

Neizrazito, evolucijsko i neuroračunarstvo: izvješće uz 6. laboratorijsku vježbu - sustav ANFIS

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Derivacije pogreške s obzirom na parametre mreže

Ukupna pogreška E je:

$$E = \frac{1}{2} \sum_{n=1}^N (o^{(n)} - t^{(n)})^2,$$

gdje je N broj pravila, $o^{(n)}$ dobiveni izlaz iz sustava, a $t^{(n)}$ očekivani izlaz iz sustava, za n -ti primjer.

n -ti izlaz $o^{(n)}$ je:

$$o^{(n)} = \sum_{m=1}^M \tilde{w}_m f_m(x^{(n)}, y^{(n)}; p_m, q_m, r_m),$$

gdje je \tilde{w}_m pripadna normirana težina, a f_m prijenosna funkcija s parametrima p_m, q_m , i r_m , m -tog pravila; $x^{(n)}$ i $y^{(n)}$ su vrijednosti n -og primjera u ovom zadatku.

Prijenosna funkcija m -og pravila f_m je, u ovom zadatku, linearna kombinacija vrijednosti $x^{(n)}$ i $y^{(n)}$ n -og ulaza:

$$f_m(x^{(n)}, y^{(n)}; p_m, q_m, r_m) = p_m x^{(n)} + q_m y^{(n)} + r_m.$$

Normirana težina m -og pravila \tilde{w}_m je:

$$\tilde{w}_m = \frac{w_m}{\sum_{k=1}^M w_k}.$$

Nenormirana težina m -og pravila w_m jednaka je t -normi mjera pripadnosti n -og ulaznog primjera paru neizrazitih skupova (A_m, B_m) . U ovom zadatku zadana t -norma je algebarski produkt (\cdot) , pa je nenormirana težina w_m :

$$w_m = \mu_{A_m}(x^{(n)}) \cdot \mu_{B_m}(y^{(n)}),$$

gdje je $\mu_{X_m}(z)$ mjera pripadnosti elementa z neizrazitom skupu X_m m -og pravila.

Mjere pripadnosti $\mu_{X_m}(z)$ u ovom zadatku modelirane su parametriziranim sigmoidalnim funkcijama s parametrima a_{X_m} i b_{X_m} :

$$\mu_{X_m}(z; a_{X_m}, b_{X_m}) = \frac{1}{1 + e^{-b_{X_m}(z-a_{X_m})}}.$$

Za m -to pravilo postoji 7 parametara koje je potrebno trenirati na zadanom skupu podataka: $p_m, q_m, r_m, a_{A_m}, a_{B_m}, b_{A_m}$, i b_{B_m} . Ukupno je $7M$ parametara, gdje je M broj pravila.

Derivacija ukupne pogreške s obzirom na izlaz iz sustava $o^{(n)}$ za n -ti primjer je:

$$\frac{\partial E}{\partial o^{(n)}} = o^{(n)} - t^{(n)}.$$

Derivacija izlaza $o^{(n)}$ za n -ti primjer po prijenosnoj funkciji f_m m -tog pravila je:

$$\frac{\partial o^{(n)}}{\partial f_m} = \tilde{w}_m.$$

Derivacije prijenosne funkcije f_m m -tog pravila po parametrima p_m, q_m , i r_m , za n -ti primjer, su:

$$\frac{\partial f_m}{\partial p_m} = x^{(n)}, \quad \frac{\partial f_m}{\partial q_m} = y^{(n)}, \quad \frac{\partial f_m}{\partial r_m} = 1.$$

Derivacije nenormirane težine w_m m -tog pravila po mjerama pripadnosti neizrazitim skupovima μ_{A_m} i μ_{B_m} , za n -ti primjer su:

$$\frac{\partial w_m}{\partial \mu_{A_m}} = \mu_{B_m}(y^{(n)}; a_{B_m}, b_{B_m}), \quad \frac{\partial w_m}{\partial \mu_{B_m}} = \mu_{A_m}(x^{(n)}; a_{A_m}, b_{A_m}),$$

Budući da je normirana težina \tilde{w}_j funkcija svih težina $\{w_m\}_{m=1}^M$, derivacije normiranih težina w_j j -tog pravila po nenormiranoj težini w_m m -tog pravila nisu 0 te ih je potrebno izračunati:

$$\begin{aligned} \frac{\partial \tilde{w}_j}{\partial w_m} &= \begin{cases} \frac{\left(\sum_{k=1}^M w_k\right) - w_m}{\left(\sum_{k=1}^M w_k\right)^2}, & m = j, \\ \frac{-w_j}{\left(\sum_{k=1}^M w_k\right)^2}, & m \neq j, \end{cases} \\ &= \frac{\delta_{m,j} \left(\sum_{k=1}^M w_k\right) - w_j}{\left(\sum_{k=1}^M w_k\right)^2}, \end{aligned}$$

uz Kroneckerov delta $\delta_{i,j}$:

$$\delta_{i,j} = \begin{cases} 1, & i = j, \\ 0, & i \neq j, \end{cases}$$

a ukupna derivacija izlaza iz sustava $o^{(n)}$ za n -ti primjer po težini w_m m -tog pravila je:

