Integrated Math III—a course cited as the gatekeeper to college and career readiness—and provide a tool to differentiate math instruction by linking Quantile measures to classroom resources and basal math textbooks.

There are over 5,000 test items that were rigorously developed to connect to a Quantile measure. When students are tested with SMI College & Career, they are tested on items that represent five content strands (Number & Operations; Algebraic Thinking, Patterns, and Proportional Reasoning; Geometry, Measurement, and Data; Statistics & Probability [Grades 6–11 only]; and Expressions & Equations, Algebra, and Functions [Grades 6–11 only]) and receive a single measure—a Quantile measure—that indicates their instructional readiness for calibrated content.

Scholastic Central

Scholastic Central puts your assessment calendar, data snapshots and news regarding student performance and usage, instructional recommendations, and professional learning resources all in one centralized location to make it easy to assess and plan instruction.

Leadership Dashboard

Administrators access the Leadership Dashboard on Scholastic Central to view high-level data snapshots and data analytics for the schools using SMI College & Career. Follow up with individual schools for appropriate intervention to increase student performance to proficiency.

You can use the Leadership Dashboard to:

- View Performance Level and Growth data snapshots, with pop-up information and a table of detailed data by school
- Filter school- and district-level data by demographics
- View resources for Implementation Success Factors
- Schedule and view reports

Rationale for and Uses of *Scholastic Math Inventory* College & Career

A comprehensive program of mathematics education includes curriculum, assessment, and instruction.

- Curriculum is the planned set of standards and materials that are intended for implementation during the academic year.
- Instruction is the enacting of curriculum; it is when students build new concepts and skills.
- Assessment should inform instruction by describing skills and concepts students are ready to learn.

The challenge for educators is meeting the demand of curriculum in a classroom of diverse learners. This challenge is met when the curriculum unfolds in a way that all students make progress. Typically, the best starting point for instruction is to identify the skills and concepts that each student is ready to learn.

SMI College & Career provides educators with information related to the difficulty of skills and concepts, as well as the increasing difficulty of content progressions. This information is found in the SMI skills and concepts database at www.scholastic.com/SMI.

An SMI College & Career test reports students' Quantile measures. In the SMI College & Career reports, student test results are aligned with specific skills and concepts that are appropriate for instruction. For example, Figure 1 depicts a report that specifies growth in the skills and concepts that a student is ready to learn and links those concepts to the Common Core State Standards identification number and other Quantile-based instructional information.

FIGURE 1. Growth Report.

GROWTH

Growth Report

CLASS: 3rd Period

SMI College™ & Career

School: Lincoln Middle School
Teacher: Sarah Foster
Grade: 5

Time Period: 12/13/14–02/22/15

		FIRST TEST		LAST TEST		
STUDENT	GRADE	DATE	QUANTILE® MEASURE/ PERFORMANCE LEVEL	DATE	QUANTILE® MEASURE/ PERFORMANCE LEVEL	GROWTH IN QUANTILE® MEASURE
Gainer, Jacquelyn	5	12/13/14	925Q P	02/22/15	1100Q A	175Q
Hartsock, Shalanda	5	12/13/14	595Q B	02/22/15	750Q B	155Q
Cho, Henry	5	12/13/14	955Q P	02/22/15	1100Q A	155Q
Cooper, Maya	5	12/13/14	700Q B	02/22/15	820Q P	120Q
Robinson, Tiffany	5	12/13/14	390Q BB	▶ 02/22/15	485Q BB	95Q
Cocanower, Jaime	5	12/13/14	640Q B	02/22/15	710Q B	70Q
Garcia, Matt	5	12/13/14	615Q B	02/22/15	680Q B	65Q
Terrell, Walt	5	12/13/14	670Q B	02/22/15	720Q B	50Q
Enoki, Jeanette	5	▶ 12/13/14	750Q P	02/22/15	800Q B	50Q
Collins, Chris	5	▶ 12/13/14	855Q P	▶ 02/22/15	890Q P	35Q
Morris, Timothy	5	12/13/14	620Q B	02/22/15	650Q B	30Q
Ramirez, Jeremy	5	▶ 12/13/14	580Q B	02/22/15	600Q B	20Q

KEY

EM Emerging Mathematician
A ADVANCED
P PROFICIENT
B BASIC
BB BELOW BASIC
▶ Test taken in less than 15 minutes

YEAR-END PROFICIENCY RANGES					
GRADE K	10Q–175Q	**GRADE 5**	**820Q–1020Q**	GRADE 9	1140Q–1325Q
GRADE 1	260Q–450Q	GRADE 6	870Q–1125Q	GRADE 10	1220Q–1375Q
GRADE 2	405Q–600Q	GRADE 7	950Q–1175Q	GRADE 11	1350Q–1425Q
GRADE 3	625Q–850Q	GRADE 8	1030Q–1255Q	GRADE 12	1390Q–1505Q
GRADE 4	715Q–950Q				

USING THE DATA	
Purpose: This report shows changes in student performance and growth on SMI over time.	***Follow-Up:*** Provide opportunities to challenge students who show significant growth. Provide targeted intervention and support to students who show little growth.

Printed by: Teacher

Page 1 of 1
Printed on: 2/22/2015

After an initial assessment with SMI College & Career, it is possible to monitor progress, predict the student's likelihood of success when instructed on mathematical skills and concepts, and report on actual student growth toward the objective of algebra completion and, by extension, college and career readiness.

Students' Quantile measures indicate their readiness for instruction on skills and concepts within a range of 50Q above and below their Quantile measure. Students should be successful at independent practice with skills and concepts that are about 150Q to 250Q below their Quantile measure. With SMI College & Career test results, educators can choose materials and resources for targeted instruction and practice.

On a school-wide or instructional level, SMI College & Career results can be used to screen for intervention and acceleration, measure progress at benchmarking intervals, group students for differentiated instruction, provide an indication of outcomes on summative assessments, provide an independent measure of programmatic success, and inform district decision making.

SMI supports school districts' efforts to accelerate the learning path of struggling students. State educational agencies (SEAs), local educational agencies (LEAs), and schools can use Title 1, Part A funds associated with the American Recovery and Reinvestment Act of 2009 (ARRA) to identify, create, and structure opportunities and strategies to strengthen education, drive school reform, and improve the academic achievement of at-risk students using funds under Title I, Part A of the Elementary and Secondary Education Act of 1965 (ESEA) (US Department of Education, 2009). Tiered intervention strategies can be used to provide support for students who are "at risk" of not meeting state performance levels that define "proficient" achievement.

One such tiered approach is Response to Intervention (RTI), which involves providing the most appropriate instruction, services, and scientifically based interventions to struggling students—with increasing intensity at each tier of instruction (Cortiella, 2005).

As an academic assessment that can be used as a universal screener of all students, SMI College & Career can also be used to identify those students who are "at risk" and provide student grouping recommendations for appropriate instruction. SMI College & Career can be administered three to five times per year to monitor students' growth. Regular monitoring of students' progress is critical in determining if a student should move from one tier of intervention to another and to determine the effectiveness of the intervention.

Another instructional approach supported by SMI College & Career is differentiated instruction in the Tier I classroom (Tomlinson, 2001). By providing direct instructional recommendations for each student or each group of students and linking those recommendations to the skills and concepts of the Quantile Framework, SMI College & Career provides data to target and pace instruction.

Limitations of *Scholastic Math Inventory* College & Career

SMI College & Career utilizes an algorithm to ensure that each assessment is targeted to measure the readiness for instruction of each student. Teachers and administrators can use the results to identify the mathematic skills and concepts that are most appropriate for their students.

However, as with any assessment, SMI College & Career is one source of evidence about a student's mathematical understandings. Obviously, impactful decisions are best made when using multiple sources of evidence. Other sources include student work such as homework and unit test results, state test data, adherence to mathematics curriculum and pacing guides, student motivation, and teacher judgment.

One measure of student performance, taken on one day, is never sufficient to make high-stakes, student-specific decisions such as summer school placement or retention.

Theoretical Foundation and Validity of the Quantile Framework for Mathematics

Theoretical Foundation and Validity of the Quantile Framework for Mathematics

The Quantile Framework is the backbone on which the mathematical skills and concepts assessed in SMI College & Career are mapped. The Quantile Framework is a scale that describes a student's mathematical achievement. Similar to how degrees on a thermometer measure temperature, the Quantile Framework uses a common metric—the Quantile—to scientifically measure a student's ability to reason mathematically, monitor a student's readiness for mathematics instruction, and locate a student on its taxonomy of mathematical skills, concepts, and applications.

The Quantile Framework uses this common metric to measure many different aspects of education in mathematics. The same metric can be applied to measure the materials used in instruction, to calibrate the assessments used to monitor instruction, and to interpret the results that are derived from the assessments. The result is an anchor to which resources, concepts, skills, and assessments can be connected.

There are dozens of mathematics tests that measure a common construct and report results in proprietary, non-exchangeable metrics. Not only are all of the tests using different units of measurement, but all use different scales on which to make measurements. Consequently, it is difficult to connect the test results with materials used in the classroom. The alignment of materials and linking of assessments with the Quantile Framework enables educators, parents, and students to communicate and improve mathematics learning. The benefits of having a common metric include being able to:

- Develop individual multiyear growth trajectories that denote a developmental continuum from the early elementary level to Algebra II and Precalculus. The Quantile scale is vertically constructed, so the meaning of a Quantile measure is the same regardless of grade level.
- Monitor and report student growth that meets the needs of state-initiated accountability systems
- Help classroom teachers make day-to-day instructional decisions that foster acceleration and growth toward algebra readiness and through the next several years of secondary mathematics

To develop the Quantile Framework, the following preliminary tasks were undertaken:

- Building a structure of mathematical performance that spans the developmental continuum from Kindergarten content through Geometry, Algebra II, and Precalculus content
- Developing a bank of items that had been field tested
- Developing the Quantile scale (multiplier and anchor point) based on the calibrations of the field-test items
- Validating the measurement of mathematics achievement as defined by the Quantile Framework

Each of these tasks is described in the sections that follow. The implementation of the Quantile Framework in the development of SMI College & Career, as well as the use of SMI College & Career and the interpretation of results, is described in later sections of this guide.

The Quantile Framework for Mathematics Taxonomy

To develop a framework of mathematical performance, an initial structure needs to be established. The structure of the Quantile Framework is organized around two guiding principles—(1) mathematics is a content area, and (2) learning mathematics is developmental in nature.

The National Mathematics Advisory Panel report (2008, p. xix) recommended the following:

> To prepare students for Algebra, the curriculum must simultaneously develop conceptual understanding, computational fluency, and problem-solving skills . . . [t]hese capabilities are mutually supportive, each facilitating learning of the others. Teachers should emphasize these interrelations; taken together, conceptual understanding of mathematical operations, fluent execution of procedures, and fast access to number combinations jointly support effective and efficient problem solving.

When developing the Quantile Framework, MetaMetrics recognized that in order to adequately address the scope and complexity of mathematics, multiple proficiencies and competencies must be assessed. The Quantile Framework is an effort to recognize and define a developmental context of mathematics instruction. This notion is consistent with the National Council of Teachers of Mathematics' (NCTM) conclusions about the importance of school mathematics for college and career readiness presented in *Administrator's Guide: Interpreting the Common Core State Standards to Improve Mathematics Education*, published in 2011.

Strands as Sub-domains of Mathematical Content

A strand is a major subdivision of mathematical content. The strands describe what students should know and be able to do. The National Council of Teachers of Mathematics (NCTM) publication *Principles and Standards for School Mathematics* (2000, hereafter NCTM Standards) outlined ten standards—five content standards and five process standards. These content standards are Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability. The process standards are Communications, Connections, Problem Solving, Reasoning, and Representation.

As of March 2014, the Common Core State Standards for Mathematics (CCSS) have been adopted in 44 states, the Department of Defense Education Activity, Washington DC, Guam, the Northern Mariana Islands, and the US Virgin Islands. The CCSS identify critical areas of mathematics that students are expected to learn each year from Kindergarten through Grade 8. The critical areas are divided into domains that differ at each grade level and include Counting and Cardinality, Operations and Algebraic Thinking, Number and Operations in Base Ten, Number and Operations—Fractions, Ratios and Proportional Relationships, the Number System, Expressions and Equations, Functions, Measurement and Data, Statistics and Probability, and Geometry. The CCSS for Grades 9–12 are organized by six conceptual categories: Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics and Probability (NGA Center & CCSSO, 2010a).

The six strands of the Quantile Framework bridge the Content Standards of the NCTM Standards and the domains specified in the CCSS.

1. **Number Sense.** Students with number sense are able to understand a number as a specific amount, a product of factors, and the sum of place values in expanded form. These students have an in-depth understanding of the base-ten system and understand the different representations of numbers.

2. **Numerical Operations.** Students perform operations using strategies and standard algorithms on different types of numbers but also use estimation to simplify computation and to determine how reasonable their results are. This strand also encompasses computational fluency.

3. **Geometry.** The characteristics, properties, and comparison of shapes and structures are covered by geometry, including the composition and decomposition of shapes. Not only does geometry cover abstract shapes and concepts, but it provides a structure that can be used to observe the world.

4. **Algebra and Algebraic Thinking.** The use of symbols and variables to describe the relationships between different quantities is covered by algebra. By representing unknowns and understanding the meaning of equality, students develop the ability to use algebraic thinking to make generalizations. Algebraic representations can also allow the modeling of an evolving relationship between two or more variables.

5. **Data Analysis and Probability.** The gathering of data and interpretation of data are included in data analysis, probability, and statistics. The ability to apply knowledge gathered using mathematical methods to draw logical conclusions is an essential skill addressed in this strand.

6. **Measurement.** The description of the characteristics of an object using numerical attributes is covered by measurement. The strand includes using the concept of a unit to determine length, area, and volume in the various systems of measurement, and the relationship between units of measurement within and between these systems.

The Quantile Skill and Concept

Within the Quantile Framework, a Quantile Skill and Concept, or QSC, describes a specific mathematical skill or concept a student can acquire. These QSCs are arranged in an orderly progression to create a taxonomy called the Quantile scale. Examples of QSCs include:

1. Know and use addition and subtraction facts to 10 and understand the meaning of equality
2. Use addition and subtraction to find unknown measures of nonoverlapping angles
3. Determine the effects of changes in slope and/or intercepts on graphs and equations of lines

The QSCs used within the Quantile Framework were developed during Spring 2003, for Grades 1–8, Grade 9 (Algebra I), and Grade 10 (Geometry). The framework was extended to Algebra II and revised during Summer and Fall 2003. The content was finally extended to include material typically taught in Kindergarten and Grade 12 (Precalculus) during the Summer and Fall 2007.

The first step in developing a content taxonomy was to review the curricular frameworks from the following sources:

- NCTM *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000)
- *Mathematics Framework for the 2005 National Assessment of Educational Progress: Prepublication Edition* (NAGB, 2005)
- North Carolina *Standard Course of Study* (Revised in 2003 for Kindergarten through Grade 12) (NCDPI, 1996)

- California Mathematics Framework and state assessment blueprints: *Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve* (2000 Revised Edition), *Mathematics Content Standards for California Public Schools: Kindergarten Through Grade Twelve* (December 1997), blueprints document for the Star Program California Standards Tests: Mathematics (California Department of Education, adopted by SBE October 9, 2002), and sample items for the California Mathematics Standards Tests (California Department of Education, January 2002).
- Florida Sunshine State Standards: *Sunshine State Standards Grade Level Expectations* for Mathematics, Grade 2 through Grade 10. The Sunshine State Standards "are the centerpiece of a reform effort in Florida to align curriculum, instruction, and assessment" (Florida Department of Education, 2007, p. 1).
- Illinois: *The Illinois Learning Standards for Mathematics.* Goals 6 through 10 emphasize the following: Number and Operations, Measurement, Algebra, Geometry, and Data Analysis and Statistics—*Mathematics Performance Descriptors, Grades 1–5 and Grades 6–12* (2002).
- Texas Essential Knowledge and Skills: *Texas Essential Knowledge and Skills for Mathematics* (TEKS) was adopted by the Texas State Board of Education and became effective on September 1, 1998. The TEKS, a state-mandated curriculum, was "specifically designed to help students to make progress . . . by emphasizing the knowledge and skills most critical for student learning" (TEA, 2002, p. 4).

The review of the content frameworks resulted in the development of a list of QSCs spanning mathematical knowledge from Kindergarten through Grade 12 (college and career readiness or precalculus). Each QSC is aligned with one of the six content strands. Currently, there are approximately 549 QSCs, which can be viewed and searched at www.scholastic.com/SMI or www.Quantiles.com.

Each QSC consists of a description of the content, a unique identification number, the grade at which it typically first appears, and the strand with which it is associated.

Quantile Item Bank

The second task in the development of the Quantile Framework for Mathematics was to develop and field-test a bank of items that could be used in future linking studies and calibration and development projects. Item bank development for the Quantile Framework went through several stages—content specification, item writing and review, field-testing and analyses, and final evaluation.

Content Specification

Each QSC developed during the design of the Quantile Framework was paired with a particular strand and identified as typically being taught at a particular grade level. The curricular frameworks from Florida, North Carolina, Texas, and California were synthesized to identify the appropriate grade level for each QSC. If a QSC was included in any of these state frameworks, it was then added to the list of QSCs for the item bank utilized in the Quantile Framework field study.

During Summer and Fall 2003, more than 1,400 items were developed to assess the QSCs associated with content extending from first grade through Algebra II. The items were written and reviewed by mathematics educators trained to develop multiple-choice items (Haladyna, 1994). Each item was associated with a strand and a QSC. In the development of the Quantile Framework item bank, the reading demand of the items was kept as low as possible to ensure that the items were testing mathematics achievement and not reading.

Item Writing and Review

Item writers were teachers of, and item-development specialists who had experience with, mathematics education at various levels. Employing individuals with a range of experiences helped to ensure that the items were valid measures. Item writers were provided with training materials concerning the development of multiple-choice items and the Quantile Framework. Included in the item-writing materials were incorrect and ineffective items that illustrated the criteria used to evaluate items, along with corrections based on those criteria. The final phase of item-writer training was a short practice session with three items.

Item writers were given additional training related to sensitivity issues. Some item-writing materials address these issues and identify areas to avoid when selecting passages and developing items. These materials were developed based on work published concerning universal design and fair access, including the issues of equal treatment of the sexes, fair representation of minority groups, and fair representation of disabled individuals.

A group of specialists representing various perspectives—test developers, editors, curriculum specialists, and mathematics specialists—reviewed and edited the items. These individuals examined each item for sensitivity issues and for the quality of the response options. During the second stage of the item review process, items were approved, approved with edits, or deleted.

Field Testing and Analyses

The next stage in the development of the Quantile item bank was the field-testing of all of the items. First, individual test items were compiled into leveled assessments distributed to groups of students. The data gathered from these assessments were then analyzed using a variety of statistical methods. The final result was a bank of test items appropriately placed within the Quantile scale, suitable for determining the mathematical achievement of students on this scale.

Assessments for Field Testing

Assessment forms were developed for 10 levels for the purposes of field-testing. Levels 2 though 8 were aligned with the typical content taught in Grades 2–8. Level 9 was aligned with the typical content taught in Algebra I. Level 10 was aligned with the typical content taught in Geometry. Finally, Level 11 was aligned with the typical content taught in Algebra II. A total of 30 test forms were developed (three per assessment level), and each test form was composed of 30 items.

Creating the taxonomy of QSCs across all grade levels involved linking the field test forms such that the concepts and difficulty levels between tests overlapped. This was achieved by designating a linking set of items for each grade level. These items were administered to the originally intended grade and were also placed on off-grade forms (above or below one grade).

With the structure of the test forms established, the forms needed to be populated with the appropriate items. First, a pool of items was formed from the items developed during the item-writing phase. The repository consisted of 66 items for each grade level, from Grade 2 to Algebra II (10 levels total). Of these, 54 items were designated on-grade-level items and would only appear on test forms for that particular grade level. The remaining 12 items were designated linking items and could appear on test forms one grade level above or below the level of the item.

The final field tests were composed of 686 unique items. Besides the 660 items mentioned above, two sets of 12 linking items were developed to serve as below-level items for Grade 2 and above-level items for Algebra II. Two additional Algebra II items were developed to ensure coverage of all the QSCs at that level.

The three test forms for each grade level were developed using a domain-sampling model in which items were randomly assigned within the QSC to a test form. To achieve the goal of linking the test forms within a grade level, as well as across grade levels, the linking items were utilized as follows: Each test form contained six items from the linking set at the same grade level as the test form. For across-grade linking, four items were added to each field-test form from the below-grade linking set, and two items were added to each field-test form from the above-grade linking set. In conclusion, the linking items were used such that test items overlapped on two forms within the same grade level and on two or more forms from different grade levels.

Linking the test levels vertically (across grades) employed a common-item test design (design in which items are used on multiple forms). In this design, multiple tests are given to nonrandom groups, and a set of common items is included in the test administration to allow some statistical adjustments for possible sample-selection bias. This design is most advantageous where the number of items to be tested (treatments) is large and the consideration of cost (in terms of time) forces the experiment to be smaller than is desired (Cochran & Cox, 1957).

The Quantile Framework Field Study

The Quantile Framework field study was conducted in February 2004. Thirty-seven schools from 14 districts across six states (California, Indiana, Massachusetts, North Carolina, Utah, and Wisconsin) agreed to participate in the study. Data were received from 34 of the schools (two elementary schools and one middle school did not return data). A total of 9,847 students in Grades 2 through 12 were tested. The number of students tested per school ranged from 74 to 920. The schools were diverse in terms of geographic location, size, and type of community (e.g., suburban; small town, small city, or rural communities; and urban). See Table 1 for information about the sample at each grade level and the total sample. See Table 2 for test administration forms by level.

Rulers were provided to students; protractors were provided to students administered test levels 3–11. Formula sheets were provided on the back of the test booklet for students administered levels 5–8, 10, and 11. The use of calculators was permitted on the second part of each test. Students administered level 5 and below could use a four-function calculator, and students administered level 6 and above could use a scientific calculator. Administration time was about 45 minutes at each grade level. Students administered the level 2 test could have the test read aloud, and mark in the test booklet, if that was the typical form of assessment in the classroom.

TABLE 1. Field study participation by grade and gender.

Grade	Sample Size *(N)*	Percent Female *(N)*	Percent Male *(N)*
2	1,283	48.1 (562)	51.9 (606)
3	1,354	51.9 (667)	48.1 (617)
4	1,454	47.7 (622)	52.3 (705)
5	1,344	48.9 (622)	51.1 (650)
6	976	47.7 (423)	52.3 (463)
7	1,250	49.8 (618)	50.2 (622)
8	1,015	51.9 (518)	48.1 (481)
9	489	52.0 (252)	48.0 (233)
10	259	48.6 (125)	51.4 (132)
11	206	49.3 (101)	50.7 (104)
12	143	51.7 (74)	48.3 (69)
Missing	74	39.1 (9)	60.9 (14)
Total	**9,847**	**49.6 (4,615)**	**50.4 (4,696)**

TABLE 2. Test form administration by level.

Test Level	*N*	Missing	Form A	Form B	Form C
2	1,283	4	453	430	397
3	1,354	7	561	387	399
4	1,454	17	616	419	402
5	1,344	3	470	448	423
6	917	13	322	293	289
7	1,309	6	462	429	411
8	1,181	16	387	391	387
9	415	4	141	136	134
10	226	5	73	77	71
11	313	10	102	101	100
Missing	51	31	9	8	3
Total	**9,847**	**116**	**3,596**	**3,119**	**3,016**

At the conclusion of the field test, complete data was available from 9,678 students. Data were deleted if the test level or the test form was not indicated on the answer sheet, or if the answer sheet was blank. These field-test data were analyzed using both the classical measurement model and the Rasch (one-parameter logistic item response theory) model. Item statistics and descriptive information (item number, field-test form and item number, QSC, and answer key) were printed for each item and attached to the item record. The item record contained the statistical, descriptive, and historical information for an item, a copy of the item as it appeared on the test forms, any comments by reviewers, and the psychometric notations. Each item had a separate item record.

Field-Test Analyses—Classical Measurement

For each item, the *p*-value (percent correct) and the point-biserial correlation between the item score (correct response) and the total test score were computed. Point-biserial correlations were also computed between each of the incorrect responses and the total score. In addition, frequency distributions of the response choices (including omits) were tabulated (both actual counts and percents).

Point-biserial correlations provide an estimate of the relationship of ability as measured by a specific item and ability as measured by the overall test. All items were retained for further analyses during the development of the Quantile scale using the Rasch item response theory model. Items with point-biserial correlations less than 0.10 were removed from the item bank for future linking studies. Table 3 displays the summary items statistics.

TABLE 3. Summary item statistics from the Quantile Framework field study.

Level	Number of Items Tested	Mean *p*-value (Range)	Mean Correct Response Point-Biserial Correlation (Range)	Mean Incorrect Responses Point-Biserial Correlation (Range)
2	90	0.583 (0.12–0.95)	0.322 (–0.15–0.56)	–0.209 (–0.43–0.12)
3	90	0.532 (0.11–0.93)	0.256 (–0.08–0.52)	–0.221 (–0.54–0.02)
4	90	0.552 (0.12–0.92)	0.242 (–0.21–0.50)	–0.222 (–0.48–0.12)
5	90	0.535 (0.12–0.95)	0.279 (–0.05–0.50)	–0.225 (–0.45–0.05)
6	90	0.515 (0.04–0.86)	0.244 (–0.08–0.45)	–0.218 (–0.46–0.09)
7	90	0.438 (0.10–0.77)	0.294 (–0.12–0.56)	–0.207 (–0.46–0.25)
8	90	0.433 (0.10–0.81)	0.257 (–0.15–0.50)	–0.201 (–0.45–0.13)
9	90	0.396 (0.10–0.79)	0.208 (–0.19–0.52)	–0.193 (–0.53–0.22)
10	90	0.511 (0.01–0.97)	0.193 (–0.26–0.53)	–0.205 (–0.55–0.18)
11	90	0.527 (0.09–0.98)	0.255 (–0.09–0.51)	–0.223 (–0.52–0.07)

Field-Test Analyses—Bias

Differential item functioning (DIF) examines the relationship between the score on an item and group membership, while controlling for achievement. The Mantel-Haenszel procedure is a widely used methodology to examine differential item functioning. (Roussos, Schnipke, & Pashley, 1999, p. 293). The Mantel-Haenszel procedure examines DIF by examining j 2×2 contingency tables, where j is the number of different levels of achievement actually accomplished by the examinees (actual total scores received on the test). The focal group is the group of interest and the reference group serves as a basis for comparison for the focal group (Camilli & Shepard, 1994; Dorans & Holland, 1993).

The Mantel-Haenszel chi-square statistic tests the alternative hypothesis that there is a linear association between the row variable (score on the item) and the column variable (group membership). The Mantel-Haenszel χ^2 distribution has one degree of freedom and is determined as:

$$Q_{MH} = (n - 1)r^2 \qquad \text{(Equation 1)}$$

where r^2 is the Pearson correlation between the row variable and the column variable (SAS Institute Inc., 1985).

The Mantel-Haenszel Log Odds Ratio statistic is used to determine the direction of DIF and can be calculated using SAS. This measure is obtained by combining the odds ratios, α_j, across levels with the formula for weighted averages (Camilli & Shepard, 1994).

For the gender analyses, males (approximately 50.4% of the population) were defined as the reference group and females (approximately 49.6% of the population) were defined as the focal group.

The results from the Quantile Framework field study were reviewed for inclusion on future linking studies. The following statistics were reviewed for each item: p-value, point-biserial correlation, and DIF estimates. Items that exhibited extreme statistics were considered biased and removed from the item bank (47 out of 685).

From the studies conducted with the Quantile Framework item bank (Palm Beach County [FL] linking study, Mississippi linking study, Department of Defense/TerraNova linking study, and Wyoming linking study), approximately 6.9% of the items in any one study were flagged as exhibiting DIF using the Mantel-Haenszel statistic and the t-statistic from Winsteps. For each linking study the following steps were used to review the items: (1) flag the items exhibiting DIF, (2) review the flagged items to determine if the content of the item is something that all students are expected to know, and (3) make a decision to retain or delete the item.

Field-Test Analyses—Rasch Item Response Theory

Classical test theory has two basic shortcomings: (1) the use of item indices whose values depend on the particular group of examinees from which they were obtained, and (2) the use of examinee achievement estimates that depend on the particular choice of items selected for a test. The basic premises of item response theory (IRT) overcome these shortcomings by predicting the performance of an examinee on a test item based on a set of underlying abilities (Hambleton & Swaminathan, 1985). The relationship between an examinee's item performance and the set of traits underlying item performance can be described by a monotonically increasing function called an item characteristic curve (ICC). This function specifies that as the level of the trait increases, the probability of a correct response to an item increases.

The conversion of observations into measures can be accomplished using the Rasch (1980) model, which states a requirement for the way item difficulties (calibrations) and observations (count of correct items) interact in a probability model to produce measures. The Rasch item response theory model expresses the probability that a person (n) answers a certain item (i) correctly by the following relationship (Hambleton & Swaminathan, 1985; Wright & Linacre, 1994):

$$P_{ni} = \frac{e^{b_n - d_i}}{1 + e^{b_n - d_i}} \qquad \text{(Equation 2)}$$

where d_i is the difficulty of item i ($i = 1, 2, \ldots$, number of items),

b_n is the achievement of person n ($n = 1, 2, \ldots$, number of persons),

$b_n - d_i$ is the difference between the achievement of person n and the difficulty of item i, and

P_{ni} is the probability that examinee n responds correctly to item i.

The Rasch measurement model assumes that item difficulty is the only item characteristic that influences the examinee's performance. In other words, all items are equally discriminating in their ability to identify low-achieving persons and high achieving persons (Bond & Fox, 2001; Hambleton, Swaminathan, & Rogers, 1991). In addition, the lower asymptote is zero, which specifies that examinees of very low achievement have zero probability of correctly answering the item. The Rasch model has the following assumptions:

(1) *unidimensionality*—only one construct is assessed by the set of items

(2) *local independence*—when abilities influencing test performance are held constant, an examinee's responses to any pair of items are statistically independent (conditional independence, i.e., the only reason an examinee scores similarly on several items is because of his or her achievement, not because the items are correlated)

The Rasch model is based on fairly restrictive assumptions, but it is appropriate for criterion-referenced assessments.

For the Quantile Framework field study, all students and items were submitted to a Winsteps analysis using a logit convergence criterion of 0.0001 and a residual convergence criterion of 0.001. Items that a student skipped were treated as missing, rather than being treated as incorrect. Only students who responded to at least 20 items were included in the analyses (22 students were omitted, 0.22%).

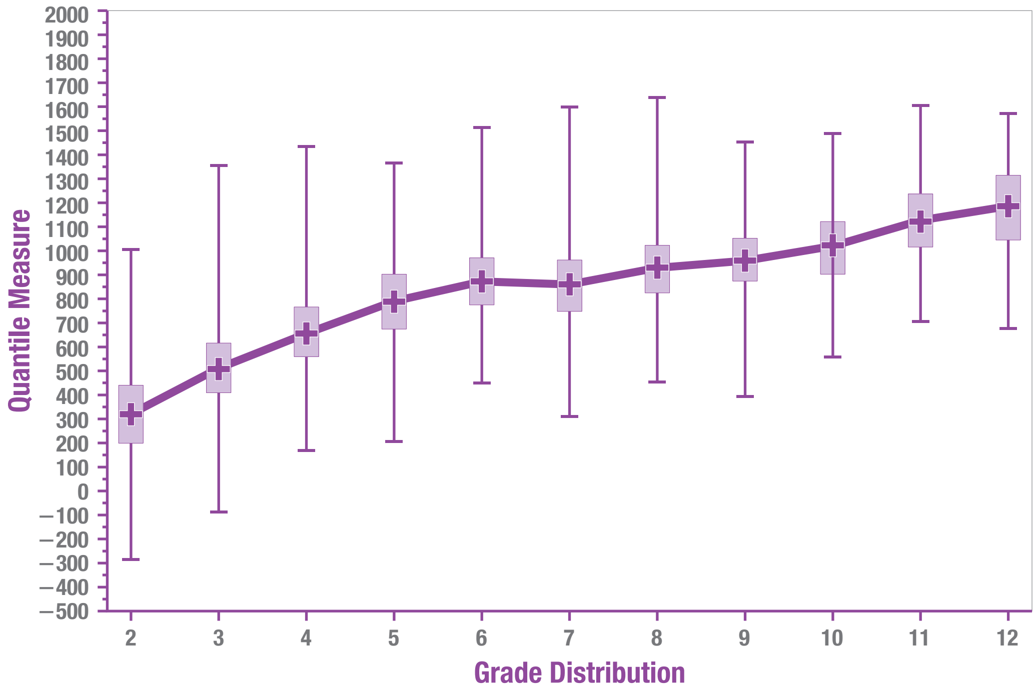
The Quantile measure comes from multiplying the logit value by 180 and is anchored at 656Q. The multiplier and the anchor point will be discussed in a later section. Table 4 shows the mean and median Quantile measures for all students with complete data at each grade level. While there is not a monotonically increasing trend in the mean and median Quantile measures (note that the measure for Grade 6 is higher than the measure for Grade 7), the measures are not significantly different. Results from other studies (e.g., *PASeries* Math) did exhibit a monotonically increasing function.

TABLE 4. Mean and median Quantile measure for $N = 9{,}656$ students with complete data.

Grade Level	*N*	Mean Quantile Measure (Standard Deviation)	Median Quantile Measure
2	1,275	320.68 (189.11)	323
3	1,339	511.41 (157.69)	516
4	1,427	655.45 (157.50)	667
5	1,337	790.06 (167.71)	771
6	959	871.82 (153.02)	865
7	1,244	860.52 (174.16)	841
8	1,004	929.01 (157.63)	910
9	482	958.69 (152.81)	953
10	251	1019.97 (162.87)	1005
11	200	1127.34 (178.57)	1131
12	138	1185.90 (189.19)	1164

Figure 2 shows the relationship between grade level and Quantile measure. The box and whisker plot shows the score progression from grade to grade (the *x* axis). Across all 9,656 students, the correlation between grade and Quantile measure was 0.76 in this initial filed study.

FIGURE 2. Rasch achievement estimates of $N = 9{,}656$ students with complete data.



All students with outfit mean square statistics greater than or equal to 1.8 were removed from further analyses. A total of 480 students (4.97%) were removed from further analyses. The number of students removed ranged from 8.47% (108) in Grade 2 to 2.29% (22) in Grade 6 with a mean percent decrease of 4.45% per grade.

All remaining students (9,176) and all items were analyzed with Winsteps using a logit convergence criterion of 0.0001 and a residual convergence criterion of 0.001. Items that a student skipped were treated as missing, rather than being treated as incorrect. Only students who responded to at least 20 items were included in the analyses. Table 5 shows the mean and median Quantile measures for the final set of students at each grade level. Figure 3 shows the results from the final set of students. The correlation between grade and Quantile measure is 0.78 for this interim field study.

TABLE 5. Mean and median Quantile measure for the final set of $N = 9{,}176$ students.

Grade Level	*N*	Median Logit Value	Mean (Median) Quantile Measure
2	1,167	–2.800	289.03 (292)
3	1,260	–1.650	502.18 (499)
4	1,352	–0.780	652.60 (656)
5	1,289	0.000	795.25 (796)
6	937	0.430	880.77 (874)
7	1,181	0.370	877.75 (863)
8	955	0.810	951.41 (942)
9	466	1.020	982.62 (980)
10	244	1.400	1044.08 (1048)
11	191	2.070	1160.49 (1169)
12	134	2.295	1219.87 (1210)

FIGURE 3. Box and whisker plot of the Rasch ability estimates (using the Quantile scale) for the final sample of students with outfit statistics less than 1.8 ($N = 9{,}176$).

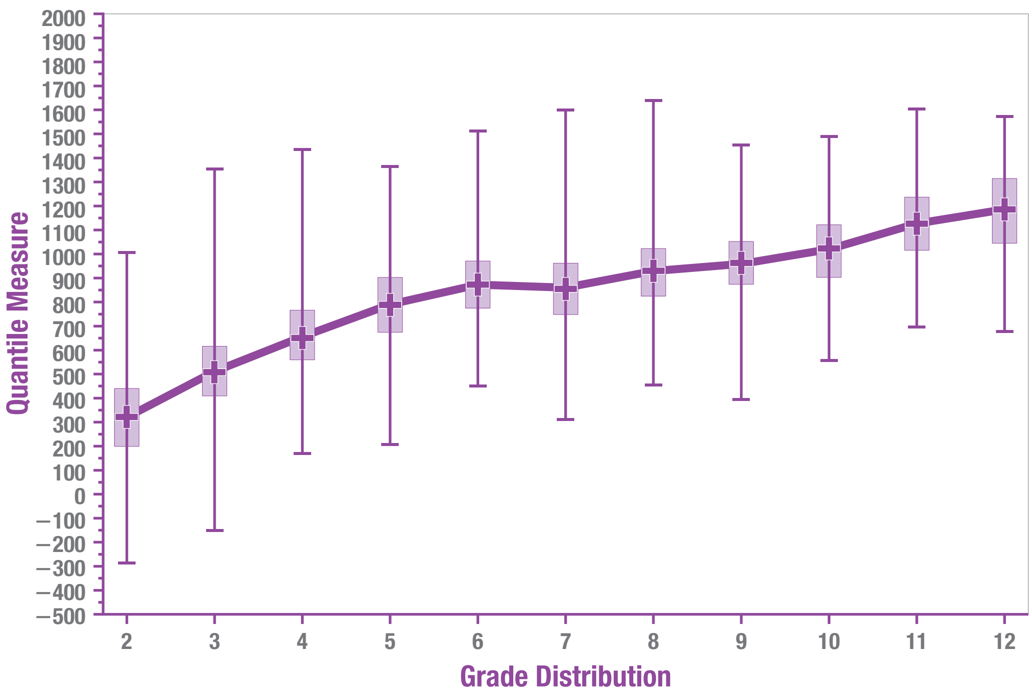
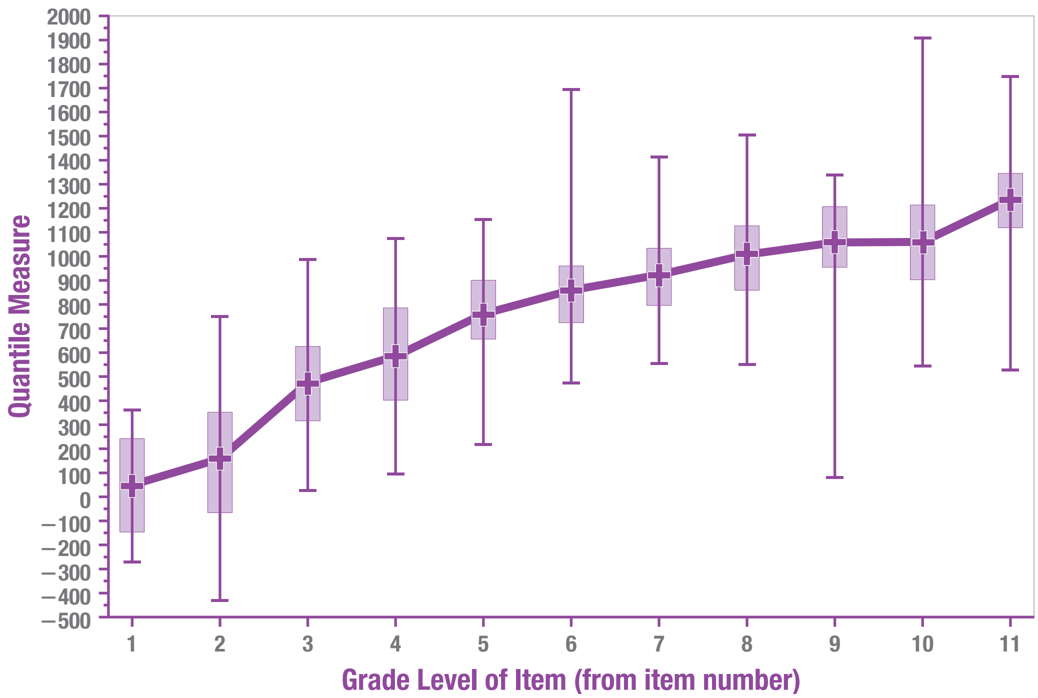


Figure 4 shows the distribution of item difficulties based on the final sample of students. For this analysis, missing data were treated as skipped items and not counted as wrong. There is a gradual increase in difficulty when items are sorted by the test level for which they were written. This distribution appears to be nonlinear, which is consistent with other studies. The correlation between grade level for which the item was written and the Quantile measure of the item was 0.80.

