Smart Refrigerator Heat Transfer Calculations Revision 2 (3/24/2025)

There are not many new calculations to review for this revision. The primary purpose of this revision is sharing the reorganized and clear version of the previous calculation.

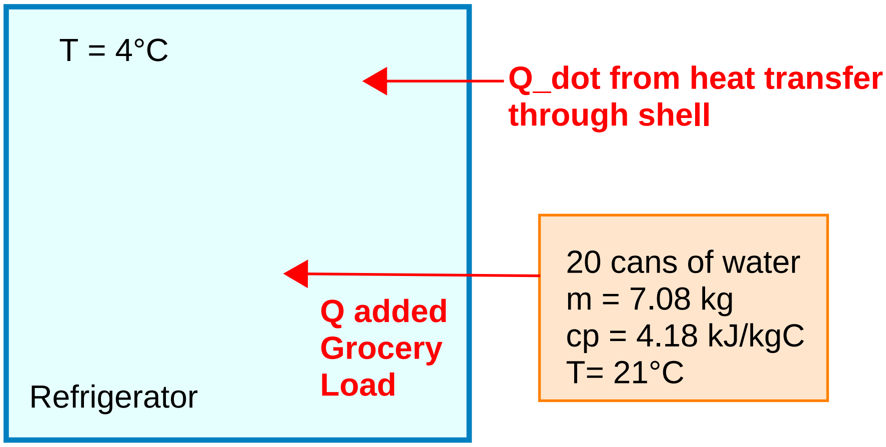
I would greatly appreciate if you could review the following:

> Briefly looking over the first revision proof of concept calculations (Page 2-5)

> Does the problem interpretation/set up look alright for calculating the outlet temperature based on given mass flow rate (Page 6-7)

> Are the areas and perimeters for the channel correct (Page 8)

Calculation Revision 1 (Adding written out equations and pictures for clarity)



STEP 1 - estimate how much heat might be added to the refrigerator during the passive cooling mode. In practice, this would be when the user loads groceries into the refrigerator.

To estimate the size of the load in this revision, I am finding the heat removed from 20 cans filled with water that going from 21 degrees (room temperature) to 4 degrees (refrigeration temperature).

This is likely an overestimate as many groceries are already below room temperature when loaded into a refrigerator.

Calculating the mass of water:

$$\left(\begin{matrix}\text{Quantity}\\ \text{of cans}\end{matrix}\right)\left(\begin{matrix}\text{oz per}\\ \text{can}\end{matrix}\right)\left(\frac{m^3}{oz}\ \text{conversion}\right) = \text{Volume } (m^3)$$

$$\text{Volume}(m^3) \times \text{density}\left(\frac{kg}{m^3}\right) = \text{mass}(kg)$$

ex with 20 cans... density of water at 20°C

$$(20\ \text{cans})\left(\frac{12\ oz}{can}\right)\left(\frac{0.0000296\ m^3}{oz}\right) = 0.0071\ m^3$$

$$0.0071\ m^3 \times 998.2\ \frac{kg}{m^3} = 7.08\ kg$$

Calculating the Grocery Load (Q)

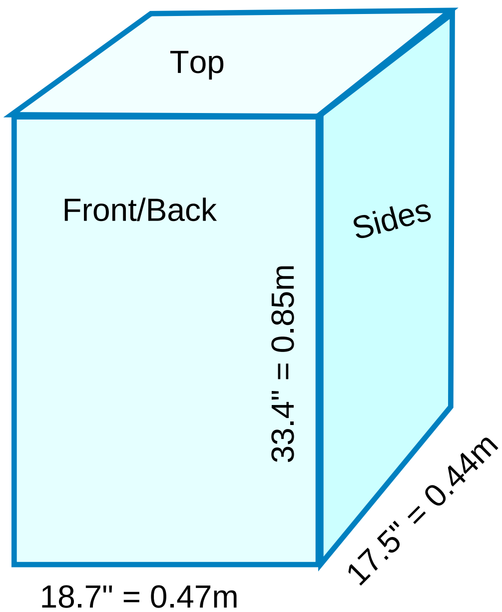
initial grocery temp; refrigeration temperature

$$Q = mc_p(T_1 - T_2)$$

$$Q = (7.08\ kg)\left(4.18\ \frac{kJ}{kgK}\right)(21°C - 4°C) = 503\ kJ$$

STEP 2 - Calculate the Surface area of refrigerator

Assumptions:
> No heat transfer through the bottom of the refrigerator
> Dimensions based on similar mini refrigerator model