$$\begin{aligned}
\frac{\partial o^{(n)}}{\partial w_m} &= \sum_{j=1}^M \frac{\partial o^{(n)}}{\partial \tilde{w}_j} \frac{\partial \tilde{w}_j}{\partial w_m} = \sum_{j=1}^M f_j(x^{(n)}, y^{(n)}; p_j, q_j, r_j) \frac{\delta_{m,j} \left(\sum_{k=1}^M w_k \right) - w_j}{\left(\sum_{k=1}^M w_k \right)^2} \\
&= \frac{\sum_{j=1}^M \delta_{m,j} \left(\left(\sum_{k=1}^M w_k \right) - w_j \right) f_j(x^{(n)}, y^{(n)}; p_j, q_j, r_j)}{\left(\sum_{k=1}^M w_k \right)^2} \\
&= \frac{\sum_{j=1}^M \delta_{m,j} \left(\sum_{k=1}^M w_k \right) f_j(x^{(n)}, y^{(n)}; p_j, q_j, r_j) - \sum_{j=1}^M w_j f_j(x^{(n)}, y^{(n)}; p_j, q_j, r_j)}{\left(\sum_{k=1}^M w_k \right)^2} \\
&= \frac{\left(\sum_{k=1}^M w_k \right) f_m(x^{(n)}, y^{(n)}; p_m, q_m, r_m) - \sum_{j=1}^M w_j f_j(x^{(n)}, y^{(n)}; p_j, q_j, r_j)}{\left(\sum_{k=1}^M w_k \right)^2} \\
&= \frac{\sum_{k=1}^M w_k (f_m(x^{(n)}, y^{(n)}; p_m, q_m, r_m) - f_k(x^{(n)}, y^{(n)}; p_k, q_k, r_k))}{\left(\sum_{k=1}^M w_k \right)^2}
\end{aligned}$$

Derivacije mjere pripadnosti μ_{X_m} elementa $z^{(n)}$ neizrazitom skupu X_m m -tog pravila po parametrima a_{X_m} i b_{X_m} su:

$$\begin{aligned}
\frac{\partial \mu_{X_m}}{\partial a_{X_m}} &= \frac{-1}{\left(1 + e^{-b_{X_m}(z^{(n)} - a_{X_m})} \right)^2} \cdot e^{-b_{X_m}(z^{(n)} - a_{X_m})} \cdot (-b) \\
&= -b \cdot \mu_{X_m}(z^{(n)}; a_{X_m}, b_{X_m}) \cdot (1 - \mu_{X_m}(z^{(n)}; a_{X_m}, b_{X_m})) \\
\frac{\partial \mu_{X_m}}{\partial b_{X_m}} &= \frac{-1}{\left(1 + e^{-b_{X_m}(z^{(n)} - a_{X_m})} \right)^2} \cdot e^{-b_{X_m}(z^{(n)} - a_{X_m})} \cdot (-(x - a)) \\
&= (x - a) \cdot \mu_{X_m}(z^{(n)}; a_{X_m}, b_{X_m}) \cdot (1 - \mu_{X_m}(z^{(n)}; a_{X_m}, b_{X_m}))
\end{aligned}$$

Konačno, tražene derivacije ukupne pogreške s obzirom na sve parametre sustava za m -to pravilo i n -ti ulazni primjer su:

$$\frac{\partial E}{\partial p_m} = \frac{\partial E}{\partial o^{(n)}} \frac{\partial o^{(n)}}{\partial f_m} \frac{\partial f_m}{\partial p_m} = (o^{(n)} - t^{(n)}) \cdot \tilde{w}_m \cdot x^{(n)},$$

$$\frac{\partial E}{\partial q_m} = \frac{\partial E}{\partial o^{(n)}} \frac{\partial o^{(n)}}{\partial f_m} \frac{\partial f_m}{\partial q_m} = (o^{(n)} - t^{(n)}) \cdot \tilde{w}_m \cdot y^{(n)},$$

$$\frac{\partial E}{\partial r_m} = \frac{\partial E}{\partial o^{(n)}} \frac{\partial o^{(n)}}{\partial f_m} \frac{\partial f_m}{\partial r_m} = (o^{(n)} - t^{(n)}) \cdot \tilde{w}_m,$$