FIGURE 4. Box and whisker plot of the Rasch ability estimates (using the Quantile scale) of the 685 Quantile Framework items for the final sample of students ($N = 9{,}176$).



The field testing of the items written for the Quantile Framework indicates a strong correlation between the grade level of the item and the item difficulty.

The Quantile Scale

For development of the Quantile scale, two features needed to be defined:

(1) the scale multiplier (conversion factor from the Rasch model)

(2) the anchor point

Once the scale is defined, it can be used to assign Quantile measures to individual Quantile Skills and Concepts, or QSCs, as well as clusters of QSCs.

Generating the Quantile Scale

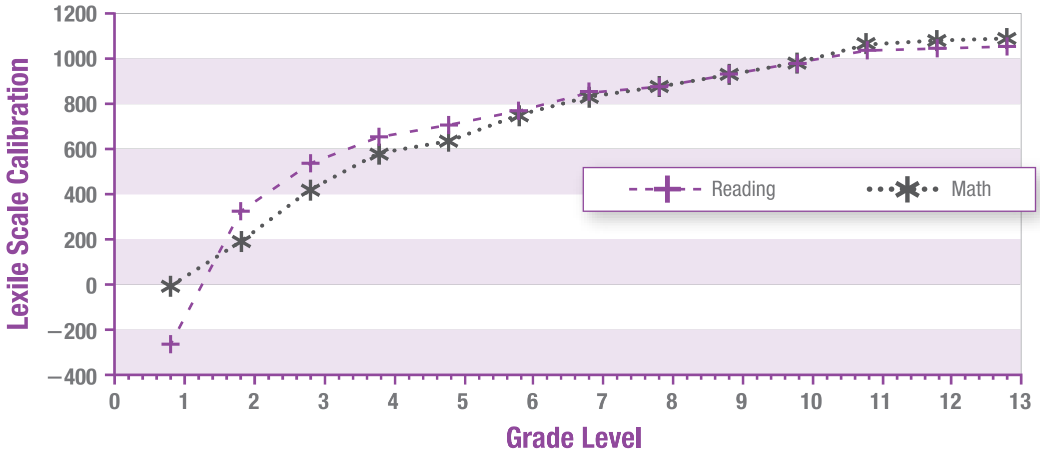
As described in the previous section, the Rasch item response theory model (Wright & Stone, 1979) was used to estimate the difficulties of items and the abilities of persons on the logit scale. The calibrations of the items from the Rasch model are objective in the sense that the relative difficulties of the items remain the same across different samples (specific objectivity). When two items are administered to the same individual, it can be determined which item is harder and which one is easier. This ordering should hold when the same two items are administered to a second person.

The problem is that the location of the scale is not known. General objectivity requires that scores obtained from different test administrations be tied to a common zero; absolute location must be sample independent (Stenner, 1990). To achieve general objectivity instead of simply specific objectivity, the theoretical logit difficulties must be transformed to a scale where the ambiguity regarding the location of zero is resolved.

The first step in developing the Quantile scale was to determine the conversion factor needed to transform logits from the Rasch model into Quantile scale units. A vast amount of research has already been conducted on the relationship between a student's achievement in reading and the Lexile® scale. Therefore, the decision was made to examine the relationship between reading and mathematics scales used with other assessments.

The median scale score for each grade level on a norm-referenced assessment linked with the Lexile scale is plotted in Figure 5 using the same conversion equation for both reading and mathematics. Based on Figure 5, it was concluded that the same conversion factor used with the Lexile scale could be used with the Quantile scale.

FIGURE 5. Relationship between reading and mathematics scale scores on a norm-referenced assessment linked to the Lexile scale in reading.

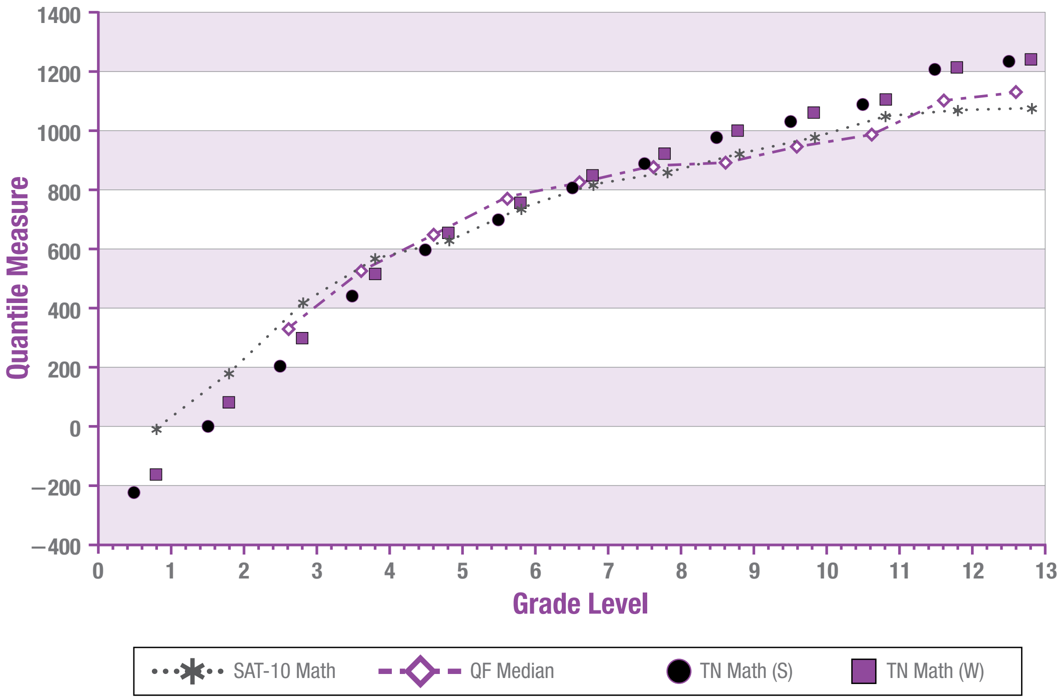


The second step in developing a Quantile scale with a fixed zero was to identify an anchor point for the scale. Given the number of students at each grade level in the field study, and the fact that state assessment programs typically test students in Grades 4 or 5, it was concluded that the scale should be anchored between Grades 4 and 5.

Median performance at the end of Grade 3 on the Lexile scale is 590L. Median performance at the end of Grade 4 on the Lexile scale is 700L. The Quantile Framework field study was conducted in February, and this point would correspond to six months (0.6 years) into the school year. To determine the location of the scale, a value of 66 Quantile scale units was added to the median performance at the end of Grade 3 to reflect the growth of students in Grade 4 prior to the field study ($700 - 590 = 110$; $110 \times 0.6 = 66$).

Therefore, the value of 656Q was used for the location of Grade 4 median performance. The anchor point was validated with other assessment data and collateral data from the Quantile Framework field study (see Figure 6).

FIGURE 6. Relationship between grade level and mathematics performance on the Quantile Framework field study and other mathematics assessments.



As a result of the above analyses, a linear equation of the form

$$[(\text{Logit} - \text{Anchor Logit}) \times 180] + 656 = \text{Quantile measure} \qquad \text{(Equation 3)}$$

was used to convert logit difficulties to Quantile measures where the anchor logit is the median for Grade 4 in the Quantile Framework field study.

Knowledge Clusters

The next step was to use the Quantile Framework to estimate the Quantile measure of each QSC. Having a measure for each QSC on the Quantile scale will then allow the difficulty of skills and concepts and the complexity of other resources to be evaluated. The Quantile measure of a QSC estimates the solvability, or a prediction of how difficult the skill or concept will be for a learner.

The QSCs also fall into Knowledge Clusters along a content continuum. Recall that the Quantile Framework is a content taxonomy of mathematical skills and concepts. Knowledge Clusters are a family of skills, like building blocks, that depend one upon the other to connect and demonstrate how understanding of a mathematical topic is founded, supported, and extended along the continuum. The Knowledge Clusters illustrate the interconnectivity of the Quantile Framework and the natural progression of mathematical skills (content trajectory) needed to solve increasingly complex problems.

The Quantile measures and Knowledge Clusters for QSCs were determined by a group of three to five subject-matter experts (SMEs). Each SME has had classroom experience at multiple developmental levels, has completed graduate-level courses in mathematics education, and understands basic psychometric concepts and assessment issues.

For the development of Knowledge Clusters, certain terminology was developed to describe relationships between the QSCs.

- **A target QSC** is the skill or concept that is the focus of instruction.
- **A prerequisite QSC** is a QSC that describes a skill or concept that provides a building block necessary for another QSC. For example, adding single-digit numbers is a prerequisite for adding two-digit numbers.
- **A supplemental QSC** is a QSC that describes associated skills or knowledge that assists and enriches the understanding of another QSC. For example, two supplemental QSCs are: multiplying two fractions and determining the probability of compound events.
- **An impending QSC** describes a skill or concept that will further augment understanding, building on another QSC. An impending QSC for using division facts is simplifying equivalent fractions.

Each target QSC was classified with prerequisite QSCs and supplemental QSCs or was identified as a foundational QSC. As a part of a taxonomy, QSCs are either a single link in a chain of skills that lead to the understanding of larger mathematical concepts, or they are the first step toward such an understanding. A QSC that is classified as foundational requires only general readiness to learn.

The SMEs examined each QSC to determine where the specific QSC comes in the content continuum based on their classroom experience, instructional resources (e.g., textbooks), and other curricular frameworks (e.g., NCTM Standards). The process called for each SME to independently review the QSC and develop a draft Knowledge Cluster. The second step consisted of the three to five SMEs meeting and reviewing the draft clusters. Through discussion and consensus, the SMEs developed the final Knowledge Cluster.

Once the Knowledge Cluster for a QSC was established, the information was used when determining the Quantile measure of a QSC, as described below. If necessary, Knowledge Clusters are reviewed and refined if the Quantile measures of the QSCs in the cluster are not monotonically increasing (steadily increasing) or there is not an instructional explanation for the pattern.

Quantile Measures of Quantile Skills and Concepts

The Quantile Framework is a theory-referenced measurement system of mathematical understanding. As such, a QSC Quantile measure represents the "typical" difficulty of all items that could be written to represent the QSC and the collection of items can be thought of as an *ensemble* of the all of the items that could be developed for a specific skill or concept. During 2002, Stenner, Burdick, Sanford, and Burdick (2006) conducted a study to explore the "ensemble" concept to explain differences across reading items with the Lexile Framework® for Reading. The theoretical Lexile measure of a piece of text is the mean theoretical difficulty of all items associated with the text. Stenner and his colleagues state that the "Lexile Theory replaces statements about individual items with statements about ensembles. The ensemble interpretation enables the elimination of irrelevant details. The extra-theoretical details are taken into account jointly, not individually, and, via averaging, are removed from the data text explained by the theory" (p. 314). The result is that when making text-dependent generalizations, text readability can be measured with high accuracy and the uncertainty in expected comprehension is largely due to the unreliability in reader measures.

While expert judgment alone could be used to scale the QSCs, empirical scaling is more replicable. Actual performance by students on an assessment was used to determine the Quantile measure of a QSC empirically. The process employed items and data from two national field studies:

- Quantile Framework field study (686 items, $N = 9{,}647$, Grades 2 through Algebra II) as described earlier in this guide
- *PASeries* Mathematics field study (7,080 items, $N = 27{,}329$, Grades 2 through 9/Algebra I), which is described in the *PASeries* Mathematics Technical Manual (MetaMetrics, 2005)

The items initially associated with each QSC were reviewed by SMEs and accepted for inclusion in the set of items, moved to another QSC, or not included in the set. The following criteria were used:

- Items must be responded to by at least 50 examinees, administered at the target grade level, and have a point-biserial correlation greater than or equal to 0.16.
- Grade levels for items must match the grade level of the introduction of the skill or concept as derived from the national review of curricular frameworks (described on pages 9 and 10 of this document).
- Items must cover only appropriate introductory material for instruction of concept (e.g., the first night's homework after introducing the topic, or the A and B level exercises in a textbook) based on consensus of the SMEs.

Once the set of items meeting the inclusion criteria was identified, the set of items was reviewed to ensure that the curricular breadth of the QSC was covered. If the group of SMEs considered the set of items to be acceptable, then the Quantile measure of the QSC was calculated empirically. The Quantile measure of a QSC is defined as the mean Quantile measure of items that met the criteria.

The final step in the process was to review the Quantile measure of the QSC in relationship to the Quantile measures of the QSCs identified as prerequisite and supplemental to the QSC. If the group of SMEs did not consider the set of items to be acceptable, then the Quantile measure of the QSC was estimated and assigned a Quantile zone. By assigning a Quantile zone instead of a Quantile measure to these QSCs, the SMEs were able to provide a valid estimate of the skill or concept's difficulty.

In 2007, with the extension of the Quantile Framework to include Kindergarten and Precalculus, the Quantile measures of the QSCs were reviewed. Where additional items had been tested and the data was available, estimated QSC Quantile measures were calculated. In 2014, a large data set from the administration of SMI was analyzed to examine the relationship between the original QSC Quantile measures and empirical QSC means from the items administered. The overall correlation between QSC Quantile measures and empirically estimated Quantile measures was 0.98 ($N = 7{,}993$ students). Based on the analyses, 12 QSCs were identified with larger-than-expected deviations given the "ensemble" interpretation of a QSC Quantile measure. Each QSC was reviewed in terms of the SMI items that generated the data, linking studies where the QSC was employed, and data from other assessments developed employing the Quantile Framework. Of the 12 QSCs identified, it was concluded that the Quantile measure of nine of the QSCs should be recalculated. Five of the QSCs are targeted for Kindergarten and Grade 1 and the current data set provided data to calculate a Quantile measure (the Quantile measure for the QSC had been previously estimated). The other four QSC Quantile measures were revised because the type of "typical" item and the technology used to assess the skill or concept had shifted from the time that the QSC Quantile measure was established in 2004 (QSCs: 79, 654, 180, and 217). Three of the QSC Quantile measures were not changed (QSC: 134, 604, 408) because (1) some of the SMI items did not reflect the intent of the QSC, or (2) not enough items were tested to indicate that the Quantile measure should be recalculated.

Validity of the Quantile Framework for Mathematics

Validity is the extent to which a test measures what its authors or users claim it measures. Specifically, test validity concerns the appropriateness of inferences "that can be made on the basis of observations or test results" (Salvia & Ysseldyke, 1998, p. 166). The 1999 *Standards for Educational and Psychological Testing* (American Educational Research Association, American Psychological Association, & National Council on Measurement in Education, 1999) state that "validity refers to the degree to which evidence and theory support the interpretations of test scores entailed in the uses of tests" (p. 9). In other words, a valid test measures what it is supposed to measure.

Stenner, Smith, and Burdick state that "[t]he process of ascribing meaning to scores produced by a measurement procedure is generally recognized as the most important task in developing an educational or psychological measure, be it an achievement test, interest inventory, or personality scale" (1983). For the Quantile Framework, which measures student understanding of mathematical skills and concepts, the most important aspect of validity that should be examined is construct-identification validity. This global form of validity encompassing content-description and criterion-prediction validity may be evaluated for the Quantile Framework for Mathematics by examining how well Quantile measures relate to other measures of mathematical achievement.

Relationship of Quantile Measures to Other Measures of Mathematical Understandings

Scores from tests purporting to measure the same construct, for example "mathematical achievement," should be moderately correlated (Anastasi, 1982). Table 6 presents the results from field studies conducted with the Quantile Framework while the Quantile scale was being developed. For each of the tests listed, student mathematics scores were strongly correlated, with correlation coefficients around 0.70, with Quantile measures from the Quantile Framework field study. This suggests that measures derived from the Quantile Framework meet the moderate-correlation requirement described by Anastasi (1982).

TABLE 6. Results from field studies conducted with the Quantile Framework.

Standardized Test	Grades in Study	*N*	Correlation between Test Score and Quantile Measure
RIT and Measures of Academic Progress (MAP by NWEA)	4 & 5	94	0.69
North Carolina End-of-Grade Tests (Mathematics)	4 & 5	341	0.73

Relationship of Quantile Framework to Other Measures of Mathematics Understanding

The Quantile Framework for Mathematics has been linked with several standardized tests of mathematics achievement. When assessment scales are linked, a common frame of reference can be used to interpret the test results. This frame of reference can be "used to convey additional normative information, test-content information, and information that is jointly normative and content-based. For many test uses . . . [this frame of reference] conveys information that is more crucial than the information conveyed by the primary score scale" (Petersen, Kolen, & Hoover, 1993, p. 222).

When two score scales are linked, the linking function can be used to provide a context for understanding the results of the assessments. It is often difficult to explain what mathematical skills a student actually understands based on the results of a mathematics test. Typical questions regarding assessment measures are:

- "If a student scores 1200 on the mathematics assessment, what does this mean?"
- "Based on my students' test results, what math concepts can they understand and do?"

Once a linkage is established with an assessment that covers specific concepts and skills, then the results of the assessment can be explained and interpreted in the context of the specific concepts a student can understand and skills the student has mastered.

Table 7 presents the results from linking studies conducted with the Quantile Framework. For each of the tests listed, student mathematics scores were reported using the test's scale, as well as by Quantile measures. This dual reporting provides a rich, criterion-related frame of reference for interpreting the standardized test scores. Each student who takes one of the standardized tests can receive, in addition to norm- or criterion-referenced test results, information related to the specific QTaxons on which he or she is ready to be instructed.

Table 7 also shows that measures derived from the Quantile Framework are more than moderately correlated to other measures of mathematical understanding. The correlation coefficients were around 0.90 for all but one of the tests studied.

TABLE 7. Results from linking studies conducted with the Quantile Framework.

Standardized Test	Grades in Study	*N*	Correlation Between Test Score and Quantile Measure
Mississippi Curriculum Test, Mathematics (MCT)	2–8	7,039	0.89
TerraNova (CTB/McGraw-Hill)	3, 5, 7, 9	6,356	0.92
Texas Assessment of Knowledge and Skills (TAKS)	3–11	14,286	0.69–0.78*
Proficiency Assessments for Wyoming Students (PAWS)	3, 5, 8, and 11	3,923	0.87
Progress in Math (PiM)	1–8	4,692	0.92
Progress Toward Standards (PTS3)	3–8 and 10	8,544	0.86–0.90*
Comprehensive Testing Program (CTP4)	3, 5, and 7	802	0.90
North Carolina End-of-Grade and North Carolina End-of-Course (NCEOG/NCEOC)	3, 5, and 7; A1, G, A2	5,069	0.88–0.90*
Comprehensive Testing Progressing (CTP4—ERB)	3, 5, and 7	953	0.87 to 0.90
Kentucky Core Content Tests (KCCT)	3–8 and 11	12,660	0.80 to 0.83*
Oklahoma Core Competency Tests (OCCT)	3–8	5,649	0.81 to 0.85*
Iowa Assessments	2, 4, 6, 8, and 10	7,365	0.92
Virginia Standards of Learning (SOL)	3–8, A1, G, and A2	12,470	0.86 to 0.89*
Kentucky Performance Rating for Educational Progress (K-PREP)	3–8	6,859	0.81 to 0.85*
North Carolina ACT	11	3,320	0.90
North Carolina READY End-of-Grade/ End-of-Course Tests (NC EOG/NC EOC)	3, 4, 6, 8, and A1/I1	10,903	0.87 to 0.90*

* Separate conversion equations were derived for each grade/course.

Multidimensionality of Quantile Framework Items

Test dimensionality is defined as the minimum number of abilities or constructs measured by a set of test items. A construct is a theoretical representation of an underlying trait, concept, attribute, process, and/or structure that a test purports to measure (Messick, 1993). A test can be considered to measure one latent trait, construct, or ability (in which case it is called unidimensional); or a combination of abilities (in which case it is referred to as multidimensional). The dimensional structure of a test is intricately tied to the purpose and definition of the construct to be measured. It is also an important factor in many of the models used in data analyses. Though many of the models assume unidimensionality, this assumption cannot be strictly met because there are always other cognitive, personality, and test-taking factors that have some level of impact on test performance (Hambleton & Swaminathan, 1985).

The complex nature of mathematics and the curriculum standards most states have adopted also contribute to unintended dimensionality. Application and process skills, the reading demand of items, and the use of calculators could possibly add features to an assessment beyond what the developers intended. In addition, the NCTM Standards, upon which many states have based curricula, describe the growth of students' mathematical development across five content standards: Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability. These standards, or sub-domains of mathematics, are useful in organizing mathematics instruction in the classroom. These standards could represent different constructs and thereby introduce more sources of dimensionality to tests designed to assess these standards.

Investigation of Dimensionality of Mathematics Assessments

A recent study conducted by Burg (2007) analyzed the dimensional structure of mathematical achievement tests aligned to the NCTM content standards. Since there is not a consensus within the measurement community on a single method to determine dimensionality, Burg employed four different methods for assessing dimensionality:

(1) exploring the conditional covariances (DETECT)

(2) assessment of essential unidimensionality (DIMTEST)

(3) item factor analysis (NOHARM)

(4) principal component analysis (WINSTEPS)

All four approaches have been shown to be effective indices of dimensional structure. Burg analyzed Grades 3–8 data from the Quantile Framework field study previously described.

Each set of on-grade items for a test form from Grades 3–8 were analyzed for possible sources of dimensionality related to the five mathematical content strands. The analyses were also used to compare test structures across grades. The results indicated that although mathematical achievement tests for Grades 3–8 are complex and exhibit some multidimensionality, the sources of dimensionality are not related to the content strands. The complexity of the data structure, along with the known overlap of mathematical skills, suggests that mathematical achievement tests could represent a fundamentally unidimensional construct. While these sub-domains of mathematics are useful for organizing instruction, developing curricular materials such as textbooks, and describing the organization of items on assessments, they do not describe a significant psychometric property of the test or impact the interpretation of the test results. Mathematics, as measured by the SMI, can be described as one construct with various sub-domains.

These findings support the NCTM Connections Standard, which states that all students (prekindergarten through Grade 12) should be able to make and use connections among mathematical ideas and see how the mathematical ideas interconnect. Mathematics can be best described as an interconnection of overlapping skills with a high degree of correlation across the mathematical topics, skills, and strands.

Furthermore, these findings support the goals of the Common Core State Standards (CCSS) for Mathematics by providing the foundations of a growth model by which a single measure can inform progress toward college and career readiness.

College and Career Preparedness in Mathematics

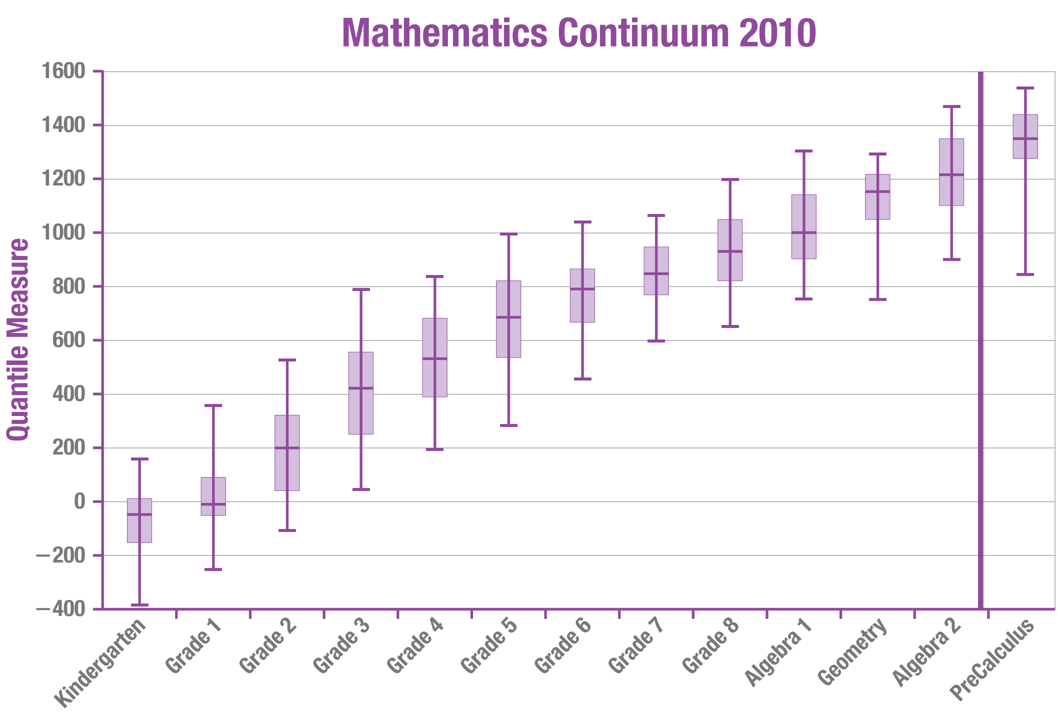
There is increasing recognition of the importance of bridging the gap that exists between K–12 and higher education and other postsecondary endeavors. Many state and policy leaders have formed task forces and policy committees such as P-20 councils.

The Common Core State Standards for Mathematics were designed to enable all students to become college and career ready by the end of high school while acknowledging that students are on many different pathways to this goal: “One of the hallmarks of the Common Core State Standards for Mathematics is the specification of content that all students must study in order to be college and career ready. This ‘college and career ready line’ is a minimum for all students” (NGA Center & CCSSO, 2010b, p. 4). The CCSS for Mathematics suggest that “college and career ready” means completing a sequence that covers Algebra I, Geometry, and Algebra II (or equivalently, Integrated Mathematics 1, 2, and 3) during the middle school and high school years; and, leads to a student’s promotion into more advanced mathematics by their senior year. This has led some policy makers to generally equate the successful completion of Algebra II as a working definition of college and career ready. Exactly how and when this content must be covered is left to the states to designate in their implementations of the CCSS for Mathematics throughout K–12.

The *mathematical demand* of a mathematical textbook (in Quantile measures) quantitatively defines the level of mathematical achievement that a student needs in order to be ready for instruction on the mathematical content of the textbook. Assigning QSCs and Quantile measures to a textbook is done through a calibration process. Textbooks were analyzed at the lesson level and the calibrations were completed by subject matter experts (SMEs) experienced with the Quantile Framework and with the mathematics taught in mathematics classrooms. The intent of the calibration process is to determine the mathematical demand presented in the materials. Textbooks contain a variety of activities and lessons. In addition, some textbook lessons may include a variety of skills. Only one Quantile measure is calculated per lesson and is obtained through analyzing the Quantile measures of the QSCs that have been mapped to the lesson. This Quantile measure represents the composite task demand of the lesson.

MetaMetrics has calibrated more than 41,000 instructional materials (e.g., textbook lessons, instructional resources) across the K–12 mathematics curriculum. Figure 7 shows the continuum of calibrated textbook lessons from Kindergarten through precalculus where the median of the distribution for precalculus is 1350Q. The range between the first quartile and the median of the first three chapters of precalculus textbooks is from 1200Q to 1350Q. This range describes an initial estimate of the mathematical achievement level needed to be ready for mathematical instruction corresponding to the "college and career readiness" standard in the Common Core State Standards for Mathematics.

FIGURE 7. A continuum of mathematical demand for Kindergarten through precalculus textbooks (box plot percentiles: 5th, 25th, 50th, 75th, and 95th).



Using SMI College & Career

Using SMI College & Career

The *Scholastic Math Inventory* College & Career (SMI College & Career) is a computer-adaptive mathematics test that provides a measure of students' readiness for mathematics instruction in the form a Quantile measure. The results of the test can be used to measure how well students understand, and are likely to be successful with various grade-appropriate mathematical skills and topics. SMI College & Career is designed to be administered three to five times during a school year.

SMI College & Career consists of a bank of more than 5,000 four-option, multiple-choice items that represent different mathematics concepts and topics. While the items span the five content standards, more than 75% of the items for Kindergarten through Grade 8 are associated with Number & Operations (K–8); Algebraic Thinking, Patterns, and Proportional Reasoning (K–8); and Expressions & Equations, Algebra, and Functions (6–8). In Grades 9 through 11 the focus shifts so that approximately 60% of the items in the Grade 9 and 11 item banks are associated with Expressions & Equations, Algebra, and Functions, and approximately 60% of the items in the Grade 10 item bank are associated with Geometry, Measurement, and Data. The weighting of content by grade was designed to reflect the priorities expressed in the CCSSM and latest state mathematics standards. The items cover a wide range of presentations, such as computational items, word problems and story problems, graphs, tables, figures, and other representations.

The *SMI Professional Learning Guide* provides suggestions for test administration and an overview of Scholastic Achievement Manager (SAM) features. It also includes information on how SMI College & Career can be implemented in a variety of instructional environments including in Response to Intervention implementations. The guide provides a detailed explanation of each SMI College & Career report and how SMI College & Career data can be used to differentiate instruction in the core curriculum classroom.

All the documentation, installation guides, technical manuals, software manuals, and all technical updates provided in the program are available for download on the Scholastic Product Support site. The address to that site is: http://edproductsupport.scholastic.com/ts/product/smi/.

After installation, the first step in using SMI College & Career is the *Scholastic Achievement Manager*, or SAM—the learning management system for all Scholastic technology programs. Educators use SAM to collect and organize student-produced data. SAM helps educators understand and implement data-driven instruction by:

- Managing student rosters
- Generating reports that capture student performance at various levels of aggregation (student, classroom, group, grade, school, and district)
- Locating helpful resources for classroom instruction and aligning the instruction to standards
- Communicating student progress to parents, teachers, and administrators

The *SMI Professional Learning Guide* also provides teachers with information on how to use the results from SMI College & Career in the classroom. Teachers can use the reported Quantile measures to determine appropriate instructional support materials for their students. Information related to best practices for test administration, interpreting reports, and using Quantile measures in the classroom is also provided.

Administering the Test

SMI College & Career can be administered multiple times during the school year. Typically, SMI College & Career should be administered three times during the school year—at the beginning, the middle, and the end—to monitor students' progress in developing mathematical understandings. Within an intervention program, SMI College & Career can be administered every eight weeks. SMI College & Career should be administered no more than three to five times per year in order to allow sufficient growth in between testing sessions. The tests are intended to be untimed, and typically students take 20 to 40 minutes to complete the test.

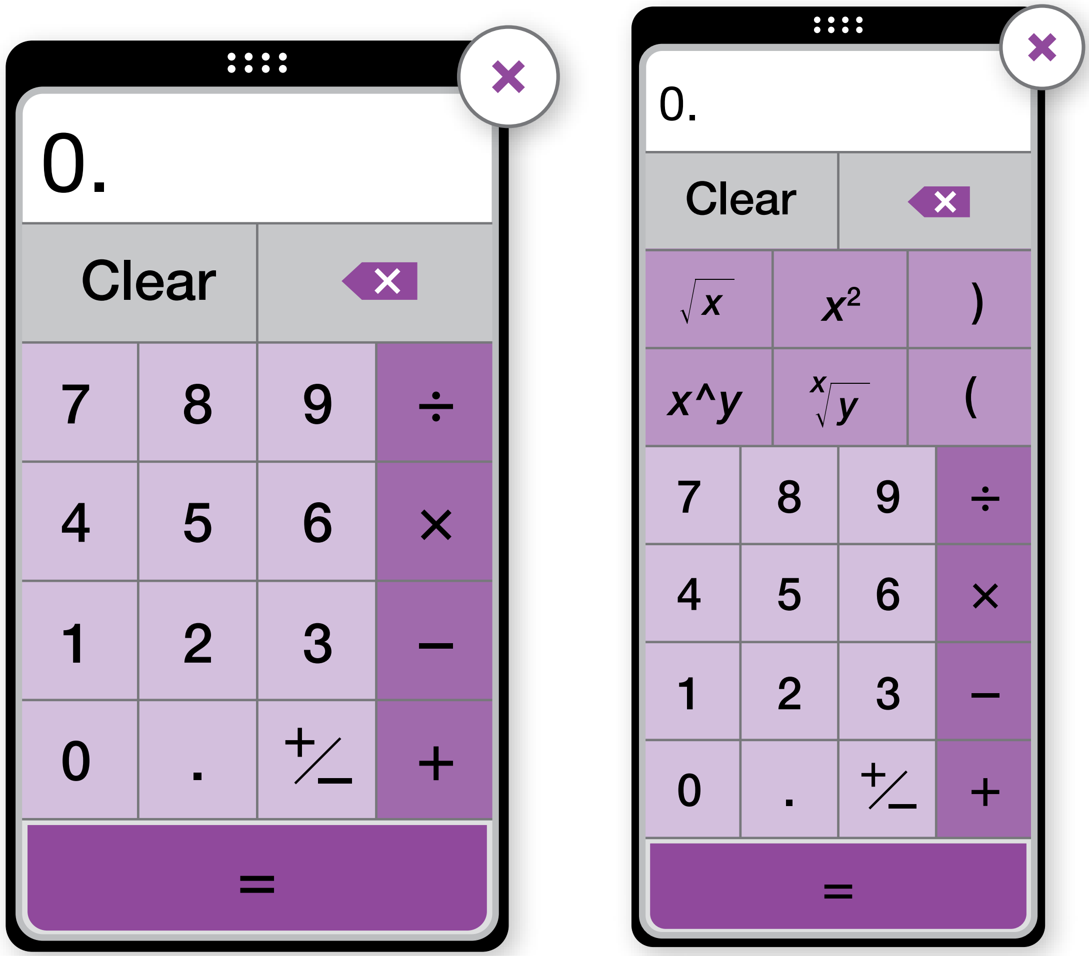
SMI College & Career can be administered in a group setting or individually—wherever computers are available. The test can also be administered on mobile devices. The setting should be quiet and free from distractions. Teachers should make sure that students have the computer skills needed to complete the test and have scratch paper and pencils.

Students log on to the program with usernames and passwords. The practice items in the assessment are provided to ensure that students understand the directions, know how to use the computer to take the test, and are not encountering server connectivity issues. In this section, students are introduced to the calculators and formula sheets that are available for certain items within SMI College & Career. Calculators and specific formula sheets are available based on the grade level of the item.

SMI College & Career includes two types of on-screen calculators depicted in Figure 8. Items written for Grade 5 and lower are supported by a four-function calculator. Items written for Grade 6 and higher are supported by a scientific calculator. Students in Grades 8 and above may use graphing calculators, which are not provided by the program. These students should be provided access to their own graphing calculators with functionality similar to that of a TI-84. When the purpose of the item is computational, SMI College & Career disables the use of the calculator automatically. The student should become familiar with the calculator while completing the Practice Test items.

Administrators can turn off the calculator globally for the assessment. This option is often selected in states where calculators are not permitted on high stakes exams. However, turning the calculator off may extend the time students take on the assessment and may impact results. For the most comparable results, it is suggested that a policy decision is made at the district level concerning calculator access in SMI College & Career.

FIGURE 8. SMI College & Career's four-function calculator (left) and scientific calculator (right).



SMI College & Career also includes three on-screen Formula Sheets that include useful equations. There is a Formula Sheet available for items written for Grades 3–5 (see Figure 9), Grades 6–8 (see Figure 10), and Grades 9–11 (see Figure 11). Items written for Grade 2 and lower do not need a Formula Sheet. The student can review the Formula Sheet before taking the Practice Test items. SAM allows an administrator to turn off the Formula Sheet. However, turning the Formula Sheet off increases the demand on the student to solve the problems, and the decision to provide access to the Formula Sheet should be determined globally at the district level.

FIGURE 9. SMI College & Career Grades 3–5 Formula Sheet.

Perimeter

Square: $P = 4 \times s$

Rectangle: $P = (2 \times l) + (2 \times w)$

Area

Square: $A = s \times s$

Rectangle: $A = l \times w$

Volume

Cube: $V = s \times s \times s$

Rectangle prism: $V = B \times h$ or $V = l \times w \times h$

FIGURE 10. SMI College & Career Grades 6–8 Formula Sheet.

Area

Triangle: $A = \frac{1}{2} bh$

Parallelogram: $A = bh$

Trapezoid: $A = \frac{1}{2}(b_1 + b_2)h$

Circle: $A = \pi r^2$

Circumference of a circle

$C = 2\pi r$ or $C = \pi d$

Surface Area

Prism: SA = sum of the areas of all faces

Pyramid: SA = sum of the areas of all faces

Cylinder: $SA = 2\pi r^2 + 2\pi rh$

Volume

Cube: $V = s^3$

Prism: $V = Bh$

Cylinder: $V = \pi r^2 h$

Cone: $V = \frac{1}{3}\pi r^2 h$

Sphere: $V = \frac{4}{3}\pi r^3$

Pi

$\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$

Simple Interest

$I = Prt$

Pythagorean Theorem

$c^2 = a^2 + b^2$

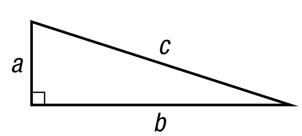


FIGURE 11. SMI College & Career Grades 9–11 Formula Sheets.

