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Homework 2

Problem A

Rectangular clipping region with min $(111, 22)$ and max $(222, 149)$ and a subject line through the vertices $P_1 = (0, 0)$ and $P_2 = (246, 333)$. The linear interpolation equation for the line P is $P(t) = P_1 + (P_2 - P_1)t$.

The edge vertices for the rectangular clipping region are as follows:

$$P_a = (x_a, y_a) = (111, 22)$$
$$P_b = (x_b, y_b) = (111, 149)$$
$$P_c = (x_c, y_c) = (222, 149)$$
$$P_d = (x_d, y_d) = (222, 22)$$

The normal will be defined below:

$$n_a = \big(-(y_b - y_a), (x_b - x_a)\big) = \big(-(149 - 22), (111 - 111)\big) = (-127, 0)$$
$$n_b = \big(-(y_c - y_b), (x_c - x_b)\big) = \big(-(149 - 149), (222 - 111)\big) = (0, 111)$$
$$n_c = \big(-(y_d - y_c), (x_d - x_c)\big) = \big(-(22 - 149), (222 - 222)\big) = (127, 0)$$
$$n_d = \big(-(y_a - y_d), (x_a - x_d)\big) = \big(-(22 - 22), (111 - 222)\big) = (0, -111)$$

The t values will be shown below:

$$t_a = \frac{n_a \cdot (P_a - P_1)}{n_a \cdot (P_2 - P_1)} = \frac{(-127, 0) \cdot \big((111, 22) - (0, 0)\big)}{(-127, 0) \cdot \big((246, 333) - (0, 0)\big)} = \frac{(-127, 0) \cdot (111, 22)}{(-127, 0) \cdot (246, 333)}$$
$$= \frac{(-127)(111) + (0)(22)}{(-127)(246) + (0)(333)} = \frac{-14{,}097}{-31{,}242} = 0.45121951$$
$$t_b = \frac{n_b \cdot (P_b - P_1)}{n_b \cdot (P_2 - P_1)} = \frac{(0, 111) \cdot \big((111, 149) - (0, 0)\big)}{(0, 111) \cdot \big((246, 333) - (0, 0)\big)} = \frac{(0, 111) \cdot (111, 149)}{(0, 111) \cdot (246, 333)}$$
$$= \frac{(0)(111) + (111)(149)}{(0)(246) + (111)(333)} = \frac{16{,}539}{36{,}963} = 0.44744745$$
$$t_c = \frac{n_c \cdot (P_c - P_1)}{n_c \cdot (P_2 - P_1)} = \frac{(127, 0) \cdot \big((222, 149) - (0, 0)\big)}{(127, 0) \cdot \big((246, 333) - (0, 0)\big)} = \frac{(127, 0) \cdot (222, 149)}{(127, 0) \cdot (246, 333)}$$
$$= \frac{(127)(222) + (0)(149)}{(127)(246) + (0)(333)} = \frac{28{,}194}{31{,}242} = 0.90243902$$

$$t_d = \frac{n_d \cdot (P_d - P_1)}{n_d \cdot (P_2 - P_1)} = \frac{(0,-111) \cdot \big((222,22) - (0,0)\big)}{(0,-111) \cdot \big((246,333) - (0,0)\big)} = \frac{(0,-111) \cdot (222,22)}{(0,-111) \cdot (246,333)}$$
$$= \frac{(0)(222) + (-111)(22)}{(0)(246) + (-111)(333)} = \frac{-2{,}442}{-36{,}963} = 0.06606607$$

Plugging in the t values back into the equation $P(t)$ yields:

$$P(0.45121951) = (0,0) + \big((246,333) - (0,0)\big)(0.45121951)$$
$$= \big((0.45121951)(246), (0.45121951)(333)\big) = (110.999999,\ \ 150.256097)$$
$$P(0.44744745) = (0,0) + \big((246,333) - (0,0)\big)(0.44744745)$$
$$= \big((0.44744745)(246), (0.44744745)(333)\big) = (110.072073,\ \ 149.000001)$$
$$P(0.90243902) = (0,0) + \big((246,333) - (0,0)\big)(0.90243902)$$
$$= \big((0.90243902)(246), (0.90243902)(333)\big) = (221.999999,\ \ 300.512194)$$
$$P(0.06606607) = (0,0) + \big((246,333) - (0,0)\big)(0.06606607)$$
$$= \big((0.06606607)(246), (0.06606607)(333)\big) = (16.2522532,\ \ 22.0000013)$$

Unfortunately, the steps taken are from the lecture notes and they appear to be of no help when trying to see which t values are appropriate for clipping to the following vertices:

$$P_a = (x_a, y_a) = (111, 22)$$
$$P_b = (x_b, y_b) = (111, 149)$$
$$P_c = (x_c, y_c) = (222, 149)$$
$$P_d = (x_d, y_d) = (222, 22)$$

So, in order to solve this problem, I will solve the problem using a method I found on YouTube which hopefully will give me an answer to this problem.

$P(t) = P_1 + (P_2 - P_1)t$ is the equation of the line. More formally, the equation is $P(t) = (246, 333)t$ since P_1 is (0, 0) which will have no effect on the values of t since it is zero for both the x and y coordinate. We can break this equation up into the x and y coordinates as shown below:

$$x(t) = 246t$$
$$y(t) = 333t$$

The only values for x and y that the line can be clipped to are 111, 222 for the x values and 22 and 149 for the y values based on points a through b shown above. Therefore, the values of t are as follows:

$$t = \frac{111}{246}$$
$$t = \frac{222}{246}$$
$$t = \frac{22}{333}$$

$$t = \frac{149}{333}$$

Plugging this back in to the original equation yields:

$$P\left(\frac{111}{246}\right) = \left(\left(246 \cdot \frac{111}{246}\right), \left(333 \cdot \frac{111}{246}\right)\right) = (111, \;\; 150.25609756097561)$$
$$P\left(\frac{222}{246}\right) = \left(\left(246 \cdot \frac{222}{246}\right), \left(333 \cdot \frac{222}{246}\right)\right) = (222, \;\; 300.51219512195122)$$
$$P\left(\frac{22}{333}\right) = \left(\left(246 \cdot \frac{22}{333}\right), \left(333 \cdot \frac{22}{333}\right)\right) = (16.252252252252252, \;\; 22)$$
$$P\left(\frac{149}{333}\right) = \left(\left(246 \cdot \frac{149}{333}\right), \left(333 \cdot \frac{149}{333}\right)\right) = (110.072072072072072, \;\; 149)$$

Comparing with the original points:

$$P_a = (x_a, y_a) = (111, 22)$$
$$P_b = (x_b, y_b) = (111, 149)$$
$$P_c = (x_c, y_c) = (222, 149)$$
$$P_d = (x_d, y_d) = (222, 22)$$

And although the results are the same as the method in the lecture notes, I am now confident my numbers are correct. It appears that the line cannot be clipped to the viewing rectangle since at every point for the values of t, the line is outside of the viewing rectangle.

Problem B

The coordinates for the four corners of the viewing rectangle are the same as in Part A:

$$P_a = (x_a, y_a) = (111, 22)$$
$$P_b = (x_b, y_b) = (111, 149)$$
$$P_c = (x_c, y_c) = (222, 149)$$
$$P_d = (x_d, y_d) = (222, 22)$$

The regions are as follows:

$$A = (81, 11) = TFFT$$
$$B = (248, 210) = FTTF$$
$$C = (44, 33) = TFFF$$

Problem C

Vertices for the polygon:

$$X_a = (10, 10)$$
$$X_b = (100, 20)$$
$$X_c = (200, 200)$$

$$X_d = (20, 150)$$

The coordinates for the four corners of the viewing rectangle are the same as in Part A:

$$P_a = (x_a, y_a) = (111, 22)$$
$$P_b = (x_b, y_b) = (111, 149)$$
$$P_c = (x_c, y_c) = (222, 149)$$
$$P_d = (x_d, y_d) = (222, 22)$$

$X(t) = X_1 + (X_2 - X_1)t$ is the equation of the line. We can break this equation up into the x and y coordinates as shown below:

$$x(t) = x_1 + (x_2 - x_1)t$$
$$y(t) = y_1 + (y_2 - y_1)t$$

The equations for t are as follows:

$$t = \frac{x(t) - x_1}{(x_2 - x_1)}$$
$$t = \frac{y(t) - y_1}{(y_2 - y_1)}$$

For the line $\boldsymbol{X_a} = (10, 10) \rightarrow \boldsymbol{X_b} = (100, 20)$

$$X(t) = X_1 + (X_2 - X_1)t$$
$$X(t) = (10, 10) + \big((100, 20) - (10, 10)\big)t$$
$$X(t) = (10, 10) + (90, 10)t$$

The only values for x and y that the line can be clipped to are 111, 222 for the x values and 22 and 149 for the y values based on points a through d from the viewing rectangle. Therefore, the values of t are as follows:

$$t = \frac{x(t) - x_1}{(x_2 - x_1)} = \frac{111 - 10}{(100 - 10)} = \frac{101}{90}$$
$$t = \frac{x(t) - x_1}{(x_2 - x_1)} = \frac{222 - 10}{(100 - 10)} = \frac{212}{90}$$
$$t = \frac{y(t) - y_1}{(y_2 - y_1)} = \frac{22 - 10}{(20 - 10)} = \frac{12}{10}$$
$$t = \frac{y(t) - y_1}{(y_2 - y_1)} = \frac{149 - 10}{(20 - 10)} = \frac{139}{10}$$

We only consider the values of t that are $0 \leq t \leq 1$, so since all of the values of t are greater than 1, this line can't be clipped to the viewing rectangle.

For the line $\boldsymbol{X_b} = (100, 20) \rightarrow \boldsymbol{X_c} = (200, 200)$

$$X(t) = X_1 + (X_2 - X_1)t$$

$$X(t) = (100, 20) + \big((200, 200) - (100, 20)\big)t$$
$$X(t) = (100, 20) + (100, 180)t$$

The only values for x and y that the line can be clipped to are 111, 222 for the x values and 22 and 149 for the y values based on points a through d from the viewing rectangle. Therefore, the values of t are as follows:

$$t = \frac{x(t) - x_1}{(x_2 - x_1)} = \frac{111 - 100}{(200 - 100)} = \frac{11}{100}$$
$$t = \frac{x(t) - x_1}{(x_2 - x_1)} = \frac{222 - 100}{(200 - 100)} = \frac{122}{100}$$
$$t = \frac{y(t) - y_1}{(y_2 - y_1)} = \frac{22 - 20}{(200 - 20)} = \frac{2}{180}$$
$$t = \frac{y(t) - y_1}{(y_2 - y_1)} = \frac{149 - 20}{(200 - 20)} = \frac{129}{180}$$

We only consider the values of t that are $0 \leq t \leq 1$, so by plugging these values back into the original equation yields:

$$X\left(\frac{11}{100}\right) = (100, 20) + (100, 180)\left(\frac{11}{100}\right) = (100, 20) + \left(11, \frac{99}{5}\right) = \left(111, \frac{199}{5}\right)$$
$$= (111, 39.8)$$
$$X\left(\frac{2}{180}\right) = (100, 20) + (100, 180)\left(\frac{2}{180}\right) = (100, 20) + \left(\frac{200}{180}, 2\right) = \left(\frac{910}{9}, 22\right)$$
$$= (101.1111111, 22)$$
$$X\left(\frac{129}{180}\right) = (100, 20) + (100, 180)\left(\frac{129}{180}\right) = (100, 20) + \left(\frac{12900}{180}, 129\right) = \left(\frac{515}{3}, 149\right)$$
$$= (171.6666666, 149)$$

Comparing with the original points:

$$P_a = (x_a, y_a) = (111, 22)$$
$$P_b = (x_b, y_b) = (111, 149)$$
$$P_c = (x_c, y_c) = (222, 149)$$
$$P_d = (x_d, y_d) = (222, 22)$$

So the line will be clipped to the vertices $F_b = (111, 39.8)$ and $F_c = (171.\overline{66}, 149)$. Note, F denotes final as in the transformed vertices.