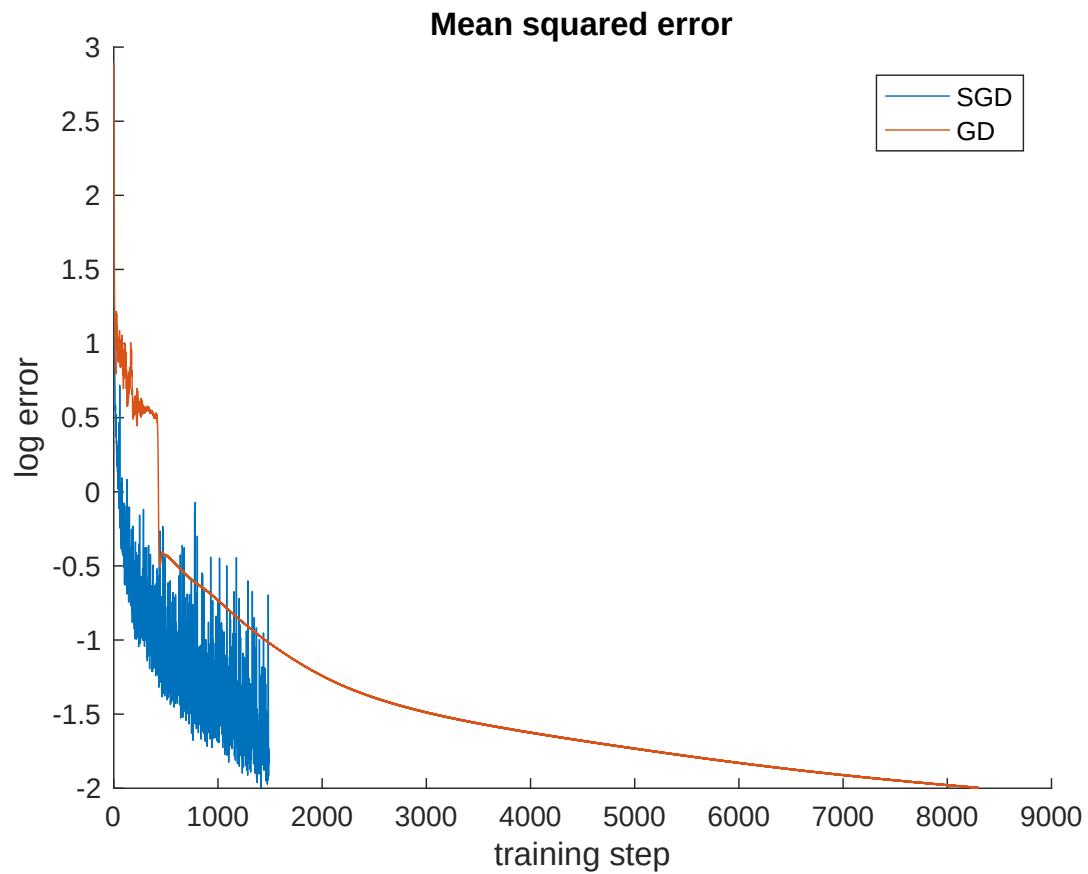
$$\begin{aligned} \frac{\partial E}{\partial a_{A_m}} &= \frac{\partial E}{\partial o^{(n)}} \frac{\partial o^{(n)}}{\partial w_m} \frac{\partial w_m}{\mu_{A_m}} \frac{\partial \mu_{A_m}}{\partial a_{A_m}}, \\ &= (o^{(n)} - t^{(n)}) \cdot \frac{\sum_{k=1}^M w_k (f_m(x^{(n)}, y^{(n)}; p_m, q_m, r_m) - f_k(x^{(n)}, y^{(n)}; p_k, q_k, r_k))}{(\sum_{k=1}^M w_k)^2} \\ &\quad \mu_{B_m}(y^{(n)}; a_{B_m}, b_{B_m}) \cdot (-b) \cdot \mu_{A_m}(x^{(n)}; a_{A_m}, b_{A_m}) \cdot (1 - \mu_{A_m}(x^{(n)}; a_{A_m}, b_{A_m})), \end{aligned}$$

$$\begin{aligned} \frac{\partial E}{\partial b_{A_m}} &= \frac{\partial E}{\partial o^{(n)}} \frac{\partial o^{(n)}}{\partial w_m} \frac{\partial w_m}{\mu_{A_m}} \frac{\partial \mu_{A_m}}{\partial b_{A_m}}, \\ &= (o^{(n)} - t^{(n)}) \cdot \frac{\sum_{k=1}^M w_k (f_m(x^{(n)}, y^{(n)}; p_m, q_m, r_m) - f_k(x^{(n)}, y^{(n)}; p_k, q_k, r_k))}{(\sum_{k=1}^M w_k)^2} \\ &\quad \mu_{B_m}(y^{(n)}; a_{B_m}, b_{B_m}) \cdot (x - a) \cdot \mu_{A_m}(x^{(n)}; a_{A_m}, b_{A_m}) \cdot (1 - \mu_{A_m}(x^{(n)}; a_{A_m}, b_{A_m})), \end{aligned}$$