Area

Triangle: $A = \frac{1}{2}bh$

Parallelogram: $A = bh$

Trapezoid: $A = \frac{1}{2}(b_1 + b_2)h$

Circle: $A = \pi r^2$

Circumference of a circle

$C = 2\pi r$ or $C = \pi d$

Volume

Prism: $V = Bh$

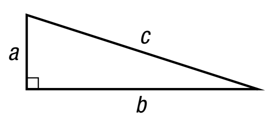
Cylinder: $V = \pi r^2 h$

Cone: $V = \frac{1}{3}\pi r^2 h$

Sphere: $V = \frac{4}{3}\pi r^3$

Pythagorean Theorem

$c^2 = a^2 + b^2$



Sum of the interior angles of a polygon with *n* sides

$S = (n - 2)(180°)$

Measure of an exterior angle of a regular polygon with *n* sides

$m\angle = \frac{360°}{n}$

Compound Interest

$A = P(1 + \frac{r}{n})^{nt}$

Exponential Growth/Decay

$A = A_0 e^{k(t - t_0)} + B_0$

Expected value

$V = p_1x_1 + p_2x_2 + \ldots + p_nx_n$

Quadratic Formula

Solution of $ax^2 + bx + c = 0$ is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Distance Formula

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Arithmetic Sequence

$a_n = a_1 + (n - 1)d$

Geometric Sequence

$a_n = a_1 r^{n-1}$

Combinations of *n* Objects Taken *r* at a Time

${}_nC_r = \frac{n!}{r!(n - r)!}$

Permutations of *n* Objects Taken *r* at a Time

${}_nP_r = \frac{n!}{(n - r)!}$

Binomial Theorem

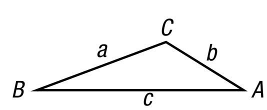
$(a + b)^n = {}_nC_0 a^n b^0 + {}_nC_1 a^{(n-1)} b^1 +$
${}_nC_2 a^{(n-2)} b^2 + \ldots + {}_nC_n a^0 b^n$

Pythagorean Identity

$\sin^2\theta + \cos^2\theta = 1$

Law of Sines

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$



Law of Cosines

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

Heron's Formula

$A = \sqrt{s(s - a)(s - b)(s - c)}$ where

$s = \frac{1}{2}(a + b + c)$

Targeting

Prior to testing, it is strongly suggested that the teacher or administrator inputs information into the SAM on the known ability of students. The categories are:

- Undetermined
- Far below level
- Below level
- On level
- Above level
- Far above level

If the student's ability is unknown, then the teacher or administrators should select undetermined. The default setting for this feature is on grade level.

This targeting information is used by the SMI College & Career algorithm to determine the starting point for the student. The value of this setting is to ensure that struggling students receive a question at a lower proficiency level. For example the following levels will provide grade-level questions that are associated with the indicated percentile:

- Undetermined—50th%
- Far below level—5th%
- Below level—25th%
- On level—50th%
- Above level—75th%
- Far above level—90th%

Targeting applies only to the first administration of SMI College & Career. The second administration of the test will start with a question at the Quantile measure received from the previous test administration.

Student Interaction With SMI College & Career

The student experience with SMI College & Career consists of three parts:

- Math Fact Screener
- Practice Test
- Scored Test

Math Fact Screener

The first part of SMI College & Career is the Math Fact Screener and is used at all grade levels. The Math Fact Screener consists of an Early Numeracy Screener for counting and quantity comparison for students in Kindergarten and Grade 1, items related to addition facts for Grades 2 and 3, and both addition and multiplication facts for Grades 4 and above. The facts presented do not change from grade to grade. The results of the Math Fact Screener are *not* used in either the SMI College & Career algorithm or in determining a student's SMI College & Career Quantile measure. The screener performs a separate assessment of a student's potential math fact knowledge and facility.

The Math Fact Screener consists of three parts (does not apply to the Early Numeracy Screener): the typing warm-up, addition facts, and multiplication facts. During the typing warm-up, students practice typing in four different values to ensure that they understand the interface used during the Math Fact Screener. Students then give the sums for 10 addition facts; and, for students in Grades 4 and above, the product for a sampling of 10 multiplication facts.

An item is visible on the screen for up to 10 seconds. If the item is not answered in 10 seconds, it is counted as incorrect and a new item is displayed. Although the item is visible for 10 seconds, students have only five seconds to correctly respond to each item in the Math Fact Screener. If an answer is correct, but is not entered within five seconds, then the question is counted as incorrect. Students do not see a time on the computer screen. There is no time limit for counting an answer correct for the Early Numeracy Screener. The program records the student answer and the time it took to respond to the fact. Students must answer 80% (eight out of 10) of the items correctly to pass the Math Fact Screener. Students in Grade 3 and below must respond correctly to 80% of the addition items to pass the Math Fact Screener. Students in Grade 4 and above must respond correctly to 80% of the addition items and 80% of the multiplication items.

The addition and multiplication sections are considered separately. A student who passes one section of the Math Fact Screener will not be administered that section again.

SAM reports the Math Fact Screener results to the administrator at both the student and group levels. These reports indicate that the student may need work on basic math facts.

Practice Test

The SMI College & Career Practice Test consists of three to five items that are significantly below the student's mathematical performance level (approximately 10th percentile for nominal grade level). The practice items are administered during the student's first experience with SMI College & Career at the beginning of each school year, unless the teacher or administrator has configured their program settings in SAM such that the practice test is a part of every test.

Practice items are designed to ensure that the student understands the directions and knows how to use the computer to take the test. It also introduces the use of the calculators and the Formula Sheets embedded within SMI College & Career. Typically, students will see three items. The program will extend the student experience to five items, however, if the student incorrectly responds to two or more of the initial three items. The student may be asked to contact his teacher to ensure that he understands how to engage with the program.

Scored SMI College & Career Test

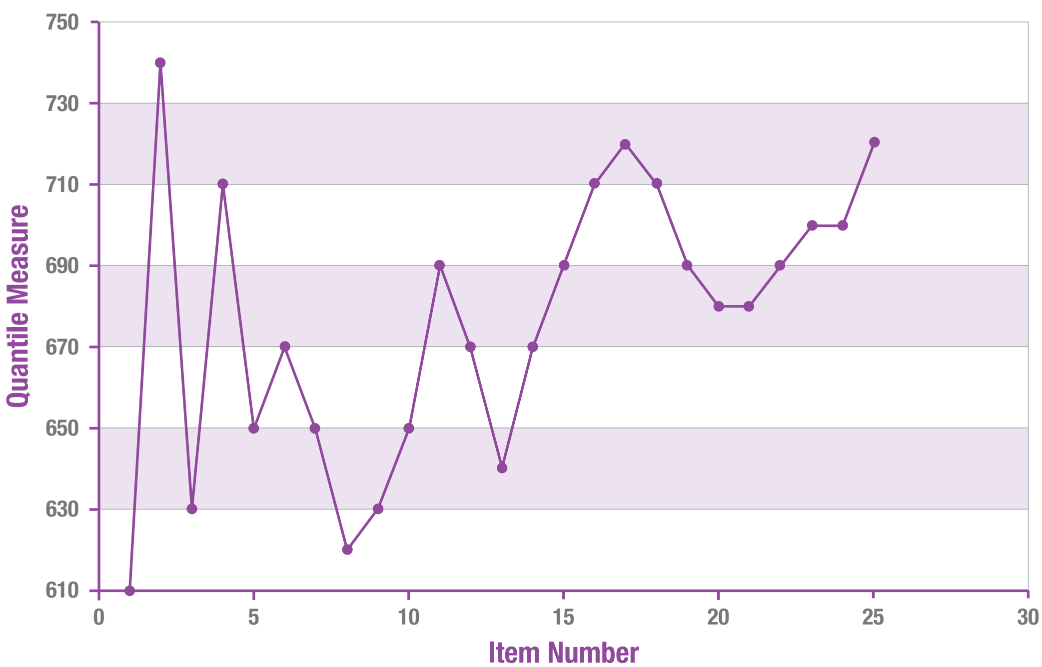
The final part of the students' interaction is the SMI College & Career test administration. The initial test item is selected for the student based on his or her grade level and the teacher's estimate of his or her mathematical ability. The first item of the first administration is one grade level below the student grade. The estimated math ability can be set only for the first administration. During the test, the SMI College & Career algorithm is designed to adapt the selection of test items according to the student's responses. After the student responds to the first question, the test then steps up or down according to the student's performance. When the test has enough information to adequately estimate the student's readiness for mathematics instruction, the test stops and the student's Quantile measure is reported.

The process described above is detailed into three phases called Start, Step, and Stop. In the Start phase, the SMI College & Career algorithm determines the best point on the Quantile scale to begin testing the student. The more information SMI College & Career has about the student, the more accurate the results. For more accurate results from the first administration, the practice of "targeting the student" is suggested. Initially, a student can be targeted using: (1) the student's grade level, and (2) the teacher's estimate of the student's ability in mathematics. For successive administrations of SMI College & Career, the student's prior Quantile measure plus an estimated amount of assumed growth based on the time in between administrations is used for targeting. While it is not necessary for a teacher to assign an estimated achievement level, assigning one will produce more accurate results; the SAM default setting is "undetermined." The teacher cannot set the math ability after the first test. For the student whose test administration is illustrated in Figure 12, the teacher entered the student's grade and an estimate of the student's mathematics achievement.

The second phase, Step, controls the selection of items presented to the student. If the only targeting information entered was the student's grade level, then the student is presented with an item that has a Quantile measure at the 100Q below the 50th percentile for his or her grade. If more information about the student's mathematical ability was entered, then the student is presented with an item more closely aligned to the student's "true" ability. If the student answers the item correctly, then the student is presented with an item that is more difficult. If the student answers the item incorrectly, then an item that is easier is presented. An SMI College & Career score (Quantile measure) for the student is updated after the student responds to each item. The SMI College & Career algorithm will always maintain a progression of items across the content strands.

Figure 12 shows how SMI College & Career may present items during a typical administration. The first item presented to a student had a Quantile measure of 610, or measured 610Q. Because the item was answered correctly, the next item was more difficult (740Q). Because this item was answered incorrectly, the third item measured 630Q. Because this item was answered correctly, the next item was harder (710Q). Note as the number of items administered increases, the differences between the Quantile measures of subsequent items decreases in order to more accurately place a student on the Quantile Framework.

FIGURE 12. Sample administration of SMI College & Career for a fourth-grade student.



The final phase, Stop, controls the termination of the test. In SMI College & Career, students will be presented with 25 to 45 items. The exact number of items a student receives depends on how accurately the student responds to the items presented. In addition, the number of items presented to the student is affected by how well the test was targeted in the beginning. Well-targeted tests begin with less measurement error, and therefore need to present the student with fewer items. In Figure 12, the student was well targeted and performed with reasonable consistency, so only 25 items were administered. It can be inferred that the experience of taking a targeted test is optimal for the students in terms of both proper challenging and maintaining motivation. A well-targeted test brings out the best in students.

Interpreting *Scholastic Math Inventory* College & Career Scores

Results from SMI College & Career are reported as scale scores (Quantile measures). This scale extends from Emerging Mathematician (below 0Q) to above 1600Q. The score is determined by the difficulty of the items a student answered both correctly and incorrectly. Scale scores can be used to report the results of both criterion-referenced tests and norm-referenced tests.

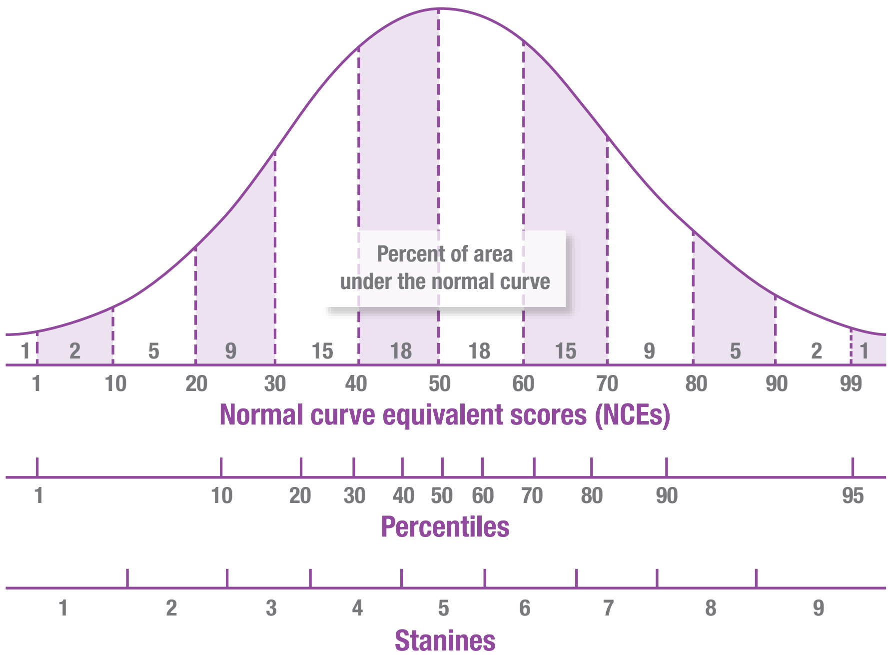
There are many reasons to use scale scores rather than raw scores to report test results. Scale scores overcome the disadvantage of many other types of scores (e.g., percentiles and raw scores) in that equal differences between scale score points represent equal differences in achievement. Each question on a test has a unique level of difficulty. Therefore, answering 23 items correctly on one form of a test requires a slightly different level of achievement than answering 23 items correctly on another form of the test. But receiving a scale score (in this case, a Quantile measure) of 675Q on one form of a test represents the same level of mathematics understanding as receiving a scale score of 675Q on another form of the test.

SMI College & Career provides both criterion-referenced and norm-referenced interpretations of the Quantile measure. Norm-referenced interpretations of test results, often required for accountability purposes, indicate how well the student's performance on the assessment compares to other, similar students' results. Criterion-referenced interpretations of test results provide a rich frame of reference that can be used to guide instruction and skills acquisition for optimal student mathematical development.

Norm-Referenced Interpretations

A norm-referenced interpretation of a test score expresses how a student performed on the test compared to other students of the same age or grade. Norm-referenced interpretations of mathematics achievement test results, however, do not provide any information about mathematical skills or topics a student has or has not mastered. For accountability purposes, percentiles, stanines, and normal curve equivalents (NCEs) are used to report test results when making comparisons (norm-referenced interpretations). For a comparison of these measures, refer to Figure 13.

FIGURE 13. Normal distribution of scores described in percentiles, stanines, and NCEs.



The percentile rank of a score indicates the percentage of scores lower than or equal to that score. Percentile ranks range from 1 to 99. For example, if a student scores at the 65th percentile, it means that she performed as well as or better than 65% of the norm group. Real differences in performance are greater at the ends of the percentile range than in the middle. Percentile ranks of scores can be compared across two or more distributions. Percentile ranks, however, cannot be used to determine differences in relative rank because the intervals between adjacent percentile ranks do not necessarily represent equal raw score intervals. Note that the percentile rank does not refer to the percentage of items answered correctly.

A normal curve equivalent (NCE) is a normalized student score with a mean of 50 and a standard deviation of 21.06. NCEs range from 1 to 99. NCEs allow comparisons between different tests for the same student or group of students and between different students on the same test. NCEs have many of the same characteristics as percentile ranks, but have the additional advantage of being based on an interval scale. That is, the difference between two consecutive scores on the scale has the same meaning throughout the scale. NCEs are required by many categorical funding agencies (for example, Title I).

A stanine is a standardized student score with a mean of 5 and a standard deviation of 2. Stanines range from 1 to 9. In general, stanines of 1 to 3 are considered below average, stanines of 4 to 6 are considered average, and stanines of 7 to 9 are considered above average. A difference of 2 between the stanines for two measures indicates that the two measures are significantly different. Stanines, like percentiles, indicate a student's relative standing in a norm group.

Normative information can be useful and is often required at the aggregate levels for program evaluation. Appendix 2 contains normative data (spring percentiles) for students in Grades K–12 at selected levels of performance.

To develop normative data, the results from a linking study with the Quantile Framework on a sample of more than 250,000 students from across the country were examined. Approximately 80% of the students attended public school, and approximately 20% attended private or parochial schools. The students in the normative population consisted of 19.8% African American, 2.7% Asian, 9.2% Hispanic, and 68.3% Other (includes White, Native American, Other, and Multiracial). Approximately 6% of the students were eligible for the free or reduced-price lunch program. Approximately half of the students attended public schools where more than half of the students were eligible for Title I funding (either school-wide or targeted assistance).

Criterion-Referenced Interpretations

An important feature of the Quantile Framework is that it also provides criterion-referenced interpretations of every measure. A criterion-referenced interpretation of a test score compares the specific knowledge and skills measured by the test to the student's proficiency with the same knowledge and skills. Criterion-referenced scores have meaning in terms of what the student knows and can do, rather than in relation to the performance of a peer group.

The power of SMI College & Career as a criterion-reference test is amplified by the design and meaning of the Quantile Framework. When the student's mathematics ability is equal to the mathematical demand of the task, the Quantile Theory forecasts that the student will demonstrate a 50% success rate on that task and is ready for instruction related to that skill or concept. When 20 such tasks are given to this student, one expects one-half of the responses to be correct. If the task is too difficult for the student, then the probability is less than 50% that the response to the task will be correct. These tasks are skills and concepts for which the student likely does not have the background knowledge required. Similarly, when the difficulty level of the task is less than a student's measure, then the probability is greater than 50% that the response will be correct. These tasks are skills and concepts are ones that the student is likely to have already mastered.

Because the Quantile Theory provides complementary procedures for measuring achievement and mathematical skills, the scale can be used to match a student's level of understanding with other mathematical skills and concepts with which the student is forecast to have a high understanding rate. Identifying skills that students are ready to learn is critical not only to developing overall mathematics learning, but also to creating a positive mathematical experience that can motivate and change attitudes about mathematics in general.

Assessment of mathematics learning is a key component in the classroom. This assessment takes on many different models and styles depending on the purpose of the assessment. It can range from asking key questions during class time, to probing critical thinking and reasoning of students' answers, to asking students to record their mathematical learning, to developing well-designed multiple-choice formats. As a progress monitoring tool, SMI College & Career provides feedback to teachers throughout the school year that can be connected with typical end-of-the-year proficiency ranges since multiple assessments are connected to the same reporting scale.

Forecasting Student Understanding and Success Rates

A student with a Quantile measure of 600Q who is to be instructed on mathematical tasks calibrated at 600Q is expected to have a 50% success rate on the tasks and a 50% understanding rate of the skills and concepts. This 50% rate is the basis for selecting tasks employing skills and concepts for instruction targeted to the student's mathematical achievement. If the mathematical demand of a task is less than the student measure, the success rate will exceed 50%. If the mathematical demand is much less, the success rate will be much greater. The difference in Quantile scale units between student achievement and mathematical demand governs understanding and success. This section gives more explicit information on predicting success rates.

If all of the tasks associated with a 400Q Quantile Skill and Concept had the same difficulty, the understanding rate resulting from the 200Q difference between the 600Q student and the 400Q mathematical demand could be determined using the Rasch model equation (see Equation 2, p. 26). This equation describes the relationship between the measure of a student's level of mathematical understanding and the difficulty of the skills and concepts. Unfortunately, understanding rates calculated using only this procedure would be biased because the difficulties of the tasks associated with a skill or concept are not all the same. The average difficulty level of the tasks *and* their variability both affect the success rate.

Figure 14 shows the general relationship between student-task discrepancy and predicted success rate. When the Student Measure and the task mathematical demand are the same, then the predicted success rate is 50% and the student is ready for instruction on the skill or concept. If a student has a measure of 600Q and the task's mathematical demand is 400Q, the difference is 200Q. According to Figure 14, a difference of +200Q (Student Measure minus task difficulty) indicates a predicted success rate of approximately 75%. Also note that a difference of –200Q indicates a predicted success rate of about 25%.

The subjective experience between 25%, 50%, and 75% understanding or success varies greatly. A student with a Quantile measure of 1000Q being instructed on QSCs with measures of 1000Q will likely have a successful instructional experience—he or she has about a 50% rate of understanding with the background knowledge needed to learn and apply the new information. Teachers working with such a student report that the student can engage with the skills and concepts that are the focus of the instruction and, as a result of the instruction, are able to solve problems utilizing those skills. In short, such students appear to understand what they are learning. A student with a measure of 1000Q being instructed on QSCs with measures of 1200Q has about a 25% understanding rate and encounters so many unfamiliar skills and difficult concepts so that the learning is frequently lost. Such students report frustration and seldom engage in instruction at this level of understanding. Finally, a student with a Quantile measure of 1000Q being instructed on QSCs with measures of 800Q has about a 75% understanding rate and reports being able to engage with the skills and concepts with minimal instruction. He or she is able to solve complex problems related to the skills and concepts, is able to connect the skills and concepts with skills and concepts from other strands, and experiences automaticity of skills.

FIGURE 14. Student-mathematical demand discrepancy and predicted success rate.

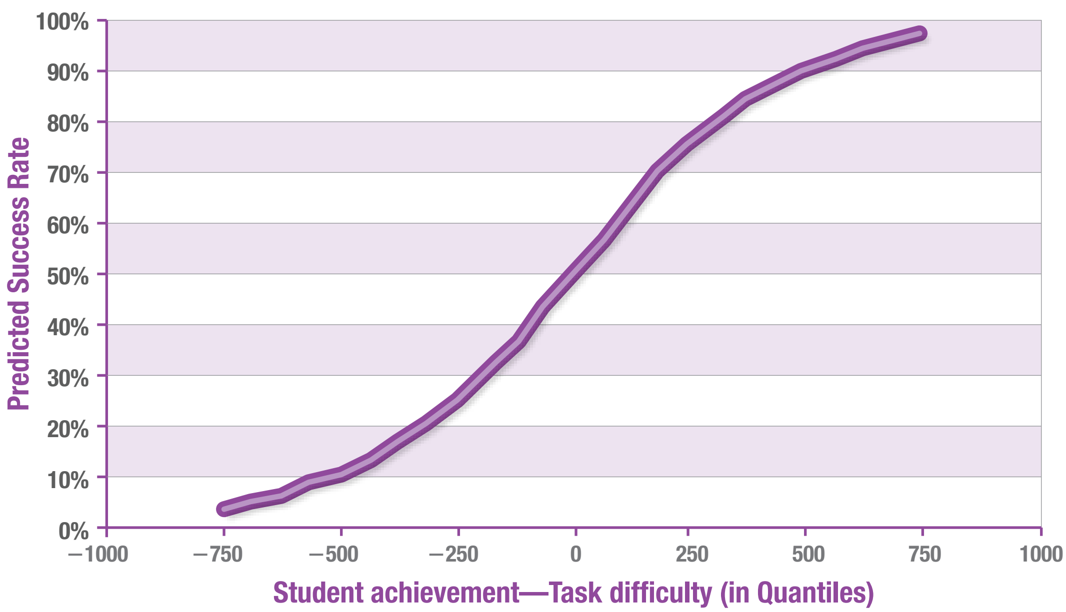


Table 8 gives an example of the predicted understanding or success rates for specific skills for a specific student. Table 9 shows success rates for one specific skill calculated for different student achievement measures.

TABLE 8. Success rates for a student with a Quantile measure of 750Q and skills of varying difficulty (demand).

Student Mathematics Achievement	Skill Demand	Skill Description	Predicted Understanding
750Q	250Q	Locate points on a number line.	90%
750Q	500Q	Use order of operations, including parentheses, to simplify numerical expressions.	75%
750Q	750Q	Translate between models or verbal phrases and algebraic expressions.	50%
750Q	1000Q	Estimate and calculate areas with scale drawings and maps.	25%
750Q	1250Q	Recognize and apply definitions and theorems of angles formed when a transversal intersects parallel lines.	10%

TABLE 9. Success rates for students with different Quantile measures of achievement for a task with a Quantile measure of 850Q.

Student Mathematics Achievement	Problems Related to "Locate points in all quadrants of the coordinate plane using ordered pairs."	Predicted Understanding
350Q	850Q	10%
600Q	850Q	25%
850Q	850Q	50%
1100Q	850Q	75%
1350Q	850Q	90%

The primary utility of the Quantile Framework is its ability to forecast what happens when students engage in mathematical tasks. The Quantile Framework makes a pointed success prediction every time a skill is chosen for a student. There is error in skill measures, student measures, and their difference modeled as predicted success rates. However, the error is sufficiently small that the judgments about the students, task demand, and success rates are useful.

Performance Levels

The SMI performance levels were originally developed in 2009 using an iterative process. Each phase of the development built upon previous discussions as well as incorporated new information as the process continued. Performance levels were set for Grades 2–9. In 2013, Scholastic began the redevelopment of SMI College & Career to align with the Common Core State Standards in Mathematics (NGA Center and CCSSO, 2010a) and expand the range of use from Grade 2 through Algebra I to Kindergarten through Algebra II/High School Integrated Math 3. One aspect of this redevelopment was to add performance standards for Kindergarten, Grade 1, Geometry/Math 2 (generally Grade 10), and Algebra II/Math 3 (generally Grade 11). In order to add these additional grades based on the CCSSM demands and to be consistent across all grade levels, the Grade 2–Algebra I standards were also modified.

The following sources of data were examined to develop the SRI College & Career performance standards:

- *Student-based standards:* North Carolina End-of-Grade and End-of-Course Math Assessments (North Carolina Department of Public Instruction 2013 Quantile linking study, Grades 3–8 and Algebra I/Integrated 1, MetaMetrics, Inc., 2013); Virginia Mathematics Standards of Learning Tests (Virginia Department of Education 2012 Quantile Linking Study, Grades 3–8, Algebra I and II, and Geometry, MetaMetrics, Inc., 2012c); Kentucky Performance Rating for Educational Progress Math Test (Kentucky Department of Education 2012 Quantile Linking Study, Grades 3–8, MetaMetrics, Inc., 2012b); National Assessment of Educational Progress—Math (National Center for Educational Statistics “Lexile/Quantile Feasibility Study,” May 2011, Grades 4, 8, and 12, MetaMetrics, Inc., 2011); and ACT Mathematics Tests administered in North Carolina (NCDPI and ACT 2012 Quantile linking study, Grade 11, MetaMetrics, Inc., 2012a)
- *Resource-based standards:* “2010 Math Text Continuum,” MetaMetrics, Inc., 2011, in “QF & CCR-2011.pdf”

The bottom of the “proficient” range for each grade level associated with the three states was examined and a regression line was developed to smooth the data. The resulting function was similar to the top of the text continuum range across the grade levels (75th percentile of lessons associated with the grade/course). This indicates that students at this level should be ready for instruction on the more mathematically demanding topics at the end of the school year, which is consistent with expectation. The top of the “proficient” range for each grade level associated with the three states was examined and a regression line was developed to smooth the data. The proposed SMI College & Career proficient range for each grade level was examined and compared with the Spring Quantile percentile tables. This information was used to extrapolate to Kindergarten and Grades 1 and 2. These results are consistent with the ranges associated with NAEP and ACT to define “college readiness.”

These proficient levels were used as starting points to define the ranges associated with the remaining three performance levels for each grade level. Setting of these performance levels combined information about the QSC/skill and concept difficulty as well as information related to the performance levels observed from previous Quantile Framework linking studies. These levels were refined further based on discussion by educational and assessment specialists. The policy descriptions for each of the performance levels used at each grade level are as follows:

- **Advanced:** Students scoring in this range exhibit superior performance on grade-level-appropriate skills and concepts and, in terms of their mathematics development, may be considered on track for college and career.
- **Proficient:** Students scoring in this range exhibit competent performance on grade-level-appropriate skills and concepts and, in terms of their mathematics development, may be considered on track for college and career.
- **Basic:** Students scoring in this range exhibit minimally competent performance on grade-level-appropriate skills and concepts and, in terms of their mathematics development, may be considered marginally on track for college and career.
- **Below Basic:** Students scoring in this range do not exhibit minimally competent performance on grade-level-appropriate skills and concepts and, in terms of their mathematics development, are not considered on track for college and career.

The final scores for each grade level and performance level used with SMI are presented in Table 10.

TABLE 10. SMI College & Career performance level ranges by grade (Spring Norms).

Grade	Below Basic	Basic	Proficient	Advanced
K	EM*400–EM185	EM190–5	10–175	180 and Above
1	EM400–60	65–255	260–450	455 and Above
2	EM400–205	210–400	405–600	605 and Above
3	EM400–425	430–620	625–850	855 and Above
4	EM400–540	545–710	715–950	955 and Above
5	EM400–640	645–815	820–1020	1025 and Above
6	EM400–700	705–865	870–1125	1130 and Above
7	EM400–770	775–945	950–1175	1180 and Above
8	EM400–850	855–1025	1030–1255	1260 and Above
9	EM400–940	945–1135	1140–1325	1330 and Above
10	EM400–1020	1025–1215	1220–1375	1380 and Above
11	EM400–1150	1155–1345	1350–1425	1430 and Above
12	EM400–1190	1195–1385	1390–1505	1510 and Above

*Emerging Mathematician

Algebra Readiness and College and Career Readiness

In addition to describe performance in relation to describing general mathematical achievement, SMI College & Career provides a Quantile measure that represents a student who is deemed ready for Algebra I. To determine this value, the following information sources were examined: state standards for Grade 8 (before Algebra I) proficiency (895Q to 1080Q), the SMI College & Career Grade 8 proficiency cutoff (1030Q), and the QSCs associated with the algebra strand in Grades 8 and 9 (Grade 8: 700Q to 1190Q, Mean = 972.5Q; Grade 9: 700Q to 1350Q, Mean = 1082.0Q). It was concluded that a Quantile measure of 1030Q could be used to describe "readiness for Algebra I."

The CCSS state that "[t]he high school portion of the Standards for Mathematical Content specifies the mathematics all students should study for college and career readiness. These standards do not mandate the sequence of high school courses. However, the organization of high school courses is a critical component to implementation of the standards.

- a traditional course sequence (Algebra I, Geometry, and Algebra II)
- an integrated course sequence (Mathematics 1, Mathematics 2, Mathematics 3) . . ."

(NGA and CCSSO, 2010a, p. 84). To provide a Quantile measure that represents a student who is deemed ready for the mathematics demands of college and career, the "Mathematics Continuum" presented in Figure 7 was examined. The interquartile range for Algebra II is from 1200Q to 1350Q. It was concluded that a Quantile measure of 1350Q could be used to describe "readiness for college and career."

Using SMI College & Career Results

SMI College & Career begins with the concept of targeted-level testing and makes a direct link with those measures to instruction. With the Quantile Framework for Mathematics as the yardstick of skill difficulty, SMI College & Career produces a measure that places skills, concepts, and students on the same scale. The Quantile measure connects each student to mathematical resources—Knowledge Clusters, specific state standards, and the Common Core State Standards for Mathematics, widely adopted basal textbooks, supplemental math materials, and math intervention programs. Because SMI College & Career provides an accurate measure of where each student is in his or her mathematical development, the instructional implications and skill success rate for optimal growth are explicit. SMI College & Career targeted testing identifies for the student the mathematical skills and topics that are appropriately challenging to him or her.

SMI College & Career provides a database that directly links Quantile measures and QSCs to all state standards including the Common Core State Standards and hundreds of widely adopted textbooks and curricular resources. This database also allows educators to target mathematical skills and concepts and unpack the Knowledge Cluster associated with each Quantile Skill and Concept. The searchable database is found on www.Scholastic.com/SMI and is one of the many of the supporting tools available at www.Quantiles.com.

SMI College & Career Reports to Support Instruction

The key benefit of SMI College & Career is its ability to generate immediate actionable data that can be used in the classroom to monitor and interpret student progress. The Scholastic Achievement Manager (SAM) organizes and analyzes the results gathered from student tests and presents this information in a series of clear, understandable, actionable reports that can help educators track growth in mathematics achievement over time, evaluate progress toward proficiency goals, and accomplish administrative tasks. SMI College & Career reports help educators effectively assess where students are now and where they need to go. *The SMI Professional Learning Guide* provides detailed descriptions of each of the SMI College & Career reports, which are designed to support four broad functions: (1) progress monitoring, (2) instructional planning, (3) school-to-home communication, and (4) growth.

One key SMI College & Career report is the Instructional Planning Report (see Figure 15), which orders students by percentile rank and places them into Performance Levels. In addition to identifying students in need of review and fluency building in basic math facts, the report also provides instructional recommendation for students in the lower Performance Levels. The instructional recommendations focus on Critical Foundations—those skills and concepts that are most essential to accelerate students to grade-level proficiencies and college and career readiness. The Critical Foundations are identified descriptively, by QSC number and description as well as the Common Core State Standard identification number. Teachers can use this information to access Knowledge Clusters and textbook alignments for intervention and differentiation purposes in the SMI Skills Database at www.scholastic.com/SMI.

FIGURE 15. Instructional Planning Report.

INSTRUCTIONAL PLANNING

Instructional Planning Report

Class: 3rd Period

SMI College™ & Career

School: Williams Middle School
Teacher: Sarah Greene
Grade: 5
Time Period: 12/13/14–02/22/15

			SCREENER					NORMATIVE DATA		
PERFORMANCE LEVEL	STUDENTS	GRADE	ADDITION	MULTI	QUANTILE® MEASURE	DATE	TEST TIME (MIN)	PERCENTILE RANK	NCE	STANINE
A	Gainer, Jacquelyn	5	●	●	1100Q	02/22/15	35	95	77	7
A	Cho, Henry	5	●	●	1100Q	02/22/15	25	95	77	7
P	▶ Collins, Chris	5	●	●	890Q	02/22/15	12	76	66	7
P	Kohlmeier, Ryan	5	●	◒	820Q	02/22/15	40	73	65	6
P	Cooper, Maya	5	●	●	820Q	02/22/15	37	73	64	6
B	Enoki, Jeanette	5	●	●	800Q	02/22/15	39	71	63	6
B	Hartsock, Shalanda	5	●	●	750Q	02/22/15	33	70	59	4
B	Terrell, Walt	5	●	●	720Q	02/22/15	26	45	56	5
B	Cocanower, Jaime	5	●	●	710Q	02/22/15	38	45	53	5
B	Garcia, Matt	5	●	◒	680Q	02/22/15	15	44	52	5
B	Dixon, Ken	5	●	◒	660Q	02/22/15	35	41	47	5
B	Morris, Timothy	5	●	◒	650Q	02/22/15	30	43	48	5
B	▶ Blume, Joy	5	●	◒	640Q	02/22/15	9	39	45	5
B	Ramirez, Jeremy	5	●	●	600Q	02/22/15	37	37	45	4
BB	▶ Robinson, Tiffany	5	◒	◒	485Q	02/22/15	9	20	40	1
BB	Williams, Anthony	5	◒	◒	EM	02/22/15	35	10	26	1

KEY

- **EM** Emerging Mathematician
- **A** ADVANCED
- **P** PROFICIENT
- **B** BASIC
- **BB** BELOW BASIC
- ◒ Student may need to develop this skill
- ● Student has acquired this skill
- ▶ Test taken in less than 15 minutes

YEAR-END PROFICIENCY RANGES		
GRADE K 10Q–175Q	**GRADE 5 820Q–1020Q**	GRADE 9 1140Q–1325Q
GRADE 1 260Q–450Q	GRADE 6 870Q–1125Q	GRADE 10 1220Q–1375Q
GRADE 2 405Q–600Q	GRADE 7 950Q–1175Q	GRADE 11 1350Q–1425Q
GRADE 3 625Q–850Q	GRADE 8 1030Q–1255Q	GRADE 12 139Q–1505Q
GRADE 4 715Q–950Q		

USING THE DATA	
Purpose: This report provides instructional recommendations for students at each SMI performance level.	***Follow-Up:*** Use instructional recommendations to plan appropriate support for students at each level.

Printed by: Teacher — Page 1 of 5 — Printed on: 2/22/2015

Aligning SMI College & Career Results With Classroom Instruction

To support teachers in the classroom in connecting the SMI College & Career results with classroom instructional practices, the QSCs associated with each of the 12 SMI College & Career content grade levels are presented in Appendix 1. (This Appendix is also available online at www.scholastic.com/SMI.) This information can be used to match instruction with student Quantile measures to provide focused intervention to support whole-class instruction.

Educators can consult the Performance Level Growth Report or the Student Progress Report and identify the Quantile measure of the Quantile Skill and Concept (QSC) to be taught to determine the likelihood that all students in the class will have the prerequisite skills necessary for instruction on the topic. This information can be used to determine how much scaffolding and support each student will need.

Development of SMI College & Career

Development of SMI College & Career

Scholastic Math Inventory College & Career was developed to assess a student's readiness for mathematics instruction and is based on the Quantile Framework for Mathematics. It is a computer-adaptive assessment and individualizes for each student. SMI College & Career is designed for students from Kindergarten through Algebra II (or High School Integrated Math III), which is commonly considered an indicator of college and career readiness. The content covered ranges from skills typically taught in Kindergarten through content introduced in high school.

Specifications of the SMI College & Career Item Bank

The specifications for the SMI College & Career item bank were defined through an iterative process of developing specifications, reviewing the specifications in relation to national curricular frameworks, and then revising the specifications to better reflect the design principles of SMI College & Career. The specifications were developed by curricular, instructional, and assessment specialists of Scholastic and MetaMetrics.

The SMI College & Career item specifications defined the items to be developed in terms of the strand covered, the QSC assessed, and the targeted grade level. In addition, several other characteristics of the items, such as context, reading demand, ethnicity, and gender were also considered to create a diverse item bank.

The SMI College & Career item bank specifications adhered to a strand variation that changed for different grade level bands. Following the philosophy of the Common Core State Standards (CCSS) for Mathematics, the greatest percentages of items in Kindergarten through Grade 5 assess topics in the Number & Operations strand. At Grade 6, the emphasis shifts to the Algebraic Thinking, Patterns, and Proportional Reasoning strand and the Expressions & Equations, Algebra, and Functions strand. Table 11 presents the design specifications for the SMI College & Career item bank.

TABLE 11. Designed strand profile for SMI: Kindergarten through Grade 11 (Algebra II).

	Number & Operations	Algebraic Thinking, Patterns, and Proportional Reasoning	Geometry, Measurement, and Data	Statistics & Probability	Expressions & Equations, Algebra, and Functions
Kindergarten	55%	25%	20%	–	–
Grade 1	50%	30%	20%	–	–
Grade 2	45%	30%	25%	–	–
Grade 3	40%	35%	25%	–	–
Grade 4	45%	35%	20%	–	–
Grade 5	65%	15%	20%	–	–
Grade 6	15%	35%	10%	10%	30%
Grade 7	15%	30%	10%	10%	35%
Grade 8	10%	5%	13%	7%	65%
Grade 9	5%	5%	10%	15%	65%
Grade 10	5%	5%	40%	10%	40%
Grade 11	5%	5%	20%	10%	60%

The QSCs previously listed for an SMI College & Career content grade level were compared with the Common Core State Standards, which have been aligned with the Quantile Framework (alignment available at www.Quantiles.com). Each standard was aligned with the appropriate QSC(s). There were several QSCs that spanned more than one grade level of the CCSS. This resulted in additions and deletions to the list of QSCs associated with each SMI College & Career content grade level.

Finally, the QSCs associated with each of the SMI College & Career grade level item banks were reviewed by SRI International for alignment with the CCSS for Mathematics. Where necessary, QSCs were reviewed and added or deleted. The QSCs associated with each of the 12 SMI College & Career content grade levels are presented in Appendix 1.

