Neizrazito, evolucijsko i neuroračunarstvo: izvješće uz 6. laboratorijsku vježbu - sustav ANFIS

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Derivacije pogreške s obzirom na parametre mreže

Ukupna pogreška E je:

$$E = \frac{1}{2} \sum_{n=1}^{N} \left(o^{(n)} - t^{(n)} \right)^2 ,$$

gdje je N broj pravila, $o^{(n)}$ dobiveni izlaz iz sustava, a $t^{(n)}$ očekivani izlaz iz sustava, za n-ti primjer.

n-ti izlaz $o^{(n)}$ je:

$$o^{(n)} = \sum_{m=1}^{M} \widetilde{w}_m f_m \left(x^{(n)}, y^{(n)}; p_m, q_m, r_m \right) ,$$

gdje je $\widetilde{w}_m$ pripadna normirana težina, a f_m prijenosna funkcija s parametrima p_m, q_m, i r_m, m-tog pravila; $x^{(n)}$ i $y^{(n)}$ su vrijednosti n-tog primjera u ovom zadatku.

Prijenosna funkcija m-tog pravila f_m je, u ovom zadatku, linearna kombinacija vrijednosti $x^{(n)}$ i $y^{(n)}$ n-tog ulaza:

$$f_m \left(x^{(n)}, y^{(n)}; p_m, q_m, r_m \right) = p_m x^{(n)} + q_m y^{(n)} + r_m .$$

Normirana težina m-tog pravila $\widetilde{w}_m$ je:

$$\widetilde{w}_m = \frac{w_m}{\sum_{k=1}^{M} w_k} .$$

Nenormirana težina m-tog pravila w_m jednaka je t-normi mjera pripadnosti n-tog ulaznog primjera paru neizrazitih skupova (A_m, B_m). U ovom zadatku zadana t-norma je algebarski produkt $(\cdot)$, pa je nenormirana težina w_m:

$$w_m = \mu_{A_m} \left(x^{(n)} \right) \cdot \mu_{B_m} \left(y^{(n)} \right) ,$$

gdje je $\mu_{X_m}(z)$ mjera pripadnosti elementa z neizrazitom skupu X_m m-tog pravila.

Mjere pripadnosti $\mu_{X_m}(z)$ u ovom zadatku modelirane su parametriziranim sigmoidalnim funkcijama s parametrima a_{X_m} i b_{X_m}:

$$\mu_{X_m}(z; a_{X_m}, b_{X_m}) = \frac{1}{1 + e^{-b_{X_m}(z - a_{X_m})}}.$$

Za m-to pravilo postoji 7 parametara koje je potrebno trenirati na zadanom skupu podataka: p_m, q_m, r_m, a_{A_m}, a_{B_m}, b_{A_m}, i b_{B_m}. Ukupno je $7M$ parametara, gdje je M broj pravila.

Derivacija ukupne pogreške s obzirom na izlaz iz sustava $o^{(n)}$ za n-ti primjer je:

$$\frac{\partial E}{\partial o^{(n)}} = o^{(n)} - t^{(n)}.$$

Derivacija izlaza $o^{(n)}$ za n-ti primjer po prijenosnoj funkciji f_m m-tog pravila je:

$$\frac{\partial o^{(n)}}{\partial f_m} = \widetilde{w}_m.$$

Derivacije prijenosne funkcije f_m m-tog pravila po parametrima p_m, q_m, i r_m, za n-ti primjer, su:

$$\frac{\partial f_m}{\partial p_m} = x^{(n)}, \qquad \frac{\partial f_m}{\partial q_m} = y^{(n)}, \qquad \frac{\partial f_m}{\partial r_m} = 1.$$

Derivacije nenormirane težine w_m m-tog pravila po mjerama pripadnosti neizrazitim skupovima μ_{A_m} i μ_{B_m}, za n-ti primjer su:

$$\frac{\partial w_m}{\partial \mu_{A_m}} = \mu_{B_m}\left(y^{(n)}; a_{B_m}, b_{B_m}\right), \qquad \frac{\partial w_m}{\partial \mu_{B_m}} = \mu_{A_m}\left(x^{(n)}; a_{A_m}, b_{A_m}\right),$$