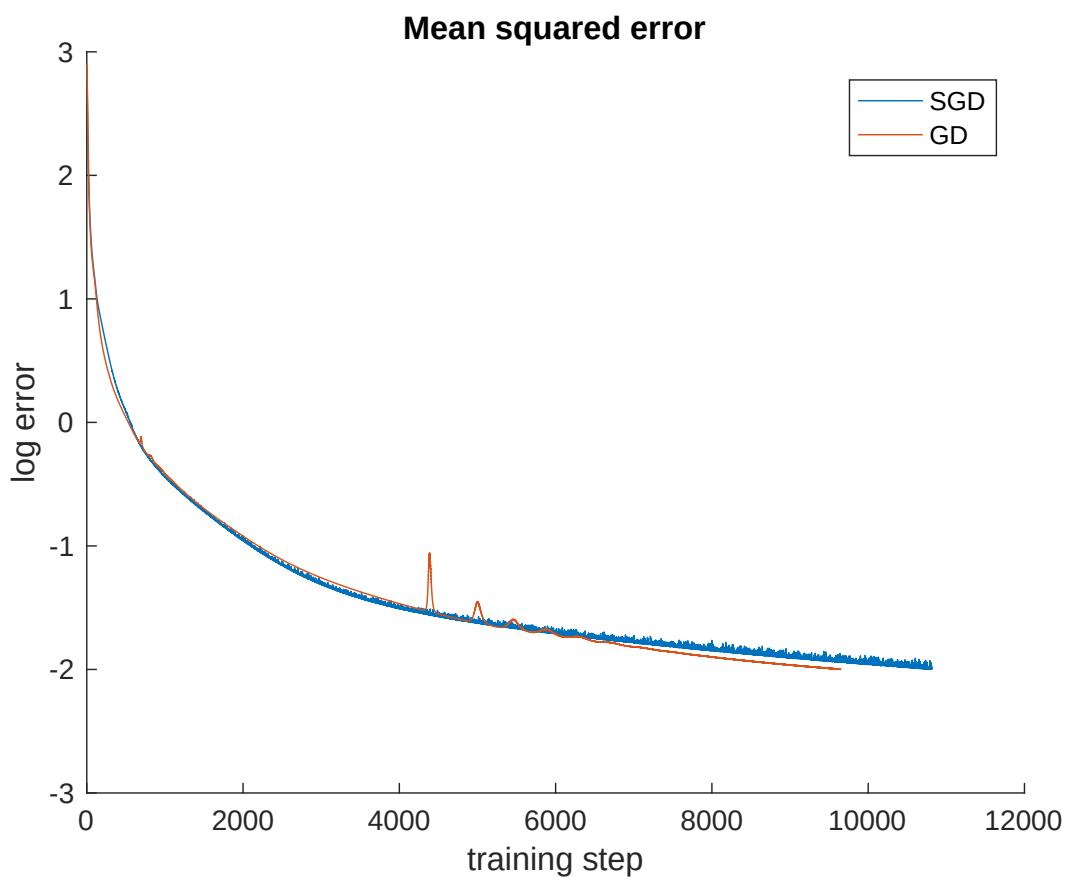
$$\begin{aligned} \frac{\partial E}{\partial a_{B_m}} &= \frac{\partial E}{\partial o^{(n)}} \frac{\partial o^{(n)}}{\partial w_m} \frac{\partial w_m}{\mu_{B_m}} \frac{\partial \mu_{B_m}}{\partial a_{B_m}}, \\ &= (o^{(n)} - t^{(n)}) \cdot \frac{\sum_{k=1}^M w_k (f_m(x^{(n)}, y^{(n)}; p_m, q_m, r_m) - f_k(x^{(n)}, y^{(n)}; p_k, q_k, r_k))}{(\sum_{k=1}^M w_k)^2} \\ &\quad \mu_{A_m}(x^{(n)}; a_{A_m}, b_{A_m}) \cdot (-b) \cdot \mu_{B_m}(y^{(n)}; a_{B_m}, b_{B_m}) \cdot (1 - \mu_{B_m}(y^{(n)}; a_{B_m}, b_{B_m})), \end{aligned}$$

$$\begin{aligned} \frac{\partial E}{\partial b_{B_m}} &= \frac{\partial E}{\partial o^{(n)}} \frac{\partial o^{(n)}}{\partial w_m} \frac{\partial w_m}{\mu_{B_m}} \frac{\partial \mu_{B_m}}{\partial b_{B_m}}, \\ &= (o^{(n)} - t^{(n)}) \cdot \frac{\sum_{k=1}^M w_k (f_m(x^{(n)}, y^{(n)}; p_m, q_m, r_m) - f_k(x^{(n)}, y^{(n)}; p_k, q_k, r_k))}{(\sum_{k=1}^M w_k)^2} \\ &\quad \mu_{A_m}(x^{(n)}; a_{A_m}, b_{A_m}) \cdot (x - a) \cdot \mu_{B_m}(y^{(n)}; a_{B_m}, b_{B_m}) \cdot (1 - \mu_{B_m}(y^{(n)}; a_{B_m}, b_{B_m})). \end{aligned}$$