Sides: 0.44m x 0.85 m = 0.377 m^2
Top: 0.47m x 0.44 m = 0.211m^2
Front/Back: 0.47m x 0.85 m = 0.403 m^2

Total surface area: 2 x Sides + 2 x Front/back + Top

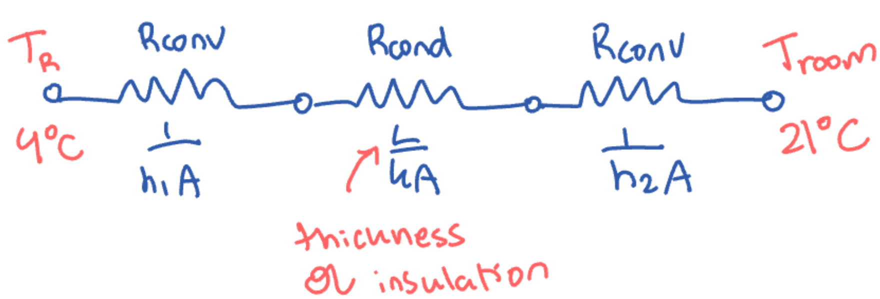
Total A = (2 x 0.377 m^2) + (2 x 0.403 m^2) + 0.211 m^2

Total A = 1.771 m^2

STEP 3 - Estimate how much heat enters the refrigerator from heat transfer through the walls.

Assumptions for this revision (must improve later)
> Ignoring the conduction through the inner and outer plastic shell of the refrigerator
> Using placeholder values for h1, k, and h2
> Surface area on the interior and exterior is assumed equal.
> The entire refrigerator is at 4C (not considering freezer portion)

Resistance Network



$$R_{TOT} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A}$$

Assumed thickness of insulation: 5 cm = 0.05 m
Assumed k (thermal conductivity of insulation): 0.36 W/mC
Assumed h1 (interior heat transfer coefficient): 10 W/m^2C
Assumed h2 (exterior heat transfer coefficient): 10 W/m^2C

Calculating the R total

$$R_{conv1} = \frac{1}{h_1 A} = \frac{1}{(10\frac{W}{m^2C})(1.771m^2)} = 0.056\frac{C}{W}$$

$$R_{cond} = \frac{L}{kA} = \frac{0.05m}{(0.36\frac{W}{mC})(1.771m^2)} = 0.078\frac{C}{W}$$

$$R_{conv2} = \frac{1}{h_2 A} = \frac{1}{(10\frac{W}{m^2C})(1.771m^2)} = 0.056\frac{C}{W}$$

$$R_T = 0.056 + 0.078 + 0.056 = 0.191\frac{C}{W}$$

Calculating the heat transfer through the walls

$$\dot{Q} = \frac{\Delta T}{R_{TOT}} = \frac{21°C - 4°C}{0.191\frac{C}{W}} = 88.9W = 0.0889kW$$

STEP 4 - Calculate the total heat that must be removed during passive cooling period

Assumptions for this revision
>The refrigerator is in passive mode for 3 hours

Converting the heat transfer from kW to kJ based on the 3 hour period

$$Q(kJ) = \dot{Q}(kW) \times Time(seconds)$$

$$Q_{kJ} = 0.0889\frac{kJ}{s} \times 3hours \times \left(\frac{3600s}{hr}\right) = 960kJ$$

Adding the heat from heat transfer through the walls to the grocery load for the total heat that must be removed during the passive cooling period

Q total = 960 kJ + 503 kJ = 1463

STEP 5 - Calculate the volume of ice that must be melted

$$Q = mL = 1463 kJ$$

$$m = \frac{Q}{L} = \frac{1463 kJ}{334 \frac{kJ}{kg}} = 4.38 kg$$

$$V_{ice} = \frac{m}{\rho} = \frac{4.38 kg}{917 \frac{kg}{m^3}} = 0.0048 m^3 = 292 in^3$$

To add perspective on how large 292 in^3 of ice is, can use a known reference such as a text book.