Budući da je normirana težina $\widetilde{w}_j$ funkcija svih težina $\{w_m\}_{m=1}^{M}$, derivacije normiranih težina w_j j-tog pravila po nenormiranoj težini w_m m-tog pravila nisu 0 te ih je potrebno izračunati:

$$\begin{aligned}\frac{\partial \widetilde{w}_j}{\partial w_m} &= \begin{cases} \dfrac{\left(\sum_{k=1}^{M} w_k\right) - w_m}{\left(\sum_{k=1}^{M} w_k\right)^2}, & m = j, \\ \dfrac{-w_j}{\left(\sum_{k=1}^{M} w_k\right)^2}, & m \neq j, \end{cases} \\ &= \frac{\delta_{m,j}\left(\sum_{k=1}^{M} w_k\right) - w_j}{\left(\sum_{k=1}^{M} w_k\right)^2},\end{aligned}$$

uz Kroneckerov delta $\delta_{i,j}$:

$$\delta_{i,j} = \begin{cases} 1, & i = j, \\ 0, & i \neq j, \end{cases}$$

a ukupna derivacija izlaza iz sustava $o^{(n)}$ za n-ti primjer po težini w_m m-tog pravila je:

$$
\begin{aligned}
\frac{\partial o^{(n)}}{\partial w_m} &= \sum_{j=1}^{M} \frac{\partial o^{(n)}}{\partial \widetilde{w}_j} \frac{\partial \widetilde{w}_j}{\partial w_m} = \sum_{j=1}^{M} f_j\left(x^{(n)}, y^{(n)}; p_j, q_j, r_j\right) \frac{\delta_{m,j}\left(\sum_{k=1}^{M} w_k\right) - w_j}{\left(\sum_{k=1}^{M} w_k\right)^2} \\
&= \frac{\sum_{j=1}^{M} \delta_{m,j}\left(\left(\sum_{k=1}^{M} w_k\right) - w_j\right) f_j\left(x^{(n)}, y^{(n)}; p_j, q_j, r_j\right)}{\left(\sum_{k=1}^{M} w_k\right)^2} \\
&= \frac{\sum_{j=1}^{M} \delta_{m,j}\left(\sum_{k=1}^{M} w_k\right) f_j\left(x^{(n)}, y^{(n)}; p_j, q_j, r_j\right) - \sum_{j=1}^{M} w_j f_j\left(x^{(n)}, y^{(n)}; p_j, q_j, r_j\right)}{\left(\sum_{k=1}^{M} w_k\right)^2} \\
&= \frac{\left(\sum_{k=1}^{M} w_k\right) f_m\left(x^{(n)}, y^{(n)}; p_m, q_m, r_m\right) - \sum_{j=1}^{M} w_j f_j\left(x^{(n)}, y^{(n)}; p_j, q_j, r_j\right)}{\left(\sum_{k=1}^{M} w_k\right)^2} \\
&= \frac{\sum_{k=1}^{M} w_k\left(f_m\left(x^{(n)}, y^{(n)}; p_m, q_m, r_m\right) - f_k\left(x^{(n)}, y^{(n)}; p_k, q_k, r_k\right)\right)}{\left(\sum_{k=1}^{M} w_k\right)^2}
\end{aligned}
$$

Derivacije mjere pripadnosti μ_{X_m} elementa $z^{(n)}$ neizrazitom skupu X_m m-tog pravila po parametrima a_{X_m} i b_{X_m} su:

$$
\begin{aligned}
\frac{\partial \mu_{X_m}}{\partial a_{X_m}} &= \frac{-1}{\left(1 + e^{-b_{X_m}\left(z^{(n)} - a_{X_m}\right)}\right)^2} \cdot e^{-b_{X_m}\left(z^{(n)} - a_{X_m}\right)} \cdot (-b) \\
&= -b \cdot \mu_{X_m}\left(z^{(n)}; a_{X_m}, b_{X_m}\right) \cdot \left(1 - \mu_{X_m}\left(z^{(n)}; a_{X_m}, b_{X_m}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \mu_{X_m}}{\partial b_{X_m}} &= \frac{-1}{\left(1 + e^{-b_{X_m}\left(z^{(n)} - a_{X_m}\right)}\right)^2} \cdot e^{-b_{X_m}\left(z^{(n)} - a_{X_m}\right)} \cdot (-(x - a)) \\
&= (x - a) \cdot \mu_{X_m}\left(z^{(n)}; a_{X_m}, b_{X_m}\right) \cdot \left(1 - \mu_{X_m}\left(z^{(n)}; a_{X_m}, b_{X_m}\right)\right)
\end{aligned}
$$

Konačno, tražene derivacije ukupne pogreške s obzirom na sve parametre sustava za m-to pravilo i n-ti ulazni primjer su:

$$
\frac{\partial E}{\partial p_m} = \frac{\partial E}{\partial o^{(n)}} \frac{\partial o^{(n)}}{\partial f_m} \frac{\partial f_m}{\partial p_m} = \left(o^{(n)} - t^{(n)}\right) \cdot \widetilde{w}_m \cdot x^{(n)},
$$

$$
\frac{\partial E}{\partial q_m} = \frac{\partial E}{\partial o^{(n)}} \frac{\partial o^{(n)}}{\partial f_m} \frac{\partial f_m}{\partial q_m} = \left(o^{(n)} - t^{(n)}\right) \cdot \widetilde{w}_m \cdot y^{(n)},
$$

