Take-Home Exam

Fall term 2018-2019

Submission deadline: January 14th 2019 at 13h00 GMT

Candidate number: ... ...

Instruction

Each of the problems **a** through **f** below has equal weight and each contributes 16 marks, 96 in total. Sub-questions inside questions **a-f** have equal weight.

The remaining 4 marks out of 100 are awarded as rounding-off' mark based upon the overall quality of the submission as follows: *0:sufficient, 1: good, 2: very good, 3: excellent and 4: outstanding*.

Submit as a pdf-file!

Exam Questions

a: A Knowledge network

In the course we looked at how knowledge can be viewed as a collection of knowledge components, in the most primitive case these were "data", connected by link representing informational connections between the nodes. Suppose we have a knowledge network with n where every node has a probability of $1/m$ to be connected by an outgoing link to any of the other nodes, and for these links to have a weight which is randomly drawn from a probability distribution of your choice which leaves the probability of the weight being negative less than 5%. There should be no self-loops!

Question: Write a command that produces such a matrix and that takes n as an argument, calculate such networks for $n = 10$ and $m = 2,$ 3 and 4 and produce a graph of these networks.

b: Axioms, raw-data and all that

Question: Use the command from **a:** to calculate such networks for $n = 250$ and $m =$ 3 and 4 and produce histograms for the distribution of in-degrees and out-degrees in these networks.

Question: Can you give an example of knowledge elements in a Science or any other research discipline whose in-degree you would expect to be 0?

Question: Can you give an example of a knowledge element in a Science or any other research discipline whose out-degree you would expect to be 0?

c: What you value and what you hold

A researcher has a portfolio of research puzzles $\vec{h}$ and assigns a relative relevance $\vec{r}$ to these puzzles.

Question: State the optimisation problem that the researcher faces in assigning relevance and making a portfolio choice.

Question: Calculate the optimal portfolio and relevance allocation for the two networks you have generated under **b:**.

Question: Produce a histogram of the relevance and of the portfolio holdings and discus you interpretations.

d: A research community in equilibrium

Suppose we have a collection of N researchers like the one in **c:** who interact with one another through random interactions during which they exchange information, data, research resources. Suppose the relevant utility function for these agents, that describes the utility of the expected priority-reward r_j of puzzle j, is given by $u[r] = \sqrt{r}$.

Question: Describe how we can view these agents as being in a statistical choice equilibrium.

Question: Compute the partition function, the Helmholtz utility of this statistical equilibrium and plot a few of the choice-probabilities as a function of $T = \beta^{-1}$, including the one(s) that survive(s) in the $T \to 0\ \infty$ limit. Explain your results.

e: Paradigms

Consider the two networks you have calculated under **b:**.

Question: Give an argument why the direct outcome of a research process might be a directed network whereas the actual underlying knowledge network might nevertheless be a symmetric network?

Question: Form the adjacency matrices of the symmetric networks, compute the possible paradigms that the networks support and, for each of the networks, produce graphs that visually identify the differences between the first and second paradigm.

Question: Explain why, in the case of paradigm 1, for researcher J the number $\sigma_J = 0$ or 1 depending on whether she has chosen an optimal portfolio $\vec{h}_J$ that is consistent with the 'true' paradigm ($\sigma_J = 1$) or that was consistent with a mistaken paradigm ($\sigma_J = 0$).

Question: Explain why, in the case of paradigm 2, for researcher J the number $\sigma_J = -1$ or 1 depending on whether she has chosen an optimal portfolio $\vec{h}_J$ that is consistent with one version of paradigm 2 ($\sigma_J = 1$) or that was consistent with the opposing version of paradigm 2 ($\sigma_J = 1$).

f: Mean-field paradigm choice

We will now consider the researchers doubting between the two possible versions of paradigm 2, i.e. with choice variables $\sigma_J = \pm 1$. Suppose we have a community of 500 researchers. Each of them has a 1/3 probability of being connected to each of the others. Assume that the preferences of researcher J concerning this choice are given by

$$u_J(\sigma_J) = E\,\sigma_J + \sum_{M=1}^{N} C_{\mathrm{JM}}\,\sigma_J\,\sigma_M \;,$$

where C_{JM} are the matrix-elements of the adjacency matrix of the community-network among the researchers and E is the evidence available to each researcher.

Question: Generate a random network among these 500 researchers, without self-loops, where the weights are either 1 or 0 according to the above description, call its adjacency matrix C1, also generate the symmetrized version of it and call its adjacency matrix CS.

Question: Use *mean field theory* and the mean out-degree of the researcher-network to analyse whether the two communities can agree on a consistent choice of σ_J .

Question: Calculate the variance of the out-degree in your random network and discuss, perhaps using mean-field theory, what the effect of this is on the critical value of T below which consensus formation starts to occur.