Assuming the textbook is 2.5 in thick and 8.5 x 11 in page...

$$V_{textbook} = 2.5 in \times 8.5 in \times 11 in = 234 in^3$$

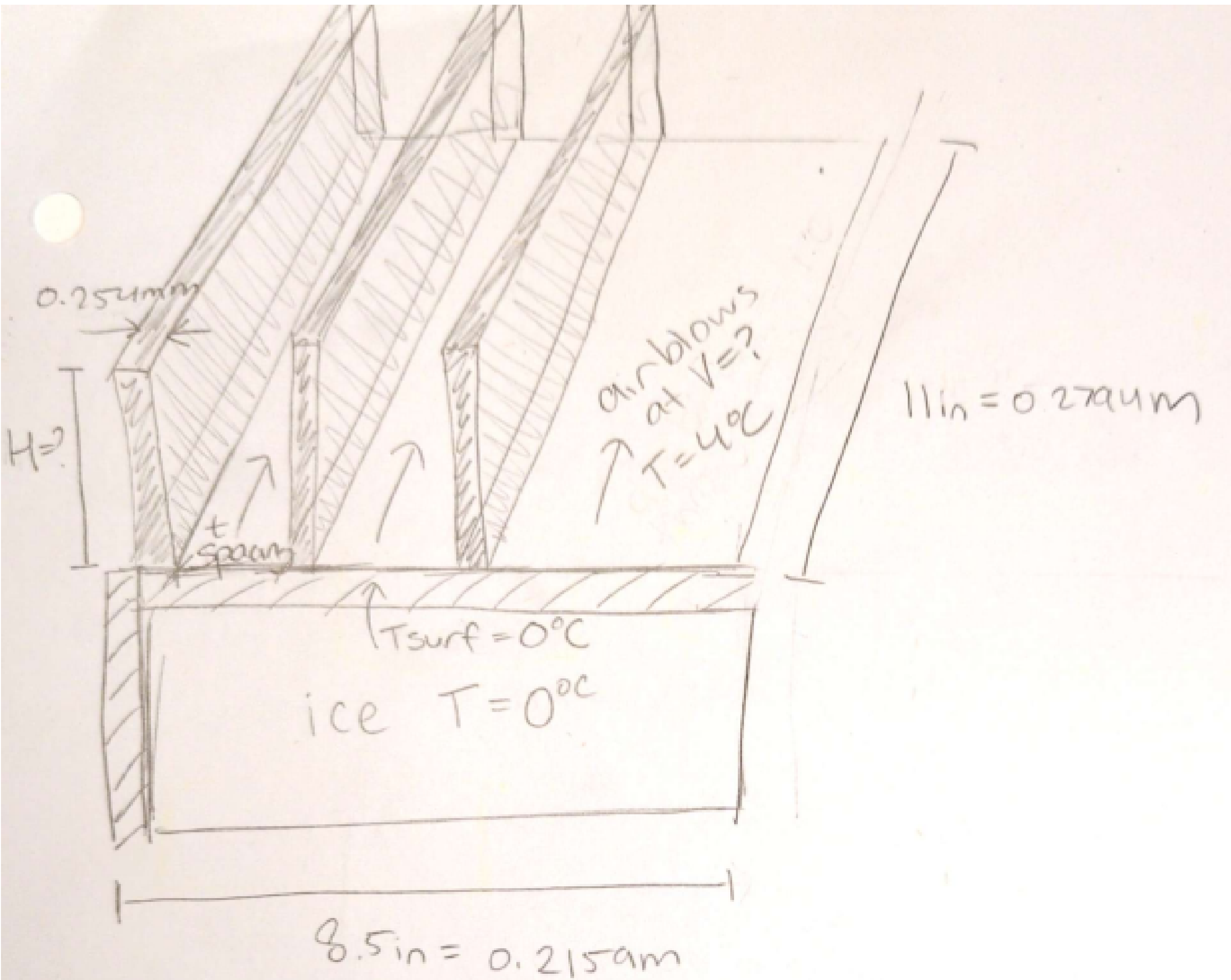
$$\begin{matrix}\text{textbooks} \\ \text{Required}\end{matrix} = \frac{V_{ice}}{V_{text}} = \frac{292 in^3}{234 in^3} = 1.2$$