$$\frac{\partial E}{\partial r_m} = \frac{\partial E}{\partial o^{(n)}} \frac{\partial o^{(n)}}{\partial f_m} \frac{\partial f_m}{\partial r_m} = \left(o^{(n)} - t^{(n)}\right) \cdot \widetilde{w}_m,$$

$$\begin{aligned}
\frac{\partial E}{\partial a_{A_m}} &= \frac{\partial E}{\partial o^{(n)}} \frac{\partial o^{(n)}}{\partial w_m} \frac{\partial w_m}{\mu_{A_m}} \frac{\partial \mu_{A_m}}{\partial a_{A_m}}, \\
&= \left(o^{(n)} - t^{(n)}\right) \cdot \frac{\sum_{k=1}^{M} w_k \left(f_m \left(x^{(n)}, y^{(n)}; p_m, q_m, r_m\right) - f_k \left(x^{(n)}, y^{(n)}; p_k, q_k, r_k\right)\right)}{\left(\sum_{k=1}^{M} w_k\right)^2} \cdot \\
&\quad \mu_{B_m} \left(y^{(n)}; a_{B_m}, b_{B_m}\right) \cdot (-b) \cdot \mu_{A_m} \left(x^{(n)}; a_{A_m}, b_{A_m}\right) \cdot \left(1 - \mu_{A_m} \left(x^{(n)}; a_{A_m}, b_{A_m}\right)\right),
\end{aligned}$$

$$\begin{aligned}
\frac{\partial E}{\partial b_{A_m}} &= \frac{\partial E}{\partial o^{(n)}} \frac{\partial o^{(n)}}{\partial w_m} \frac{\partial w_m}{\mu_{A_m}} \frac{\partial \mu_{A_m}}{\partial b_{A_m}}, \\
&= \left(o^{(n)} - t^{(n)}\right) \cdot \frac{\sum_{k=1}^{M} w_k \left(f_m \left(x^{(n)}, y^{(n)}; p_m, q_m, r_m\right) - f_k \left(x^{(n)}, y^{(n)}; p_k, q_k, r_k\right)\right)}{\left(\sum_{k=1}^{M} w_k\right)^2} \cdot \\
&\quad \mu_{B_m} \left(y^{(n)}; a_{B_m}, b_{B_m}\right) \cdot (x - a) \cdot \mu_{A_m} \left(x^{(n)}; a_{A_m}, b_{A_m}\right) \cdot \left(1 - \mu_{A_m} \left(x^{(n)}; a_{A_m}, b_{A_m}\right)\right),
\end{aligned}$$

$$\begin{aligned}
\frac{\partial E}{\partial a_{B_m}} &= \frac{\partial E}{\partial o^{(n)}} \frac{\partial o^{(n)}}{\partial w_m} \frac{\partial w_m}{\mu_{B_m}} \frac{\partial \mu_{B_m}}{\partial a_{B_m}}, \\
&= \left(o^{(n)} - t^{(n)}\right) \cdot \frac{\sum_{k=1}^{M} w_k \left(f_m \left(x^{(n)}, y^{(n)}; p_m, q_m, r_m\right) - f_k \left(x^{(n)}, y^{(n)}; p_k, q_k, r_k\right)\right)}{\left(\sum_{k=1}^{M} w_k\right)^2} \cdot \\
&\quad \mu_{A_m} \left(x^{(n)}; a_{A_m}, b_{A_m}\right) \cdot (-b) \cdot \mu_{B_m} \left(y^{(n)}; a_{B_m}, b_{B_m}\right) \cdot \left(1 - \mu_{B_m} \left(y^{(n)}; a_{B_m}, b_{B_m}\right)\right),
\end{aligned}$$

$$\begin{aligned}
\frac{\partial E}{\partial b_{B_m}} &= \frac{\partial E}{\partial o^{(n)}} \frac{\partial o^{(n)}}{\partial w_m} \frac{\partial w_m}{\mu_{B_m}} \frac{\partial \mu_{B_m}}{\partial b_{B_m}}, \\
&= \left(o^{(n)} - t^{(n)}\right) \cdot \frac{\sum_{k=1}^{M} w_k \left(f_m \left(x^{(n)}, y^{(n)}; p_m, q_m, r_m\right) - f_k \left(x^{(n)}, y^{(n)}; p_k, q_k, r_k\right)\right)}{\left(\sum_{k=1}^{M} w_k\right)^2} \cdot \\
&\quad \mu_{A_m} \left(x^{(n)}; a_{A_m}, b_{A_m}\right) \cdot (x - a) \cdot \mu_{B_m} \left(y^{(n)}; a_{B_m}, b_{B_m}\right) \cdot \left(1 - \mu_{B_m} \left(y^{(n)}; a_{B_m}, b_{B_m}\right)\right).
\end{aligned}$$

Mjere pripadnosti

Pogreške na primjerima

Ukupna pogreška tokom treniranja