Within a content grade level of the SMI College & Career item bank, QSCs were weighted according to the amount of classroom time or importance that a particular topic or skill is typically given in that grade based on the alignment with the CCSS. A weight of 1 (small amount of time/minor topic), 2 (moderate amount of time/important topic), or 3 (large amount of time/critical topic) was assigned to each QSC within a grade by a subject matter expert (SME) and reviewed by another SME. Within a grade, QSCs with a weight of 1 included fewer items than QSCs designated with a weight of 3.

In addition to the mathematical content of the QSC, other features were considered in the SMI College & Career item bank specifications as well. The item bank was designed so a range of items within a single QSC and grade would be administered. Specifically this required the use of both computational problems as well as context/applied/word/story problems. An emphasis was placed on having more computational items in comparison to the number of context/applied/word problems. This emphasis was designed to minimize the importance of reading level and other factors that might influence performance on the assessment, so that only mathematical achievement is measured. Calculator availability was also determined by QSC. Some QSCs allow students the use of an online or personal calculator, while for other QSCs the online calculator is not available or should not be used.

SMI College & Career Item Development

The SMI College & Career item bank is comprised of four-option, multiple-choice items. It is a familiar item type, versatile and efficient in testing all levels of achievement, and most useful in computer-based testing (Downing, 2006). When properly written, this item type directly assesses specific student understandings for a particular objective. That is, every item was written to assess one QSC and one standard only.

The item consists of a question (stem) and four options (responses). All the information required to answer a question is contained in the stem and any associated graphic(s). Most stems are phrased "positively," but a few items use a "negative" (e.g., the use of the word *not*) format. The number of negative items is minimal particularly in the lower grades. When used, the word *not* is placed in bold/italics to emphasize its presence to the student. Stems also incorporate several other formats, such as word problems; incomplete number sentences; solving equations; and reading or interpreting figures, graphs, charts, and tables. Word problems require a student to read a short context before answering a question. The reading demand is intentionally kept lower than the grade of the item to assess the mathematical knowledge of the student and not his or her reading skills. All figures, graphs, charts, and tables include titles and other descriptive information as appropriate.

Each item contains four responses (A, B, C, or D). Three of the responses are considered foils or distractors, and one, and only one, response is the correct or best answer. Items were written so that the foils represented typical errors, misconceptions, or miscalculations. Item writers were encouraged to write foils based on their own classroom experiences and/or common error patterns documented in texts such as R.B. Ashlock's book *Error Patterns in Computation* (2010). Item writers were required to write rationales for each distractor.

In keeping with the findings and recommendation of the National Mathematics Advisory Panel, items were developed with minimal "nonmathematical sources of influence on student performance" (2008, p. xxv). Unnecessary context was avoided where possible and anything that could be considered culturally or economically biased was removed.

Item writers for SMI College & Career were classroom teachers and other educators who had experience with the everyday mathematics achievement of students at various levels and national mathematics curricular standards. Using individuals with classroom teaching experience helped to ensure that the items were valid measures of mathematics achievement. This ensured that items included not only the appropriate content but also considered the appropriate grade level and typical errors used to develop plausible distractors.

The development of the SMI item bank consisted of four phases. Phase 1 occurred in Fall and Winter 2008, Phase 2 occurred in Fall 2011, Phase 3 occurred in Fall and Winter 2013, and Phase 4 occurred in the Summer of 2015.

Phase 1: Twenty-six individuals from nine different states developed items for Phase 1 of the SMI College & Career item bank. The mean number of years of teaching experience for the item writers was 15.3 years. Over 60% of the writers had a master's degree, and all but one was currently certified in teaching. Approximately 70% of the writers listed their current job title as "teacher," and the other item writers were either curriculum specialists, administrators, or retired. One writer was a professional item writer. The majority of the item writers were Caucasian females, but 25% of the writers were male and 14% of the item writers described themselves as African American, Asian, or Hispanic.

Phase 2: Four individuals developed items for Phase 2 of the SMI College & Career item bank. These individuals were curriculum specialists at MetaMetrics with expertise in elementary school (1), middle school (2), and high school (1). The number of years of classroom experience ranged from 2 to 30 years, and the number of years as a MetaMetrics' curriculum specialist ranged from 1 to 8 years. The four individuals had experience in developing multiple mathematics assessments.

Phase 3: Eleven individuals developed items for Phase 3 of the SMI College & Career item bank. The mean number of years of teaching experience for the item writers was 15.9 years. Over 45% of the writers had a master's degree, and all but one was currently certified in teaching. Approximately 37% of the writers listed their current job title as "teacher," and the other item writers were either curriculum specialists, administrators, or retired. One writer was a professional item writer. The majority of the item writers were Caucasian females, but 27% of the writers were male.

Phase 4: All items that were added to SMI passed through several editorial rounds of review that were conducted by an internal team of content experts. After being reviewed and edited in-house, the items were assigned QSCs and Quantile measures by MetaMetrics Inc. in order to align to the Quantile Framework. All items were then reviewed by an external team of teachers and content experts, who evaluated whether the content of the items and the language used were appropriate for the targeted grade levels.

In addition, most new items being introduced into SMI have been field tested with small samples of SMI students. This item pilot helped identify items that were more or less difficult than anticipated, with those items identified either being removed or modified depending upon the results. All new items were also reviewed by teams of teachers who evaluated whether the content of the items and the language used were appropriate for the targeted grade level.

In all phases of item development, item writers were required to participate in a training that focused on guidelines for writing SMI College & Career multiple-choice items and an introduction to the Quantile Framework. In addition, each item writer was provided copies of the following:

- Webinar presentation (i.e., guidelines for item development)
- Mathematical Framework (NAEP, 2007)
- Calculator literacy information
- *Standards for Evaluating Instructional Materials for Social Content* (California Department of Education, 2000)
- Universal Design Checklist (Pearson Educational Measurement)
- List of names by gender and ethnicity identities as developed by Scholastic Inc.

Scholastic specified that item context represent a diverse population of students. In particular, if an item used a student name, then there should be equal representation of males and females. Scholastic also provided guidelines and specific names such that the names used in items would reflect ethnic and cultural diversity. The list of names provided represented approximately 30% African American names, 30% Hispanic names, 25% European (not Hispanic) names, 5% Asian names, and 10% Native American or other names.

Item writers were also given extensive training related to sensitivity issues. Part of the item writing materials addressed these issues and identified areas to avoid when writing items. The following areas were covered: violence and crime, depressing situations/death, offensive language, drugs/alcohol/tobacco, sex/attraction, race/ethnicity, class, gender, religion, supernatural/magic, parent/family, politics, topics that are location specific, and brand names or junk food. These materials were developed based on standards published by CTB/McGraw-Hill for universal design and fair access—equal treatment of the sexes, fair representation of minority groups, and the fair representation of disabled individuals (CTB/McGraw Hill, 1983).

Item writers were initially asked to develop and submit 10 items. The items were then reviewed for content alignment to the SMI College & Career curricular framework (QSC and, in the case of Phase 3 and 4 item development, the CCSS), item format, grammar, and sensitivity. Based on this review, item writers received feedback. Most item writers were then able to start writing assignments, but a few were required to submit additional items for acceptance before an assignment was made.

All items were subjected to a multistep review process. First, items were reviewed by curriculum experts and edited according to the item writing guidelines, QSC content, grade appropriateness, and sensitivity guidelines. The content expert reviews also added detailed art specifications. Some items were determined to be incompatible with the QSC and, during Phase 3 item development, the Common Core State Standards. These items were deemed unsuitable and therefore rewritten. Whenever possible, items were edited and maintained in the item bank.

The next several steps in the item review process included a review of the items by a group of specialists representing various perspectives. Test developers and editors examined each item for sensitivity issues, CCSS alignment, QSC alignment, and grade match, as well as the quality of the response options. Upon the updating of all edits and art specifications, items were presented to other reviewers to "cold solve." That is, individuals who had not participated in the review process thus far read and answered each item. Their answers were checked with the correct answer denoted with the item. Any inconsistencies or suggested edits were reviewed and made when appropriate.

At this point in the process, items were then submitted to Scholastic for review. Scholastic then sent the items for external review to ensure that the item aligned with the QSC and also with the CCSS. Items were either "approved" or "returned" with comments and suggestions for strengthening the item. Returned items were edited, reviewed again, and then resubmitted unless the changes were extensive. Items with extensive changes were deleted and another item was submitted.

SMI College & Career Review of Existing Item Bank

As part of the item development process for SMI College & Career, all previously developed SMI items were reviewed. Each item was examined for its alignment with the QSC, alignment with the CCSS, grade appropriateness, mathematical terminology, and language. All items that were added to SMI passed through several editorial rounds of review that were conducted by an internal team of content experts. After being reviewed and edited in-house, the items were assigned QSCs and Quantile measures in order to align to the Quantile Framework. All items were subsequently reviewed by an external team of teachers and cognitive experts, who evaluated whether the content of the items and the language used were appropriate for the targeted grade levels.

In addition, most new items being introduced into SMI have been field tested with small samples of SMI students matched by grade level. This item field study helped identify items that were more or less difficult than anticipated, with those items identified either being removed or modified depending upon the results. All new items were also reviewed by teams of teachers who evaluated whether the content of the items and the language used were appropriate for the targeted grade level.

SMI College & Career Final Item Bank Specifications

The final SMI College & Career item bank has a total of over 5,000 items. Following this extensive review process of new and existing items, the item bank resulted in the final strand profiles shown in Table 12.

TABLE 12. Actual strand profile for SMI after item writing and review.

	Number & Operations	Algebraic Thinking, Patterns, and Proportional Reasoning	Geometry, Measurement, & Data	Statistics & Probability	Expressions & Equations, Algebra, and Functions
Kindergarten	55%	15%	30%	–	–
Grade 1	40%	40%	20%	–	–
Grade 2	31%	28%	41%	–	–
Grade 3	37%	33%	30%	–	–
Grade 4	49%	21%	30%	–	–
Grade 5	70%	12%	18%	–	–
Grade 6	20%	25%	8%	13%	34%
Grade 7	22%	24%	21%	19%	14%
Grade 8	5%	3%	41%	12%	39%
Grade 9	4%	10%	14%	11%	61%
Grade 10	5%	2%	57%	6%	30%
Grade 11	2%	13%	15%	8%	62%

SMI College & Career Computer-Adaptive Algorithm

School-wide tests are often administered at grade level to large groups of students in order to make decisions about students and schools. Consequently, since all students in a grade are administered the same test, each test must include a wide range of items to cover the needs of both low-achieving and high-achieving students. These wide-range tests are often unable to measure some students as precisely as a more focused assessment could.

To provide the most accurate measure of a student's mathematics developmental level, it is important to assess the student's current mathematical achievement. One method is to use as much background information as possible to target a specific test level for each student. This information can consist of the student's grade level and a teacher's judgment concerning the mathematical achievement of the student. This method requires the test administrator to administer multiple test forms during one test session, which can be cumbersome and may introduce test security problems.

With the widespread availability of computers in classrooms and schools, another, more efficient method is to administer a test tailored to each student—computer-adaptive testing (CAT). Computer-adaptive testing is conducted individually with the aid of a computer algorithm to select each item so that the greatest amount of information about the student's achievement is obtained before the next item is selected. SMI College & Career employs such a methodology for testing online.

Many benefits of computer-adaptive testing have been described in educational literature (Stone & Lunz, 1994; Wainer et al., 1990; Wang & Vispoel, 1998). Each test is tailored to the individual student and item selection is based on the student's achievement and responses to each question. The benefits also include the following:

- Increased efficiency—through reduced testing time and targeted testing
- Immediate scoring—a score can be reported as soon as the student finishes the test
- Control over item bank—because the test forms do not have to be physically developed, printed, shipped, administered, or scored, a broader range of forms can be used

In addition, studies conducted by Hardwicke and Yoes (1984) and Schinoff and Steed (1988) provide evidence that below-level students tend to prefer computer-adaptive tests because they do not discourage students by presenting a large number of items that are too hard for them (cited in Wainer, 1993).

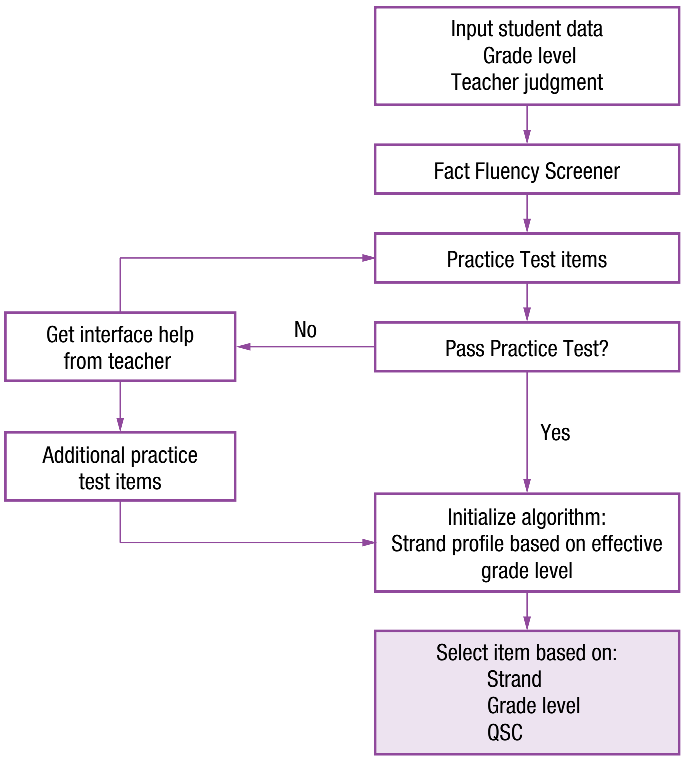
The computer-adaptive algorithm used with SMI College & Career is also based on the Rasch (one-parameter) item response theory (IRT) model. An extensive discussion of this model was provided earlier in this guide, in connection with the field test analyses used to develop the Quantile Framework itself. The same procedure used to determine the Quantile measure of the students in the field study is used to determine the Quantile measure of a student taking the SMI College & Career test. In short, the Rasch model uses an iterative procedure to calculate a student's score based on the differences between the desired score and the difficulty level of the items on the test and the performance of the student on each item. The Quantile measure, based on this convergence within the Rasch model, is recomputed with the addition of each new data point.

As described earlier, SMI College & Career uses a three-phase approach to assess a student's level of mathematical understanding—Start, Step, Stop. During test administration, the computer adapts the test continually according to the student's responses to the questions. The student starts the test; the test steps up or down according to the student's performance; and, when the computer has enough information about the student's mathematical achievement, the test stops.

The first phase, Start, determines the best point on the Quantile scale to begin testing the student. Figure 16 presents a flow chart of the Start phase of SMI College & Career. The algorithm requires several parameters before it can begin selecting items. One requirement is a Student Measure, an initial value for a student's understandings, which is used as a starting point. For a student who is taking SMI College & Career for the first time, the student's initial Quantile measure will be based on either the teacher's estimate of the student's understanding or the default value (the proficient level for the student's grade). If a student has previously taken SMI College & Career, the algorithm will use the Quantile measure from the last SMI administration.

Another required parameter is the effective grade level for each student. The effective grade level is typically the grade level entered into the SAM system. However, if a student is at either the extreme high or low end of the performance level, the algorithm will adjust the strand profile and the items to create an assessment that is better targeted to the student, at one grade level above or below.

FIGURE 16. The Start phase of the SMI College & Career computer-adaptive algorithm.



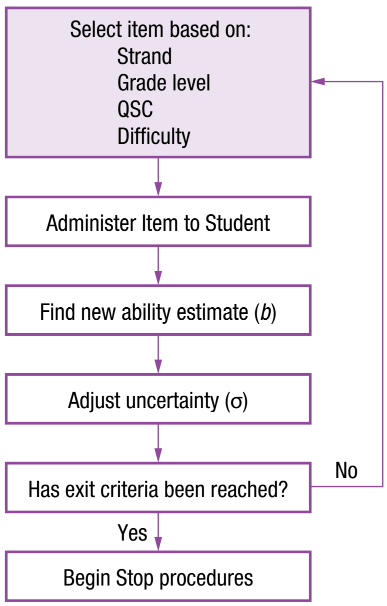
The second phase, Step, identifies which item a student will see. As shown in Figure 17, an item is selected according to specific criteria and administered to a student. A Student Measure, or estimate of student understanding, is then recalculated based on the student's response. The algorithm selects another item based on the student's performance on the previous item, as well as other criteria meant to ensure coverage across content strands.

Items are selected using several criteria: strand, grade level, QSC, and difficulty. The effective grade level (typically the actual grade level of the student) determines which strand profile and order is used throughout the test administration. Once a strand is determined, the algorithm will select an appropriate QSC within that strand based on a comparison of the Student Measure and the Skill Measure of the QSC. In other words, the algorithm searches to find an item that matches both the strand designation and the performance level of the student. The number of times a QSC is shown on a test is limited. In addition, the algorithm will screen items to prevent a student from seeing the same item during consecutive test administrations.

The strand profile, which ensures that students respond to items from each of the five content strands, varies slightly across grades. Using the SMI College & Career strand distribution described in Table 12, the first 13–14 items cover all strands proportionally and then the process is repeated. For example, in Grade 3, students are administered five items from the Number & Operations strand; five items from the Algebraic Thinking, Patterns & Proportional Reasoning strand; and three items from the Geometry, Measurement, and Data strand. In Grade 9, students are administered one item from each of the following strands: Numbers & Operations; Algebraic Thinking, Patterns & Proportional Reasoning; and Geometry, Measurement, and Data. Then, students are administered two items from the Statistics & Probability strand and nine items from the Expressions & Equations, Algebra, and Functions strand.

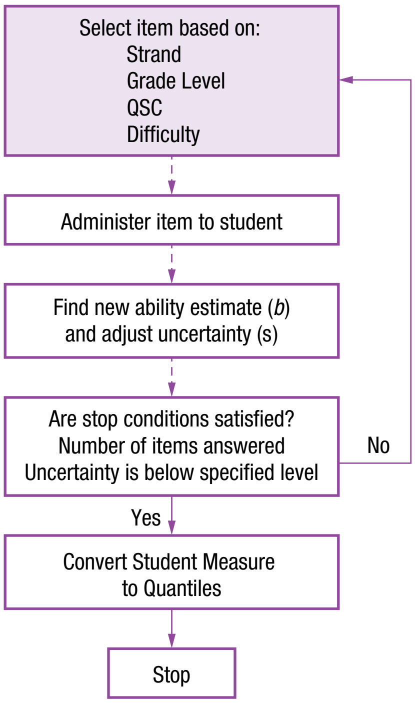
It is difficult to ascertain the ideal starting point for a student's subsequent testing experience. The final score from the previous administration provides a good initial reference point. However, it is also important that a student gain some measure of early success during any test administration—that is one of the reasons why many fixed form tests begin with relatively easier items. Scholastic has analyzed score decline data that indicated that students with higher starting Student Scores (Quantile measure) tend to underperform on the early items in the test. This pattern places greater stress on the algorithm's ability to converge on a student's "true" ability estimate. In addition, a lack of success at the beginning of the assessment can also lead to motivational issues in some students. A series of simulation studies conducted by Scholastic have shown that student score declines can be reduced significantly by adjusting the starting item Quantile measure. At the beginning of any SMI College & Career administration, the first item is presented at approximately 100Q below the student's last estimated achievement level. It is believed this early success will also set a positive tone for the remainder of the student's testing session.

FIGURE 17. The Step phase of the SMI College & Career computer-adaptive algorithm.



During the last phase, Stop, the SMI College & Career algorithm evaluates the exit criteria to determine if the algorithm should end and the results should be reported. Figure 18 presents a flow chart of the Stop phase of SMI College & Career. The program requires that students answer a minimum of 25 items, with 45 items being administered on the initial administration of SMI College & Career. On successive administrations of SMI College & Career, if the algorithm has enough information with 25 items to report a Student Measure with a small amount of uncertainty, then the program ends. If more information is needed to minimize the measurement error, then up to 20 more items are administered. Test reliability is influenced by many factors including the quality of the items, testing conditions, and the student taking the test. In addition, after controlling for these factors reliability can also be positively impacted by increasing the number of items on the test (Anastasi & Urbina, 1997). Scholastic has conducted a series of simulation studies varying test length and found that increasing the test length significantly decreases SEM. For most students the algorithm requires significantly less than 45 items to obtain an accurate estimate of a student's math achievement. However, for some students additional items may be needed in order to obtain a stable and accurate estimate of their achievement. Extending the potential number of items that a student might receive to 45 allows the algorithm a greater level of flexibility and improved accuracy for this group of students.

FIGURE 18. The Stop phase of the SMI College & Career computer-adaptive algorithm.



Reliability

Reliability

Reliability can be defined as "the degree to which test scores for a group of test takers are consistent over repeated applications of a measurement procedure and hence are inferred to be dependable and repeatable for an individual test taker" (Berkowitz, Wolkowitz, Fitch, & Kopriva, 2000). In reality, all test scores include some measure of error (or level of uncertainty). This measurement error is related to many factors, such as the statistical model used to compute the score, the items used to determine the score and the condition of the test taker when the test was administered.

Reliability is a major consideration in evaluating any assessment procedure. Four sources of measurement error should be examined for SMI College & Career:

(1) The proportion of test performance that is not due to error (marginal reliability)

(2) The consistency of test scores over time (alternate form/test-retest reliability)

(3) The error associated with a QSC Quantile measure, and

(4) The error associated with a student (standard error of measurement)

The first two sources of measurement error are typically used at the district level to describe the consistency and comparability of scores. These studies will be conducted during the 2014–2015 school year. The last two sources of measurement error are more associated with the interpretation and use of individual student results. By quantifying the measurement error associated with these sources, the reliability of the test results can also be quantified.

QSC Quantile Measure—Measurement Error

In a study of reading items, Stenner, Burdick, Sanford, and Burdick (2006) defined an ensemble to consist of all of the items that could be developed from a selected piece of text. This hierarchical theory (items and their use nested within the passage) is based on the notion of an ensemble as described by Einstein (1902) and Gibbs (1902). Stenner and his colleagues investigated the ensemble differences across items, and it was determined that the Lexile measure of a piece of text is equivalent to the mean difficulty of the items associated with the passage.

The Quantile Framework is an extension of this ensemble theory and defines the ensemble to consist of all of the items that could be developed from a selected QSC at an introductory level. Each item that could be developed for a QSC will have a slightly different level of difficulty from other items developed for the same QSC when tested with students. These differences in difficulty can be due to such things as the wording in the stem, the level of the foils, how diagnostic the foils are, the extent of graphics utilized in the item, etc. The Quantile measure of an item within SMI College & Career is the mean difficulty level of the QSC ensemble.

Error may also be introduced when a QSC included at a certain grade level is not covered, or not covered at the same grade level, in a particular state curriculum. Although the grade level objectives and expectations are very similar across state curriculums, there are a handful of discrepancies that result in the same QSC being introduced at different grade levels. For example, basic division facts are introduced in Grade 3 in some states while other state curriculums consider it a Grade 4 topic.

SMI College & Career Standard Error of Measurement

One source of uncertainty in SMI College & Career scores is related to the individual student. Replication theory describes the impact of retesting student performance using a different set of items (method) on a different occasion (moment). Method and moment are random facets and are expected to vary with each replication of the measurement process. Any calibrated set of items given on a particular day is considered interchangeable with any other set of items given on another day within a two-week period.

The interchangeability of the item sets suggest there is no a priori basis for believing that one particular method-moment combination will yield a higher or lower measure than any other. That is not to say that the resulting measures are expected to be the same. On the contrary, they are expected to be different. It is unknown which method-moment combination will in practice result in a more difficult testing situation. The anticipated variance among replications due to method-moment combinations and their interactions is one source of measurement error.

A better understanding of how error due to replication comes about can be gained by describing some of the behavior factors that may vary from administration to administration. Characteristics of the moment and context of measurement can contribute to variation in replicate measures. Suppose, unknown to the test developer, scores increase with each replication due to the student's familiarity with the items and the format of the test, and therefore the results may not be truly indicative of the student's progress. This "occasion main effect" would be treated as error.

The mental state of the student at the time the test is administered can also be a source of error. Suppose Jessica eats breakfast and rides the bus on Tuesdays and Thursdays, but on other days Jessica gets no breakfast and must walk one mile to school. Some of the test administrations occur on what Jessica calls her "good days" and some occur on her "bad days." Variation in her mathematics performance due to these context factors contributes to error. (For more information related to why scores change, see the paper entitled "Why Do Scores Change?" by Gary L. Williamson (2004), available at www.Lexile.com.)

The best approach to attaching uncertainty to a student's measure is to replicate the item response record (i.e., simulate what would happen if the reader were actually assessed again). Suppose eight-year-old José takes two 30-item SMI College & Career tests one week apart. The occasions (the two different days) and the 30 items nested within each occasion can be independently replicated (two-stage replication), and the resulting two measures averaged for each replicate. One thousand replications would result in a distribution of replicate measures. The standard deviation of this distribution is the replicated standard error measurement, and it describes uncertainty in measurement of José's mathematics understandings by treating methods (items), moment (occasion and context), and their interactions as error. Furthermore, in computing José's mathematics measure and the uncertainty in that measure, he is treated as an individual without reference to the performance of other students. This replication procedure allows psychometricians to estimate an individual's measurement error.

There is always some uncertainty associated with a student's score because of the measurement error associated with test unreliability. This uncertainty is known as the standard error of measurement (SEM). The magnitude of the SEM of an individual student's score depends on the following characteristics of the test (Hambleton et al., 1991):

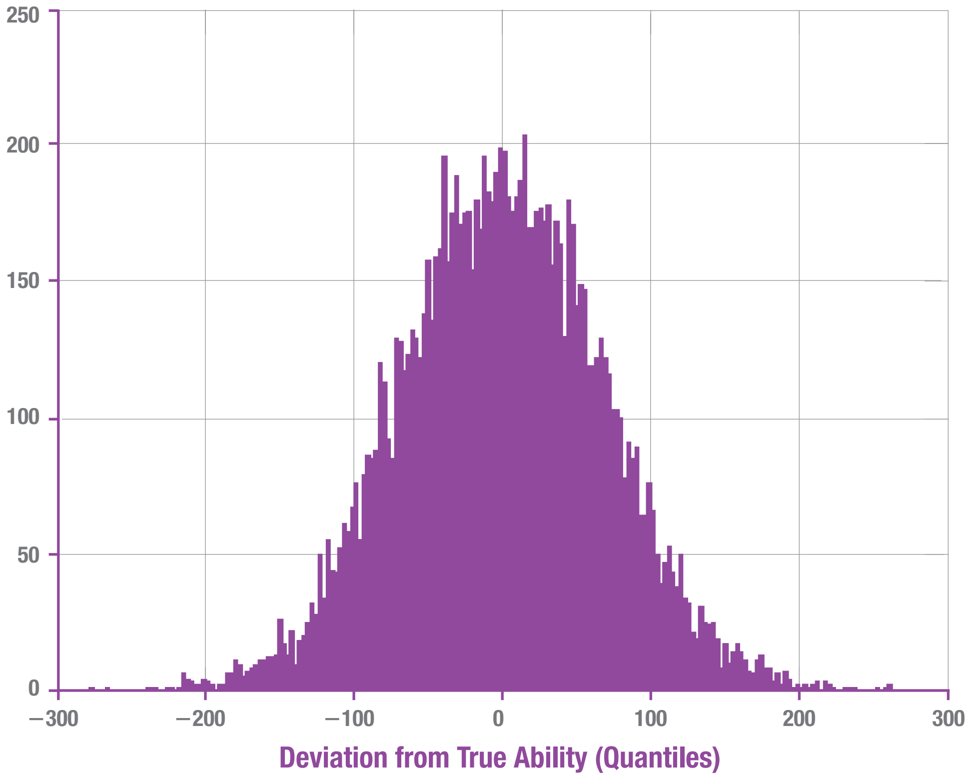
- The number of test items—smaller standard errors are associated with longer tests
- The quality of the test items—in general, smaller standard errors are associated with highly discriminating items for which correct answers cannot be obtained by guessing
- The match between item difficulty and student achievement—smaller standard errors are associated with tests composed of items with difficulties approximately equal to the achievement of the student (targeted tests)

SMI College & Career was developed using the Rasch one-parameter item response theory model to relate a student's ability to the difficulty of the items. There is a certain amount of measurement error due to model misspecification (violation of model assumptions) associated with each score on SMI College & Career. The computer algorithm that controls the administration of the assessment uses a Bayesian procedure to estimate each student's mathematical ability. This procedure uses prior information about students to control the selection of items and the recalculation of each student's understanding after responding to an item.

Compared to a fixed-form test where all students answer the same questions, a computer-adaptive test produces a different test for every student. When students take a computer-adaptive test, they all receive approximately the same raw score or number of items correct. This occurs because all students are answering questions that are targeted for their unique ability level—the questions are neither too easy nor too hard. Because each student takes a unique test, the error associated with any one score or student is also unique.

To examine the standard measurement error of SMI College & Career, a sample of four thousand Grade 5 students was simulated. Every student had the same true ability of 700Q, and each student's start ability was set in the range of 550Q to 850Q. The test length was set uniformly to 30 items, and no tests were allowed to end sooner.

FIGURE 19. Distribution of SEMs from simulations of student SMI College & Career scores, Grade 5.



From the simulated test results in Figure 19, it can be seen that most of the score errors were small. Using the results of the simulation, the initial standard error for an SMI College & Career score is estimated to be approximately 70Q. This means that on average, if a student takes the SMI College & Career three times, two out of three of the student's scores will be within 70 points of the student's true readiness for mathematics instruction.

Validity

Validity

The validity of a test is the degree to which the test actually measures what it purports to measure. Validity provides a direct check on how well the test fulfills its purpose. “The process of ascribing meaning to scores produced by a measurement procedure is generally recognized as the most important task in developing an educational or psychological measure, be it an achievement test, interest inventory, or personality scale” (Stenner, Smith, & Burdick, 1983). The appropriateness of any conclusion drawn from the results of a test is a function of the test’s validity. According to Kane (2006), “to validate a proposed interpretation or use of test scores is to evaluate the rationale for this interpretation or use” (p. 23).

Historically, validity has been categorized in three areas: content-description validity, criterion-prediction validity, and construct-identification validity. Although the current argument-based approach to validity (Kane, 2006) reflects principles inherent in all these areas, it is often convenient to organize discussions around the three areas separately. Initially, the primary source of validity evidence for SMI College & Career comes from the examination of the content and the degree to which the assessment could be said to measure mathematical understandings (construct-identification validity evidence). As more data are collected and more studies are completed, additional validity evidence can be described.

Content Validity

The content validity of a test refers to the adequacy with which relevant content has been sampled and represented in the test. The content validity of SMI College & Career is based on the alignment between the content of the items and the curricular framework used to develop SMI College & Career. Within SMI College & Career, each item was aligned with a specific QSC in the Quantile Framework and with a specific standard in the Common Core State Standards (CCSS) for Mathematics. The development of the Common Core State Standards Initiative (CCSSI) has established a clear set of K–12 standards that will enable all students to become increasingly more proficient in understanding and utilizing mathematics—with steady advancement to college and career readiness by high school graduation.

The mathematics standards stress both procedural skill and conceptual understanding to prepare students for the challenges of their postsecondary pursuits, not just to pass a test. They lay the groundwork for K–5 students to learn about whole numbers, operations, fractions, and decimals, all of which are required to learn more challenging concepts and procedures. The middle school standards build on the concepts and skills learned previously to provide logical preparation for high school mathematics. The high school standards then assemble the skills taught in the earlier grades challenging students to continue along productive learning progressions in order to develop more sophisticated mathematical thinking and innovative problem-solving methods. Students who master the prescribed mathematical skills and concepts through Grade 7 will be well prepared for algebra in Grade 8.

The mathematics standards outline eight practices that each student should develop in the early grades and then master as they progress through middle and high school:

1. Make sense of problems and persevere in solving.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The Quantile Framework places the mathematics curriculum, teaching materials, and students on a common, developmental scale, enabling educators to match students with instructional materials by readiness level, forecast their understanding, and monitor their progress. To see the alignment, visit www.scholastic.com/SMI or www.Quantiles.com.

Content validity was also built into SMI during its development. SMI was designed to measure readiness for mathematical instruction. To this end, the tests were constructed with content skills in mind. All items were written and reviewed by experienced classroom teachers to ensure that the content of the items was developmentally appropriate and representative of classroom experiences.

For more information on the content validity of SMI and the Quantile Framework, please refer to the other sections of this guide (section entitled "QSC Descriptions and Standards Alignment" in Appendix 1). SMI and the Quantile Framework are the result of rigorous research and development by a large team of educational experts, mathematicians, and assessment specialists.

Construct-Identification Validity

The construct-identification validity of a test is the extent to which the test may be said to measure a theoretical construct or trait, such as readiness for mathematics instruction. It is expected that scores from a valid test of mathematics skills should show expected:

1. Differences by age and/or grade
2. Differences among groups of students that traditionally show different or similar patterns of development in mathematics (e.g., differences in socioeconomic levels, gender, ethnicity, etc.)
3. Relationships with other measures of mathematical understanding

Construct-identification validity is the most important aspect of validity related to SMI College & Career. SMI College & Career is designed to measure the development of mathematical abilities; therefore, how well it measures mathematical understanding and how well it measures the development of these mathematical understandings must be examined.

Construct Validity From SMI Enterprise Edition

Evidence for the construct validity of SMI College & Career is provided by the body of research supporting SMI Enterprise Edition collected between 2009 and 2011. SMI College & Career employs many of the items developed for SMI and utilizes the same computer-adaptive testing algorithm and scoring and reporting protocols as were initially developed for SMI.

Information and results of the validity studies conducted in three phases can be found in Appendix 4. The following results were observed:

- Students classified as needing math intervention services scored significantly lower than students not classified as needing math intervention services.
- Students classified as Gifted and Talented scored significantly higher than students not classified as Gifted and Talented.
- Students classified as requiring Special Education services scored significantly lower than students not requiring Special Education services.
- Student scores on SMI rose rapidly in elementary grades and leveled off in middle school depending on the program being implemented (e.g., whole-class instruction versus remediation program). The developmental nature of mathematics was demonstrated in these results.
- Student scores on SMI exhibited moderate correlations with state assessments of mathematics. The within-grade correlations and the overall across-grades correlation (where appropriate) were moderate as expected given the different mode of administration between the two tests (fixed, constant form for all students within a grade on the state assessments, as compared to the SMI, which is a computer-adaptive assessment that is tailored to each student's level of achievement).
- Growth across a school year was constant across Grades 2–6 (approximately 0.6Q per day or approximately 108Q per year). With a small sample of students where data was collected over two years, a negative correlation was observed between the student's initial SMI Quantile measure and the amount grown over the two school years. This negative correlation is consistent with the interpretation that lower-performing students typically grow more than higher-performing students.
- For gender, there was no clear pattern in the differences in performance of males and females.

- For Race/Ethnicity, a significant difference was observed for most of the sites with the differences between the mean differences as expected.
- For bilingual status, while a significant difference was observed for one site, the level of significance was not strong and the differences were as expected with students not classified as bilingual scoring higher. For ELL, ESL, and LEP status, a significant difference due to language proficiency classification was observed for three of the sites, and the differences between the mean differences were as expected, with students classified as needing EL services scoring lower.
- For economically disadvantaged classification, a significant difference due to FRPL status was observed for one of the sites. The differences between the mean SMI Quantile measures were as expected with the "No" classification scoring higher.

Construct Validity From the Quantile Framework

Evidence for the construct validity of SMI College & Career is provided by the body of research supporting the Quantile Framework for Mathematics. The development of SMI College & Career utilized the Quantile Framework and the calibration of items specified to previously field-tested and analyzed items. Item writers for SMI College & Career were provided training on item development that matched the training used during the development of the Quantile item bank, and item reviewers had access to all items from the Quantile item bank. These items had been previously calibrated to the Quantile scale to ensure that items developed for SMI were theoretically consistent with other items calibrated to the Quantile scale, and that they maintained their individual item calibrations.

Prior research has shown that test scores derived from items calibrated from the Quantile field study are highly correlated with other assessments of mathematics achievement. The section in this technical report entitled "The Theoretical Framework of Mathematics Achievement and the Quantile Framework for Mathematics" provides a detailed description of the framework and the construct validity of the framework. The section also includes evidence to support the fact that tests based upon the framework can accurately measure mathematics achievement.

Conclusion

The *Scholastic Math Inventory* and its reports of students' readiness for mathematics instruction can be a powerful tool for educators. However, it is imperative to remain cognizant of the fact that no one test should be the sole determinant when making high-stakes decisions about students (e.g., summer school placement or retention). The student's background experiences, the curriculum in the prior grade or course, the textbook used, as well as direct observation of each student's mathematical achievement are all factors to take into consideration when making these kinds of decisions.

References

References

American Educational Research Association, American Psychological Association, & National Council on Measurement in Education (1999). *Standards for educational and psychological testing*. Washington, DC: American Psychological Association.

Anastasi, A. (1982). *Psychological testing* (fifth edition). New York, NY: Macmillan Publishing Company, Inc.

Anastasi, A., & Urbina, S. (1997). *Psychological testing* (seventh edition), Upper Saddle River, NJ: Prentice Hall.

Ashlock, R.B. (2010). *Error patterns in computation: Using error patterns to help each student learn* (tenth edition). Boston, MA: Allyn & Bacon.

Berkowitz, D., Wolkowitz, B., Fitch, R., & Kopriva, R. (2000). The use of tests as part of high-stakes decision-making for students: A resources guide for educators and policy-makers. Washington, DC: US Department of Education. [Available online: http://www.ed.gov/offices/OCR/testing/]

Bond, T.G., & Fox, C.M. (2001). *Applying the Rasch model: Fundamental measurement in the human sciences*. Mahwah, NJ: Lawrence Erlbaum Associates.

Burg, S.S. (2007). *An investigation of dimensionality across grade levels and effects on vertical linking for elementary grade mathematics achievement tests*. University of North Carolina, Chapel Hill.

California Department of Education (CDE). (1997, December). *Mathematics content standards for California public schools: Kindergarten through grade twelve.* Sacramento, CA: Author.

California Department of Education (CDE). (2000). *Mathematics framework for California public schools: Kindergarten through grade twelve* (Revised Edition). Sacramento, CA: Author.

California Department of Education (CDE). (2000). *Standards for evaluating instructional materials for social content*. Sacramento, CA: Author.

California Department of Education (CDE). (2002, January). *Sample items for the California Mathematics Standards Tests.* Sacramento, CA: Author.

California Department of Education (CDE). (2002, October). *Blueprints document for the Star Program California Standards Tests: Mathematics.* Sacramento, CA: Author.

Camilli, G., & Shepard, L.A. (1994). *Methods for identifying biased test items*. Thousand Oaks, CA: Sage Publications, Inc.

Cochran, W., & Cox, G. (1957). *Experimental designs* (second edition). New York, NY: John Wiley & Sons, Inc.

