Bifurcational Analysis and Results of Varied Cell Length and GTPase Activity

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Overview

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In this talk, we will go over a cellular model and how different bifurcations can lead to very interesting features, such as changes in the number of steady states and/or activity of the steady state(s), homoclinic orbits, and the SNIC-H orbit, which we are trying to compute.

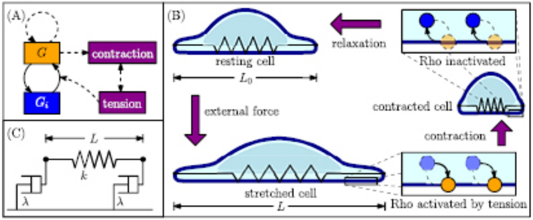

Figure: An image detailing the cellular process to be discussed (Zmurchok, 2018)

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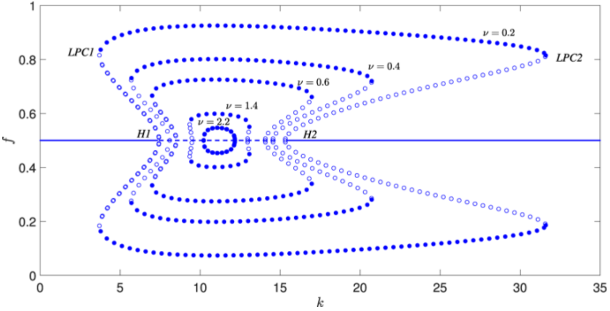

Figure: Bifurcation diagram with ν as the changing parameter (Russo, 2019)

The Equations

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Equation 1

$\frac{dG}{dt} = \left(b + f(T) + \gamma \frac{G^n}{1+G^n}\right)(G_T - G) - G$, where
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We focus on the parameters b and β.

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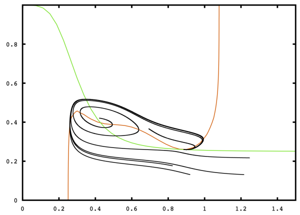

Figure: $b = 0.137$, $\beta = 0.17278$

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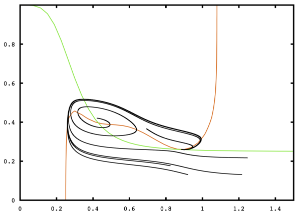

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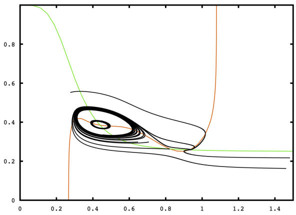

Figure: $b = 0.146$, $\beta = 0.16309$

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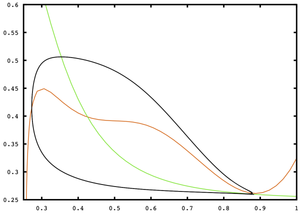

Figure: $b = 0.14$, $\beta = 0.16546$

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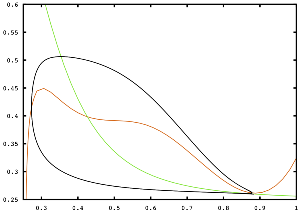

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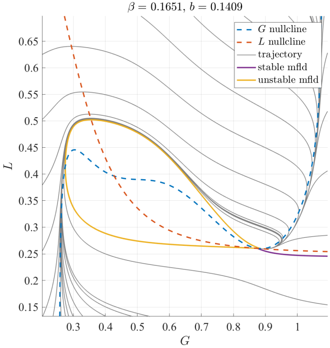

Figure: $b = 0.1409$, $\beta = 0.1651$. (credit to Prof. Cytrynbaum)

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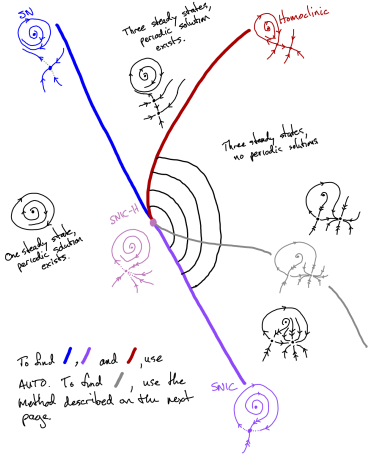
A SNIC-H (Saddle Node on Invariant Circle-Homoclinic) bifurcation occurs when there are two steady states, one of which is an unstable spiral, and the other a saddle node. The saddle node has a homoclinic bifurcation.

A Diagram for Simplicity (credit to Prof. Cytrynbaum)



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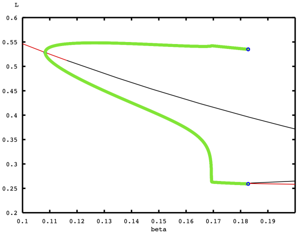
The blue clusters of points signify unstable orbits, while the green clusters signify stable orbits.

