Assignment 1

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Gaussian function:

$$G(x, y; \sigma) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$

Scale-normalized Laplacian of Gaussian:

$$LoG = \sigma^2 \nabla^2 G$$

Difference of Gaussian:

$$DoG = G(x, y; k\sigma) - G(x, y; \sigma)$$

First, we can compute partial differential according to the Gaussian function

$$\frac{\partial^2 G}{\partial x^2} = \frac{x^2 - \sigma^2}{2\pi\sigma^6} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$

$$\frac{\partial^2 G}{\partial y^2} = \frac{y^2 - \sigma^2}{2\pi\sigma^6} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$

$$\frac{\partial G}{\partial \sigma} = \frac{x^2 + y^2 - 2\sigma^2}{2\pi\sigma^5} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$

Then we can see that

$$\nabla^2 G = \frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2} = \frac{x^2 + y^2 - 2\sigma^2}{2\pi\sigma^6} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$

And therefore

$$LoG = \sigma^2 \nabla^2 G = \frac{x^2 + y^2 - 2\sigma^2}{2\pi\sigma^4} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right) = \sigma \frac{\partial G}{\partial \sigma}$$

We know that

$$\frac{\partial G}{\partial \sigma} \approx \frac{G(x, y; k\sigma) - G(x, y; \sigma)}{k\sigma - \sigma} = \frac{DoG}{(k-1)\sigma}$$

Finally, we can get that

$$DoG \approx (k-1)\sigma \frac{\partial G}{\partial \sigma} = (k-1) LoG$$

So the conclusion is DoG can be a good approximation of LoG.