Matt Kennedy
CS 678

Homework Chapter 3

Bayesian Decision Theory

1. The microfilaria test is an imperfect indicator of heartworm disease in dogs (for example, if only male microfilaria are present, the test will give a False Negative). Suppose a veterinarian has seen the following results in her practice this year:

Microfilaria test	Heartworm?
⊕	Yes
Neg	No
Neg	No
Neg	No
⊕	No
Neg	No
Neg	Yes
⊕	No

- Calculate the *posterior probability* of having heartworm for a dog who has tested ⊕ for microfilaria at this clinic.

HW = has heartworm
$P(HW) = \frac{1}{4}$
$P(\text{No HW}) = \frac{3}{4}$
$P(⊕ \mid HW) = \frac{1}{2}$
$P(⊕ \mid \text{No HW}) = \frac{1}{3}$

$$P(HW \mid ⊕) = \frac{P(⊕ \mid HW) P(HW)}{P(⊕)}$$

$$P(⊕) = \sum \begin{matrix} P(⊕ \mid HW) P(HW) = (\frac{1}{2})(\frac{1}{4}) = \frac{1}{8} \\ P(⊕ \mid \text{No HW}) P(\text{No HW}) = (\frac{1}{3})(\frac{3}{4}) = \frac{1}{4} \end{matrix}$$

$$P(HW \mid ⊕) = \frac{(\frac{1}{2})(\frac{1}{4})}{(\frac{1}{8} + \frac{1}{4})} = \boxed{\frac{1}{3}}$$

2. According to Bayes' equation, P(C|x) decreases as P(x) increases. Why?

If we look at this example of dogs with heartworm, ⊕ is the number of positives. If the number of dogs with heartworm stays constant, but more dogs are diagnosed positively, then the probability a dog actually has heartworm given a positive test result decreases.

An increase in $P(x)$ will decrease $P(C \mid x)$ when $P(C)$ stays constant.