Calculation Revision 2 - Planning for Ice to Air heat transfer model

The heat transfer between the ice and the air should probably be in the form of fins + forced convection in the channels

The unknown parameters:

> Outlet temperature of the air
> Mass flow rate (Speed of the air)
> Size and quantity of fins



Please note that this model is upside down -> the fins will be underneath the ice

The GOAL: Ultimately, we want to determine the required size of fins. First, we can find the required outlet temperature (The temperature at which the air reenters the refrigerator) based on a given mass flow rate.

The outlet temperature will always be dependent on the mass flow rate, so it will be impossible to solve one without the other. It is more important for our design specifications to control the mass flow rate of the air.

STEP 1

We can research small fan specifications, choose one that requires little power and is of appropriate size and convert the output cfm to mass flow rate.

STEP 2

We can determine the required Qdot based on the load of the refrigerator

>Q_dot total = Q dot from heat transfer through shell + Q dot from added grocery load

>For the grocery load: To provide a heat transfer calculation rather than heat, we can assume that 1 can is put in the refrigerator every X minutes.

STEP 3

To determine the resulting outlet temperature, we can use the following equation:

Qdot = mdot x cp x (Toutlet - Tinlet)

Note: cp is for air, not water

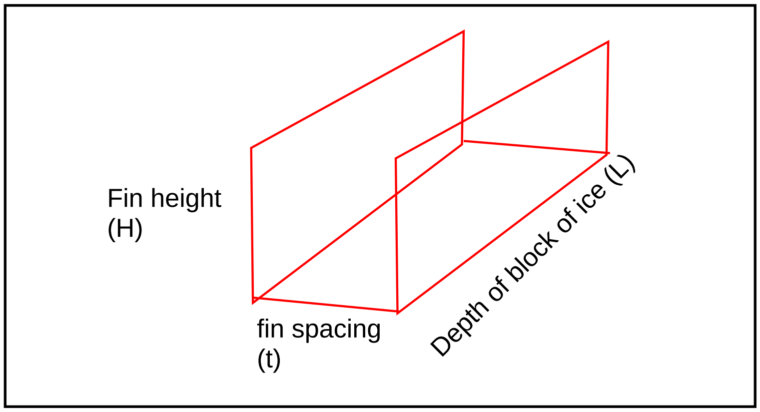
STEP 4

Then, we can experiment with different fin heights and spacing using the equation for internal convection with constant surface temperature

$$\ln \frac{T_s - T_e}{T_s - T_i} = -\frac{hA_s}{\dot{m}c_p}$$

My next step is to work with this equation (And the equations/table for Nusselt number) to calculate the required ratio between the fin height and spacing.

Calculate the hydraulic diameter (Dh)



N = number of fins

Calculate Dh = 4Ac/p using the following equations for Ac and P

Ac = fin spacing x fin height x number of fins = t x H x N

p = 2 x fin height + fin spacing = 2 x H + t

Feedback requested: Is this the correct Ac and Perimeter?

Calculate the surface area

As = Number of fins x (2 x side area) + bottom area = [2x(HxL) + (t x L)] N

Feedback requested: Is this the correct As?

NOTE:

I am planning on first assuming that the flow is laminar and

In order to calculate Reynolds and find Nusselt number with table or equation, we need the mass flow rate.