<u>**For the line**</u> $\boldsymbol{X_c} = (200, 200) \rightarrow \boldsymbol{X_d} = (20, 150)$

$$X(t) = X_1 + (X_2 - X_1)t$$
$$X(t) = (200, 200) + \big((20, 150) - (200, 200)\big)t$$
$$X(t) = (200, 200) + (-180, -50)t$$

The only values for x and y that the line can be clipped to are 111, 222 for the x values and 22 and 149 for the y values based on points a through d from the viewing rectangle. Therefore, the values of t are as follows:

$$t = \frac{x(t) - x_1}{(x_2 - x_1)} = \frac{111 - 200}{(20 - 200)} = \frac{89}{180}$$
$$t = \frac{x(t) - x_1}{(x_2 - x_1)} = \frac{222 - 200}{(20 - 200)} = -\frac{11}{90}$$
$$t = \frac{y(t) - y_1}{(y_2 - y_1)} = \frac{22 - 200}{(150 - 200)} = \frac{89}{25}$$
$$t = \frac{y(t) - y_1}{(y_2 - y_1)} = \frac{149 - 200}{(150 - 200)} = \frac{51}{50}$$

We only consider the values of t that are $0 \leq t \leq 1$, so by plugging these values back into the original equation yields:

$$X\left(\frac{89}{180}\right) = (200, 200) + (-180, -50)\left(\frac{89}{180}\right) = (200, 200) + \left(-89, -\frac{445}{18}\right) = \left(111, \frac{3155}{18}\right)$$
$$= (111, 175.27777778)$$

Comparing with the original points:

$$P_a = (x_a, y_a) = (111, 22)$$
$$P_b = (x_b, y_b) = (111, 149)$$
$$P_c = (x_c, y_c) = (222, 149)$$
$$P_d = (x_d, y_d) = (222, 22)$$

So the line will be clipped to the vertices $F_c = (171.\overline{66}, 149)$ and $F_d = (111, 149)$. Note, F denotes final as in the transformed vertices. The reason F_d is clipped to a y value of 149 Is because since 175 is out of bounds, it only makes sens to clip t to the highest y-value 149.

<u>For the line</u> $\boldsymbol{X_d} = (20, 150) \rightarrow \boldsymbol{X_a} = (10, 10)$

Since both of these vertices are outside the viewing polygon, they are clipped entirely out of the viewing rectangle.

<u>Final Vertices</u>

The final polygon is clipped to the vertices $F_b = (111, 39.8)$, $F_c = (171.\overline{66}, 149)$, and $F_d = (111, 149)$ resulting in a triangle.

Problem D

Part A

X(t) will be equal to an array of three values cooresponding to r g and b values. The question description is really hard to understand so in order to solve this in a way that I undertand it, t will be a 2 value vector corresponding to t values for the x and y coordinates. The top of the square will be equal to $t_y = 1$ and the bottom ov the square will be equal to $t_y = 0$. For the x direction, the left of the square will be $t_x = 0$ and the right side of the square will be $t_x = 1$.

$$x(t) = \left\{\left[\max\left((1-t_y),(t_x)\right)\right],(1-t_y),\left[\min\left((1-t_y),(1-t_x)\right)\right]\right\}$$

Part B

$$t_x = \frac{xCoord}{ScreenWidth}$$
$$t_y = \frac{yCoord}{ScreenWidth}$$
$$setColor\left(\left\{\left[\max\left((1-t_y),(t_x)\right)\right],(1-t_y),\left[\min\left((1-t_y),(1-t_x)\right)\right]\right\}\right)$$
$$drawPixel(xCoord, yCoord)$$

Problem E

Part A

There are 9 indepent **constant** parameters. One of the constant parameters is the t value for the spline. The other 8 come from the x and y coordinates for the 4 points in the spline.

Part B

The second derivative always matches at junctions in a B-spline cubic.

If this were a Bezier spline, the second and third control points determine the shape of the curve but aren't part of the curve. Derivative won't match at junctions either.

If this were a Catmill-Rom spline, the start and end points determine the shape of the curve but aren't part of the curve and the 1st derivative will always match at junctions and is determined by previous and successive control points.