Cortiella, C. (2005). *A parent's guide to response-to-intervention [Parent Advocacy Brief]*. Retrieved April 22, 2009, from http://www.ncld.org/images/stories/downloads/parent_center/rti_final.pdf

CTB/McGraw-Hill. (1983). *Guidelines for bias-free publishing*. Monterey, CA: Author.

Dorans, N.J., & Holland, P.W. (1993). DIF detection and description: Mantel-Haenszel and standardization. In P.W. Holland and H. Wainer (Eds.), *Differential item functioning* (pp. 35–66). Hillsdale, NJ: Lawrence Erlbaum.

Downing, S.M. (2006). Selected-response item formats in test development. In S.M. Downing & T.M. Haladyna (Eds.), *Handbook of test development*. Mahwah, NJ: Lawrence Erlbaum Associates.

Einstein, A. (1902). Ann. d. Phys., IV. *Folge, 9*, 417.

Florida Department of Education (FL DOE). (2007). *Sunshine State Standards: Mathematics.* Tallahassee, FL: Author.

Gibbs, J.W. (1902). *Elementary principles in statistical mechanics.* New Haven, CT: Yale University Press.

Green, B.F., Bock, R.D., Humphreys, L.G., Linn, R.L., & Reckase, M.D. (1984). Technical guidelines for assessing computer adaptive tests. *Journal of Educational Measurement*, *21*(4), 347–360.

Haladyna, T.M. (1994). *Developing and validating multiple-choice test items*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Hambleton, R.K., & Swaminathan, H. (1985). *Item response theory: Principles and applications*. Boston, MA: Kluwer-Nijhoff.

Hambleton, R.K., Swaminathan, H., & Rogers, H.J. (1991). *Fundamentals of item response theory*. Newbury Park, CA: Sage Publications, Inc.

Hardwicke, S.B., & Yoes, M.E. (1984). *Attitudes and performance on computerized adaptive testing*. San Diego, CA: Rehab Group.

Illinois State Board of Education (ISBE). (2002). *Mathematics performance descriptors, grades 1-5 and grades 6-12.* Springfield, IL: Author.

Kane, M.T. (2006). Validation. In R.L. Brennan (Ed.), *Educational measurement* (fourth edition, pp. 17–64). Westport, CT: Praeger.

Linacre, J.M. (2010). A user's guide to Winsteps ministep Rasch-model computer programs [Computer software and manual]. Chicago: Winsteps.

Messick, S. (1993). Validity. In R.L. Linn (Ed.), *Educational measurement* (third edition, pp. 13–104). Washington, DC: American Council on Education.

MetaMetrics, Inc. (2005). *PASeries* Mathematics technical manual. Iowa City, IA: Pearson Educational Measurement.

MetaMetrics, Inc. (2011). *Lexile/Quantile Feasibility Study: A study to link NAEP Reading with The Lexile® Framework for Reading and NAEP Mathematics with The Quantile® Framework for Mathematics.* Durham, NC: Author.

MetaMetrics, Inc. (2012a). *Linking the ACT® Mathematics with the Quantile® Framework: A study to link the ACT® Mathematics Test administered in North Carolina with The Quantile® Framework for* Mathematics. Durham, NC: Author.

MetaMetrics, Inc. (2012b). *Linking the K-PREP Math Tests with the Quantile® Framework: A study to link the Kentucky Performance Rating for Educational Progress Math Test with The Quantile® Framework for Mathematics.* Durham, NC: Author.

MetaMetrics, Inc. (2012c). *Linking the Mathematics SOL Tests with the Quantile® Framework: A study to link the Virginia Mathematics Standards of Learning Tests with The Quantile® Framework for Mathematics.* Durham, NC: Author.

MetaMetrics, Inc. (2013). *Linking the NC READY EOG Math/EOC Algebra I/Integrated I with the Quantile® Framework: A study to link the North Carolina READY EOG Math/EOC Algebra I/Integrated I with The Quantile® Framework for Mathematics.* Durham, NC: Author.

National Assessment Governing Board (2005). *Mathematics framework for the 2005 National Assessment of Educational Progress* (prepublication edition). Washington, DC: Author.

National Assessment Governing Board (NAGB). (2007, May). *National Assessment of Educational Progress: Mathematics framework for 2009* (Pre-publication Edition). Washington, DC: National Assessment Governing Board and U.S. Department of Education.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics (2011). *Administrator's guide: Interpreting the Common Core State Standards to improve mathematics education*. Reston, VA: Author.

National Governors Association (NGA) and Council of Chief State School Officers (CCSSO). (2010a, June). Common Core State Standards Initiative. Retrieved from http://www.corestandards.org/the-standards/

National Governors Association Center for Best Practices (NGA Center) & the Council of Chief State School Officers (CCSSO). (2010b). *Common Core State Standards for mathematics: Appendix A.* Retrieved from http://www.corestandards.org/assets/CCSSI_Mathematics_Appendix_A.pdf

National Mathematics Advisory Panel (2008). *The final report of the National Mathematics Advisory Panel*: US Department of Education.

North Carolina Department of Public Instruction (NCDPI). (1996). *North Carolina Standard Course of Study -- Mathematics*. Raleigh, NC: Author.

Pearson Educational Measurement. (2003). Universal design checklist.

Petersen, N.S., Kolen, M.J., & Hoover, H.D. (1993). Scaling, norming, and equating. In R.L. Linn (Ed.), *Educational measurement* (third edition). Phoenix, AZ: Oryx Press.

Rasch, G. (1980). *Probabilistic models for some intelligence and attachment tests*. Chicago, IL: The University of Chicago Press.

Roussos, L., Schnipke, D., & Pashley, P. (1999). A generalized formula for the Mantel-Haenszel differential item functioning parameter. *Journal of Behavioral and Educational Statistics, 24*, 293–322.

Salvia, J., & Ysseldyke, J.E. (1998). *Assessment* (seventh edition). Boston, MA: Houghton Mifflin Company.

SAS Institute, Inc. (1985). The FREQ procedure. In *SAS users guide: Statistics, version 5 edition*. Cary, NC: Author.

Schinoff, R.B., & Steed, L. (1988). The CAT program at Miami-Dade Community College. In D. Doucette (Ed.), *Computerized adaptive testing: The state of the art in assessment at three community colleges* (pp. 25–36). Laguna Hills, CA: League for Innovation in the Community College.

Stenner, A.J. (1990). Objectivity: Specific and general. *Rasch Measurement Transactions, 4*(111).

Stenner, A.J., Burdick, H., Sanford, E.E., & Burdick, D.S. (2006). How accurate are Lexile text measures? *Journal of Applied Measurement, 7*(3), 307–322.

Stenner, A.J., Smith, M.C., & Burdick, D.S. (1983). Toward a theory of construct definition. *Journal of Educational Measurement, 20*(4), 305–315.

Stone, G.E., & Lunz, M.E. (1994). The effect of review on the psychometric characteristics of computerized adaptive tests. *Applied Measurement in Education, 7*, 211–222.

Texas Education Agency (TEA). (2002). *Texas Essential Knowledge and Skills for mathematics.* Austin, TX: Author.

Tomlinson, C.A., (2001). How to differentiate instruction in mixed-ability classrooms, second edition, *Association for Supervision and Curriculum Development.*

US Department of Education, Office of Elementary and Secondary Education. (2009). GUIDANCE The American Recovery and Reinvestment Act of 2009 (ARRA): Using Title I, Part A ARRA funds for grants to local educational agencies to strengthen education, drive reform, and improve results for students. Retrieved May 14, 2014, from http://www2.ed.gov/policy/gen/leg/recovery/guidance/titlei-reform.pdf

Wainer, H. (1993). Some practical considerations when converting a linearly administered test to an adaptive format. *Educational Researchers*, *12*(1), 15–20.

Wainer, H., Dorans, N.J., Flaugher, R., Green, B.F., Mislevy, R.J., Steinburg, L., & Thissen, D. (1990). *Computerized adaptive testing: A primer*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Wang, T., & Vispoel, W.P. (1998). Properties of ability estimation methods in computerized adaptive testing. *Journal of Educational Measurement*, *35*, 109–135.

Williamson, G.L. (2004). Why do scores change? Durham, NC: MetaMetrics, Inc.

Williamson, G.L. (2006). What is expected growth? Durham, NC: MetaMetrics, Inc.

Wright, B.D., & Linacre, J.M. (1994). The Rasch model as a foundation for the Lexile Framework. Durham, NC: MetaMetrics, Inc.

Wright, B.D., & Stone, M.H. (1979). *Best Test Design*. Chicago: MESA Press.

Appendices

Appendix 1: QSC Descriptions and Standards Alignment

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
EM260	Model the concept of addition for sums to 10.	Number & Operations	K.OA.1, K.OA.2, K.OA.4	36
EM210	Read and write numerals using one-to-one correspondence to match sets of 0 to 10.	Number & Operations	K.CC.3, K.CC.4.a, K.CC.4.b, K.CC.5	4
EM200	Use directional and positional words.	Geometry, Measurement & Data	K.G.1	15
EM180	Describe likenesses and differences between and among objects.	Geometry, Measurement & Data	K.G.4	16
EM150	Create and identify sets with greater than, less than, or equal number of members by matching.	Number & Operations	K.CC.6	7
EM150	Describe, compare, and order objects using mathematical vocabulary.	Geometry, Measurement & Data	K.G.3, K.MD.1, K.MD.2, 1.MD.1, 1.MD.2	14
EM150	Know and use addition and subtraction facts to 10 and understand the meaning of equality.	Algebraic Thinking, Patterns & Proportional Reasoning	K.OA.3, K.OA.4, K.OA.5, 1.OA.7	41
EM150	Measure length using nonstandard units.	Geometry, Measurement & Data	K.MD.1, 1.MD.1, 1.MD.2	581
EM130	Recognize the context in which addition or subtraction is appropriate, and write number sentences to solve number or word problems.	Algebraic Thinking, Patterns & Proportional Reasoning	K.OA.1, K.OA.2, 1.OA.1	39
EM110	Organize, display, and interpret information in concrete or picture graphs.	Geometry, Measurement & Data	K.MD.3	20
EM110	Identify missing addends for addition facts.	Algebraic Thinking, Patterns & Proportional Reasoning	K.OA.4, 1.OA.1	75
EM100	Read and write numerals using one-to-one correspondence to match sets of 11 to 100.	Number & Operations	K.CC.3, K.CC.4.a, K.CC.4.b, K.CC.5, 1.NBT.1	25
EM100	Identify, draw, and name basic shapes such as triangles, squares, rectangles, hexagons, and circles.	Geometry, Measurement & Data	K.G.1, K.G.2, K.G.3, K.G.5, 2.G.1	536

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
EM100	Tell time to the nearest hour and half-hour using digital and analog clocks.	Geometry, Measurement & Data	1.MD.3	1005
EM90	Group objects by 2s, 5s, and 10s in order to count.	Number & Operations	1.OA.5	30
EM80	Rote count beginning at 1 or at another number by 1s, and rote count by 2s, 5s, and 10s to 100 beginning at 2, 5, or 10.	Number & Operations	K.CC.1, K.CC.2, 1.NBT.1	24
EM80	Add 3 single-digit numbers in number and word problems.	Algebraic Thinking, Patterns & Proportional Reasoning	1.OA.2	76
EM80	Identify and name spheres and cubes.	Geometry, Measurement & Data	K.G.1, K.G.2, K.G.3	537
EM80	Know and use related addition and subtraction facts.	Algebraic Thinking, Patterns & Proportional Reasoning	1.OA.4	1003
EM60	Rote count 101 to 1,000.	Number & Operations	1.NBT.1	65
EM60	Use models to determine properties of basic solid figures (slide, stack, and roll).	Geometry, Measurement & Data	K.G.4	627
EM50	Sort a set of objects in one or more ways; explain.	Geometry, Measurement & Data	K.MD.3	54
EM50	Read and write word names for whole numbers from 101 to 999.	Number & Operations	2.NBT.3	68
EM40	Use addition and subtraction facts to 20.	Number & Operations	1.OA.1, 1.OA.6, 2.OA.2	78
EM40	Use counting strategies to add and subtract within 100 that include counting forward, counting backward, grouping, ten frames, and hundred charts.	Algebraic Thinking, Patterns & Proportional Reasoning	1.NBT.4, 1.NBT.6, 1.OA.2, 1.OA.5, 2.OA.1	617
EM20	Model the concept of subtraction using numbers less than or equal to 10.	Number & Operations	K.OA.1, K.OA.2	37
EM20	Identify and make figures with line symmetry.	Geometry, Measurement & Data	4.G.3	85
EM10	Represent numbers up to 100 in a variety of ways such as tallies, ten frames, and other models.	Number & Operations	K.CC.5, K.NBT.1, 1.NBT.2.a, 1.NBT.2.b	33
EM10	Organize, display, and interpret information in picture graphs and bar graphs using grids.	Geometry, Measurement & Data	1.MD.4, 2.MD.10	61

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
EM10	Add 2- and 3-digit numbers with and without models for number and word problems that do not require regrouping.	Number & Operations	1.NBT.4, 1.NBT.5, 2.MD.5, 2.MD.6, 2.NBT.5, 2.NBT.6, 2.NBT.7, 2.NBT.8, 3.NBT.2, 3.MD.1, 3.MD.2	79
10	Use place value with ones and tens.	Number & Operations	K.NBT.1, 1.NBT.2.a, 1.NBT.2.b, 1.NBT.2.c, 1.NBT.3, 1.NBT.4, 1.NBT.6	35
10	Use relationships between minutes, hours, days, weeks, months, and years to describe time. Recognize the meaning of a.m. and p.m. for the time of day.	Geometry, Measurement & Data	2.MD.7, 3.MD.1	618
10	Compare and order sets and numerals up to 20, including using symbol notation (>, <, =).	Number & Operations	K.CC.4.c, K.CC.7, K.MD.3	1001
20	Measure weight using nonstandard units.	Geometry, Measurement & Data	K.MD.1	582
20	Measure capacity using nonstandard units.	Geometry, Measurement & Data	K.MD.1	583
20	Find the unknown in an addition or subtraction number sentence.	Algebraic Thinking, Patterns & Proportional Reasoning	1.OA.8	1004
30	Use place value with hundreds.	Number & Operations	2.NBT.1.a, 2.NBT.1.b, 2.NBT.3, 2.NBT.4, 2.NBT.5, 2.NBT.6, 2.NBT.7, 3.NBT.1, 3.NBT.2	71
40	Combine two- and three- dimensional simple figures to create a composite figure.	Geometry, Measurement & Data	K.G.6, 1.G.2	542
50	Compare and order sets and numerals from 21 to 100, including using symbol notation (>, <, =).	Number & Operations	1.NBT.3	26

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
50	Use models and appropriate vocabulary to determine properties of basic plane figures (open or closed, number of sides and vertices or corners).	Geometry, Measurement & Data	K.G.4, 1.G.1	1002
60	Identify odd and even numbers using objects.	Algebraic Thinking, Patterns & Proportional Reasoning	2.OA.3	70
70	Answer comparative and quantitative questions about charts and graphs.	Geometry, Measurement & Data	1.MD.4, 2.MD.10	59
70	Determine the value of sets of coins.	Geometry, Measurement & Data	2.MD.8	105
70	Subtract 2- and 3-digit numbers with and without models for number and word problems that do not require regrouping.	Number & Operations	1.NBT.5, 1.NBT.6, 2.MD.4, 2.MD.5, 2.MD.6, 2.NBT.5, 2.NBT.7, 2.NBT.8, 3.NBT.2, 3.MD.1, 3.MD.2	599
80	Represent a number in a variety of numerical ways.	Algebraic Thinking, Patterns & Proportional Reasoning	K.NBT.1, K.OA.3, K.OA.4, K.CC.3, 1.OA.2, 1.OA.6, 2.OA.1	663
90	Add 2- and 3-digit numbers with and without models for number and word problems that require regrouping.	Number & Operations	1.NBT.4, 2.MD.5, 2.MD.6, 2.NBT.5, 2.NBT.6, 2.NBT.7, 3.NBT.2, 3.MD.1, 3.MD.2	598
100	Model the division of sets or the partition of figures into two, three, or four equal parts (fair shares).	Number & Operations	1.G.3, 2.G.3	38
100	Measure lengths in inches/centimeters using appropriate tools and units.	Geometry, Measurement & Data	3.MD.4	99
110	Identify odd and even numbers.	Algebraic Thinking, Patterns & Proportional Reasoning	2.OA.3	113
110	Skip count by 2s, 5s, and 10s beginning at any number.	Number & Operations	2.NBT.2	1007

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
120	Indicate the value of each digit in any 2- or 3-digit number.	Number & Operations	2.NBT.1.b, 2.NBT.3	73
140	Relate standard and expanded notation to 3- and 4-digit numbers.	Number & Operations	2.NBT.3, 2.NBT.4	110
150	Find the value of an unknown in a number sentence.	Algebraic Thinking, Patterns & Proportional Reasoning	2.MD.5, 2.OA.1, 3.OA.3, 3.OA.4, 3.OA.6, 4.OA.2, 4.OA.3	549
160	Identify and name basic solid figures: rectangular prism, cylinder, pyramid, and cone; identify in the environment.	Geometry, Measurement & Data	K.G.1, K.G.2	46
160	Identify or generate numerical and geometric patterns; correct errors in patterns or interpret pattern features.	Algebraic Thinking, Patterns & Proportional Reasoning	4.OA.5	91
160	Read and write word names for numbers from 1,000 to 9,999.	Number & Operations	4.NBT.2	109
180	Use multiplication facts through 144.	Algebraic Thinking, Patterns & Proportional Reasoning	3.OA.3, 3.OA.4, 3.OA.7	121
190	Represent fractions concretely and symbolically, including representing whole numbers as fractions.	Number & Operations	3.NF.1, 3.NF.2.a, 3.NF.2.b, 3.NF.3.c, 3.G.2	114
200	Organize, display, and interpret information in line plots and tally charts.	Geometry, Measurement & Data	1.MD.4, 2.MD.9	60
210	Compare and order numbers less than 10,000.	Number & Operations	2.NBT.4	111
210	Tell time at the five-minute intervals.	Geometry, Measurement & Data	2.MD.7	541
210	Estimate, measure, and compare capacity using appropriate tools and units in number and word problems.	Geometry, Measurement & Data	3.MD.2, 4.MD.2	650

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
220	Use the commutative and associative properties to add or multiply numerical expressions.	Algebraic Thinking, Patterns & Proportional Reasoning	1.NBT.4, 1.NBT.6, 1.OA.3, 2.NBT.5, 2.NBT.6, 2.NBT.7, 3.OA.5, 3.OA.9, 3.NBT.2, 3.NBT.3, 4.NBT.5, 4.NF.3.c, 5.MD.5.a, 5.NBT.7	161
240	Model multiplication in a variety of ways including grouping objects, repeated addition, rectangular arrays, skip counting, and area models.	Algebraic Thinking, Patterns & Proportional Reasoning	2.G.2, 2.OA.4, 3.OA.1, 3.OA.3, 4.NBT.5	118
240	Estimate, measure, and compare length using appropriate tools and units in number and word problems.	Geometry, Measurement & Data	2.MD.1, 2.MD.2, 2.MD.3, 2.MD.4, 2.MD.5, 2.MD.9, 4.MD.2	649
250	Recognize the 2-dimensional elements of 3-dimensional figures.	Geometry, Measurement & Data	K.G.3	52
250	Identify, draw, and name shapes such as quadrilaterals, trapezoids, parallelograms, rhombi, and pentagons.	Geometry, Measurement & Data	2.G.1	83
250	Locate points on a number line.	Number & Operations	2.MD.6, 3.NF.2a, 3.NF.2.b, 3.NF.3a, 3.MD.1	97
250	Subtract 2- and 3-digit numbers with and without models for number and word problems that require regrouping.	Number & Operations	2.MD.4, 2.MD.5, 2.MD.6, 2.NBT.5, 2.NBT.7, 3.NBT.2, 3.MD.1, 3.MD.2	117
270	Tell time to the nearest minute using digital and analog clocks.	Geometry, Measurement & Data	3.MD.1	1011

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
280	Use the identity properties for addition and multiplication and the zero property for multiplication.	Algebraic Thinking, Patterns & Proportional Reasoning	1.NBT.4, 1.NBT.6, 2.NBT.5, 2.NBT.6, 2.NBT.7, 3.NBT.2, 3.NBT.3, 3.OA.5, 3.OA.9	119
300	Make different sets of coins with equivalent values.	Geometry, Measurement & Data	2.MD.8	106
300	Compare fractions with the same numerator or denominator concretely and symbolically.	Number & Operations	3.NF.3.d	538
300	Write an addition or a subtraction sentence that represents a number or word problem; solve.	Algebraic Thinking, Patterns & Proportional Reasoning	2.MD.5, 2.OA.1	544
300	Multiply a 1-digit number by a 2-digit multiple of 10.	Number & Operations	3.NBT.3	1010
310	Understand that many whole numbers factor in different ways.	Algebraic Thinking, Patterns & Proportional Reasoning	4.OA.4	163
320	Model division in a variety of ways including sharing equally, repeated subtraction, rectangular arrays, and the relationship with multiplication.	Algebraic Thinking, Patterns & Proportional Reasoning	3.OA.2, 3.OA.3, 3.OA.7, 4.NBT.6, 5.NBT.6, 5.NBT.7, 5.NF.3	120
320	Identify combinations of fractions that make one whole.	Number & Operations	2.G.3, 3.G.2, 4.NF.3.a	540
330	Compare decimals (tenths and hundredths) with and without models.	Number & Operations	4.NF.7	156
330	Multiply a multidigit whole number by a 1-digit whole number or a 2-digit multiple of 10.	Number & Operations	4.NBT.5	165
350	Know and use division facts related to multiplication facts through 144.	Algebraic Thinking, Patterns & Proportional Reasoning	3.OA3, 3.OA.4, 3.OA.6, 3.OA.7	162
360	Describe and demonstrate patterns in skip counting and multiplication; continue sequences beyond memorized or modeled numbers.	Algebraic Thinking, Patterns & Proportional Reasoning	3.OA.9	129
360	Estimate, measure, and compare weight using appropriate tools and units in number and word problems.	Geometry, Measurement & Data	3.MD.2, 4.MD.2	651

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
360	Write an addition and subtraction sentence that represents a two-step word problem; solve.	Algebraic Thinking, Patterns & Proportional Reasoning	2.OA.1	1006
370	Locate a point in Quadrant I of a coordinate grid given an ordered pair; name the ordered pair for a point in Quadrant I of a coordinate grid.	Geometry, Measurement & Data	5.G.1, 5.G.2, 5.OA.3, 6.RP.3.a	138
390	Organize, display, and interpret information in tables and graphs (frequency tables, pictographs, and line plots).	Geometry, Measurement & Data	3.MD.3	137
390	Write a multiplication or a division sentence that represents a number or word problem; solve.	Algebraic Thinking, Patterns & Proportional Reasoning	3.OA.3, 4.OA.1, 4.OA.2	607
390	Write a ratio or rate to compare two quantities.	Algebraic Thinking, Patterns & Proportional Reasoning	6.RP.1, 6.RP.2	654
400	Determine perimeter using concrete models, nonstandard units, and standard units in number and word problems.	Geometry, Measurement & Data	3.MD.8	146
400	Identify and draw intersecting, parallel, skew, and perpendicular lines and line segments. Identify midpoints of line segments.	Geometry, Measurement & Data	4.G.1	176
400	Model the concept of percent and relate to the value in decimal or fractional form.	Algebraic Thinking, Patterns & Proportional Reasoning	6.RP.3.c	626
400	Write number sentences using any combination of the four operations that represent a two-step word problem; solve.	Algebraic Thinking, Patterns & Proportional Reasoning	3.OA.8, 3.MD.3, 3.MD.8	1008
400	Use models to represent a fraction as a product of a whole number and a unit fraction in number and word problems.	Number & Operations	4.NF.4.a, 4.NF.4.b, 4.NF.4.c	1017
410	Round whole numbers to a given place value.	Number & Operations	3.NBT.1, 4.NBT.3	660
420	Apply appropriate type of estimation for sums and differences.	Number & Operations	3.OA.8, 4.OA.3	153
440	Describe the probability of an chance event using a fraction or ratio.	Statistics & Probability	7.SP.5, 7.SP.8.a	185
450	Use benchmark numbers (zero, one-half, one) and models to compare and order fractions.	Number & Operations	3.NF.3.d, 4.NF.2	115
450	Divide using single-digit divisors with and without remainders.	Number & Operations	4.NBT.6	166

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
450	Use manipulatives, pictorial representations, and appropriate vocabulary (e.g., polygon, side, angle, vertex, diameter) to identify and compare properties of plane figures.	Geometry, Measurement & Data	2.G.1, 3.G.1	174
450	Determine the area of rectangles, squares, and composite figures using nonstandard units, grids, and standard units in number and word problems.	Geometry, Measurement & Data	3.MD.5.a, 3.MD.5.b, 3.MD.6, 3.MD.7.b, 3.MD.7.d, 3.MD.8, 3.G.2, 4.MD.3	192
450	Use models to develop the relationship between the total distance around a figure and the formula for perimeter; find perimeter using the formula in number and word problems.	Geometry, Measurement & Data	4.MD.3	1018
460	Determine the value of sets of coins and bills using cent sign and dollar sign appropriately. Create equivalent amounts with different coins and bills.	Geometry, Measurement & Data	2.MD.8, 4.MD.2	147
460	Read, write, and compare whole numbers from 10,000 to less than one million using standard and expanded notation.	Number & Operations	4.NBT.2	152
470	Describe data using the mode.	Statistics & Probability	6.SP.2	135
470	Estimate and compute the cost of items greater than $1.00; make change.	Geometry, Measurement & Data	2.MD.8	148
470	Rewrite and compare decimals to fractions (tenths and hundredths) with and without models and pictures.	Number & Operations	4.NF.6	157
470	Use concepts of positive numbers, negative numbers, and zero (e.g., on a number line, in counting, in temperature, in "owing") to describe quantities in number and word problems.	Number & Operations	6.NS.5, 6.NS.6.a	169
470	Read and write word names for rational numbers in decimal form to the hundredths place or the thousandths place.	Number & Operations	5.NBT.3.a	648
470	Apply appropriate types of estimation for number and word problems that include estimating products and quotients.	Algebraic Thinking, Patterns & Proportional Reasoning	3.OA.8, 4.OA.3	1009
470	Indicate and compare the place value of each digit in a multidigit whole number or decimal.	Number & Operations	4.NBT.1, 5.NBT.1	1014
470	Add multidigit numbers with regrouping in number and word problems.	Number & Operations	4.NBT.4	1015
480	Organize, display, and interpret information in bar graphs.	Geometry, Measurement & Data	3.MD.3	134

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
480	Organize, display, and interpret information in graphs containing scales that represent multiple units.	Geometry, Measurement & Data	3.MD.3	136
480	Use a coordinate grid to solve number and word problems. Describe the path between given points on the plane.	Geometry, Measurement & Data	5.G.2	547
480	Model the concept of the volume of a solid figure using cubic units.	Geometry, Measurement & Data	5.MD.3.a, 5.MD.3.b, 5.MD.4, 5.MD.5.a, 6.G.2	630
480	Use addition and subtraction to find unknown measures of nonoverlapping angles.	Geometry, Measurement & Data	4.MD.7, 7.G.5	1019
490	Subtract multidigit numbers with regrouping in number and word problems.	Number & Operations	4.NBT.4	1016
500	Use order of operations including parentheses and other grouping symbols to simplify numerical expressions.	Algebraic Thinking, Patterns & Proportional Reasoning	5.OA.1	167
520	Organize, display, and interpret information in line plots with a horizontal scale in fractional units.	Geometry, Measurement & Data	3.MD.4, 4.MD.4, 5.MD.2, S.ID.1	1012
530	Estimate and compute products of whole numbers with multidigit factors.	Number & Operations	4.NBT.5, 5.NBT.5	170
530	Use manipulatives, pictorial representations, and appropriate vocabulary (e.g., face, edge, vertex, and base) to identify and compare properties of solid figures.	Geometry, Measurement & Data	2.G.1	175
530	Identify and draw angles (acute, right, obtuse, and straight).	Geometry, Measurement & Data	4.G.1	202
530	Use reasoning with equivalent ratios to solve number and word problems.	Algebraic Thinking, Patterns & Proportional Reasoning	6.RP.3.d	551
550	Graph or identify simple inequalities using symbol notation $>$, $<$, $\leqslant$, $\geqslant$, and $\neq$ in number and word problems.	Expressions & Equations, Algebra, Functions	6.EE.8	604
560	Use grids to develop the relationship between the total numbers of square units in a rectangle and the length and width of the rectangle (l x w); find area using the formula in number and word problems.	Geometry, Measurement & Data	3.MD.7.a, 3.MD.7.c, 3.OA.5, 5.NF.4.b	191

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
560	Use the distributive property to represent and simplify numerical expressions.	Algebraic Thinking, Patterns & Proportional Reasoning	3.OA.5, 3.MD.7.c, 4.NBT.5, 4.NBT.6, 5.NBT.6, 5.NBT.7, 6.NS.4	578
560	Identify the nets for prisms, pyramids, cylinders, and cones in geometric and applied problems.	Geometry, Measurement & Data	6.G.4	645
580	Estimate and compute sums and differences with decimals.	Number & Operations	5.NBT.7, 6.NS.3, 7.EE.3	201
580	Identify the number of lines of symmetry in a figure and draw lines of symmetry.	Geometry, Measurement & Data	4.G.3	615
580	Solve multistep number and word problems using the four operations.	Number & Operations	4.OA.3	1013
590	Add and subtract decimals using models and pictures to explain the process and record the results.	Number & Operations	5.NBT.7	158
590	Construct or complete a table of values to solve problems associated with a given relationship.	Algebraic Thinking, Patterns & Proportional Reasoning	4.MD.1, 4.OA.5	180
590	Write equivalent fractions with smaller or larger denominators.	Number & Operations	4.NF.5, 5.NF.5.b	668
600	Find the fractional part of a whole number or fraction with and without models and pictures.	Number & Operations	5.NF.4.a, 5.NF.4.b, 5.NF.6.	160
600	Round decimals to a given place value; round fractions and mixed numbers to a whole number or a given fractional place value.	Number & Operations	5.NBT.4	164
600	Read, write, and compare numbers with decimal place values to the thousandths place or numbers greater than one million.	Number & Operations	5.NBT.3.a, 5.NBT.3.b	195
600	Use exponential notation and repeated multiplication to describe and simplify exponential expressions.	Expressions & Equations, Algebra, Functions	6.EE.1	220
600	Estimate products and quotients of decimals or of mixed numbers.	Expressions & Equations, Algebra, Functions	7.EE.3	669
610	Find multiples, common multiples, and the least common multiple of numbers; explain.	Algebraic Thinking, Patterns & Proportional Reasoning	4.OA.4, 6.NS.4	221
610	Distinguish between a population and a sample and draw conclusions about the sample (random or biased).	Statistics & Probability	7.SP.1, S.IC.1, S.IC.3	314

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
610	Model and identify mixed numbers and their equivalent fractions.	Number & Operations	4.NF.3.b, 4.NF.3.c, 5.NF.3	546
610	Identify and classify triangles according to the measures of the interior angles and the lengths of the sides; relate triangles based upon their hierarchical attributes.	Geometry, Measurement & Data	4.G.2, 5.G.3, 5.G.4	624
610	Estimate sums and differences with fractions and mixed numbers.	Number & Operations	5.NF.2, 7.EE.3	675
610	Recognize that a statistical question is one that will require gathering data that has variability.	Statistics & Probability	6.SP.1	1033
620	Identify the place value of each digit in a multidigit numeral to the thousandths place.	Number & Operations	5.NBT.1, 5.NBT.3a	154
620	Translate between models or verbal phrases and numerical expressions.	Algebraic Thinking, Patterns & Proportional Reasoning	5.OA.2	1020
620	Add and subtract fractions and mixed numbers using models and pictures to explain the process and record the results in number and word problems.	Number & Operations	4.NF.3.d, 4.NF.5, 5.NF.2	1023
630	Use models to write equivalent fractions, including using composition or decomposition or showing relationships among halves, fourths, and eighths, and thirds and sixths.	Number & Operations	3.NF.3.a, 3.NF.3.b, 4.NF.1	116
640	Calculate distances from scale drawings and maps.	Algebraic Thinking, Patterns & Proportional Reasoning	7.G.1	317
650	Solve one-step linear equations and inequalities and graph solutions of the inequalities on a number line in number and word problems.	Expressions & Equations, Algebra, Functions	6.EE.7	208
650	Recognize and use patterns in powers of ten (with or without exponents) to multiply and divide whole numbers and decimals.	Number & Operations	5.NBT.2	633
670	Add and subtract fractions and mixed numbers with like denominators (without regrouping) in number and word problems.	Number & Operations	4.MD.4, 4.NF.3.a, 4.NF.3.b, 4.NF.3.c, 4.NF.3.d	199
680	Identify, draw, and name: points, rays, line segments, lines, and planes.	Geometry, Measurement & Data	4.G.1, 4.MD.5.a, G.CO.1	173
680	Use models or points in the coordinate plane to illustrate, recognize, or describe rigid transformations (translations, reflections, and rotations) of plane figures.	Geometry, Measurement & Data	G.CO.2, G.CO.3	178

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
680	Name polygons by the number of sides. Distinguish quadrilaterals based on properties of their sides or angles; relate quadrilaterals based upon their hierarchical attributes.	Geometry, Measurement & Data	3.G.1, 4.G.2, 5.G.3, 5.G.4	620
690	Estimate and solve division problems with multidigit divisors; explain solution.	Number & Operations	5.NBT.6, 6.NS.2	171
690	Find factors, common factors, and the greatest common factor of numbers; explain.	Algebraic Thinking, Patterns & Proportional Reasoning	4.OA.4, 6.NS.4	222
690	Solve two-step linear equations and inequalities and graph solutions of the inequalities on a number line.	Expressions & Equations, Algebra, Functions	7.EE.4.a, 7.EE.4.b, 7.G.5	275
700	Use geometric models and equations to investigate the meaning of the square of a number and the relationship to its positive square root. Know perfect squares to 625.	Algebraic Thinking, Patterns & Proportional Reasoning	8.EE.2, N.RN.1	265
700	Multiply or divide two decimals or a decimal and a whole number in number and word problems.	Number & Operations	5.NBT.7, 6.NS.3	608
700	Determine the complement of an event.	Statistics & Probability	S.CP.1	646
710	Compare and order fractions using common numerators or denominators.	Number & Operations	4.NF.2	155
710	Convert fractions and terminating decimals to the thousandths place to equivalent forms without models; explain the equivalence.	Number & Operations	7.NS.2.d, 8.NS.1	196
710	Use remainders in problem-solving situations and interpret the remainder with respect to the original problem.	Number & Operations	4.OA.3	266
710	Represent division of a unit fraction by a whole number or a whole number by a unit fraction using models to explain the process in number and word problems.	Number & Operations	5.NF.7.a, 5.NF.7.b, 5.NF.5.c	1026
720	Read, write, or model numbers in expanded form using decimal fractions or exponents.	Number & Operations	5.NBT.3.a	226
720	Write a proportion to model a word problem; solve proportions.	Algebraic Thinking, Patterns & Proportional Reasoning	7.RP.3	263
720	Estimate the square root of a number between two consecutive integers with and without models. Use a calculator to estimate the square root of a number.	Number & Operations	8.NS.2	297
720	Describe a data set by its number of observations, what is being measured, and the units of measurement.	Statistics & Probability	6.SP.5.a, 6.SP.5.b	1035

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
720	Indicate the probability of a chance event with or without models as certain, impossible, more likely, less likely, or neither likely nor unlikely using benchmark probabilities of 0, 1/2, and 1.	Statistics & Probability	7.SP.5, 7.SP.7.b	1045
740	Identify prime and composite numbers less than 100.	Algebraic Thinking, Patterns & Proportional Reasoning	4.OA.4	223
750	Describe the effect of operations on size and order of numbers.	Algebraic Thinking, Patterns & Proportional Reasoning	5.OA.2, 5.NF.5.a, 5.NF.5.b	168
750	Identify and label the vertex, rays, and interior and exterior of an angle. Use appropriate naming conventions to identify angles.	Geometry, Measurement & Data	G.CO.1	203
750	Translate between models or verbal phrases and algebraic expressions.	Expressions & Equations, Algebra, Functions	6.EE.2.a, N.Q.2, A.SSE.1.a, A.SSE.1.b	218
750	Determine the sample space for an event using counting strategies (include tree diagrams, permutations, combinations, and the Fundamental Counting Principle).	Statistics & Probability	7.SP.8.a, 7.SP.8.b	251
750	Describe or compare the relationship between corresponding terms in two or more numerical patterns or tables of ratios.	Algebraic Thinking, Patterns & Proportional Reasoning	5.OA.3, 6.RP.3.a	1021
750	Multiply and divide decimals using models and pictures to explain the process and record the results.	Number & Operations	5.NBT.7	1022
770	Simplify numerical expressions that may contain exponents.	Expressions & Equations, Algebra, Functions	6.EE.2.c	236
770	Identify corresponding parts of similar and congruent figures.	Geometry, Measurement & Data	8.G.2, 8.G.4, 8.G.5	241
780	Analyze graphs, identify situations, or solve problems with varying rates of change.	Expressions & Equations, Algebra, Functions	8.F.5	209
780	Identify additive inverses (opposites) and multiplicative inverses (reciprocals, including zero) and use them to solve number and word problems.	Number & Operations	6.NS.6.a, 7.NS.1.a, 7.NS.1.b	623
780	Use geometric models and equations to investigate the meaning of the cube of a number and the relationship to its cube root.	Algebraic Thinking, Patterns & Proportional Reasoning	8.EE.2, N.RN.1	1048
790	Add and subtract fractions and mixed numbers with unlike denominators in number and word problems.	Number & Operations	4.NF.5, 5.MD.2, 5.NF.1, 5.NF.2	231
790	Compare and order integers with and without models.	Number & Operations	6.NS.7.b	235