The Actual One-Parameter Diagrams ($b = 0.132$)



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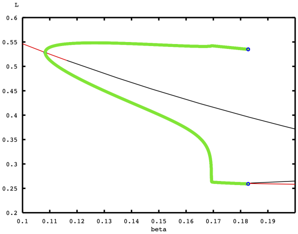

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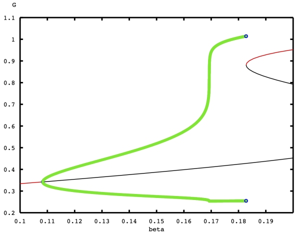

Figure: G vs. β

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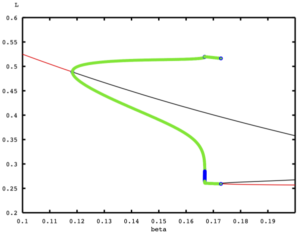

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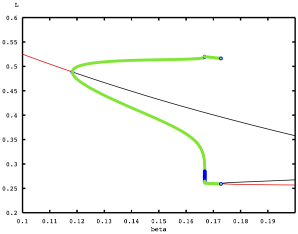

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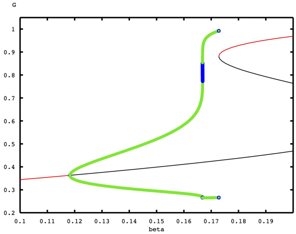

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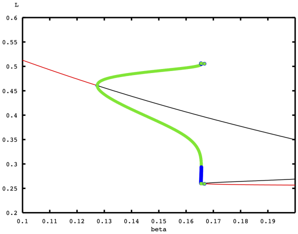

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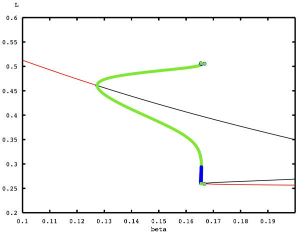

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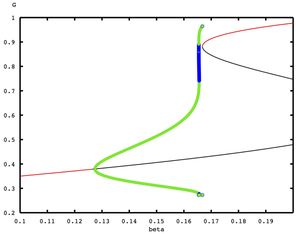

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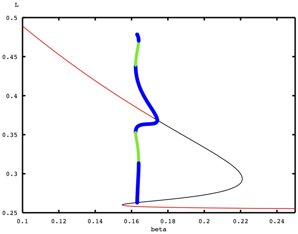

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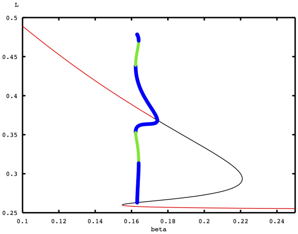

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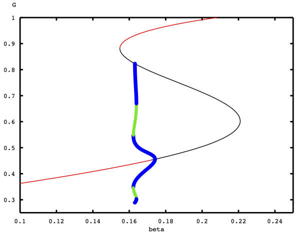

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Movie Time!

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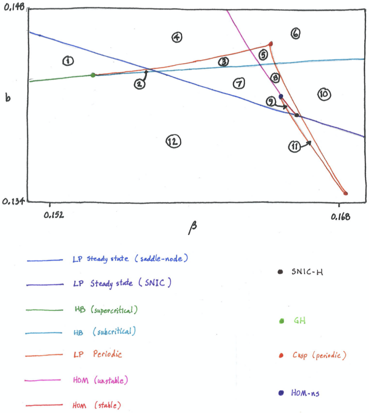

Figure: Expectation of Two-Parameter Diagram (credit to Prof. Nagata)

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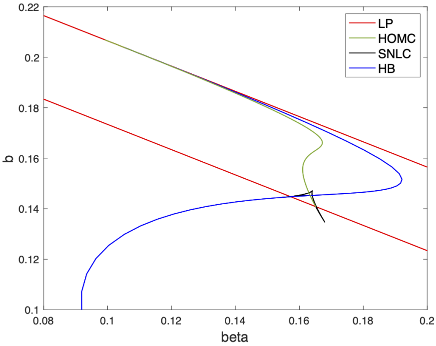

Figure: The Two-Parameter Diagram Obtained through Continuation Software

A Closer Look

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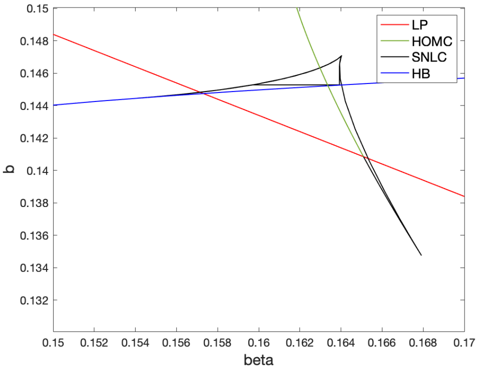

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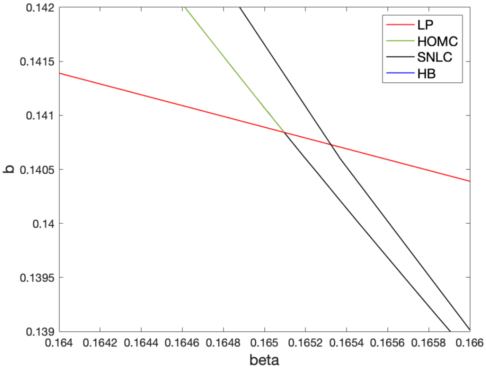

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References

1. Russo, L., Spiliotis, K., Giannino, F. et al. *Bautin bifurcations in a forest-grassland ecosystem with human-environment interactions*. Sci Rep 9, 2665 (2019) 4, https://doi.org/10.1038/s41598-019-39296-x

2. Zmurchok, C., Bhaskar, D., Edelstein-Keshet, L. *Coupling mechanical tension and GTPase signaling to generate cell and tissue dynamics*. Phys. Biol. 15 046004 (2018) 3, https://doi.org/10.1088/1478-3975/aab1c0

Thank You for Your Time!