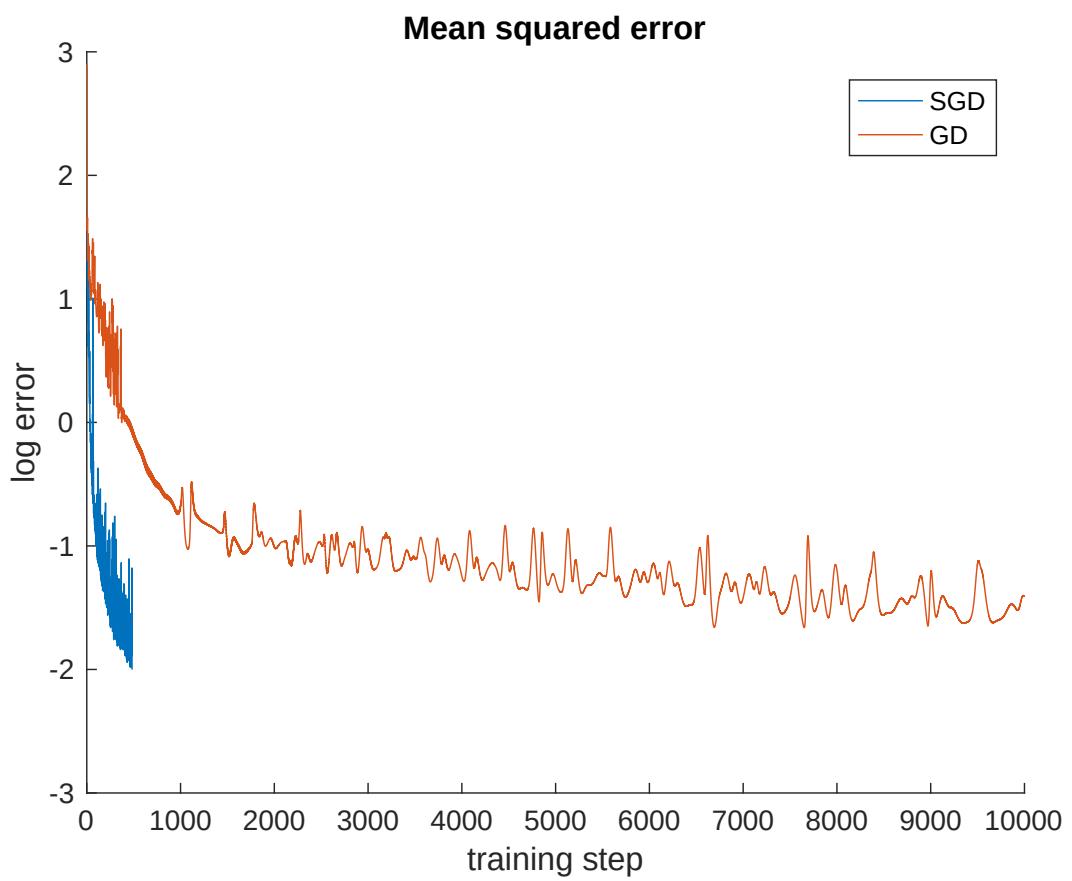
Pogreška tijekom treniranja



Slika 1: Pogreška za vrijeme treniranja uz umjerenu stopu učenja $\eta = 0.01$.

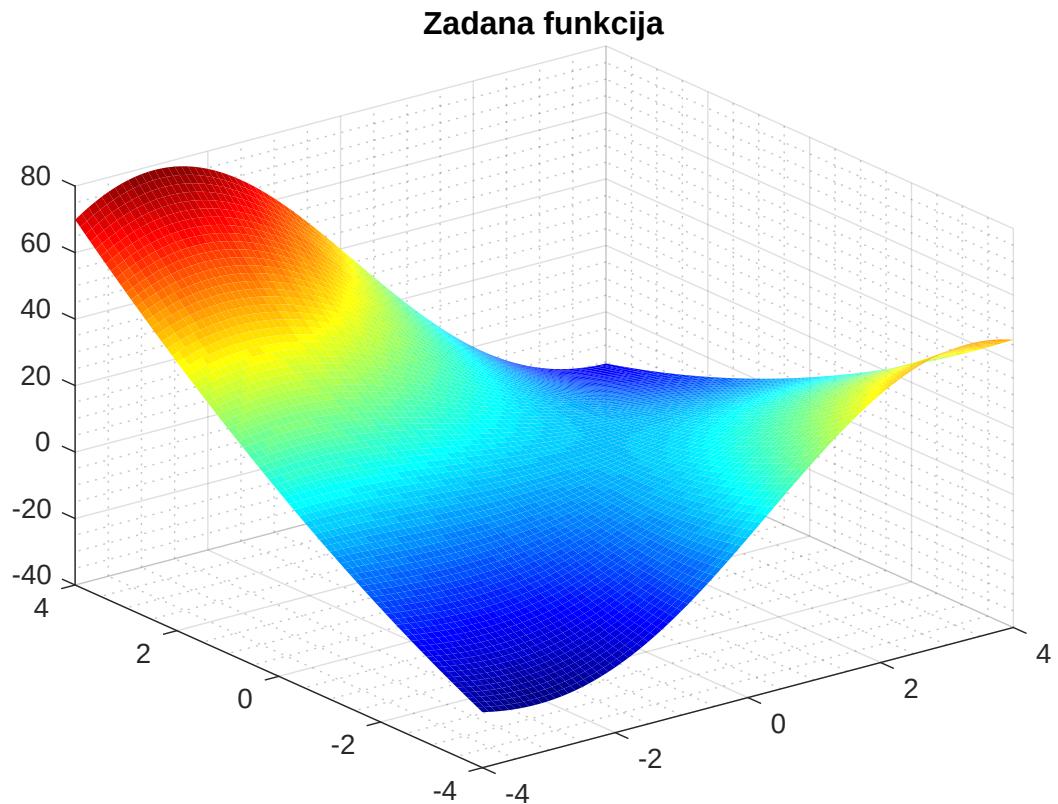


Slika 2: Pogreška za vrijeme treniranja uz nisku stopu učenja $\eta = 0.001$.



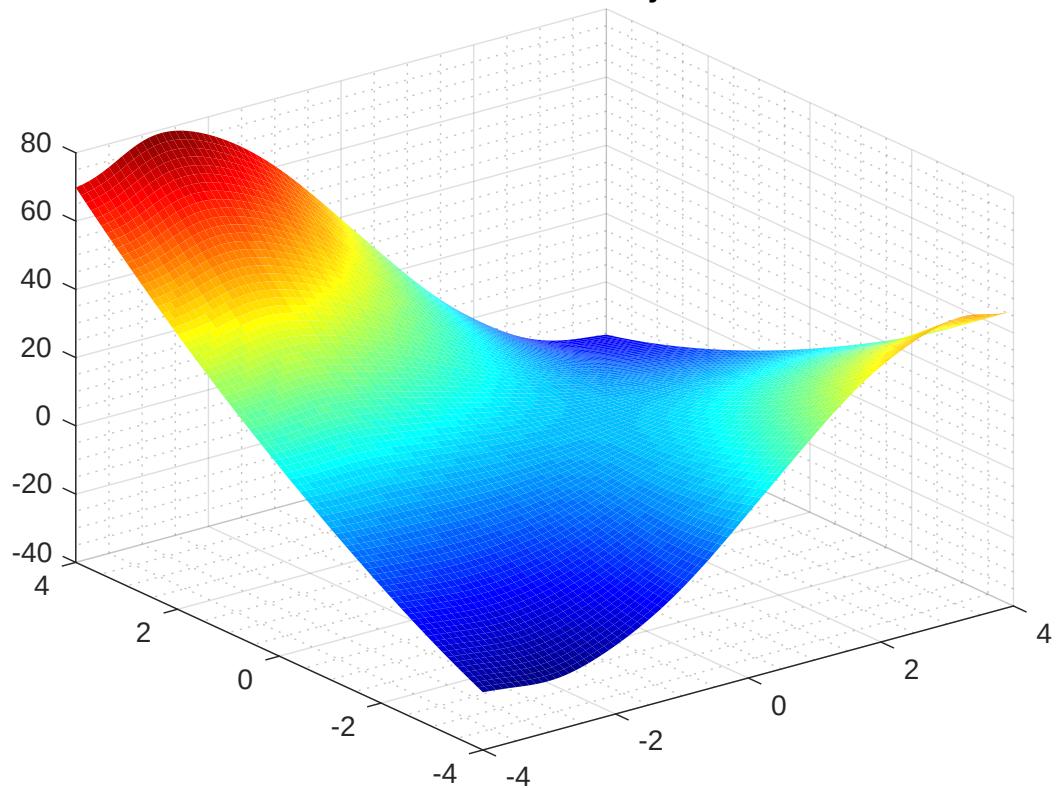
Slika 3: Pogreška za vrijeme treniranja uz visoku stopu učenja $\eta = 0.05$.

Pogreške na primjerima

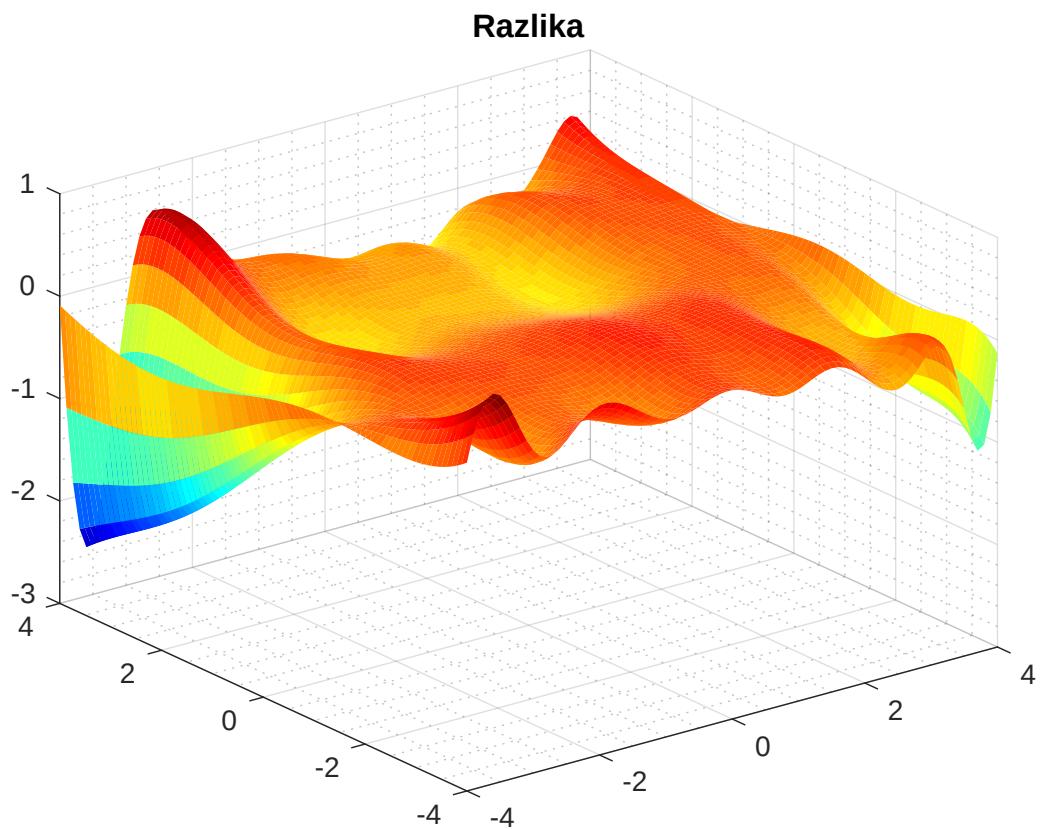


Slika 4: Zadana funkcija $((x - 1)^2 - (y + 2)^2 - 5xy + 3) \cdot \cos^2\left(\frac{\pi}{5}\right)$.

Naučena funkcija

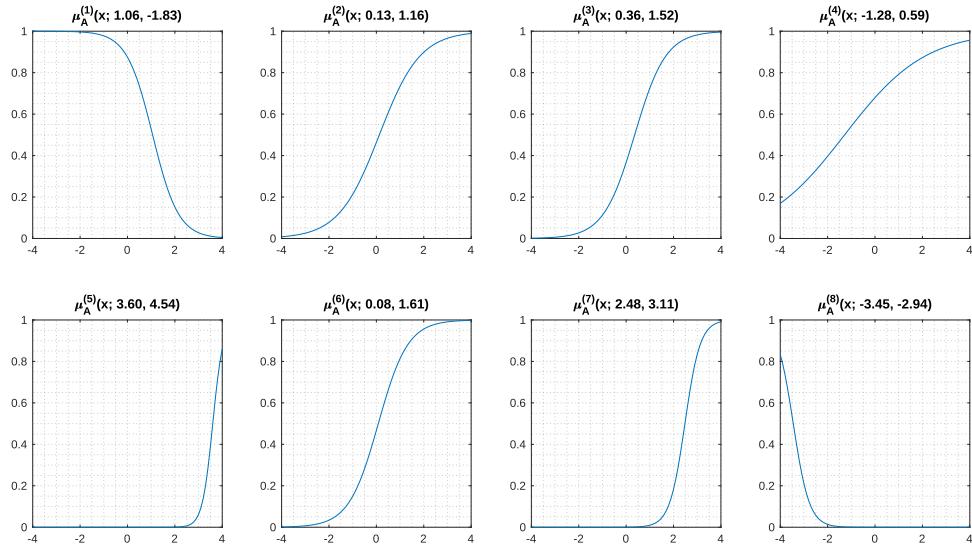


Slika 5: Naučena funkcija.

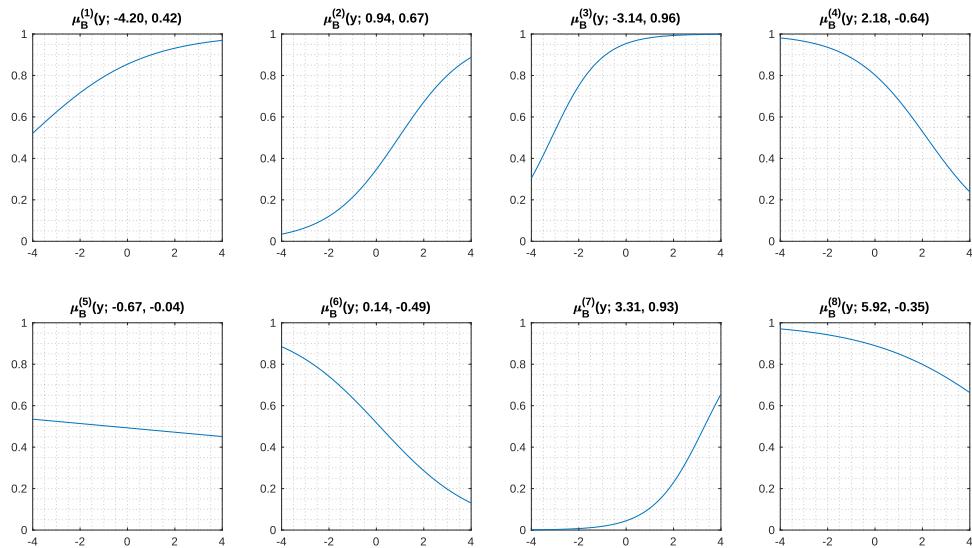


Slika 6: Razlika zadane i naučene funkcije.

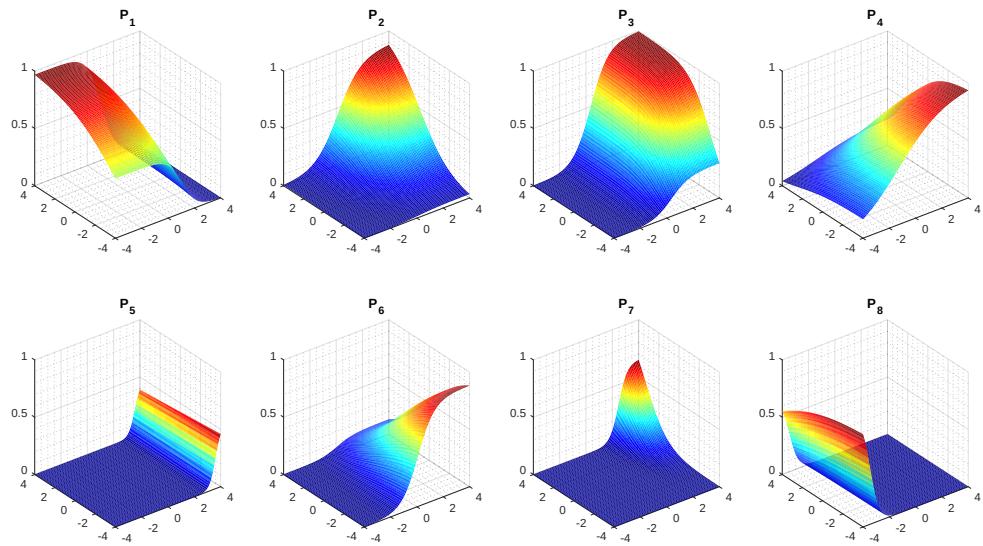
Mjere pripadnosti u naučenom sustavu



Slika 7: Mjere pripadnosti neizrazitom skupu A .

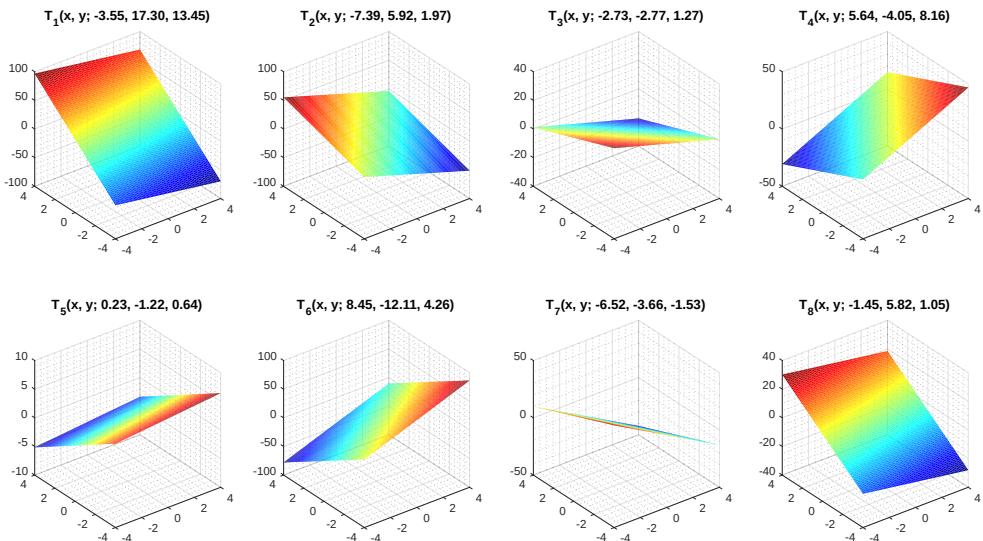


Slika 8: Mjere pripadnosti neizrazitom skupu B .

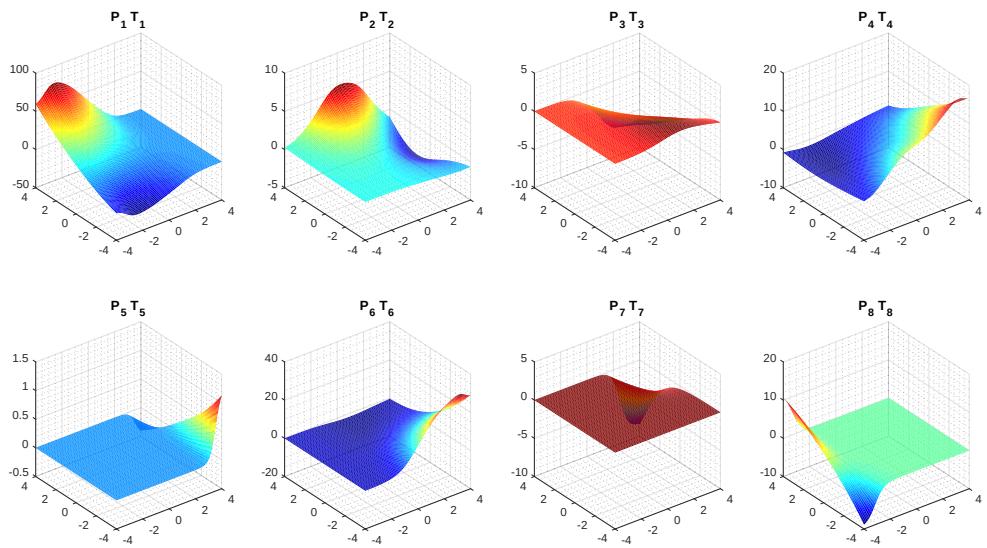


Slika 9: t -norma (algebarski produkt) mjera pripadnosti neizrazitim skupovima A i B .

Prijenosne funkcije u naučenom sustavu



Slika 10: Prijenosne funkcije.



Slika 11: Prijenosne funkcije pomnožene pripadnim normiranim težinama.