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
790	Organize, display, and interpret information in histograms.	Statistics & Probability	6.SP.4, S.ID.1	278
800	Describe data using the median.	Geometry, Measurement & Data	6.SP.2, 6.SP.5.c	183
800	Given a list of ordered pairs in a table or graph, identify either verbally or algebraically the rule used to generate and record the results.	Expressions & Equations, Algebra, Functions	6.EE.9	244
800	Model or compute with integers using addition or subtraction in number and word problems.	Number & Operations	7.NS.1.b, 7.NS.1.c	261
800	Write a linear equation or inequality to represent a given number or word problem; solve.	Expressions & Equations, Algebra, Functions	6.EE.7, 7.EE.4.a, 7.EE.4.b, A.CED.1, A.CED.3	276
800	Organize, display, and interpret information in scatter plots. Approximate a trend line and identify the relationship as positive, negative, or no correlation.	Statistics & Probability	8.SP.1	311
800	Represent division of whole numbers as a fraction in number and word problems.	Number & Operations	5.NF.3	1024
800	Represent division of fractions and mixed numbers with and without models and pictures in number and word problems; describe the inverse relationship between multiplication and division.	Number & Operations	6.NS.1	1028
800	Identify parts of a numerical or algebraic expression.	Expressions & Equations, Algebra, Functions	6.EE.2.b	1031
800	Interpret probability models for data from simulations or for experimental data presented in tables and graphs (frequency tables, line plots, bar graphs).	Statistics & Probability	7. SP.7.a, 7.SP.7.b	1046
800	Identify and use appropriate scales and intervals in graphs and data displays.	Geometry, Measurement & Data	N.Q.1	1057
810	Write an equation to describe the algebraic relationship between two defined variables in number and word problems, including recognizing which variable is dependent.	Expressions & Equations, Algebra, Functions	6.EE.9, F.BF.1.a	210
810	Draw circles; identify and determine the relationships between the radius, diameter, chord, center, and circumference.	Geometry, Measurement & Data	G.CO.1	237
810	Model or compute with integers using multiplication or division in number and word problems.	Number & Operations	7.NS.2.a, 7.NS.2.b	262
810	Identify linear and nonlinear relationships in data sets.	Statistics & Probability	8.SP.1	572

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
810	Determine and interpret the components of algebraic expressions including terms, factors, variables, coefficients, constants, and parts of powers in number and word problems.	Algebraic Thinking, Patterns & Proportional Reasoning	A.SSE.1.a, A.SSE.1.b	1055
820	Multiply two fractions or a fraction and a whole number in number and word problems.	Number & Operations	5.NF.6, 5.MD.2	224
820	Convert measures of length, area, capacity, weight, and time expressed in a given unit to other units in the same measurement system in number and word problems.	Geometry, Measurement & Data	4.MD.1, 4.MD.2, 5.MD.1, 6.RP.3.d	258
820	Rewrite or simplify algebraic expressions including the use of the commutative, associative, and distributive properties, and inverses and identities in number and word problems.	Expressions & Equations, Algebra, Functions	6.EE.3, 6.EE.4, 7.EE.1, 7.EE.2	300
820	Locate, given the coordinates of, and graph points which are the results of rigid transformations in all quadrants of the coordinate plane; describe the path of the motion using geometric models or appropriate terms.	Geometry, Measurement & Data	6.NS.6.b, G.CO.2, G.CO.4	616
820	Solve number and word problems using percent proportion, percent equation, or ratios.	Algebraic Thinking, Patterns & Proportional Reasoning	6.RP.3.c, 7.RP.3	622
820	Determine the degree of a polynomial and indicate the coefficients, constants, and number of terms in the polynomial.	Expressions & Equations, Algebra, Functions	A.SSE.1.a	639
820	Use the commutative, associative, and distributive properties, and inverses and identities to solve number and word problems with rational numbers.	Algebraic Thinking, Patterns & Proportional Reasoning	7.NS.1.d, 7.NS.2.a, 7.NS.2.b, 7.NS.2.c	1039
830	Calculate unit rates in number and word problems, including comparison of units rates.	Algebraic Thinking, Patterns & Proportional Reasoning	6.RP.2, 6.RP.3.b	233
830	Determine the probability from experimental results or compare theoretical probabilities and experimental results.	Statistics & Probability	7.SP.6	249
830	Use the definition of rational numbers to convert decimals and fractions to equivalent forms.	Number & Operations	7.NS.2.d, 8.NS.1	1040
840	Compare and order rational numbers with and without models.	Number & Operations	6.NS.7.a, 6.NS.7.b	260
840	Evaluate algebraic expressions in number and word problems.	Expressions & Equations, Algebra, Functions	6.EE.2.c	274

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
840	Use models to find volume for prisms and cylinders as the product of the area of the base (B) and the height. Calculate the volume of prisms in number and word problems.	Geometry, Measurement & Data	5.MD.5.a, 5.MD.5.b, 6.G.2, 7.G.6, G.GMD.1	289
850	Describe data using the mean.	Statistics & Probability	6.SP.2, 6.SP.5.c	214
850	Draw and measure angles using a protractor. Understand that a circle measures 360 degrees.	Geometry, Measurement & Data	4.MD.5.a, 4.MD.5.b, 4.MD.6	217
850	Locate points in all quadrants of the coordinate plane using ordered pairs in number and word problems.	Statistics & Probability	6.NS.6.b, 6.NS.6.c, 6.NS.8, 6.G.3	247
850	Describe, use, and compare real numbers. Use the definition of rational numbers to derive and distinguish irrational numbers.	Number & Operations	8.NS.1, 8.NS.2	564
850	Identify relations as directly proportional, linear, or nonlinear using rules, tables, and graphs.	Algebraic Thinking, Patterns & Proportional Reasoning	7.RP.2.a, 8.F.3, 8.F.5	567
850	Use the discriminant to determine the number and nature of the roots of a quadratic equation.	Expressions & Equations, Algebra, Functions	A.REI.4.b	591
860	Make predictions based on theoretical probabilities or experimental results.	Statistics & Probability	7.SP.6	316
860	Identify from a set of numbers which values satisfy a given equation or inequality.	Expressions & Equations, Algebra, Functions	6.EE.5, 6.EE.6, 6.EE.8	1032
870	Divide two fractions or a fraction and a whole number in number or word problems.	Number & Operations	5.MD.2, 6.NS.1	230
870	Calculate or estimate the percent of a number including discounts, taxes, commissions, and simple interest.	Algebraic Thinking, Patterns & Proportional Reasoning	6.RP.3.c, 7.RP.3	264
870	Represent multiplication or division of mixed numbers with and without models and pictures.	Number & Operations	5.NF.6	1025
880	Describe cross-sectional views of three-dimensional figures.	Geometry, Measurement & Data	7.G.3, G.GMD.4	556
880	Calculate unit rates of ratios that include fractions to make comparisons in number and word problems.	Algebraic Thinking, Patterns & Proportional Reasoning	7.RP.1	1037
880	Construct and interpret a two-way table to display two categories of data from the same source.	Statistics & Probability	8.SP.4	1054
880	Use set notation to describe domains, ranges, and the intersection and union of sets. Identify cardinality of sets, equivalent sets, disjoint sets, complement, or subsets.	Expressions & Equations, Algebra, Functions	S.CP.1	1072

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
890	Write equations to represent direct variation and use direct variation to solve number and word problems.	Algebraic Thinking, Patterns & Proportional Reasoning	7.RP.2.c, A.CED.2	362
890	Perform multistep operations with rational numbers (positive and negative) in number and word problems.	Number & Operations	7.EE.3, 7.NS.1.b, 7.NS.1.c, 7.NS.1.d, 7.NS.2.a, 7.NS.3	642
890	Make predictions based on results from surveys and samples.	Statistics & Probability	7.SP.2	1043
900	Solve number and word problems involving percent increase and percent decrease.	Algebraic Thinking, Patterns & Proportional Reasoning	7.RP.3	295
900	Graphically solve systems of linear equations.	Expressions & Equations, Algebra, Functions	8.EE.8.a, 8.EE.8.b, 8.EE.8.c, A.REI.6	309
900	Use the distance formula to find the distance between two points. Use the midpoint formula to find the coordinates of the midpoint of a segment.	Geometry, Measurement & Data	G.CO.11, G.SRT.1.b, G.GPE.4, G.GPE.7	483
900	Use pictorial representations and appropriate vocabulary to identify relationships with circles (e.g., tangent, secant, concentric circles, inscribe, circumscribe, semicircles, and minor and major arcs) in number and word problems.	Geometry, Measurement & Data	G.C.2, G.C.3, G.MG.1	519
900	Determine the absolute value of a number with and without models in number and word problems.	Number & Operations	6.NS.7.c	636
900	Given a proportional relationship represented by tables, graphs, models, or algebraic or verbal descriptions, identify the unit rate (constant of proportionality).	Algebraic Thinking, Patterns & Proportional Reasoning	7.RP.2.b, 7.RP.2.d	1038
910	Write whole numbers in scientific notation; convert scientific notation to standard form; investigate the uses of scientific notation.	Expressions & Equations, Algebra, Functions	8.EE.3, 8.EE.4	259
910	Write a problem given a simple linear equation or inequality.	Expressions & Equations, Algebra, Functions	7.EE.4.a, 7.EE.4.b	277
910	Describe, extend, and analyze a wide variety of geometric and numerical patterns, such as Pascal's triangle or the Fibonacci sequence.	Algebraic Thinking, Patterns & Proportional Reasoning	A.APR.5	308
920	Determine the probability of compound events (with and without replacement).	Statistics & Probability	7.SP.8.a, 7.SP.8.c	285

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
920	Use proportions to express relationships between corresponding parts of similar figures.	Expressions & Equations, Algebra, Functions	8.EE.6	292
920	Determine the quartiles or interquartile range for a set of data.	Statistics & Probability	6.SP.3, 6.SP.5.c, S.ID.2	559
920	Multiply or divide with mixed numbers in number and word problems.	Number & Operations	5.NF.6	609
920	Determine the mean absolute deviation (MAD) for one or more sets of data. Describe the meaning of MAD for given data sets.	Statistics & Probability	6.SP.3, 6.SP.5.c	1000
920	Determine the volume of composite figures in number and word problems.	Geometry, Measurement & Data	5.MD.5.c	1027
920	Contrast statements about absolute values of integers with statements about integer order.	Number & Operations	6.NS.7.d	1030
920	Use frequency tables, dot plots, and other graphs to determine the shape, center, and spread of a data distribution.	Statistics & Probability	6.SP.2, 6.SP.3, 6.SP.4, 6.SP.5.a, 6.SP.5.b	1034
930	Generate a set of ordered pairs using a rule which is stated in verbal, algebraic, or table form; generate a sequence given a rule in verbal or algebraic form.	Algebraic Thinking, Patterns & Proportional Reasoning	5.OA.3, 6.RP.3.a	243
930	Investigate and determine the relationship between the diameter and the circumference of a circle and the value of pi; calculate the circumference of a circle.	Geometry, Measurement & Data	7.G.4	254
930	Use ordered pairs derived from tables, algebraic rules, or verbal descriptions to graph linear functions.	Expressions & Equations, Algebra, Functions	7.RP.2.a, 8.EE.5, 8.F.3, 8.F.4, A.REI.10, F.IF.7.a	562
940	Recognize and extend arithmetic sequences and geometric sequences. Identify the common difference or common ratio.	Algebraic Thinking, Patterns & Proportional Reasoning	F.BF.2, F.LE.1.a, F.LE.2	656
940	Determine a simulation, such as random numbers, spinners, and coin tosses, to model frequencies for compound events.	Statistics & Probability	7.SP.8.c, S.IC.2, S.MD.6	1047
950	Describe data using or selecting the appropriate measure of central tendency; choose a measure of central tendency based on the shape of the data distribution.	Statistics & Probability	6.SP.3, 6.SP.5.d	281

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
950	Solve linear equations using the associative, commutative, distributive, and equality properties and justify the steps used.	Expressions & Equations, Algebra, Functions	7.EE.4.a, 8.EE.7.a, 8.EE.7.b, A.CED.1, A.REI.1, A.REI.3	332
950	Use dimensional analysis to rename quantities or rates.	Geometry, Measurement & Data	N.Q.1, N.Q.2, G.MG.2	671
960	Organize, display, and interpret information in box-and-whisker plots.	Statistics & Probability	6.SP.4, S.ID.1	310
970	Approximate a linear model that best fits a set of data; use the linear model to make predictions.	Statistics & Probability	8.SP.2, 8.SP.3, S.ID.6.a, S.ID.6.c	565
970	Determine whether a linear equation has one solution, infinitely many solutions, or no solution.	Expressions & Equations, Algebra, Functions	8.EE.7.a	1049
970	Use appropriate units to model, solve, and estimate multistep word problems.	Geometry, Measurement & Data	N.Q.1, N.Q.2	1056
980	Model and solve linear inequalities using the properties of inequality in number and word problems.	Expressions & Equations, Algebra, Functions	7.EE.4.b, A.CED.1, A.REI.3	644
990	Determine and use scale factors to reduce and enlarge drawings on grids to produce dilations.	Geometry, Measurement & Data	7.G.1, 8.G.3, G.SRT.1.b	287
990	Determine precision unit, accuracy, and greatest possible error of a measuring tool. Apply significant digits in meaningful contexts.	Geometry, Measurement & Data	N.Q.3	322
990	Evaluate absolute value expressions.	Numbers & Operations	6.NS.8, 7.NS.1.c	323
990	Write and solve systems of linear equations in two or more variables algebraically in number and word problems.	Expressions & Equations, Algebra, Functions	8.EE.8.b, 8.EE.8.c, A.REI.6, A.CED.3	333
1000	Use rules of exponents to simplify numeric and algebraic expressions.	Expressions & Equations, Algebra, Functions	8.EE.1, A.SSE.1.b, A.SSE.2, A.SSE.3.c	296
1000	Estimate and calculate using numbers expressed in scientific notation.	Expressions & Equations, Algebra, Functions	8.EE.3, 8.EE.4	298
1000	Identify and interpret the intercepts of a linear relation in number and word problems.	Expressions & Equations, Algebra, Functions	8.F.4, 8.SP.3, F.IF.4, F.IF.7.a, S.ID.7	307
1000	Recognize and apply algebra techniques to solve rate problems including distance, work, density, and mixture problems.	Expressions & Equations, Algebra, Functions	F.LE.1.b, A.REI.3, G.MG.2	574

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
1000	Estimate and calculate areas with scale drawings and maps.	Algebraic Thinking, Patterns & Proportional Reasoning	7.G.1	585
1000	Solve a literal equation for an indicated variable.	Expressions & Equations, Algebra, Functions	A.CED.4, A.REI.3	659
1000	Recognize conditions of side lengths that determine a unique triangle, more than one triangle, or no triangle.	Geometry, Measurement & Data	7.G.2	1041
1000	Recognize closure of number systems under a collection of operations and their properties with and without models; extend closure to analogous algebraic systems.	Algebraic Thinking, Patterns & Proportional Reasoning	A.APR.1, N.RN.3, A.APR.7	1062
1010	Define and identify alternate interior, alternate exterior, corresponding, adjacent and vertical angles.	Geometry, Measurement & Data	7.G.5	240
1010	Use models to develop formulas for finding areas of triangles, parallelograms, trapezoids, and circles in number and word problems.	Geometry, Measurement & Data	6.G.1	256
1010	Use models to investigate the concept of the Pythagorean Theorem.	Geometry, Measurement & Data	8.G.6	271
1020	Define and identify complementary and supplementary angles.	Geometry, Measurement & Data	7.G.5	239
1020	Graph quadratic functions. Identify and interpret the intercepts, maximum, minimum, and the axis of symmetry.	Expressions & Equations, Algebra, Functions	F.IF.4, F.IF.7.a, A.CED.2	335
1020	Solve quadratic equations using properties of equality.	Expressions & Equations, Algebra, Functions	A.CED.1, A.REI.4.b, N.CN.7	374
1030	Locate, given the coordinates of, and graph plane figures which are the results of translations or reflections in all quadrants of the coordinate plane.	Geometry, Measurement & Data	8.G.1.a, 8.G.1.b, 8.G.1.c, 8.G.3, G.CO.4, G.CO.5	270
1040	Calculate the areas of triangles, parallelograms, trapezoids, circles, and composite figures in number and word problems.	Geometry, Measurement & Data	6.G.1, 7.G.4, 7.G.6	257
1040	Use nets or formulas to find the surface area of prisms, pyramids, and cylinders in number and word problems.	Geometry, Measurement & Data	6.G.4, 7.G.6	318
1040	Convert between different representations of relations and functions using tables, the coordinate plane, and algebraic or verbal statements.	Expressions & Equations, Algebra, Functions	A.REI.10, F.IF.4, F.LE.2, A.CED.2	366
1040	Use properties, definitions, and theorems of angles and lines to solve problems related to angle bisectors, segment bisectors, and perpendicular bisectors.	Geometry, Measurement & Data	G.CO.9	491

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
1040	Use properties, definitions, and theorems to determine the congruency or similarity of polygons in order to solve problems.	Geometry, Measurement & Data	G.CO.6, G.SRT.2	497
1040	Use inverse, combined, and joint variation to solve problems.	Expressions & Equations, Algebra, Functions	A.CED.2, A.CED.3	571
1040	Use the definition of a logarithm to convert between logarithmic and exponential forms; evaluate logarithmic expressions.	Expressions & Equations, Algebra, Functions	F.LE.4	1068
1050	Use the Pythagorean Theorem and its converse to solve number and word problems, including finding the distance between two points.	Geometry, Measurement & Data	8.G.7, 8.G.8, G.SRT.4, G.SRT.8, G.GPE.1, G.GPE.7	302
1050	Determine algebraically or graphically the solutions of a linear inequality in two variables.	Expressions & Equations, Algebra, Functions	A.REI.3, A.REI.12	306
1050	Add, subtract, and multiply polynomials.	Algebraic Thinking, Patterns & Proportional Reasoning	A.APR.1	325
1050	Evaluate expressions and use formulas to solve number and word problems involving exponential functions; classify exponential functions as exponential growth or decay.	Expressions & Equations, Algebra, Functions	A.SSE.3.c, A.CED.1, F.IF.8.b, F.LE.1.c	339
1050	Determine the effects of changes in slope and/or intercepts on graphs and equations of lines.	Expressions & Equations, Algebra, Functions	F.BF.1.b, F.BF.3	350
1050	Identify outliers and determine their effect on the mean, median, and range of a set of data.	Statistics & Probability	6.SP.3, 6.SP.5.c, 6.SP.5.d, S.ID.3	561
1050	Select the appropriate measure of variability; choose a measure of variability based on the presence of outliers, clusters, and the shape of the data distribution.	Statistics & Probability	6.SP.5.d	1036
1060	Use models to investigate the relationship of the volume of a cone to a cylinder and a pyramid to a prism with the same base and height.	Geometry, Measurement & Data	G.GMD.1, G.GMD.3	319
1070	Use a variety of triangles, quadrilaterals, and other polygons to draw conclusions about the sum of the measures of the interior angles.	Geometry, Measurement & Data	7.G.2, 8.G.5, G.CO.10	204
1070	Locate, given the coordinates of, and graph plane figures which are the results of rotations (multiples of 90 degrees) with respect to a given point.	Geometry, Measurement & Data	8.G.1.a, 8.G.1.b, 8.G.1.c, 8.G.3, G.CO.4, G.CO.5	303

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
1070	Calculate the volume of cylinders, pyramids, and cones in number and word problems.	Geometry, Measurement & Data	7.G.6, 8.G.9, G.GMD.3	320
1070	Use properties of triangles to solve problems related to altitudes, perpendicular bisectors, angle bisectors, and medians.	Geometry, Measurement & Data	G.CO.10	506
1070	Solve equations involving powers and roots by using inverse relationships.	Expressions & Equations, Algebra, Functions	A.REI.4.b	569
1070	Use combinations and permutations to determine the sample space of compound events.	Statistics & Probability	S.CP.9	588
1070	Make inferences about a population based on a sample and compare variation in multiple samples.	Statistics & Probability	7.SP.2	1042
1070	Verify how properties and relationships of geometric figures are maintained or how they change through transformations.	Geometry, Measurement & Data	8.G.1.a, 8.G.1.b, 8.G.1.c, 8.G.2, 8.G.3, G.CO.4, G.CO.6	1050
1070	Identify outliers and clusters in bivariate data in tables and scatter plots.	Statistics & Probability	8.SP.1	1053
1080	Write the equation of and graph linear relationships given the slope and *y*-intercept.	Expressions & Equations, Algebra, Functions	8.F.4, A.CED.2	345
1090	Find and interpret the maximum, the minimum, and the intercepts of a quadratic function.	Expressions & Equations, Algebra, Functions	F.IF.8.a, A.SSE.3.b, F.IF.4	375
1090	Find indicated terms, the common ratio, or the common difference using recursive sequence formulas; write recursive sequence formulas.	Expressions & Equations, Algebra, Functions	F.IF.3, F.BF.1.a, F.BF.2	464
1090	Describe or graph plane figures which are the results of a sequence of transformations.	Geometry, Measurement & Data	8.G.2, 8.G.4, G.CO.3, G.CO.5, G.CO.7, G.SRT.2, G.C.1	1051
1100	Derive a linear equation that models a set of data (line of best fit) using calculators. Use the model to make predictions.	Statistics & Probability	S.ID.6.a, S.ID.6.b	342
1100	Determine the measure of an angle in degree mode or in radian mode.	Geometry, Measurement & Data	G.C.5, F.TF.1, F.TF.2	424
1100	Compare data and distributions of data, numerical and contextual, to draw conclusions, considering the measures of center and measures of variability.	Statistics & Probability	7.SP.3, 7.SP.4, S.ID.2, S.ID.3	1044

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
1100	Calculate the surface area and volume of a sphere in number and word problems.	Geometry, Measurement & Data	8.G.9, G.GMD.3	1052
1100	Describe three-dimensional figures generated by rotations of plane figures in space.	Geometry, Measurement & Data	G.GMD.4	1064
1100	Express data in a two-way table. Calculate the marginal distribution, marginal and conditional probabilities, or basic probabilities.	Statistics & Probability	S.ID.5, S.CP.4, S.MD.7	1069
1110	Write the equation of and graph linear relationships given two points on the line.	Expressions & Equations, Algebra, Functions	F.LE.2, A.CED.2	347
1110	Use slopes to determine if two lines are parallel or perpendicular.	Geometry, Measurement & Data	G.GPE.4, G.GPE.5, G.CO.11, G.SRT.1.a	532
1120	Divide polynomials by monomial divisors.	Expressions & Equations, Algebra, Functions	A.APR.6	326
1120	Graph absolute value functions and their corresponding inequalities.	Expressions & Equations, Algebra, Functions	F.IF.4, F.IF.7.b	398
1120	Use properties, definitions, and theorems of quadrilaterals (parallelograms, rectangles, rhombi, squares, trapezoids, kites) to solve problems.	Geometry, Measurement & Data	G.CO.11	500
1120	Transform (translate, reflect, rotate, dilate) polygons in the coordinate plane; describe the transformation in simple algebraic terms.	Geometry, Measurement & Data	8.G.3, G.SRT.1.a, G.SRT.1.b, G.SRT.2	534
1120	Use properties, definitions, and theorems to solve problems about rigid transformations and dilations of plane figures.	Geometry, Measurement & Data	G.SRT.1.a, G.SRT.1.b, G.SRT.2, G.SRT.3	1063
1120	Find the coordinates of a point on a segment between given endpoints that partitions the segment by a given ratio.	Geometry, Measurement & Data	G.GPE.6	1065
1130	Factor quadratic polynomials, including special products.	Expressions & Equations, Algebra, Functions	A.SSE.2	327
1130	Write the equation of and graph linear relationships given the slope and one point on the line.	Expressions & Equations, Algebra, Functions	A.CED.2	346
1140	Find the slope of a line given two points on a line, a table of values, the graph of the line, or an equation of the line in number and word problems.	Expressions & Equations, Algebra, Functions	8.EE.6, 8.F.4, F.IF.6	343
1140	Describe the slope of a line given in the context of a problem situation; compare rates of change in linear relationships represented in different ways.	Expressions & Equations, Algebra, Functions	8.EE.5, 8.F.2, 8.F.4, 8.F.5, 8.SP.3, F.IF.4, F.LE.1.b, S.ID.7	344

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
1140	Solve quadratic equations by graphing.	Expressions & Equations, Algebra, Functions	A.CED.1, A.REI.11	370
1140	Use properties of circles to solve number and word problems involving arcs formed by central angles or inscribed angles.	Geometry, Measurement & Data	G.C.5, G.MG.1	523
1150	Use properties of right triangles to solve problems using the relationships in special right triangles.	Geometry, Measurement & Data	G.SRT.4	514
1150	Use measures of arcs or central angles to find arc length or sector area of a circle.	Geometry, Measurement & Data	G.C.5, F.TF.1	529
1150	Interpret and compare properties of linear functions, graphs, and equations.	Expressions & Equations, Algebra, Functions	8.F.2, 8.F.5, F.IF.4, F.IF.7.a	568
1150	Solve systems of linear inequalities.	Expressions & Equations, Algebra, Functions	A.REI.12	674
1160	Use properties, definitions, and theorems of angles and lines to solve problems related to adjacent, vertical, complementary, supplementary, and linear pairs of angles.	Geometry, Measurement & Data	G.CO.9	489
1160	Determine whether a system of equations has one solution, multiple solutions, infinitely many solutions, or no solution using graphs, tables, and algebraic methods; compare solutions of systems of equations.	Expressions & Equations, Algebra, Functions	A.REI.5, A.REI.6, G.GPE.5	1058
1170	Use properties of triangles to solve problems related to similar triangles and the relationships of their corresponding parts.	Geometry, Measurement & Data	G.CO.10, G.SRT.2, G.SRT.3, G.SRT.4, G.SRT.5, G.SRT.6	503
1170	Use properties of triangles to solve problems related to isosceles and equilateral triangles.	Geometry, Measurement & Data	G.CO.10, G.SRT.4	505
1170	Describe and simplify imaginary and complex numbers.	Number & Operations	N.CN.1, A.REI.4.b	595
1180	Divide one polynomial by another of a lower degree using either synthetic division or the division algorithm.	Expressions & Equations, Algebra, Functions	A.APR.6	358
1180	Use and interpret function notation in number and word problems; determine a value of the function given an element of the domain.	Expressions & Equations, Algebra, Functions	F.IF.1, F.IF.2, F.IF.5	593
1180	Add, subtract, multiply, and divide functions.	Expressions & Equations, Algebra, Functions	F.BF.1.b	594
1190	Define and distinguish between relations and functions, dependent and independent variables, and domain and range; identify whether relations are functions numerically and graphically.	Expressions & Equations, Algebra, Functions	8.F.1, F.IF.1	330

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
1190	Use properties of triangles to solve problems related to congruent triangles and their corresponding parts.	Geometry, Measurement & Data	G.CO.8, G.CO.10, G.SRT.5	504
1190	Given a specific interval, find the average rate of change of a function using a table, graph, or algebraic description.	Expressions & Equations, Algebra, Functions	F.IF.6	1060
1200	Identify and interpret zeros of a quadratic function using factoring in algebraic and word problems.	Expressions & Equations, Algebra, Functions	F.IF.8.a, A.SSE.3.a, A.REI.4.b	336
1200	Solve exponential equations by rewriting expressions with like bases.	Expressions & Equations, Algebra, Functions	A.REI.3	354
1200	Solve quadratic equations using the quadratic formula.	Expressions & Equations, Algebra, Functions	A.REI.4.a, A.REI.4.b, N.CN.7, A.CED.1	373
1200	Write the equation of and graph exponential equations or functions, including $f(x) = ab^x$ and $f(x) = a(1 + r)^x$, in number and word problems; identify and interpret critical values.	Expressions & Equations, Algebra, Functions	A.CED.2, F.IF.4, F.IF.7.e, F.LE.2, A.CED.1	400
1200	Use properties, definitions, and theorems of angles and lines to solve problems related to the segment addition postulate and the angle addition postulate.	Geometry, Measurement & Data	G.CO.1, G.CO.9, G.GPE.6	490
1200	Use properties of circles to solve problems related to the equation of a circle, its center, and radius length.	Geometry, Measurement & Data	G.GPE.1, G.GPE.4	518
1200	Write the equation of a line parallel or perpendicular to a given line through a given point.	Expressions & Equations, Algebra, Functions	G.GPE.5, A.CED.2	533
1200	Describe the intervals for which a function is increasing or decreasing.	Expressions & Equations, Algebra, Functions	F.IF.4	1059
1210	Perform basic operations with complex numbers and graph complex numbers.	Number & Operations	N.CN.2	355
1210	Find the sum of a finite series and of an infinite geometric series in number and word problems.	Expressions & Equations, Algebra, Functions	A.SSE.4	466
1210	Use properties of right triangles to solve problems using the geometric mean.	Geometry, Measurement & Data	G.SRT.4	512
1210	Use rules of exponents to rewrite or simplify expressions with rational exponents or radicals and interpret their meaning.	Number & Operations	A.SSE.3.c, N.RN.1, N.RN.2, F.IF.8.b., A.SSE.1.b	631
1210	Use theorems about congruent chords and arcs and the relationships of a radius of a circle to a tangent to solve problems.	Geometry, Measurement & Data	G.C.2	653

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
1210	Complete the square to identify characteristics of relations or functions and verify graphically.	Expressions & Equations, Algebra, Functions	A.REI.4.a, A.SSE.3.b, G.GPE.1	658
1210	Distinguish between types of events (conditional, mutually exclusive, independent, dependent, etc.). Use the appropriate formula to determine probabilities of random phenomena using the addition rule, multiplication rule, or Venn diagrams.	Statistics & Probability	S.CP.2, S.CP.3, S.CP.4, S.CP.5, S.CP.6, S.CP.7, S.CP.8, S.CP.9	1073
1220	Solve systems of equations or inequalities algebraically and graphically that include nonlinear relationships.	Expressions & Equations, Algebra, Functions	A.REI.7, A.REI.11, A.CED.2	441
1220	Use properties, definitions, and theorems of polygons to solve problems related to the interior and exterior angles of a convex polygon.	Geometry, Measurement & Data	G.CO.11, G.C.3	496
1220	Use trigonometric ratios to represent relationships in right triangles to solve number and word problems.	Geometry, Measurement & Data	G.SRT.6, G.SRT.7, G.SRT.8, G.MG.1, G.MG.2, G.MG.3, G.SRT.10, F.TF.8	515
1220	Identify transformations on nonlinear parent functions using function notation, algebraic equations, or graphs.	Expressions & Equations, Algebra, Functions	F.BF.1.b, F.BF.3	637
1230	Recognize the effect of scale factors or ratios on areas and volumes of similar geometric figures; use formulas to solve number and word problems.	Geometry, Measurement & Data	G.MG.3, G.GMD.1	530
1230	Classify functions as linear, quadratic, rational, etc., based on their tabular, graphical, verbal, or algebraic description; compare properties of two or more functions represented in different ways.	Expressions & Equations, Algebra, Functions	F.IF.9, F.BF.2, F.LE.1.a, F.LE.1.b, F.LE.1.c, F.LE.3, F.IF.4	1061
1230	Compare theoretical probabilities to results from any simulations or experiments (may also include Law of Large Numbers).	Statistics & Probability	S.IC.2, S.MD.6, S.MD.7	1074
1240	Examine the graph of a polynomial function to identify properties including end behavior, real and non-real zeros, odd and even degree, and relative maxima or minima. Use the zeros and other properties to graph the polynomial function.	Expressions & Equations, Algebra, Functions	A.APR.3, F.IF.4, F.IF.7.c	377
1240	Derive a quadratic or exponential function that models a set of data (curve of best fit) using calculators. Use the model to make predictions.	Statistics & Probability	S.ID.6.a, S.ID.6.b	408

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
1240	Describe and use the symmetry of a graph and determine whether a function is even, odd, or neither.	Expressions & Equations, Algebra, Functions	F.BF.3, F.IF.4	432
1240	Define and use the normal distribution curve to model a set of data; estimate the area under the curve.	Statistics & Probability	S.ID.4	479
1240	Use coordinate geometry to confirm properties of plane figures.	Geometry, Measurement & Data	G.GPE.4, G.GPE.7	499
1240	Use various methods, including trigonometric relationships or Heron's Formula, to find the area of a triangle in number and word problems.	Geometry, Measurement & Data	G.SRT.8, G.SRT.9	1071
1250	Describe graphically, algebraically, and verbally real-world phenomena as functions; identify the independent and dependent variables and any constraints of the domain or range.	Expressions & Equations, Algebra, Functions	F.IF.5, F.BF.1.a, F.LE.5	365
1250	Graph a radical relation, function, or inequality. State the domain and range.	Expressions & Equations, Algebra, Functions	F.IF.7.b, A.CED.2, F.IF.4	388
1250	Use summation notation to describe the sums in a series to solve number and word problems.	Expressions & Equations, Algebra, Functions	A.SSE.4	465
1250	Use definitions and theorems of angles formed when a transversal intersects parallel lines.	Geometry, Measurement & Data	8.G.5, G.CO.9	492
1250	Use theorems related to the segments formed by chords, secants, and tangents to solve number and word problems.	Geometry, Measurement & Data	G.C.2, G.MG.1	524
1250	Write and solve quadratic inequalities graphically or algebraically.	Expressions & Equations, Algebra, Functions	A.CED.1	589
1250	Identify the undefined values of rational algebraic expressions.	Expressions & Equations, Algebra, Functions	A.APR.7, A.REI.2	638
1250	Determine or calculate residuals of a distribution.	Statistics & Probability	S.ID.6.b	1070
1250	Examine, interpret, or apply probability or game theory strategies to determine the fairness of outcomes of various situations, including games, economics, political science, computer science, biology, etc.	Statistics & Probability	S.MD.7	1075
1270	Solve quadratic equations by completing the square.	Expressions & Equations, Algebra, Functions	A.REI.4.a, A.REI.4.b, F.IF.8.a, N.CN.7, A.CED.1	372
1280	Use properties of triangles to solve problems related to the segments parallel to one side of a triangle, including segments joining the midpoints of two sides of a triangle (midsegments).	Geometry, Measurement & Data	G.CO.10, G.SRT.4	509
1280	Use properties of parallel lines to solve problems related to segments divided proportionally.	Geometry, Measurement & Data	G.CO.9	510

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
1280	Write a quadratic equation or quadratic function given its zeros.	Expressions & Equations, Algebra, Functions	A.CED.1, A.CED.2	662
1280	Describe the nature of the zeros of polynomial functions using Descartes's rule of signs, multiplicity, and the Fundamental Theorem of Algebra using graphic and algebraic methods.	Expressions & Equations, Algebra, Functions	N.CN.9	1076
1290	Determine measures of spread (standard deviation).	Statistics & Probability	S.ID.2, S.ID.4	477
1290	Expand binomial expressions that are raised to positive integer powers using the binomial theorem.	Algebraic Thinking, Patterns & Proportional Reasoning	A.APR.5	586
1300	Analyze a function by decomposing it into simpler functions.	Expressions & Equations, Algebra, Functions	F.IF.8.a	439
1300	Use the Law of Sines and Law of Cosines to solve number and word problems involving triangles.	Geometry, Measurement & Data	G.SRT.10, G.SRT.11	664
1300	Factor a polynomial using grouping techniques by recognizing quadratic form and forms of special products, including factors with complex numbers.	Expressions & Equations, Algebra, Functions	N.CN.8, A.SSE.2, A.APR.4, F.IF.8.a	1066
1300	Calculate the reference angle from an angle in standard position. Determine coterminal angles.	Geometry, Measurement & Data	F.TF.1, F.TF.2	1078
1310	Find sums, differences, products, and quotients of rational algebraic expressions.	Algebraic Thinking, Patterns & Proportional Reasoning	A.APR.7	360
1320	Use graphs or compositions to establish the inverse relationship between exponential and logarithmic functions.	Expressions & Equations, Algebra, Functions	F.IF.7.e	401
1320	Find the inverse of a function or relation. Verify that two functions are inverses using their graphs or composition of functions.	Expressions & Equations, Algebra, Functions	F.BF.4.a	440
1320	Estimate or calculate the margin of error; determine the size of the sample necessary for a desired margin of error.	Statistics & Probability	S.IC.4	1080
1330	Write and solve rational equations; identify extraneous solutions, including checking the solution in the original equation.	Expressions & Equations, Algebra, Functions	A.REI.2, A.CED.1	384
1340	Use the Rational Zero Theorem and the Remainder Theorem to determine the rational solutions and factors of a polynomial function.	Expressions & Equations, Algebra, Functions	A.APR.2, A.APR.3	378
1340	Compare distributions of univariate data or bivariate data to draw conclusions; evaluate statements based on data.	Statistics & Probability	S.IC.5, S.IC.6	1081

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
1350	Write a polynomial equation given its solutions.	Expressions & Equations, Algebra, Functions	A.CED.1, A.CED.2	381
1350	Write and solve radical equations and inequalities; identify extraneous solutions, including checking the solution in the original equation.	Expressions & Equations, Algebra, Functions	A.REI.2, A.CED.1	389
1350	Write and graph special functions (step, constant, and piecewise) and identify the domain and range.	Expressions & Equations, Algebra, Functions	F.IF.4, F.IF.5, F.IF.7.b	670
1350	Use the definition of a parabola to identify characteristics, write an equation, and graph the relation.	Geometry, Measurement & Data	G.GPE.2	673
1350	Use rules for the mean and rules for the standard deviation of random variables in order to determine the effect linear operations have on the shape, center, and spread of a data set.	Statistics & Probability	S.IC.5	1079
1360	Determine the area and volume of figures using right triangle relationships, including trigonometric relationships in number and word problems.	Geometry, Measurement & Data	G.MG.1, G.MG.2, G.MG.3	672
1380	Identify asymptotes, intercepts, holes, domain, and range of a rational function and sketch the graph.	Expressions & Equations, Algebra, Functions	F.IF.4	383
1380	Write and solve rational inequalities; identify extraneous solutions, including checking the solution in the original equation.	Expressions & Equations, Algebra, Functions	A.CED.1	386
1380	Use a unit circle to define trigonometric functions and evaluate trigonometric functions for a given angle.	Geometry, Measurement & Data	F.TF.2	420
1390	Find algebraically or approximate graphically or numerically solutions of equations of the form $f(x) = g(x)$ where $f(x)$ and $g(x)$ are linear, polynomial, rational, radical, absolute value, exponential, logarithmic, or trigonometric functions.	Expressions & Equations, Algebra, Functions	A.REI.11	1067
1400	Use models to make predictions and interpret the correlation coefficient of the model; distinguish between correlation and causation.	Statistics & Probability	S.ID.8, S.ID.9	341
1410	Decompose rational expressions or rational functions, including writing as partial fractions.	Expressions & Equations, Algebra, Functions	A.SSE.2, A.APR.6	1077
1420	Graph sine and cosine functions and identify the domain, range, period, amplitude, midline, and phase shift of the function.	Expressions & Equations, Algebra, Functions	F.IF.4, F.IF.7.e, F.TF.5	451
1460	Model periodic phenomena using trigonometric functions.	Expressions & Equations, Algebra, Functions	F.TF.5	472
1480	Use trigonometric identities to verify relationships.	Expressions & Equations, Algebra, Functions	F.TF.8	419

Quantile Measure	QSC Description	Strand	CCSS ID	QSC ID
1510	Graph tangent, cotangent, secant, and cosecant functions and identify the domain, range, period, and asymptotes of the function.	Expressions & Equations, Algebra, Functions	F.IF.4, F.IF.7.e	453
1540	Identify maximum and minimum points in terms of local and global behavior; identify end behavior and other critical values in functions and their graphs.	Expressions & Equations, Algebra, Functions	F.IF.4, F.IF.7.e	431

Appendix 2: Norm Reference Table (spring percentiles)

	K	1	2	3	4	5	6	7	8	9	10	11	12
1	EM400Q	EM310Q	EM140Q	30Q	70Q	135Q	240Q	280Q	300Q	350Q	365Q	520Q	565Q
5	EM400Q	EM195Q	EM10Q	175Q	260Q	325Q	415Q	440Q	490Q	515Q	540Q	630Q	670Q
10	EM400Q	EM115Q	65Q	250Q	350Q	415Q	505Q	525Q	590Q	610Q	655Q	730Q	760Q
25	EM270Q	15Q	175Q	375Q	480Q	550Q	645Q	665Q	730Q	760Q	810Q	890Q	910Q
35	EM210Q	75Q	230Q	430Q	535Q	610Q	700Q	730Q	795Q	830Q	880Q	960Q	980Q
50	EM125Q	140Q	290Q	495Q	605Q	690Q	775Q	815Q	880Q	915Q	970Q	1055Q	1075Q
65	EM45Q	210Q	360Q	560Q	670Q	760Q	845Q	885Q	965Q	1000Q	1055Q	1145Q	1165Q
75	10Q	260Q	405Q	605Q	720Q	815Q	895Q	945Q	1020Q	1065Q	1115Q	1205Q	1235Q
90	125Q	375Q	515Q	715Q	820Q	915Q	1000Q	1055Q	1140Q	1190Q	1240Q	1330Q	1370Q
95	180Q	455Q	585Q	790Q	885Q	980Q	1070Q	1115Q	1210Q	1270Q	1310Q	1400Q	1435Q

Appendix 3: Reliability Studies

To be useful, a piece of information should be reliable—stable, consistent, and dependable. Reliability can be defined as "the degree to which test scores for a group of test takers are consistent over repeated applications of a measurement procedure and hence are inferred to be dependable and repeatable for an individual test taker" (Berkowitz, Wolkowitz, Fitch, & Kopriva, 2000). In reality, all test scores include some measure of error (or level of uncertainty).

Reliability is a major consideration in evaluating any assessment procedure. Two studies have been conducted with an earlier version of SMI that, while not directly applicable to SMI College & Career, may be indicative of the level of reliability that can be expected.

SMI Marginal Reliability

For a computer-adaptive test where there are no "fixed forms" (established test forms) and the items and tests are calibrated using item response theory, the traditional measures of reliability are not appropriate (Green, Bock, Humphreys, Linn, & Reckase, 1984). Fortunately, item response theory provides an index of reliability for an entire test that does not require all children to be administered the same exact items. The marginal reliability is computed by determining the proportion of test performance that is not due to error (i.e., the true score). Technically, the marginal reliability is computed by subtracting the total variability in estimated ability by an error term, and dividing this difference by the total estimated ability. As with traditional reliability (e.g., Cronbach alpha), the marginal reliability is a coefficient between 0 and 1 that measures the proportion of the instrument score that is attributed to the actual ability levels of the participants rather than aberrant "noise." Thus, a marginal reliability that exceeds 0.80 provides evidence that the scores on a math test accurately separate or discriminate among test taker's math ability.

Within Winsteps item analysis program (Linacre, 2010), the marginal reliability is calculated as the model reliability. The model reliability estimate describes the upper bound of the "true" reliability of person ordering and is dependent on sample ability variance, length of the test, number of categories per item, and sample-item targeting.

A study was conducted to examine the marginal reliability of SMI test results. SMI was administered to 6,384 students across 13,630 administrations of the SMI in Grades 2–8 in six school districts across the nation. The data was analyzed using Winsteps and the marginal (model) reliabilities are reported in Table 13. (For more information on the sample, see the descriptions of the districts in Appendix 4, Phase II).

TABLE 13. SMI Marginal reliability estimates, by district and overall.

	Grades	Number of Students (N)	Number of SMI Administrations (N)	Number of SMI Items Tested (N)	Marginal Reliability
Alief (TX)	2–8	3,576	5,747	4,638	0.97
Brevard (FL)	2–6	381	1,591	3,297	0.98
Cabarrus (NC)	2–5	993	4,027	2,999	0.97
Clark (NV)	2–5	428	1,188	2,789	0.97
Harford (MD)	2–5	256	1,077	2,849	0.97
Kannapolis (NC)	5–6	750	1,751	3,087	0.96
All Students	2–8	6,384	13,630	4,721	0.97

Based upon these marginal reliability estimates, SMI is able to consistently order students and these estimates provide an upper bound for all other estimates of the reliability of the SMI. The marginal reliability estimate does not include “variability due to short-run random variation of the trait being measured or situational variance in the testing conditions” (Green, Bock, Humphreys, Linn, & Reckase, 1984, p. 353). In order to examine variation in test scores due to these sources of error, empirical studies need to be conducted. The next section describes a study designed to examine the consistency of scores by re-administering the SMI on successive days with the criteria that items presented to students are sampled without replacement.

SMI Test-Score Consistency

Test-retest reliability examines the extent to which two administrations of the same test yield similar results. The closer the results, the greater the test-retest reliability of the assessment. A shortcoming of test-retest reliability studies is the "practice effect" where respondents "learn" to answer the same questions in the first test and this affects their responses in the next test. Alternate-form reliability examines the extent to which two equivalent forms of an assessment yield the same results (i.e., students' scores have the same rank order on both tests). In this case, comparable (just not the same) items are administered at a common time. When taken together, alternate-form reliability and test-retest reliability are estimates of test score consistency. Generally, correlation coefficients estimating the reliability of an assessment ranging from 0.70 to 0.80 are considered as satisfactory or good.

A study was conducted to examine the stability and consistency of SMI test results. In the context of data collected with a computer-adaptive assessment over a relatively short time period, correlations between test scores are reflective of the test-retest reliability of alternate forms. SMI was administered twice to 223 students in Grades 4, 6, and 8 over a one-week period (April 29–30, 2010, and May 6, 2010) with 197 students completing both administrations. Prior information used within the SMI scoring algorithm was reset after the first administration of SMI. Of the matched sample, 54% were female ($N = 106$), 21% were classified as black ($N = 41$), 10% as Hispanic ($N = 20$), 7% as multiracial ($N = 13$), and 62% as white ($N = 123$).

Table 14 shows the test-retest reliability estimates for each grade level and across all grades over a one-week period. The overall correlation between the two SMI Quantile measures was 0.78. Test-retest reliability coefficients ranging from 0.70 to 0.80 are considered satisfactory.

TABLE 14. SMI test-retest reliability estimates over a one-week period, by grade.

Grade	*N*	SMI Test 1 Mean (SD)	SMI Test 2 Mean (SD)	Test-Retest Correlation
4	77	695.32 (154.63)	681.95 (189.07)	0.79
6	66	801.52 (183.82)	812.20 (210.21)	0.70
8	54	887.78 (219.75)	920.74 (239.31)	0.73
Total	**197**	**783.65 (199.23)**	**791.04 (231.22)**	**0.78**

Appendix 4: Validity Studies

The validity of a test is the degree to which the test actually measures what it purports to measure. Validity provides a direct check on how well the test fulfills its purpose. The appropriateness of any conclusion drawn from the results of a test is a function of the test's validity. According to Kane (2006), "to validate a proposed interpretation or use of test scores is to evaluate the rationale for this interpretation or use" (p. 23).

Initially, the primary source of validity evidence came from the examination of the content of SMI and the degree to which the assessment could be said to measure mathematical understandings (construct-identification validity evidence). Currently, data has been collected to further describe the validity (criterion-prediction and construct-identification) of SMI. While not directly applicable to SMI College & Career, these studies may be indicative of the level of reliability that can be expected.

SMI Validation Study—Phase I

Phase I of the SMI validation field study was conducted by Scholastic during summer and fall 2009, and data were collected from four school districts across the United States. The validation study was designed to provide criterion and construct validity for the SMI as a measure of a student's mathematical understanding and provide answers to the research questions. Next, the districts and their participation in the study are described.

Decatur Public School District 61 (Illinois)

Students attending a summer school program in 2009 from two middle schools in Decatur, Illinois, participated in a study where the SMI was administered twice, approximately three weeks apart. This district also submitted scores from its Statewide Assessment program, known as the Illinois Standards Achievement Test (ISAT), from 2008 and 2009 (administered in March).

Students in Grades 7 and 8 were placed in summer school if they had not met or exceeded ISAT scores from the previous year in reading and/or math or if they failed one or more subjects in school. The district's demographics include: 65% economically disadvantaged; 45% white, 45% black, 2% Hispanic, 1% Asian, 8% multiracial.

In graphing the SMI Quantile 1 and Quantile 2 student data, several outliers were observed. Outliers were determined by looking at the absolute rank difference between the SMI Quantile 1 and Quantile 2 measures. Those students with difference scores of fewer than 40 points remained in the study.

Harford County Public Schools (Maryland)

Students from one elementary school in Harford County Public Schools, Maryland, were enrolled in an extended-day intervention program. All but one student were assessed with the SMI in November 2009. The criteria used to determine participation in the extended-day intervention included, but was not limited to, the SMI Quantile measure. Student demographics in the district include 40% free or reduced-price lunch, but this varies by school due to district SES diversity. There is a high military population, which affected the free or reduced-price lunch percentage, as more students are eligible for free or reduced-price lunch since their families are not penalized for free housing. The district ethnicity distribution is as follows: 40% African American, 40% Caucasian, 15% Latino, and 5% other (including mixed ethnicities).

Raytown Consolidated School District No. 2 (Missouri)

Students in three middle schools in Raytown School District, Missouri, participated in a summer school program in 2009. Because there were so few students in Grades 1–6 and Grade 8, only Grade 7 remained in the analysis.

The district chose to use SMI with two different groups of students. The first group consisted of students who used the *Math in Context* program, the district's standard summer school math program. In addition, seventh graders in this cohort used *FASTT Math* every day for 10 minutes. The second group consisted of at-risk summer school students who were simultaneously enrolled in the *Math Academy, Math in Context,* and *FASTT Math*. These students were placed in *Math Academy* based on their grades, teacher recommendation, and STAR assessment.

Lexington Public Schools (Massachusetts)

One elementary and one middle school participated in this special study in the Lexington School District, Massachusetts. The only variables available for this school district were the SMI scores and the state test from 2008 and 2009. The SMI was administered in October 2009, and the state tests were administered in March of each year. The students participated in *Everyday Math* K–5 (Core) and in an intervention strategies program for Grades 6–8.

The following table summarizes all the locations with the number of students tested and the mean and standard deviation on the SMI. A dash (-) indicates that data are not available. There were three weeks between the data collected in the Decatur, Illinois, school district for the Quantile 1 and 2 measures.

TABLE 15. SMI Validation Study—Phase I: Descriptive statistics for the SMI Quantile measures.

Grade	2		3		4		5	
	N	Mean (SD)	*N*	Mean (SD)	*N*	Mean (SD)	*N*	Mean (SD)
Decatur, IL, SMI Q1	-	-	-	-	-	-	-	-
Decatur, IL, SMI Q2	-	-	-	-	-	-	-	-
Harford, MD	65	234.00 (151.94)	68	414.12 (155.08)	76	576.97 (152.23)	69	654.78 (178.22)
Raytown, MO	-	-	-	-	-	-	-	-
Lexington, MA	-	-	-	-	17	578.82 (182.96)	-	-

Grade	6		7		8	
	N	Mean (SD)	*N*	Mean (SD)	*N*	Mean (SD)
Decatur, IL, SMI Q1	-	-	56	527.70 (217.26)	39	583.59 (255.31)
Decatur, IL, SMI Q2	-	-	46	600.98 (181.42)	37	633.11 (215.99)
Harford, MD	-	-		-	-	-
Raytown, MO	-	-	268	658.60 (186.69)	-	-
Lexington, MA	21	865.00 (201.41)	-	-	16	1119.06 (87.54)

Note: Values based on five or fewer students have been excluded.

In each location, the following data were requested for each student:

1. SMI Quantile measure from Summer to November 2009 administration
2. State test scale scores and performance levels from Spring 2008 and Spring 2009
3. Student demographics (e.g., gender, grade, race/ethnicity, ELL status, free or reduced-price lunch status, and math intervention status and program)

The following tables provide a demographic summary and the associated means and standard deviations of Quantile measures for the associated students at the schools where data is available. Lexington School District did not provide any demographic information.

TABLE 16. SMI Validation Study—Phase I: Means and standard deviations for the SMI Quantile measures, by gender.

Gender	Female		Male	
	N	Mean (SD)	*N*	Mean (SD)
Decatur, IL, SMI Q1	42	562.38 (231.93)	53	541.34 (237.34)
Decatur, IL, SMI Q2	38	591.71 (171.52)	45	635.22 (216.06)
Harford, MD	117	479.96 (217.00)	160	471.72 (230.95)
Raytown, MO	122	670.08 (181.12)	144	649.69 (192.55)

TABLE 17. SMI Validation Study—Phase I: Means and standard deviations for the SMI Quantile measures, by race/ethnicity.

Race/Ethnicity	African American		American Indian/ Native American		Asian		Caucasian	
	N	Mean (SD)	*N*	Mean (SD)	*N*	Mean (SD)	*N*	Mean (SD)
Decatur, IL, SMI Q1	58	561.66 (239.68)	-	-	-	-	30	553.50 (231.77)
Decatur, IL, SMI Q2	53	624.43 (179.18)	-	-	-	-	26	607.88 (240.62)
Harford, MD	95	481.05 (232.76)	-	-	-	-	139	480.54 (218.13)
Raytown, MO	139	633.31 (189.53)	-	-	-	-	109	680.69 (186.64)

Race/Ethnicity	Pacific Islander		Hispanic		Multiracial	
	N	Mean (SD)	*N*	Mean (SD)	*N*	Mean (SD)
Decatur, IL, SMI Q1	-	-	-	-	7	447.14 (196.04)
Decatur, IL, SMI Q2	-	-	-	-	-	-
Harford, MD	-	-	38	437.50 (237.58)	-	-
Raytown, MO	-	-	15	712.67 (126.53)	-	-

Note: Values based on five or fewer students have been excluded.

SMI Validation Study—Phase II

Phase II of the SMI validation field study was conducted by Scholastic during the 2009–2010 school year, and data were collected from six school districts across the United States. The validation study was designed to provide criterion-prediction and construct-identification validity for the SMI as a measure of a student's readiness for mathematics instruction and provide answers to the research questions. Next, the districts and their participation is described.

Alief Independent School District (Texas)

Students in Grades 2–6 from four schools (Grades 2–4, three schools; Grades 5–6, one school) were administered the SMI in December, February, and May of the 2009–2010 school year. In addition, students in Grades 3–6 were administered the mathematics subtest of the Texas Assessment of Knowledge and Skills (TAKS) on April 26, 2010. For Grades 2–4, bilingual students in one school were omitted from testing. For Grades 5–6, bilingual classrooms were omitted from testing, and classes with lower-performing students and/or students classified as Special Education were targeted for SMI testing.

Brevard Public Schools (Florida)

All students in Grades 2–6 from one school were administered the SMI in January, March, and May 2010. In addition, students in Grades 3–6 were administered the mathematics subtest of the Florida Comprehensive Assessment Test (FCAT) between March 9 and March 19, 2010. The mathematics instructional program consisted of *FASTT Math* (used school-wide) and the Macmillan basal mathematics program, *SuccessMaker® Math* (used with students who need additional support). Students were targeted for *SuccessMaker Math* instruction based upon their Quantile measures and FCAT scores (lowest 25%). In addition, formative assessment was used in the classrooms with *FASTT Math* software fluency data and CORE Math curriculum-based measures.

Cabarrus County Schools (North Carolina)

All students in Grades 2–5 from two schools were administered the SMI in February and April or May 2010. In addition, students in Grades 3–5 were administered the mathematics subtest of the North Carolina End-of-Grade Test (NCEOG) during the last three weeks of the school year (typically in May). The mathematics instructional program consisted of *Number Worlds, Everyday Math, First in Math,* and *Real Math* (45-minute blocks per day). In addition, formative assessment was used in the classrooms with *AIMSWeb® M-CBM* (Mathematics Curriculum-Based Measurement). Students who were identified through the quarterly benchmarking assessment received support from Title 1 staff. There was also a district-wide initiative for exploring Professional Learning Communities (PLCs). In their PLCs, staff reviewed the quarterly assessments, identified students' problem areas, and remediated at the classroom level according to the common, formative assessment standards, and then intervened at the student level accordingly.

Clark County School District (Nevada)

All students in Grades 2–5 from one school were administered the SMI in November, January, February, and May of the 2009–2010 school year, and scores were used to build an intervention program. In addition, students in Grades 3–5 were administered the mathematics subtest of the criterion-referenced tests (CRT) between February 16 and March 16. The mathematics intervention program consisted of *FASTT Math* and *Do the Math*. The school also uses interim assessments that align with district benchmarks.

Harford County Public Schools (Maryland)

Students in Grades 2–5 from one school were administered the SMI in November, February, and May of the 2009–2010 school year. In addition, students in Grades 3–5 were administered the mathematics subtest of the Maryland School Assessment (MSA) between March 8 and March 23, 2010. The mathematics instructional program consisted of *Dreambox*, with *Do the Math* beginning in January 2010. All students used *First in Math* (mainly outside the school day), and students participated in an extended-day program three times per week for a 45-minute mathematics block. Students were able to do *First in Math* for an additional 15 minutes when class schedules permitted. Intervention programs were designated as supplemental, but each classroom teacher completed targeted instruction once per week as part of the *Everyday Math* program. In addition, formative assessment was used in the classrooms with a modified version of the Unit Assessments in *Everyday Math*.

Kannapolis City Schools (North Carolina)

All students in Grades 5 and 6 from one school were administered the SMI in January, in March, and then for a third time in Spring 2010. In addition, students were administered the mathematics subtest of the North Carolina End-of-Grade Test (NCEOG) during the last three weeks of the school year (typically in May). The mathematics instructional program was the standard curriculum, with SMI administered for assessment purposes only. Teacher teams created a common assessment that could be used for formative assessment every four weeks. SMI Quantile measures (along with NCEOG scores) were used to help teachers differentiate instruction, inform students' Personal Education Plans, and identify students in need of remediation. Student interventions included after-school tutoring programs and teacher-developed interventions.

The following table summarizes all the locations, with the number of students tested and the mean and standard deviation on the SMI. A dash (-) indicates that the data are not available.

TABLE 18. SMI Validation Study—Phase II: Descriptive statistics for the SMI Quantile measures (spring administration).

Grade	2		3		4	
	N	Mean (SD)	*N*	Mean (SD)	*N*	Mean (SD)
Alief, TX	315	269.68 (149.72)	400	467.91 (159.78)	547	602.73 (158.14)
Brevard, FL	5	320.00 (136.70)	101	371.53 (129.59)	94	583.67 (155.26)
Cabarrus, NC	154	287.18 (143.42)	283	481.96 (172.24)	279	604.43 (176.90)
Clark, NV	105	361.62 (139.76)	99	508.94 (197.22)	107	637.38 (147.00)
Harford, MD	64	360.31 (139.76)	59	555.93 (188.74)	67	698.81 (146.10)
Kannapolis, NC	-	-	-	-	-	-

Grade	5		6	
	N	Mean (SD)	*N*	Mean (SD)
Alief, TX	282	646.35 (160.14)	283	744.58 (201.49)
Brevard, FL	90	679.56 (145.82)	91	787.09 (166.94)
Cabarrus, NC	277	711.70 (182.40)	-	-
Clark, NV	117	704.87 (180.96)	-	-
Harford, MD	66	761.59 (197.26)	-	-
Kannapolis, NC	368	678.29 (181.50)	382	803.17 (207.94)

In each location, data were requested for each student:

1. SMI Quantile measure from the 2009–2010 school year (fall, winter, and spring) administration
2. State test scale scores and performance levels from Spring 2008, Spring 2009, and Spring 2010
3. Student demographics (e.g., gender, grade, race/ethnicity, ELL status, free or reduced-price lunch status, math intervention status and program)

The following tables provide a demographic summary and the associated means and standard deviations of Quantile measures for the associated students at the schools where data is available. A dash (-) indicates that the data are not available.

TABLE 19. SMI Validation Study—Phase II: Means and standard deviations for the SMI Quantile measures, by gender (spring administration).

Gender	Female		Male	
	N	Mean (SD)	*N*	Mean (SD)
Alief, TX	799	545.71 (229.09)	827	538.95 (221.40)
Brevard, FL	183	604.59 (218.72)	186	594,73 (215.42)
Cabarrus, NC	491	537.01 (218.23)	502	563.20 (227.40)
Clark, NV	139	614.28 (197.27)	173	633.50 (191.98)
Harford, MD	110	608.64 (215.14)	146	589.01 (239.72)
Kannapolis, NC	317	763.52 (179.75)	312	763.40 (195.98)

TABLE 20. SMI Validation Study—Phase II: Means and standard deviations for the SMI Quantile measures, by race/ethnicity (spring administration).

Race/Ethnicity	African American		American Indian/ Native American		Asian		Caucasian	
	N	Mean (SD)	*N*	Mean (SD)	*N*	Mean (SD)	*N*	Mean (SD)
Alief, TX	-	-	-	-	-	-	-	-
Brevard, FL	12	535.42 (252.12)	-	-	9	602.78 (207.49)	317	599.31 (214.83)
Cabarrus, NC	204	504.41 (213.74)	-	-	34	684.41 (227/80)	621	581.13 (216.27)
Clark, NV	15	496.67 (132.55)	-	-	5	704.00 (87.06)	154	695.94 (164.36)
Harford, MD	90	594.17 (232.32)	-	-	4	596.25 (211.83)	130	610.31 (224.42)
Kannapolis, NC	182	732.25 (184.69)	-	-	-	-	304	800.26 (182.90)

Race/Ethnicity	Pacific Islander		Hispanic		Multiracial	
	N	Mean (SD)	*N*	Mean (SD)	*N*	Mean (SD)
Alief, TX	-	-	-	-	-	-
Brevard, FL	-	-	18	587.78 (257.34)	8	713.13 (161.80)
Cabarrus, NC	-	-	92	421.96 (225.61)	38	484.34 (196.03)
Clark, NV	-	-	16	656.88 (169.01)	-	-
Harford, MD	-	-	30	542.33 (240.38)	-	-
Kannapolis, NC	-	-	143	724.93 (188.46)	-	-

SMI Validation Study—Phase III

Phase III of the SMI validation field study was conducted by Scholastic during the 2010–2011 school year. Data were collected from seven school districts across the United States. The validation study was designed to provide criterion-prediction and construct-identification validity for the SMI as a measure of a student's readiness for mathematics instruction and provide answers to the research questions. Next, the districts and their participation are described.

Alief Independent School District (Texas)

A sample of 5,073 students in Grades 2–8 from six schools (Grades 3–5, four schools; Grades 5–7, one school; and Grades 7–8, one school) were administered the SMI in October, February, and May of the 2010–2011 school year. In addition, students in Grades 3–8 were administered the mathematics subtest of the Texas Assessment of Knowledge and Skills (TAKS) on April 1, 2011.

Brevard Public Schools (Florida)

A sample of 671 students in Grades 2–6 from one elementary school were administered the SMI in September, January, and May of the 2010–2011 school year. In addition, students in Grades 3–6 were administered the mathematics subtest of the Florida Comprehensive Assessment Test (FCAT) on April 13, 2011.

Cabarrus County Schools (North Carolina)

A sample of 1,148 students in Grades 2–5 from one elementary school were administered the SMI in late September/early October, January, and late April/early May of the 2010–2011 school year. In addition, students in Grades 3–5 were administered the mathematics subtest of the North Carolina End-of-Grade Test (NCEOG) on May 11, 2011.

Clark County School District (Nevada)

A sample of 419 students in Grades 2–5 from one elementary school were administered the SMI in October, late January/early February, and May of the 2010–2011 school year, and scores were used to build an intervention program. In addition, students in Grades 3–5 were administered the mathematics subtest of the criterion-referenced tests (CRT) on March 7, 2011.

Harford County Public Schools (Maryland)

A sample of 20,195 students in Grades 2–8 from 42 schools (33 elementary schools and 9 middle schools) were administered the SMI in October/early November, January/February, and/or May/June of the 2010–2011 school year. In addition, students in Grades 3–8 were administered the mathematics subtest of the Maryland School Assessment (MSA) on March 15, 2011. Students in one elementary school (Grades 2–5) had data only for SMI and are, therefore, not included in analyses for the complete sample for Harford County Public Schools.

Kannapolis City Schools (North Carolina)

A sample of 776 students in Grades 5 and 6 from one school were administered the SMI in February and May of the 2010–2011 school year. In addition, students were administered the mathematics subtest of the North Carolina End-of-Grade Test (NCEOG) on May 1, 2011.

Killeen Independent School District (Texas)

A sample of 12,475 students in Grades 2–5 from 32 elementary schools were administered the SMI in late September/early October, January/early February, and May of the 2010–2011 school year. In addition, students in Grades 3–6 were administered the mathematics subtest of the Texas Assessment of Knowledge and Skills (TAKS) on April 1 or May 1, 2011.

The following table summarizes all the locations, with the number of students tested and the mean and standard deviation on the SMI. A dash (-) indicates that the data are not available.

TABLE 21. SMI Validation Study—Phase III: Descriptive statistics for the SMI Quantile measures (spring administration).

Grade	2		3		4	
	N	Mean (SD)	*N*	Mean (SD)	*N*	Mean (SD)
Alief, TX	143	333.11 (130.73)	228	490.99 (170.78)	293	621.45 (142.38)
Brevard, FL	93	362.26 (113.57)	101	568.66 (167.03)	98	665.61 (149.81)
Cabarrus, NC	476	375.80 (148.21)	408	568.60 (203.25)	474	693.19 (193.52)
Clark, NV	75	340.40 (102.15)	101	544.80 (151.32)	103	644.27 (171.71)
Harford, MD	2,586	408.75 (166.18)	2,789	603.46 (201.76)	2,600	694.96 (166.61)
Kannapolis, NC	-	-	-	-	-	-
Killeen, TX	64	334.45 (147.92)	3,002	531.44 (147.61)	2,977	654.73 (135.79)

TABLE 21. SMI Validation Study—Phase III: Descriptive statistics for the SMI Quantile measures (spring administration). *(continued)*

Grade	5		6		7		8	
	N	Mean (SD)	*N*	Mean (SD)	*N*	Mean (SD)	*N*	Mean (SD)
Alief, TX	89	678.09 (135.17)	94	758.14 (182.21)	286	773.02 (172.33)	152	762.11 (226.12)
Brevard, FL	88	802.44 (151.12)	92	863.32 (194.38)	-	-	-	-
Cabarrus, NC	506	802.59 (225.53)	-	-	-	-	-	-
Clark, NV	61	730.41 (144.83)	-	-	-	-	-	-
Harford, MD	2,784	799.88 (182.29)	2,182	850.01 (203.07)	2,161	877.64 (220.03)	2,219	939.06 (230.49)
Kannapolis, NC	321	712.49 (216.68)	301	815.93 (196.17)	-	-	-	-
Killeen, TX	2,786	740.24 (138.02)	-	-	-	-	-	-

In each location, data were requested for each student:

1. SMI Quantile measure from the 2009–2010 school year (fall, winter, and spring) administration
2. State test scale scores and performance levels from Spring 2008, Spring 2009, Spring 2010, and Spring 2011
3. Student demographics (e.g., gender, grade, race/ethnicity, ELL status, free or reduced-price lunch status, math intervention status and program)

The following tables provide a demographic summary and the associated means and standard deviations of Quantile measures for the associated students at the schools where data is available. A dash (-) indicates that the data are not available.

TABLE 22. SMI Validation Study—Phase III: Means and standard deviations for the SMI Quantile measures, by gender (spring administration).

Gender	Female		Male	
	N	Mean (SD)	*N*	Mean (SD)
Alief, TX	542	654.66 (181.30)	525	631.69 (197.24)
Brevard, FL	173	727.31 (195.74)	183	725.82 (202.48)
Cabarrus, NC*	289	679.36 (247.15)	306	656.91 (261.15)
Clark, NV	138	608.91 (180.53)	122	643.20 (163.07)
Harford, MD	8,280	717.69 (257.34)	8,868	730.23 (261.03)
Kannapolis, NC	331	773.47 (200.88)	346	750.06 (224.54)
Killeen, TX	4,350	636.57 (163.33)	4,479	638.35 (169.47)

* Minimum of five students per group to be reported

TABLE 23. SMI Validation Study—Phase III: Means and standard deviations for the SMI Quantile measures, by race/ethnicity (spring administration).*

Race/Ethnicity	African American		American Indian/ Native American		Asian		Caucasian	
	N	Mean (SD)	*N*	Mean (SD)	*N*	Mean (SD)	*N*	Mean (SD)
Alief, TX	492	598.53 (203.86)	-	-	89	758.43 (185.19)	33	640.61 (200.94)
Brevard, FL	12	690.83 (251.14)	-	-	-	-	306	723.14 (200.60)
Cabarrus, NC**	114	606.01 (245.59)	-	-	-	-	422	670.98 (247.11)
Clark, NV	12	571.25 (191.41)	-	-	6	680.83 (146.27)	185	646.11 (165.01)
Harford, MD	3,084	647.60 (250.16)	54	714.26 (266.82)	549	797.20 (272.23)	11,669	746.84 (255.79)
Kannapolis, NC	188	716.06 (208.09)	-	-	8	848.13 (274.07)	296	798.26 (217.41)
Killeen, TX	3,180	616.98 (166.69)	114	651.14 (179.44)	347	690.78 (168.00)	2,762	659.11 (162.59)

Race/Ethnicity	Pacific Islander		Hispanic		Multiracial	
	N	Mean (SD)	*N*	Mean (SD)	*N*	Mean (SD)
Alief, TX	11	570.45 (163.09)	738	677.99 (184.95)	-	-
Brevard, FL	9	802. 22 (184.56)	17	750.00 (172.39)	11	774.55 (146.57)
Cabarrus, NC	-	-	-	-	-	-
Clark, NV	5	652.00 (222.78)	34	508.38 (184.47)	15	663.33 (124.77)
Harford, MD	34	702.79 (242.99)	54	714.26 (266.82)	1,734	686.99 (262.83)
Kannapolis, NC	-	-	145	751.93 (199.35)	39	717.69 (203.42)
Killeen, TX	123	647.44 (147.66)	2,303	630.57 (166.07)	-	-

*Minimum of five students per group to be reported

**Omitted students classified as “3,” “4,” and “5” (N = 59)

Construct-Identification Validity

The construct-identification validity of a test is the extent to which the test may be said to measure a theoretical construct or trait, such as readiness for mathematics instruction. It is anticipated that scores from a valid test of mathematics skills should show expected:

1. Differences by age and/or grade
2. Differences among groups of students that traditionally show different or similar patterns of development in mathematics (e.g., differences in socioeconomic levels, gender, ethnicity, etc.)
3. Relationships with other measures of mathematical understanding

Construct-identification validity is the most important aspect of validity related to SMI. SMI is designed to measure the development of mathematical abilities; therefore, how well it measures mathematical understanding and how well it measures the development of these mathematical understandings must be examined.

Further Construct-Identification Validity

Convergent validity looks at the relationships between test scores and other criterion variables that the scores should be related to (e.g., student characteristics, mathematical achievement grade equivalent, and remediation programs).

Intervention Program

Because targeted mathematics intervention programs are specifically designed to improve students' mathematical achievement, an effective intervention would be expected to improve students' mathematics test scores. Participation in a math intervention program was collected from two of the Phase I sites (Decatur and Raytown). An ANOVA was conducted for the data from each site, and a significant difference due to math intervention program was observed for both of the sites (Harford, $p < .05$; Raytown, $p < .0001$). The differences between the mean SMI Quantile measures are as expected.

TABLE 24. Harford County Public Schools—Intervention study means and standard deviations for the SMI Quantile measures.

Yes		No	
N	Mean (SD)	*N*	Mean (SD)
26	561.92 (127.15)	252	467.42 (231.22)

$p < 0.5$

TABLE 25. Raytown Consolidated School District No. 2—*FASTT Math* intervention program participation means and standard deviations for the SMI Quantile measures.

Yes		No	
N	Mean (SD)	*N*	Mean (SD)
60	564.50 (211.09)	206	686.58 (170.76)

$p < .0001$

During Phase III, information related to inclusion in a math intervention program was collected from four of the seven sites (Alief, Brevard, Cabarrus, and Killeen) for students in Grades 2–6. As expected, students classified as needing math intervention services scored significantly lower than students not classified as needing math intervention services.

TABLE 26. Inclusion In math intervention program means and standard deviations for the SMI Quantile measures.

School District	Number of Students Classified as Needing Math Intervention	Mean (SD) of Math Intervention Students	Number of Students *Not* Classified as Needing Math Intervention	Mean (SD) of Regular Education Students
Alief ISD	49	462.04 (140.60)	887	604.66 (173.45)
Brevard	70	578.93 (149.21)	199	716.23 (172.97)
Cabarrus County	67	524.93 (186.56)	528	685.95 (256.34)
Killeen ISD	2,737	569.67 (165.52)	6,092	667.93 (157.68)

$p < .0001$

Gifted and Talented Classification

Gifted and Talented (GT) classification information was collected in one of the Phase I sites (Harford) and three of the Phase II sites (Alief, Harford, and Kannapolis). An ANOVA was conducted for the data from each site, and a significant difference due to GT classification was observed for all of the sites ($p < .0001$). The differences between the mean SMI Quantile measures are as expected.

TABLE 27. Gifted and Talented status means and standard deviations for the SMI Quantile measures.

School District	Number of Students Classified as Gifted and Talented	Mean (SD) of Gifted and Talented Students	Number of Students *Not* Classified as Gifted and Talented	Mean (SD) of Regular Education Students
Alief ISD	125	665.20 (186.95)	1500	531.98 (225.17)
Harford Co. (Phase I)	21	746.19 (160.55)	257	454.20 (215.27)
Harford Co. (Phase II)	18	854.72 (100.64)	238	577.98 (224.53)
Kannapolis City	57	977.54 (122.64)	572	742.12 (179.71)

$p < .0001$

During Phase III, Gifted and Talented classification information was collected from four of the seven sites (Alief, Brevard, Clark, and Killeen) for students in Grades 2–6. As expected, students classified as Gifted and Talented scored significantly higher than students not classified as Gifted and Talented.

TABLE 28. Gifted and Talented status means and standard deviations for the SMI Quantile measures.

School District	Number of Students Classified as Gifted and Talented	Mean (SD) of Gifted and Talented Students	Number of Students *Not* Classified as Gifted and Talented	Mean (SD) of Regular Education Students
Alief ISD	103	720.19 (79.73)	833	581.99 (177.28)
Brevard	8	814.38 (45.63)	261	676.40 (178.39)
Clark County	22	768.41 (128.02)	243	613.31 (171.18)
Killeen ISD	425	784.38 (134.03)	8,404	630.04 (164.50)

$p < .0300$

Special Education Classification

Special Education status was collected in two of the Phase I sites (Decatur and Harford) and five of the Phase II sites (Alief, Brevard, Cabarrus, Clark, and Kannapolis). An ANOVA was conducted for the data from each site, and a significant difference due to Special Education status was observed for four of the sites (Alief, Cabarrus, Clark, and Kannapolis; $p < .0001$). The differences between the mean SMI Quantile measures are as expected.

TABLE 29. Special Education status means and standard deviations for the SMI Quantile measures.

School District	Number of Students Classified as Special Education	Mean (SD) of Special Education Students	Number of Students *Not* Classified as Special Education	Mean (SD) of Non-Special Education Students
Alief ISD	89	452.64 (224.86)	1,364	575.74 (218.44)
Cabarrus County	94	423.03 (210.47)	899	563.55 (220.38)
Clark County	31	491.45 (227.79)	281	639.66 (184.85)
Kannapolis City	44	660.00 (204.64)	558	774.76 (183.40)

$p < .0001$

During Phase III, Special Education status information was collected from three of the seven sites (Alief, Brevard, and Killeen) for students in Grades 2–6. As expected, students classified as requiring Special Education services scored significantly lower than students not requiring Special Education services.

TABLE 30. Special Education status means and standard deviations for the SMI Quantile measures.

School District	Number of Students Classified as Special Education	Mean (SD) of Special Education Students	Number of Students *Not* Classified as Special Education	Mean (SD) of Non-Special Education Students
Alief ISD	44	411.93 (218.19)	892	606.33 (167.23)
Brevard	48	596.15 (215.03)	221	698.82 (163.02)
Killeen ISD	954	522.77 (189.50)	7,875	651.37 (157.91)

$p < .0002$

Development of Mathematics Understandings

Mathematical understandings generally increase as a student progresses through school. They increase rapidly during elementary school because students are specifically instructed in mathematics. In middle school, mathematical achievement tends to grow at a slower rate because students begin to develop at various levels. SMI was designed to assess the developmental nature of mathematical understandings.

Figure 20 shows the median performance on SMI for students at each location and grade level from the SMI Validation Study—Phase I conducted during the 2008–2009 school year.

Figures 21 and 22 show the mean performance on SMI for students at each location and grade level from the SMI Validation Study—Phase II conducted during the 2009–2010 school year.

Figures 23 and 24 show the mean performance on SMI for students at each location and grade level from the SMI Validation Study—Phase III conducted during the 2010–2011 school year.

As predicted, student scores on SMI climb rapidly in elementary grades and level off in middle school depending on the program being implemented (e.g., whole-class instruction versus remediation program). The developmental nature of mathematics is demonstrated in these results. Graphing the multiple sets of results in each chart displays that, in every study, the SMI scores are monotonically increasing as students move from grade to grade. This important characteristic supports the property of how the SMI Quantile scale was created as well as the progressive nature of mathematics.

FIGURE 20. SMI Validation Study, Phase I: SMI Quantile measures displayed by location and grade.

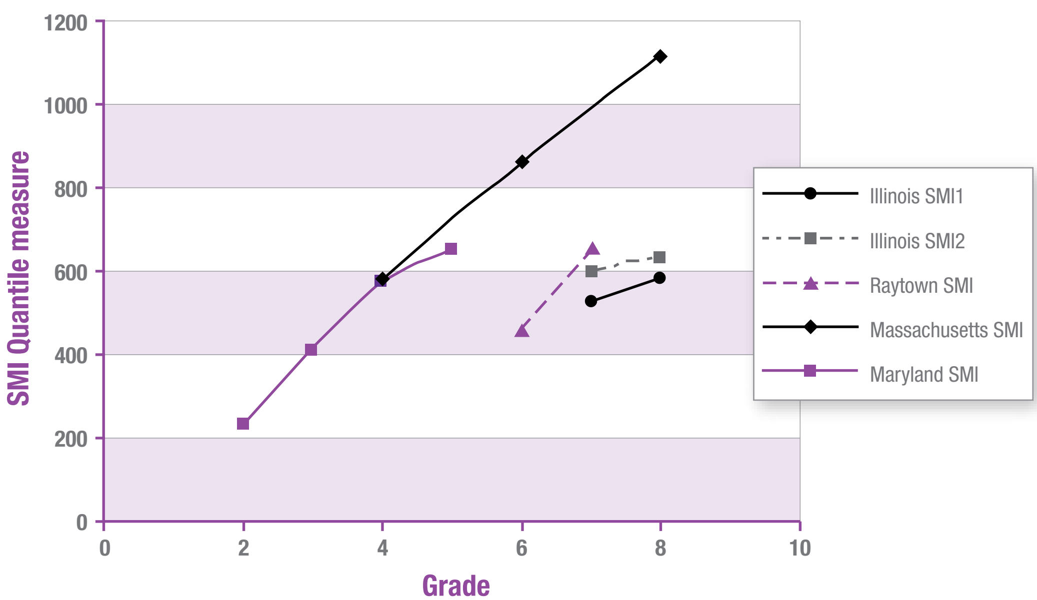


FIGURE 21. SMI Validation Study, Phase II: SMI Quantile measures displayed by location and grade.

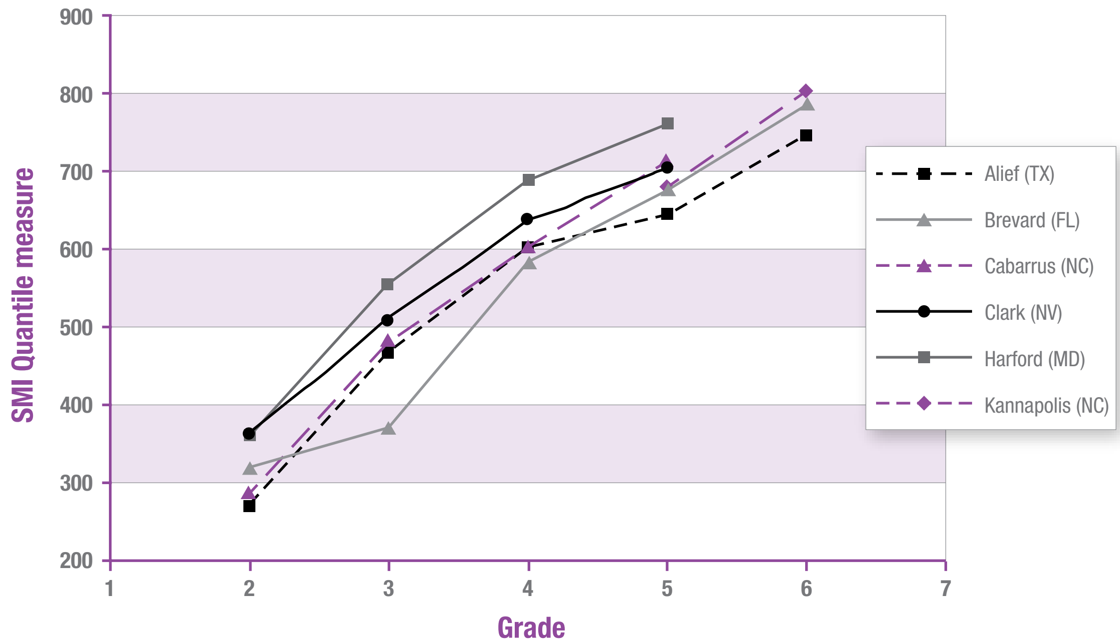


FIGURE 22. SMI Validation Study, Phase II: SMI Quantile measures displayed by grade.

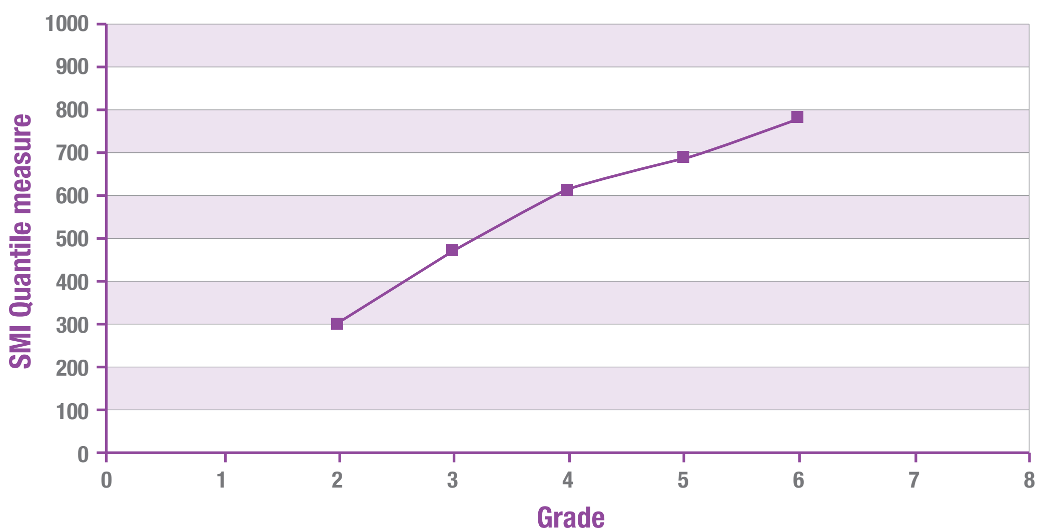


FIGURE 23. SMI Validation Study, Phase III: SMI Quantile measures displayed by location and grade.

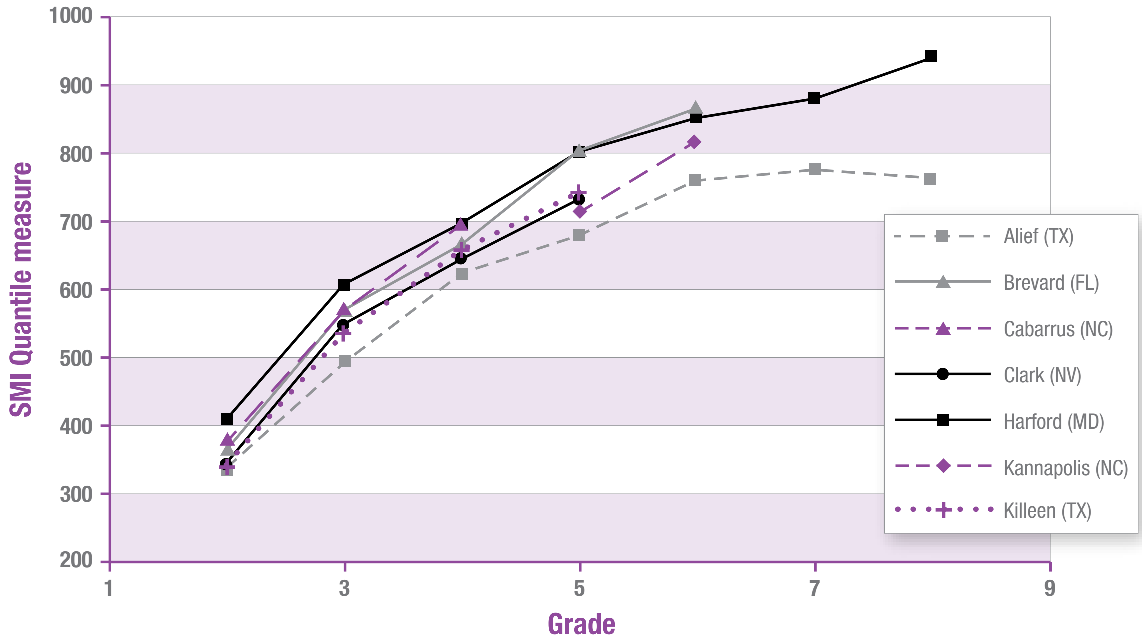
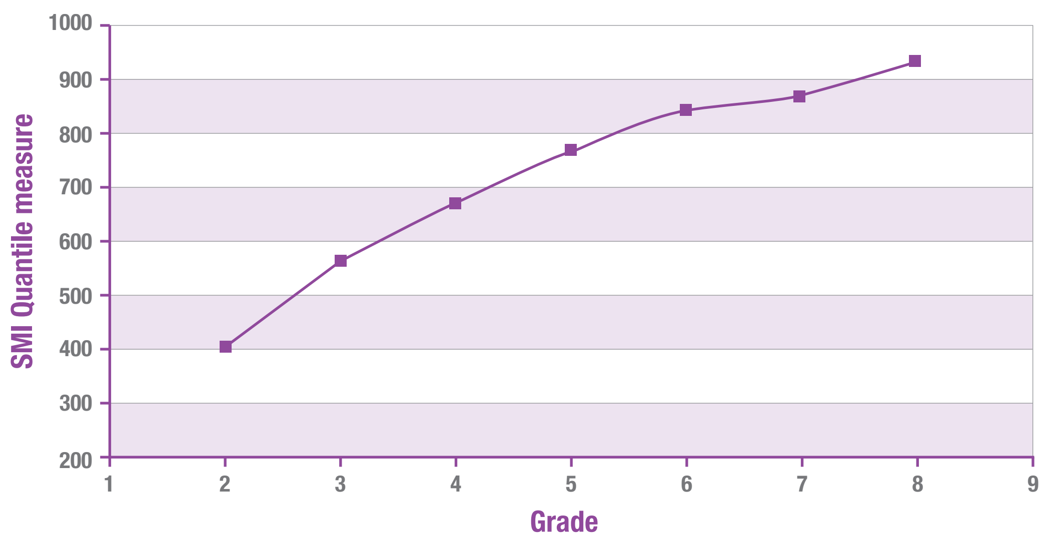


FIGURE 24. SMI Validation Study, Phase III: SMI Quantile measures displayed by grade.



The Relationship Between SMI Scores and State Math Assessment Scores

State mathematics assessment data was collected in two of the Phase I sites (Decatur and Harford) and all six of the Phase II sites (Alief, Brevard, Cabarrus, Clark, Harford, and Kannapolis). In general, SMI exhibited moderate correlations ($r = 0.60$ to 0.70) with state assessments of mathematics. The within-grade correlations and the overall across-grades correlation (where appropriate) are moderate, as expected, given the different mode of administration between the two tests (fixed, constant form for all students within a grade on the state assessments, as compared to the SMI, which is a computer-adaptive assessment that is tailored to each student's level of achievement).

Based on the data from Decatur (IL) Public School District described previously, the concurrent validity of SMI was examined. Because ISAT is on a vertical scale, the data were collapsed across grades for analysis. The following correlations indicate a strong relationship with the two administrations of the SMI Quantile measures. The ISAT 2008 and 2009 showed a moderate relationship with the SMI Quantile measures.

TABLE 31. Correlations among the Decatur Public School District test scores.

	SMI Quantile 1	SMI Quantile 2
SMI Quantile 1	-	-
SMI Quantile 2	0.81	-
ISAT 2008	0.60	0.61
ISAT 2009	0.60	0.62

During Phase I of the SMI Validation Study, in the Harford County Public Schools, the Maryland School Assessment (MSA) is administered to all students in Grades 3–8 in March. It is not on a vertical scale, so the data for the state test results could not be combined across grades. Both the mathematics and reading scores from the state test were available. For the state tests, Grades 2–3 did not have scores. Grade 4 had state scores for 2009, and Grade 5 had scores for both 2008 and 2009. The correlation between the SMI Quantile measures and the MSA mathematics scores are presented in Table 32.

TABLE 32. Correlations between SMI Quantile measures and Harford County Public Schools test scores.

	Grade 4	Grade 5
MSA 2008	-	0.67
MSA 2009	0.67	0.73

In the Alief Independent School District (Texas), students in Grades 3–6 were administered the Texas Assessment of Knowledge and Skills (TAKS) mathematics test on April 26, 2010, and students in Grades 3–6 also completed the SMI within six weeks (89.1% of students).

TABLE 33. Alief Independent School District—descriptive statistics for SMI Quantile measures and 2010 TAKS mathematics scores, by grade.

Grade	*N*	2010 TAKS Math Scale Scores Mean (SD)	SMI Quantile Measures Mean (SD)	*r*
3	281	541.97 (72.29)	455.98 (143.77)	0.61
4	383	608.84 (72.52)	573.50 (137.76)	0.53
5	204	646.81 (75.70)	641.96 (148.29)	0.52
6	224	687.73 (70.04)	723.19 (187.44)	0.63
All	**1,092**	**614.91 (88.96)**	**586.75 (179.11)**	**0.70**

Again during the spring of the 2010–2011 school year, students in Grades 3–8 in the Alief ISD were administered the Texas Assessment of Knowledge and Skills (TAKS) mathematics test, on April 4, 2011 (Grades 5 and 8), and April 26, 2011 (Grades 3, 4, 6, and 7), and students in Grades 3–8 also completed the SMI.

TABLE 34. Alief Independent School District—descriptive statistics for SMI Quantile measures and 2011 TAKS mathematics scores, by grade.

Grade	*N*	2011 TAKS Math Scale Scores Mean (SD)	SMI Quantile Measures Mean (SD)	*r*
3	348	576.10 (96.41)	493.54 (160.53)	0.69
4	401	648.60 (96.88)	629.98 (138.37)	0.63
5	85	658.64 (115.33)	683.74 (129.93)	0.37
6	90	717.41 (104.94)	761.96 (182.25)	0.70
7	283	698.04 (85.81)	774.31 (172.32)	0.58
8	134	709.91 (127.98)	766.24 (224.79)	0.38
All	**1,341**	**664.50 (115.69)**	**652.25 (197.63)**	**0.68**

In the Brevard Public Schools (Florida), students in Grades 3–6 were administered the mathematics part of the Florida Comprehensive Assessment Test (FCAT) between March 9 and 19, 2010, and students also completed the SMI within six weeks (97.0% of students).

TABLE 35. Brevard Public Schools—descriptive statistics for SMI Quantile measures and 2010 FCAT mathematics scores, by grade.

Grade	*N*	2010 FCAT Math Scale Scores Mean (SD)	SMI Quantile Measures Mean (SD)	*r*
3	5	1159.00 (270.07)	413.00 (101.71)	0.70
4	88	1550.00 (226.44)	522.44 (152.64)	0.63
5	87	1626.00 (187.80)	652.01 (129.03)	0.57
6	87	1738.00 (225.00)	782.01 (160.61)	0.69
All	**267**	**1629.00 (235.97)**	**647.19 (183.39)**	**0.69**

Again during the 2010–2011 school year, students in the Brevard Public Schools (Florida) in Grades 3–6 were administered the mathematics part of the Florida Comprehensive Assessment Test (FCAT) between April 11 and 20, 2011, and students also completed the SMI.

TABLE 36. Brevard Public Schools—descriptive statistics for SMI Quantile measures and 2011 FCAT mathematics scores, by grade.

Grade	*N*	2011 FCAT Math Scale Scores Mean (SD)	SMI Quantile Measures Mean (SD)	*r*
4	93	1543.00 (289.38)	579.41 (165.76)	0.80
5	93	1582.00 (237.55)	665.54 (151.65)	0.68
6	83	1759.00 (213.74)	810.54 (131.48)	0.79
7	88	1810.00 (247.78)	867.22 (194.91)	0.75
All	**357**	**1669.00 (273.13)**	**726.53 (198.68)**	**0.80**

In the Cabarrus County Schools (North Carolina), students in Grades 3–5 were administered the mathematics part of the North Carolina End-of-Grade Tests (NCEOG) during the last three weeks of the school year, and students also completed the SMI within six weeks (99.9% of students). The within-grade correlation for Grade 4 was lower than expected.

TABLE 37. Cabarrus County Schools—descriptive statistics for SMI Quantile measures and 2010 NCEOG mathematics scores, by grade.

Grade	*N*	2010 NCEOG Math Scale Scores Mean (SD)	SMI Quantile Measures Mean (SD)	*r*
3	283	347.20 (10.98)	583.83 (201.38)	0.56
4	279	351.54 (10.25)	655.34 (184.50)	0.38
5	276	358.19 (9.30)	793.12 (196.33)	0.74
All	**838**	**352.24 (11.16)**	**676.57 (212.53)**	**0.63**

Again during the 2010–2011 school year, students in the Cabarrus County Schools in Grades 3–5 were administered the mathematics part of the North Carolina End-of-Grade Tests (NCEOG) during the last three weeks of the school year, and students also completed the SMI.

TABLE 38. Cabarrus County Schools—descriptive statistics for SMI Quantile measures and 2011 NCEOG mathematics scores, by grade.

Grade	*N*	2011 NCEOG Math Scale Scores Mean (SD)	SMI Quantile Measures Mean (SD)	*r*
3	125	349.18 (10.29)	621.72 (193.29)	0.75
4	159	355.44 (9.75)	741.35 (180.15)	0.78
5	154	361.40 (8.56)	885.03 (202.70)	0.70
All	**438**	**355.81 (10.67)**	**667.82 (254.48)**	**0.80**

In the Clark County School District (Nevada), students in Grades 3–5 were administered the mathematics part of the criterion-referenced tests (CRT) between February 16 and March 16, 2010, and students also completed the SMI within seven to nine weeks (97.8% of students). The CRT is not reported on a vertical scale, so scores from Grades 3–5 cannot be combined for an overall correlation between CRT scale scores and SMI Quantile measures.

TABLE 39. Clark County School District—descriptive statistics for SMI Quantile measures and 2010 CRT mathematics scores, by grade.

Grade	*N*	2010 CRT Math Scale Scores Mean (SD)	SMI Quantile Measures Mean (SD)	*r*
3	95	323.76 (64.36)	445.58 (173.49)	0.60
4	102	331.30 (46.03)	608.04 (146.37)	0.66
5	115	351.96 (84.52)	698.00 (173.57)	0.74

During Phase II, in the Harford County Public Schools (Maryland), students in Grades 3–5 were administered the mathematics part of the Maryland School Assessment (MSA) between March 8 and 23, 2010, and students also completed the SMI within six weeks (100.0% of students). The MSA is not reported on a vertical scale, so scores from Grades 3–5 cannot be combined for an overall correlation between MSA scale scores and SMI Quantile measures.

Again during the 2010–2011 school year, students in the Clark County School District in Grades 3–5 were administered the mathematics part of the criterion-referenced tests (CRT) on March 7, 2011, and students also completed the SMI.

TABLE 40. Clark County School District—descriptive statistics for SMI Quantile measures and 2011 CRT mathematics scores, by grade.

Grade	*N*	2011 CRT Math Scale Scores Mean (SD)	SMI Quantile Measures Mean (SD)	*r*
3	101	346.27 (64.33)	544.80 (151.32)	0.66
4	103	328.45 (50.68)	644.27 (171.71)	0.73
5	61	345.43 (84.95)	730.41 (144.83)	0.46

TABLE 41. Harford County Public Schools—descriptive statistics for SMI Quantile measures and 2010 MSA mathematics scores, by grade.

Grade	*N*	2010 MSA Math Scale Scores Mean (SD)	SMI Quantile Measures Mean (SD)	*r*
3	58	416.45 (34.15)	515.52 (167.28)	0.63
4	64	435.83 (33.40)	656.56 (140.56)	0.56
5	65	435.65 (37.57)	705.00 (178.80)	0.80

During Phase III (2010–2011 school year) in the Harford County Public Schools, students in Grades 3–5 were administered the mathematics part of the Maryland School Assessment (MSA) between March 8 and 16, 2011, and students also completed the SMI.

TABLE 42. Harford County Public Schools—descriptive statistics for SMI Quantile measures and 2011 MSA mathematics scores, by grade.

Grade	*N*	2011 MSA Math Scale Scores Mean (SD)	SMI Quantile Measures Mean (SD)	*r*
3	2,768	424.04 (37.07)	603.43 (201.73)	0.75
4	2,531	435.04 (37.87)	694.20 (165.71)	0.72
5	2,711	427.67 (31.84)	799.74 (182.14)	0.73
6	2,024	436.07 (37.24)	849.04 (203.77)	0.75
7	2,086	427.48 (34.89)	877.80 (219.64)	0.79
8	1,965	432.71 (35.56)	944.60 (230.80)	0.80

As in Cabarrus County Schools, students in Grades 5–6 in Kannapolis City Schools (North Carolina) were administered the mathematics part of the North Carolina End-of-Grade Tests (NCEOG) during the last three weeks of the school year, and students also completed the SMI within six weeks (93.0% of students).

TABLE 43. Kannapolis City Schools—descriptive statistics for SMI Quantile measures and 2010 NCEOG mathematics scores, by grade.

Grade	*N*	2010 NCEOG Math Scale Scores Mean (SD)	SMI Quantile Measures Mean (SD)	*r*
5	309	352.86 (8.51)	693.01 (172.44)	0.64
6	320	358.06 (8.32)	814.16 (174.10)	0.69
All	**629**	**355.51 (8.80)**	**754.64 (183.45)**	**0.70**

During Phase III of the SMI Validation Study (2010–2011 school year), students in Grades 5–6 in Kannapolis City Schools were administered the mathematics part of the North Carolina End-of-Grade Tests (NCEOG) during the last three weeks of the school year, and students also completed the SMI.

TABLE 44. Kannapolis City Schools—descriptive statistics for SMI Quantile measures and 2011 NCEOG mathematics scores, by grade.

Grade	*N*	2011 NCEOG Math Scale Scores Mean (SD)	SMI Quantile Measures Mean (SD)	*r*
5	331	353.15 (8.76)	711.01 (214.20)	0.75
6	321	357.44 (8.28)	813.37 (200.14)	0.75
All	**652**	**355.25 (8.79)**	**761.51 (213.46)**	**0.77**

During the spring of the 2010–2011 school year, students in Grades 3–8 in Killeen ISD (Texas) were administered the Texas Assessment of Knowledge and Skills (TAKS) mathematics test on April 4, 2011 (Grades 5 and 8), and April 26, 2011 (Grades 3, 4, 6, and 7), and students also completed the SMI.

TABLE 45. Killeen Independent School District—descriptive statistics for SMI Quantile measures and 2011 TAKS mathematics scores, by grade.

Grade	*N*	2011 TAKS Math Scale Scores Mean (SD)	SMI Quantile Measures Mean (SD)	*r*
3	2,801	581.88 (88.86)	531.44 (147.61)	0.65
4	2,713	649.67 (89.31)	654.73 (135.79)	0.61
5	2,483	708.15 (83.89)	740.24 (138.02)	0.53
All	**7,997**	**644.54 (101.56)**	**637.47 (166.46)**	**0.71**

Growth in Mathematical Understanding

"In the simplest terms, growth is change over time. To study growth, we measure a thing repeatedly on successive occasions and draw conclusions about how it has changed" (Williamson, 2006). Growth in mathematical understanding can be determined by examining the changes in SMI Quantile measures (an equal-interval scale).

Using the data collected during the SMI Validation Study—Phase II from six school districts, a data panel was created consisting of students with two or more SMI Quantile measures. A total of 4,116 students were included in the dataset. Table 46 describes the composition of the whole sample and Table 47 describes the composition of the sample with three or more SMI Quantile measures.

TABLE 46. Description of longitudinal panel across districts, by grade.

Grade	*N*	Mean Number of SMI Administrations	Mean Number of Days Between SMI Administrations
2	533	2.6	85.3
3	843	2.6	80.6
4	1,001	2.6	82.6
5	1,062	2.4	91.4
6	677	2.7	80.0
All	**4,116**	**2.6**	**84.2**

TABLE 47. Description of longitudinal panel across districts, by grade for students with at least three Quantile measures.

Grade	*N*	Mean Number of SMI Administrations	Mean Number of Days Between SMI Administrations
2	293	3.0	84.3
3	462	3.0	78.0
4	594	3.0	79.7
5	431	3.1	76.4
6	471	3.0	70.4
All	**2,251**	**3.0**	**77.4**

Given the short time span between initial and final assessments, a linear regression model was employed to examine growth in mathematical understanding from the SMI Quantile measures. Of the panel, students with three or more SMI Quantile measures ($N = 2{,}251$, 51.4%) were used in the regression analyses. The slope of the linear regression for each student describes the amount of growth in mathematical understanding per day. The growth estimated from the SMI Quantile measures ranges from 0.5529 to 0.6790, with a mean (weighted) of 0.6138; or, the expected growth in mathematical understanding, as measured by the SMI, is about 0.6Q per day (see Table 48). Across grade levels, the R^2 statistics describing the percent of variance in SMI Quantile measures accounted for by the linear model ranged from 52.3% in Grade 5 to 62.9% in Grade 2. Across Grades 2–6, the mean slope estimate is significantly different from zero ($p < 0.0001$).

TABLE 48. Results of regression analyses for longitudinal panel, across grades.

Grade	N	Slope of Linear Regression Mean (SD)	R^2
2	293	0.6227 (0.7929)	0.6288
3	465	0.5853 (0.8690)	0.5603
4	594	0.6241 (0.8018)	0.5564
5	431	0.5529 (0.9106)	0.5231
6	471	0.6790 (1.0994)	0.5531

The results in Table 48 describe the amount of growth that can be expected across a school year, which is consistent across Grades 2–6 (approximately 0.6Q per day or approximately 108Q per year). These results are consistent with other research conducted by MetaMetrics, with the Quantile Framework showing a near-linear trend between grade level and mathematics achievement as measured on the Quantile scale.

During Phase III of the SMI Validation Study, growth was examined during the 2010–2011 school year. Data collected during the fall administration of SMI was matched with data collected during the spring administration for a sample of 10,178 students in Grades 2–6 from the seven sites. The average growth per day was estimated as 0.67Q (or 120.6Q per year) and ranged from 0.44Q per day in Grade 5 to 0.90Q per day in Grade 3 (the average growth during the school year was 155.998Q and ranged from 101Q to 208Q).

A subsample of 752 students in Grades 2–6 from four sites (Alief, Brevard, Clark, and Harford) had data from both the 2009–2010 school year and the 2010–2011 school year. This group of students grew, on average, 242Q over the two school years (102Q in Year 1 and 153Q in Year 2). As expected, a negative correlation was observed between the students' initial status (SMI Quantile measure in Winter 2009) and the amount grown over the two school years ($r = -.486$, $p < .0001$). This negative correlation is consistent with the interpretation that lower-performing students typically grow more than higher-performing students.

Additional Aspects of Construct-Identification Validity

Discriminate validity looks at the relationships between test scores and other criterion variables that the scores should not be related to (e.g., gender, race/ethnicity). Scores on assessments of mathematics ability are expected to fluctuate according to some demographic characteristics of the students taking the test.

Gender

Gender information was collected in three of the Phase I sites (Decatur, Harford, and Raytown) and six of the Phase II sites (Alief, Brevard, Cabarrus, Clark, Harford, and Kannapolis). An ANOVA was conducted for the data from each site and a significant difference due to gender was not observed for any of the sites.

During Phase III of the SMI Validation Study, gender data were collected in the seven sites. An ANOVA was conducted for the data from each site (results are presented in Table 22) and a significant difference due to gender was observed in two of the seven sites (Alief, $p < .0478$; and Harford, $p < .0015$). The differences between the means ranged from +22.97 (females performing higher than males) to −12.54 (males performing higher than females). As expected, there is no clear pattern in the differences in performance of males and females on SMI.

Race/Ethnicity

Race/Ethnicity information was collected in three of the Phase I sites (Decatur, Harford, and Raytown) and five of the Phase II sites (Brevard, Cabarrus, Clark, Harford, and Kannapolis). An ANOVA was conducted for the data from each site (results are presented in Table 20) and a significant difference due to race/ethnicity was observed for three of the sites (Cabarrus, $p < .0001$; Clark, $p < .0001$; and Kannapolis, $p < .0001$). The differences between the mean SMI Quantile measures are as expected.

During Phase III of the SMI Validation Study, race/ethnicity information was collected in all seven of the sites. An ANOVA was conducted for the data from each site (results are presented in Table 23) and a significant difference due to race/ethnicity was observed for six of the seven sites (Alief, $R^2 = 0.058$, $p < .0001$; Cabarrus, $R^2 = 0.057$, $p < .0001$; Clark, $R^2 = 0.085$, $p < .0009$; Harford, $R^2 = 0.026$, $p < .0001$; Kannapolis, $R^2 = 0.031$, $p < .0009$; and Killeen, $R^2 = 0.015$, $p < .0001$). The differences between the mean SMI Quantile measures are as expected.

Language Proficiency

Language proficiency is defined by participation in a bilingual instruction program or classification as English language learner (ELL), English as a second language (ESL), or limited English proficiency (LEP). Data was collected in one of the Phase I sites (Harford) and four of the Phase II sites (Alief, Cabarrus, Clark, and Kannapolis). For bilingual status, an ANOVA was conducted for the data from Alief and a significant difference was not observed.

For bilingual status, an ANOVA was conducted for the Phase II data from Alief and a significant difference was not observed. During Phase III, bilingual status was collected from two sites (Alief and Brevard) for students in Grades 2–6. While a significant difference was observed for Alief, the level of significance is not strong and the differences between the mean SMI Quantile measures are as expected.

TABLE 49. Alief Independent School District—bilingual means and standard deviations for the SMI Quantile measures.

Yes		No	
N	Mean (SD)	*N*	Mean (SD)
249	577.83 (155.12)	687	604.21 (180.92)

$p < .0412$

For ELL, ESL, and LEP status, an ANOVA was conducted for the data from each site and a significant difference due to language proficiency classification was observed for four of the sites (ELL: Cabarrus, $p < .0001$; ESL: Alief, $p < .01$; LEP: Alief, Kannapolis, $p < .0001$; Clark, $p < .01$). The differences between the mean SMI Quantile measures are as expected.

TABLE 50. Cabarrus County Schools—English language learners (ELL) means and standard deviations for the SMI Quantile measures.

Yes		No	
N	Mean (SD)	*N*	Mean (SD)
75	393.67 (205.27)	918	563.04 (219.81)

$p < .0001$

TABLE 51. Alief ISD—English as a second language (ESL) means and standard deviations for the SMI Quantile measures.

0		1	
N	Mean (SD)	*N*	Mean (SD)
1,260	570.48 (220.42)	375	529.67 (212.43)

$p < .01$

TABLE 52. Alief ISD—limited English proficiency (LEP) status means and standard deviations for the SMI Quantile measures.

Not-LEP		LEP		Exited LEP—Year 1		Exited LEP—Year 2	
N	Mean (SD)	*N*	Mean (SD)	*N*	Mean (SD)	*N*	Mean (SD)
120	748.12 (168.07)	414	501.87 (186.68)	130	632.19 (158.50)	105	755.05 (134.55)

$p < .0001$

TABLE 53. Clark County School District—limited English proficiency (LEP) status means and standard deviations for the SMI Quantile measures.

Yes		No	
N	Mean (SD)	*N*	Mean (SD)
5	398.00 (307.73)	307	628.63 (190.44)

$p < .01$

TABLE 54. Kannapolis City Schools—limited English proficiency (LEP) status means and standard deviations for the SMI Quantile measures.

Yes		No	
N	Mean (SD)	*N*	Mean (SD)
66	660.45 (194.23)	544	776.16 (184.62)

$p < .0001$

For ELL, ESL, and LEP status during Phase III, an ANOVA was conducted for the Grades 2–6 data from each site, and a significant difference due to language proficiency classification was observed for three of the sites (ELL: Harford, $p < .0001$; LEP: Kannapolis, $p < .0152$; Killeen, $p < .0001$). The differences between the mean SMI Quantile measures are as expected.

TABLE 55. Harford County Public Schools—English language learners (ELL) means and standard deviations for the SMI Quantile measures.

Yes		No	
N	Mean (SD)	*N*	Mean (SD)
287	586.24 (239.28)	12,446	666.93 (239.62)

$p < .0001$

TABLE 56. Kannapolis City Schools—limited English proficiency (LEP) means and standard deviations for the SMI Quantile measures.

Yes		No	
N	Mean (SD)	*N*	Mean (SD)
109	716.10 (206.92)	568	770.22 (213.77)

$p < .0152$

TABLE 57. Killeen ISD—limited English proficiency (LEP) means and standard deviations for the SMI Quantile measures.

Yes		No	
N	Mean (SD)	*N*	Mean (SD)
794	587.69 (168.30)	8,035	642.39 (165.48)

$p < .0001$

Economic Status

Economic status of students was defined by being classified as economically disadvantaged in the free or reduced-price lunch (FRPL) program.

FRPL status was collected in one of the Phase I sites (Decatur) and two of the Phase II sites (Alief and Clark). An ANOVA was conducted for the data from each site, and a significant difference due to FRPL status was observed for one of the sites (Alief, $p < .01$). The differences between the mean SMI Quantile measures are as expected.

TABLE 58. Alief ISD—Economically disadvantaged means and standard deviations for the SMI Quantile measures.

Free		Reduced		No	
N	Mean (SD)	*N*	Mean (SD)	*N*	Mean (SD)
1,299	551.55 (218.30)	127	617.64 (200.96)	209	586.24 (229.15)

$p < .01$

Economically disadvantaged classification information was collected from two Phase II sites (Brevard and Harford). An ANOVA was conducted for the data from each site and a significant difference due to FRPL status was observed for one of the sites (Harford, $p < .01$). The differences between the mean SMI Quantile measures are as expected.

TABLE 59. Harford County Public Schools—Economically disadvantaged means and standard deviations for the SMI Quantile measures.

Yes		No	
N	Mean (SD)	*N*	Mean (SD)
105	564.29 (225.34)	151	620.50 (229.86)

$p < .01$

During Phase III of the SMI Validation Study, economically disadvantaged classification information was collected from three of the seven sites (Brevard, Harford, and Killeen). An ANOVA was conducted for the data from each site, and a significant difference due to FRPL status was observed for one of the sites (Killeen, $p < .01$). The differences between the mean SMI Quantile measures are as expected.

TABLE 60. Killeen Independent School District—Economically disadvantaged means and standard deviations for the SMI Quantile measures.

Yes		No	
N	Mean (SD)	*N*	Mean (SD)
4,431	620.68 (165.35)	4,398	654.39 (165.89)

$p < .01$

Relationship Between SMI Quantile Measures and Assessments of Reading Comprehension

State mathematics assessment data was collected in two of the Phase I sites (Decatur and Harford) and all six of the Phase II sites (Alief, Brevard, Cabarrus, Clark, Harford, and Kannapolis).

Interim reading assessment information was collected in one site during Phase I of the validation study. The Maryland School Assessment (MSA) is administered to all students in Grades 3–8 in March. Additionally, Harford County Public Schools had the *Scholastic Reading Inventory* (SRI) test scores available for this study. The reporting score for the SRI is the Lexile measure. This score is on a vertical scale so these analyses were computed across grades. The correlations among the SMI Quantile measures and SRI Lexile measures from 2009 ranged from 0.42 to 0.62 (Grade 2, $r = 0.62$; Grade 3, $r = 0.42$; Grade 4, $r = 0.47$; and Grade 5, $r = 0.47$), respectively, indicating that the scores are minimally correlated.

State reading assessment results were collected from five of the six sites during Phase II of the validation study. Table 61 presents the correlations between TAKS reading scale scores and SMI Quantile measures in Alief Independent School District. Table 62 presents the correlations between FCAT reading scale scores and SMI Quantile measures in Brevard Public Schools. Tables 63 and 64 present the correlations between NCEOG reading scale scores and the SMI Quantile measures. As expected, all of the correlations are low and are lower than the correlations between state math assessments results and SMI Quantile measures.

The one exception to this trend is the result for Clark County School District. Table 65 presents the correlations between CRT reading scale scores and SMI Quantile measures for Clark County School District. The CRT is not reported on a vertical scale, so scores from Grades 3 through 5 cannot be combined for an overall correlation between CRT scale scores and SMI Quantile measures. The correlations between the CRT reading scale scores and the SMI Quantile measures are higher than expected and closely approximate the correlations between the CRT math scale scores and the SMI Quantile measures.

TABLE 61. Alief Independent School District—descriptive statistics for SMI Quantile measures and 2010 TAKS reading scores, by grade.

Grade	*N*	2010 TAKS Reading Scale Scores Mean (SD)	SMI Quantile Measures Mean (SD)	*r*
3	309	600.28 (84.34)	485.32 (145.67)	0.53
4	448	626.45 (77.64)	607.96 (142.87)	0.43
5	222	683.59 (61.94)	677.73 (142.89)	0.39
6	236	716.76 (80.93)	750.83 (186.38)	0.45
All	**1,215**	**647.78 (88.83)**	**617.26 (178.72)**	**0.58**

TABLE 62. Brevard Public Schools—descriptive statistics for SMI Quantile measures and 2010 FCAT reading scores, by grade.

Grade	*N*	2010 FCAT Reading Scale Scores Mean (SD)	SMI Quantile Measures Mean (SD)	*r*
3	5	917.60 (282.50)	413.00 (101.71)	0.50
4	88	1466.00 (240.14)	522.44 (152.64)	0.50
5	87	1656.00 (240.76)	652.01 (129.03)	0.31
6	87	1720.00 (277.79)	782.01 (160.61)	0.56
All	**267**	**1600.00 (290.21)**	**647.19 (183.39)**	**0.59**

TABLE 63. Cabarrus County Schools—descriptive statistics for SMI Quantile measures and 2010 NCEOG reading scores, by grade.

Grade	*N*	2010 NCEOG Reading Scale Scores Mean (SD)	SMI Quantile Measures Mean (SD)	*r*
3	282	344.47 (11.08)	583.37 (201.59)	0.32
4	279	347.84 (9.09)	655.34 (184.50)	0.20
5	276	352.19 (9.34)	793.12 (196.33)	0.63
All	**837**	**348.12 (10.37)**	**676.52 (212.66)**	**0.45**

TABLE 64. Kannapolis City Schools—descriptive statistics for SMI Quantile measures and 2010 NCEOG reading scores, by grade.

Grade	*N*	2010 NCEOG Reading Scale Scores Mean (SD)	SMI Quantile Measures Mean (SD)	*r*
5	309	348.28 (8.51)	693.01 (172.44)	0.53
6	320	354.09 (7.80)	814.16 (174.10)	0.56
All	**629**	**351.24 (8.65)**	**754.64 (183.45)**	**0.59**

TABLE 65. Clark County School District—descriptive statistics for SMI Quantile measures and 2010 CRT reading scores, by grade.

Grade	*N*	2010 CRT Reading Scale Scores Mean (SD)	SMI Quantile Measures Mean (SD)	*r*
3	95	330.32 (60.23)	445.58 (173.49)	0.61
4	102	348.19 (76.64)	608.04 (146.37)	0.60
5	115	324.32 (60.19)	698.00 (173.57)	0.59

State reading assessment results were collected from one of the seven sites during Phase III of the validation study (Alief). Table 66 presents the correlations between TAKS reading scale scores and SMI Quantile measures in Alief Independent School District for a sample of Grades 3 and 4 students. As expected, the correlations are low and are lower than the correlations between state math assessments results and SMI Quantile measures.

TABLE 66. Alief Independent School District—descriptive statistics for SMI Quantile measures and 2011 TAKS reading scores, by grade.

Grade	*N*	2011 TAKS Reading Scale Scores Mean (SD)	SMI Quantile Measures Mean (SD)	*r*
3	110	593.83 (98.92)	493.87 (160.53)	0.57
4	115	646.32 (103.96)	629.98 (138.37)	0.40
All	**225**	**620.00 (104.67)**	**652.25 (197.63)**	**0.54**