Calculate Reynolds number using the following Equation

Reynolds number for flow in a circular tube

$$\mathrm{Re} = \frac{V_{avg} D}{\nu} = \frac{\rho V_{avg} D}{\mu} = \frac{\rho D}{\mu}\left(\frac{\dot{m}}{\rho \pi D^2/4}\right) = \frac{4\dot{m}}{\mu \pi D}$$

Under most practical conditions, the flow in a pipe is laminar for $\mathrm{Re} < 2300$, fully turbulent for $\mathrm{Re} > 10{,}000$, and transitional in between.

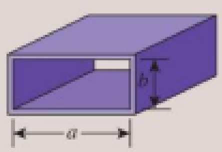
Entry Lengths

$$L_{h,\text{laminar}} \approx 0.05\,\mathrm{Re}\,D$$

$$L_{t,\text{laminar}} \approx 0.05\,\mathrm{Re}\,\mathrm{Pr}\,D = \mathrm{Pr}\,L_{h,\text{laminar}}$$

$$L_{h,\text{turbulent}} \approx L_{t,\text{turbulent}} \approx 10D$$

If the flow is laminar and fully developed: Use Table 8-1 for Ts = const

		Nusselt Number		
Tube Geometry	a/b or $\theta°$	T_s = Const.	$\dot{q}_s$ = Const.	Friction Factor f
Rectangle 	a/b			
	1	2.98	3.61	56.92/Re
	2	3.39	4.12	62.20/Re
	3	3.96	4.79	68.36/Re
	4	4.44	5.33	72.92/Re
	6	5.14	6.05	78.80/Re
	8	5.60	6.49	82.32/Re
	∞	7.54	8.24	96.00/Re

Calculate the heat transfer coefficient using the Nusselt Number

Nu = hDh/k --> h = Nuk/Dh

The GOAL: find the required mass flow rate of the air per assumed fin size.

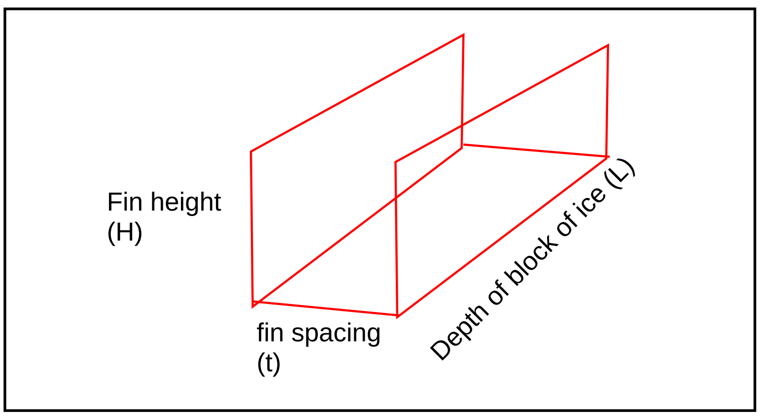
I can see two ways of solving this:
1. Determining the required Q dot

1. Using the constant temperature equation, finding Dh, Re, assuming a mass flowrate, calculating the h given that mass flow rate. Then, plug it back into the original and iterating until the correct m dot is found.

This is a constant temperature problem since the block of ice will keep the plate beneath it at 0C. We know the surface temperature, Tin, Tout. We can use the following equation. Cp can be found with tables.

STEP 1

We can calculate As and Dh based on assumptions on the fin size



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Feedback requested: Is this the correct As?

Calculate Dh = 4Ac/p using the following equations for Ac and P

Ac = fin spacing x fin height x number of fin channels = t x H x N

p = 2 x fin height + fin spacing = 2 x H + t

Feedback requested: Is this the correct Ac and Perimeter?

STEP 2: Determine if flow is developed or developing and laminar or turbulent

In order to calculate Reynolds and find Nusselt number with table or equation, we need the mass flow rate.

Calculate Reynolds number using the following Equation

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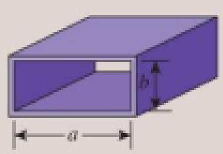
Rectangle 	a/b			
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To convert from the volumetric flow of the fan from the specifications to mass flow rate use the density of the air at the inlet and the cross sectional area of the inlet.

Then find entry length using: