Project Report: Analyzing Kalman Filter Performance on Different Flights

Introduction

This report presents the results from applying Kalman filters for tracking airplanes using simulated radar data. The study aims to identify which types of flights pose more challenges for the tracking system and determine the flight segments that are harder to track, and to assess the benefits of smoothing techniques on the predicted flight path.

Methodology

Data Preparation and Initial Exploration

Our data consisted of flight information obtained from the `get_ground_truth_data()` function, encompassing various flight paths and characteristics. We first extracted a list of flight IDs and relevant flight data. Preliminary exploration was done involving examining flight duration, altitude, and route complexity based using custom helper functions to gain deeper insights into each flight's nature, and to understand the tracking challenges posed by different flight profiles.

Kalman Filter Implementation

Description of the model

To accurately describe the model parameters for the Kalman filter, the following matrices were used for its specifications.

1. **State Vector**

$$x_k = \begin{pmatrix} \mathrm{x} \\ \mathrm{y} \\ \dot{\mathrm{x}} \\ \dot{\mathrm{y}} \end{pmatrix}$$

The state vector is represented by a 4-vector with x and y representing the x and y coordinates of the aircraft while $\dot{x}$ and $\dot{y}$ represents the velocity of the aircraft in the x and y coordinates respectively.

2. **Transition Matrix**

$$F = \begin{pmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 10 & 0 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The transition matrix is represented by a 4 x 4 matrix. It updates the state of the previous state by updating the x-coordinates by the change in distance in the x direction (velocity in x multiplied by the delta time). Similarly, it updates the state of the y-coordinates by the change in distance in the y direction. As it was assumed that the velocity of the aircraft would remain constant in this problem, we will not make any changes to the velocities of the new state.

3. Covariance Matrix for Process Noise

$$Q = \begin{pmatrix} 0.25\Delta t^4 & 0 & 0.5\Delta t^3 & 0 \\ 0 & 0.25\Delta t^4 & 0 & 0.5\Delta t^3 \\ 0.5\Delta t^3 & 0 & \Delta t^2 & 0 \\ 0 & 0.5\Delta t^3 & 0 & \Delta t^2 \end{pmatrix} \sigma_p^2$$

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The covariance matrix for process noise is represented by a 4 x 4 matrix. It considers the process covariance noise of each state. That is the x position is affected by the noise from the variance of the positions $\frac{1}{4}\Delta t^4 \sigma_p^2$ and the combined covariance of the position and velocity caused by the x-velocity $\frac{1}{2}\Delta t^3 \sigma_p^2$. Similarly, this is applied to the y position. The x-velocity is affected by the variance of the velocities $\Delta t^2 \sigma_p^2$ and the combined covariance $\frac{1}{2}\Delta t^3 \sigma_p^2$. This is similarly applied to the y-velocity.

4. Observation Vector

$$z_k = \begin{pmatrix} x \\ y \end{pmatrix}$$

The observation vector is represented by a 2-vector. Since in this problem, the radar is only able to observe a measured position of the aircraft and not the velocity, the vector only contains 2 components, the x and y position of the measured position of the aircraft.

5. Observation Matrix

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

The observation matrix is represented by a 2 x 4 matrix. Since we only wish to extract out the x and y position of the state vector, we have the following matrix that would transform the state vector to only contain two components, the x and y position.

6. Covariance Matrix for Observation Noise

$$R = \begin{pmatrix} \sigma_o^2 & 0 \\ 0 & \sigma_o^2 \end{pmatrix}$$

The covariance matrix for observation noise is represented by a 2 x 4 matrix. The components of the observation vector are affected by the variance of the measured radar position, σ_o.

Experiment Setup and Results

The filter was applied to both simulated radar data and actual flight paths to generate filtered and smooth state estimates for comparison.

We conducted experiments on a randomly selected set of flights, analyzing the performance across different flight types and segments.
These are the experiment parameters:

1. The dataset was filtered to randomly select five flights to ensure a diverse range of conditions and behaviors to analyze.
2. The filtered was applied on the entire flight duration

Experiment 1: Testing the filter with the provided noise parameters and smoothing
The five randomly chosen flights for this experiment were:

1. ADA4_025
2. CALIBRA_026
3. VOR05_034
4. REGA1_018
5. CALIBRA_021

To evaluate how much better the smoothed tracks are compared to the filtered ones, we need to compare the average and maximum distances from the actual flight path for both the filtered and smoothed estimates. Shown below is the result of this comparison:

Flight ID	Process Noise (Default)	Observation Noise (Default)	Mean Filtered Distance (metres)	Max Filtered Distance (metres)	Mean Smoothed Distance (metres)	Max Smoothed Distance (metres)
ADA4_025	1.5	100	12412.97	32720.31	12394.57	32603.87
CALIBRA_026	1.5	100	11019.48	29236.86	11005.59	29052.69
VOR05_034	1.5	100	23044.24	41095.80	23037.57	40881.12
REGA1_018	1.5	100	6131.77	10139.52	6132.58	10083.07
CALIBRA_021	1.5	100	14946.48	53414.09	14930.24	53309.69
Overall Average	-	-	13510.99	33321.32	13500.11	33186.09

Interpretation of this result:

1. Flight ID: ADA4_025

The average and maximum distance after applying smoothing techniques are nearly identical to the filtered mean, indicating that smoothing did not significantly alter the average distance. However, the maximum distance being over 32km indicate that there may be specific points along the flight path where the actual and filtered data vary significantly, potentially due to abrupt manoeuvres or complex route. Refer to figure 1.

2. Flight ID: CALIBRA_026

Similar to the previous flight, the filtering and smoothing results are quite close, indicating consistent processing. This flight has slightly shorter mean and maximum distances compared to ADA4_025, suggesting a less complex route. Refer to figure 2.

3. Flight ID: VOR05_034

Shows significantly higher mean and maximum distances compared to the first two, indicating a longer or more varied and complex flight path. Refer to figure 3.

4. Flight ID: REGA1_018

Has the shortest mean and maximum distances, suggesting a very simple flight route. Refer to figure 4.

5. Flight ID: CALIBRA_021

This dataset shows the highest discrepancies between maximum filtered and smoothed distances compared to other flights, indicating a possible long-distance flight or a highly complex route. Refer to figure 5.

<u>Overall Averages</u>

1. Average Mean Distance for Filtered Data

The overall average mean distance between the actual flight paths and their filtered counterparts for all analysed flights is approximately 13.51 km. This metric indicates the average level of deviation from the actual paths after the filtering process, providing an insight into the general accuracy of the filtering.

2. Average Max Distance for Filtered Data

The average maximum distance between the actual and filtered data across all flights is about 33.34 km. This represents the largest deviation observed in each flight's data set after filtering, highlighting areas where the filtering may have oversimplified or significantly diverged from the actual flight path.

3. Average Mean Distance for Smoothed Data

This is very similar to the average filtered mean distance, with a value of approximately 13.51 km, suggesting that the smoothing process maintains the overall trajectory accuracy established by the filtering, with minor adjustments for a more regularised path representation.

4. Average Max Distance for Smoothed Data

Almost identical to the average filtered max distance, with a value around 33.33 km, which underscores the similarity in the extent of maximum deviations between the smoothed and filtered data sets. This similarity indicates that the smoothing process does not significantly improve the filtered results.

Summary

In summary, our results have also shown that flights with smoother trajectories and straighter paths showed smaller tracking errors. In contrast, flights with irregular patterns or experiencing significant manoeuvres exhibited higher errors. As such, takeoff and landing segments were harder to track due to rapid altitude changes and acceleration. This result is aligned with the Kalman Filter assumption that the transition model and sensor model have to be both linear.

The data indicates variability in flight distances, with REGA1_018 having notably shorter routes and CALIBRA_021 showing significantly longer or more varied paths. The small differences between filtered and smoothed distances suggest that the smoothing process retains the general characteristics of the original data while likely removing noise or anomalies. Our results have shown that smoothing techniques are more effective particularly in segments with significant altitude changes or more complex routes as evidenced by flight CALIBRA_021 where the max smoothed distance is about 100 metres lesser than the max smoothed distance.

The results indicated an improvement in the estimation of the airplane's position, as evidenced by the generated plots comparing the original and estimated paths (see figure 6). Still referring to figures 6,7 and 8, the actual data appears to be quite volatile, with many fluctuations and a high degree of variance throughout the flight's duration, whereas the filtered data seems to have a smoother trajectory, resulting in a line that fluctuates less sharply. The path also appears shorter than the actual path, indicating that noise in the data has been removed, resulting in a more direct and shorter path.

The large discrepancies between the maximum and mean distance between the two data sets can suggest a few things:

- Complex routes

Flights that navigate complex routes, such as those avoiding geographic obstacles (mountains, large bodies of water), entering and exiting busy airspaces, or making multiple turns, can exhibit significant differences between the filtered (or smoothed) data and the actual data. The complexity adds variability to the flight path, increasing the likelihood of larger deviations.

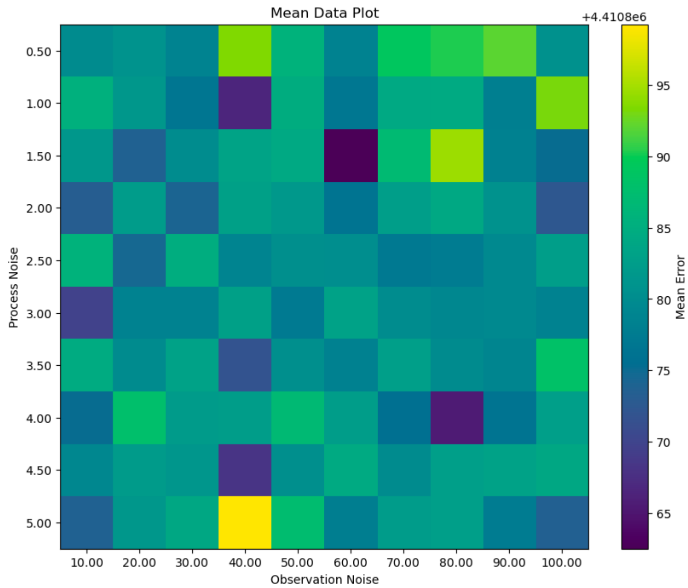
- Effectiveness of filtering

The filtering process aims to remove noise and anomalies from raw data, but they cannot perfectly replicate the true flight trajectories, especially over longer distances or for flight paths with complex routes.

Experiment 2: Testing the filter with varied process noise and observation noise parameters

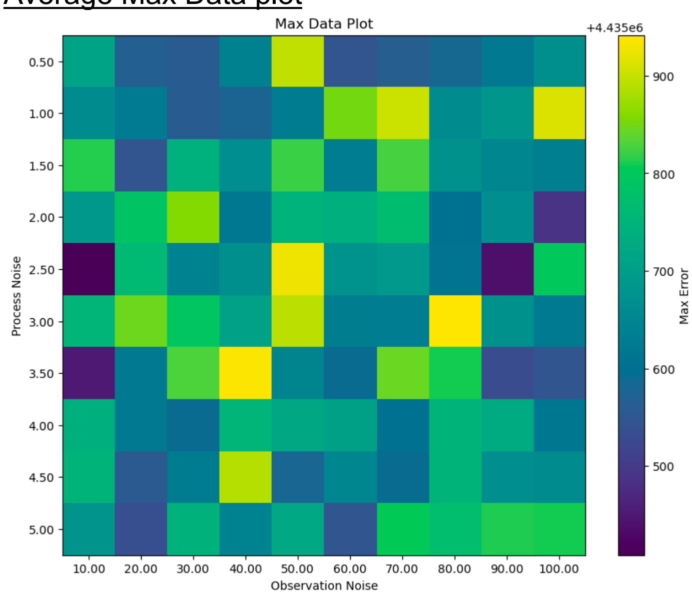
We plotted a noise table to show the relationship between the average mean error, and average maximum errors of fights ADA_025, CALIBRA_026, VOR05_034, REGA1_018, CALIBRA_021 against two parameters: observation noise and process.

<u>Average Mean Data plot</u>



Looking at the plot above, it can be seen that the optimal values for observation noise and process noise are 60 and 1.50 respectively.

<u>Average Max Data plot</u>



Looking at the plot above, it can be seen that the optimal values for observation noise and process noise are 10.00 and 2.50 respectively.

Experiment 3: Changing the model to include altitude and vertical velocity, and measure its accuracy.

<u>Description of the model</u>

1. **State Vector**

$$x_k = \begin{pmatrix} \mathrm{x} \\ \mathrm{y} \\ \mathrm{z} \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix}$$

The state vector is represented by a 6-vector with x, y and z representing the x, y and z coordinates of the aircraft while $\dot{x}$, $\dot{y}$, and $\dot{z}$ represents the velocity of the aircraft in the x, y and z coordinates respectively.

2. **Transition Matrix**

$$F = \begin{pmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

3. **Covariance Matrix for Process Noise**

$$Q = \begin{pmatrix} 0.25\Delta t^4 & 0 & 0 & 0.5\Delta t^3 & 0 & 0 \\ 0 & 0.25\Delta t^4 & 0 & 0 & 0.5\Delta t^3 & 0 \\ 0 & 0 & 0.25\Delta t^4 & 0 & 0 & 0.5\Delta t^3 \\ 0.5\Delta t^3 & 0 & 0 & \Delta t^2 & 0 & 0 \\ 0 & 0.5\Delta t^3 & 0 & 0 & \Delta t^2 & 0 \\ 0 & 0 & 0.5\Delta t^3 & 0 & 0 & \Delta t^2 \end{pmatrix} \sigma_p^2$$

4. Observation Vector

$$z_k = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

The observation vector is represented by a 3-vector.

5. Observation Matrix

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

6. Covariance Matrix for Observation Noise

$$R = \begin{pmatrix} \sigma_o^2 & 0 & 0 \\ 0 & \sigma_o^2 & 0 \\ 0 & 0 & \sigma_o^2 \end{pmatrix}$$

Results

Flight ID	Process Noise (Default)	Observation Noise (Default)	Mean Filtered Distance (metres)	Max Filtered Distance (metres)
ADA4_025	1.5	100	993.40	4294.28
CALIBRA_026	1.5	100	690.44	2860.33
VOR05_034	1.5	100	2636.42	9404.87
REGA1_018	1.5	100	663.04	1533.04
CALIBRA_021	1.5	100	710.29	2549.34
Overall Average	-	-	1138.71	7871.83

The mean filtered distance is relatively low compared to the 2D model, which suggests that the 3D model can predict the position of the aircraft with better accuracy. This is expected as the 3D filter takes into account altitude changes along with its horizontal movement, which is a more accurate representation of actual flight paths as the planes move in three dimensions, and the 3D filter captures more of the flight dynamics. The higher errors in the 2D filter suggest that ignoring the altitude dimensions leads to less accurate position estimates (by a factor of 10). This could be because changes in altitude have a significant impact on the radar measurements and thus on the position estimates.

Appendix

Figure 1: Flight data for flight ADA4_025

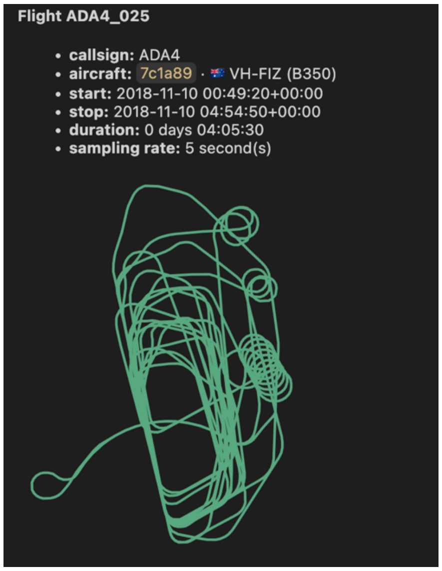


Figure 2: Flight data for flight CALIBRA_026

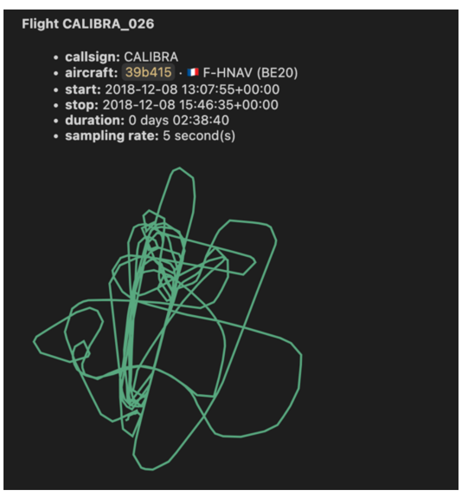


Figure 3: Flight data for flight VOR05_034

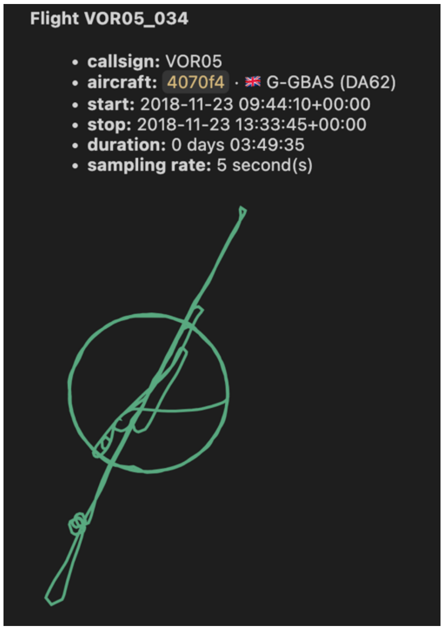


Figure 4: Flight data for flight REGA1_018

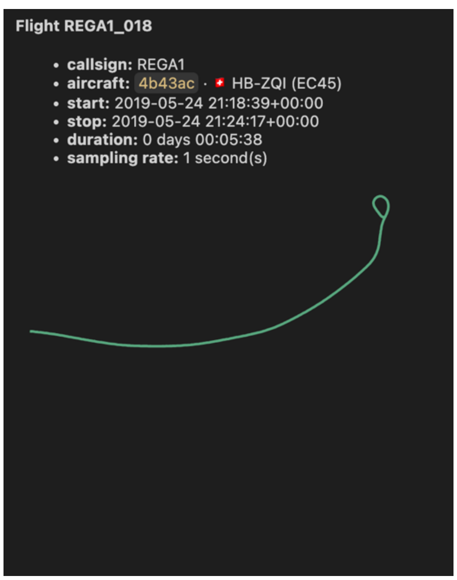


Figure 5: Flight data for flight CALIBRA_021

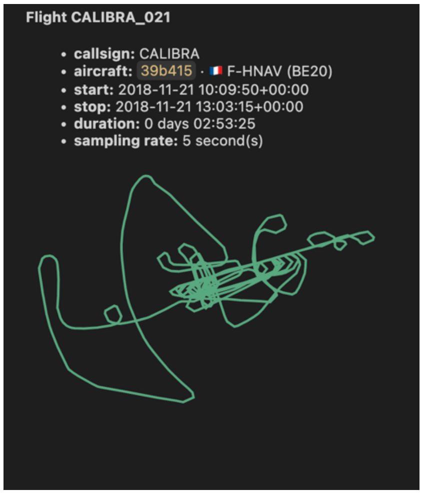


Figure 6: Plot of actual latitude and filtered latitude over time

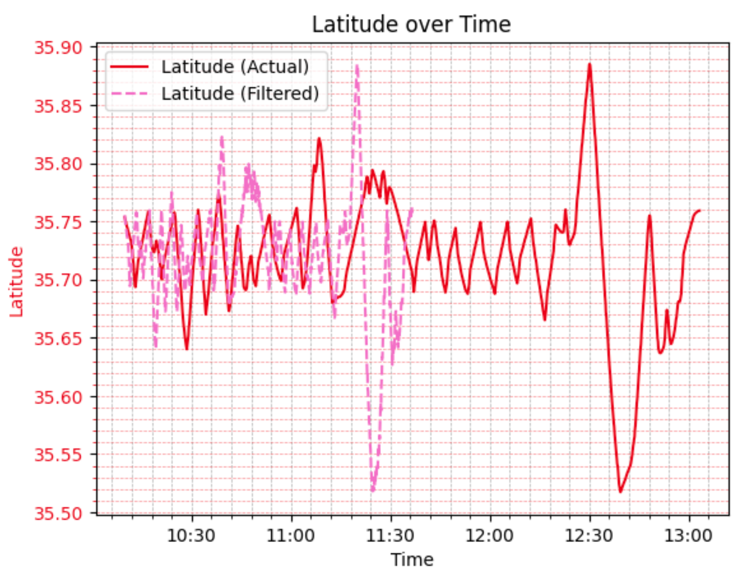


Figure 7: Plot of actual longitude and filtered longitude over time

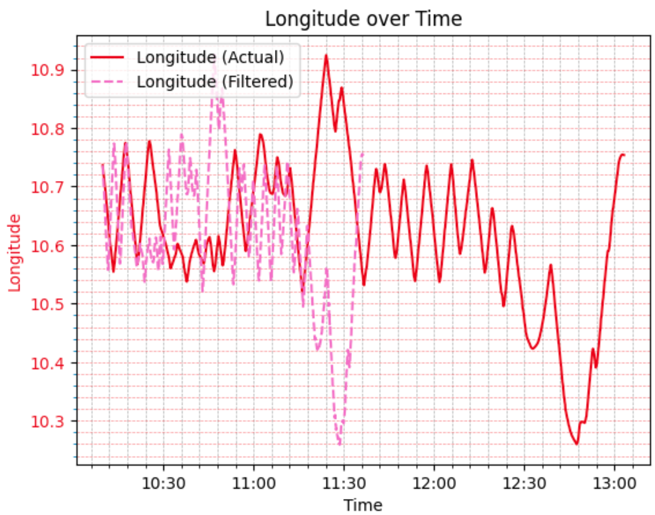


Figure 8: Plot of actual longitude, actual latitude, filtered longitude and filtered latitude over time

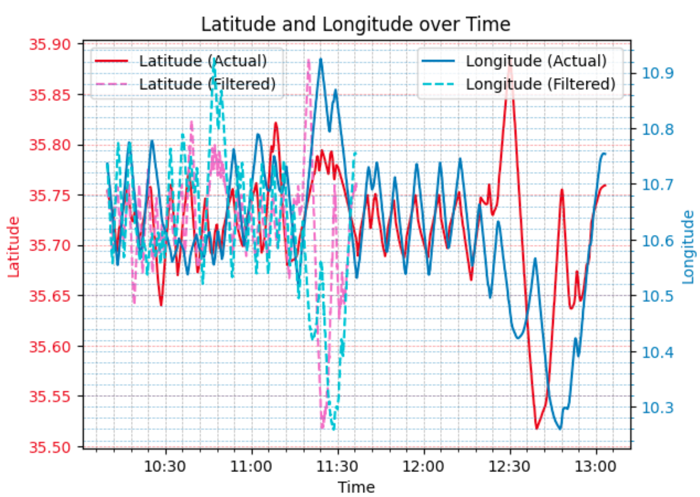


Figure 9: Plot of all the flights' duration, maximum altitude, average altitude, and route complexity

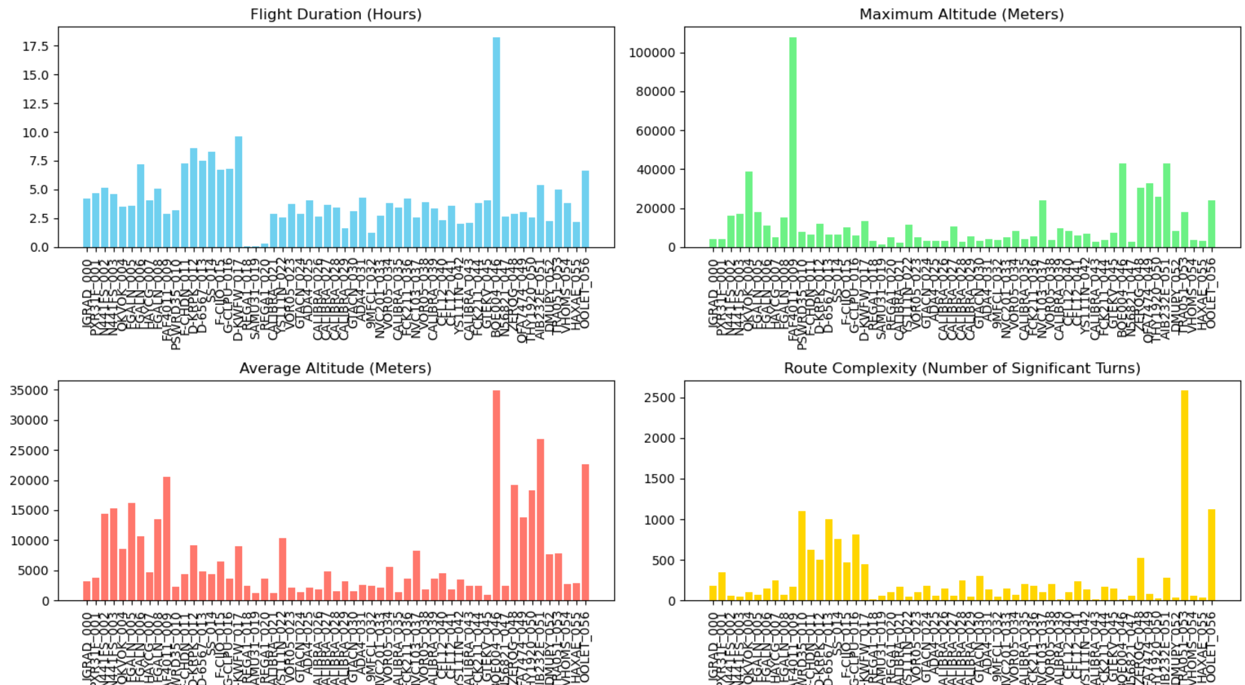


Figure 10: Plot of Latitude vs Longitude of actual path, filtered path and smoothed path for ADA4_025

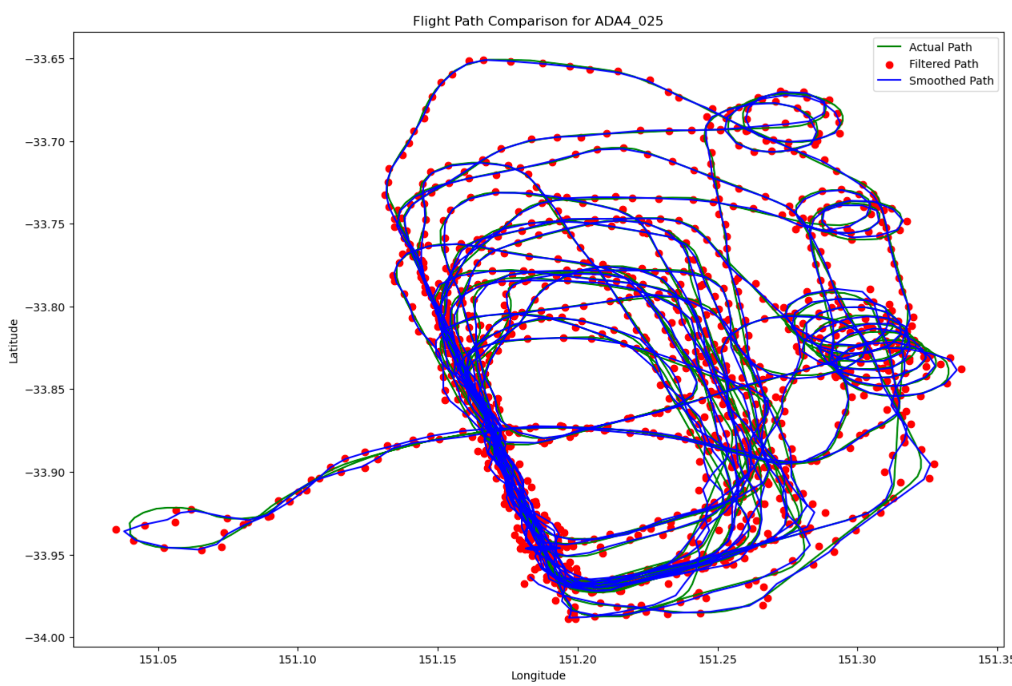


Figure 11: Plot of Latitude vs Longitude of actual path, filtered path and smoothed path for CALIBRA_026

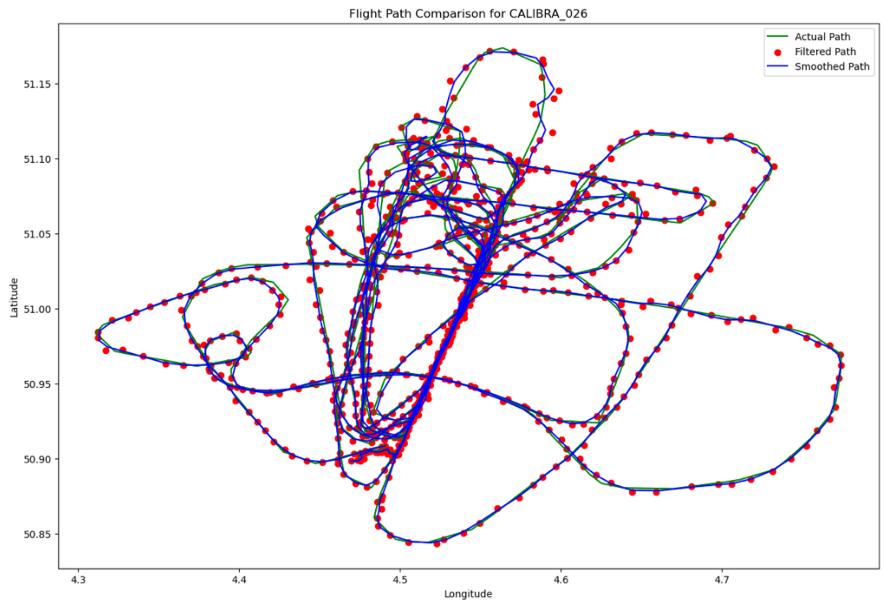


Figure 12: Plot of Latitude vs Longitude of actual path, filtered path and smoothed path for VOR05_034

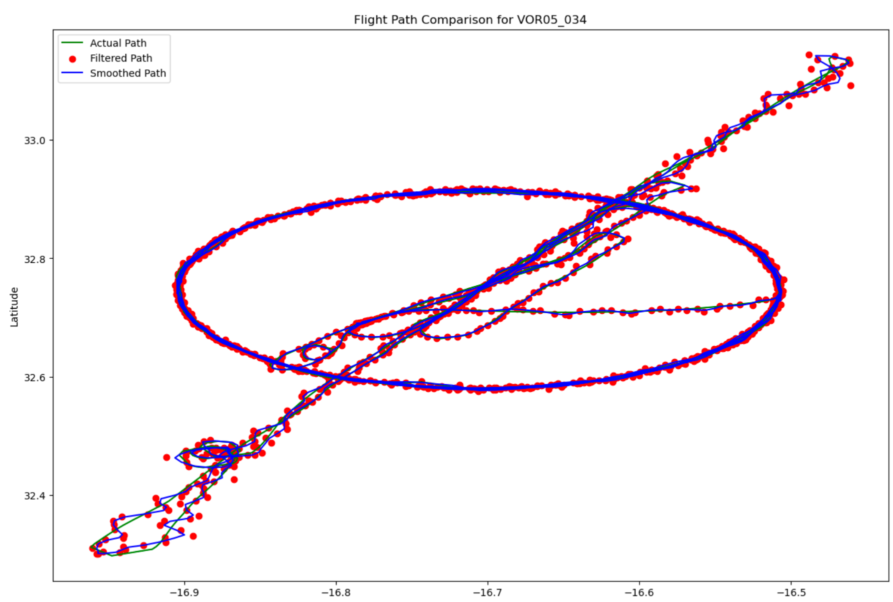


Figure 13: Plot of Latitude vs Longitude of actual path, filtered path and smoothed path for REGA1_018

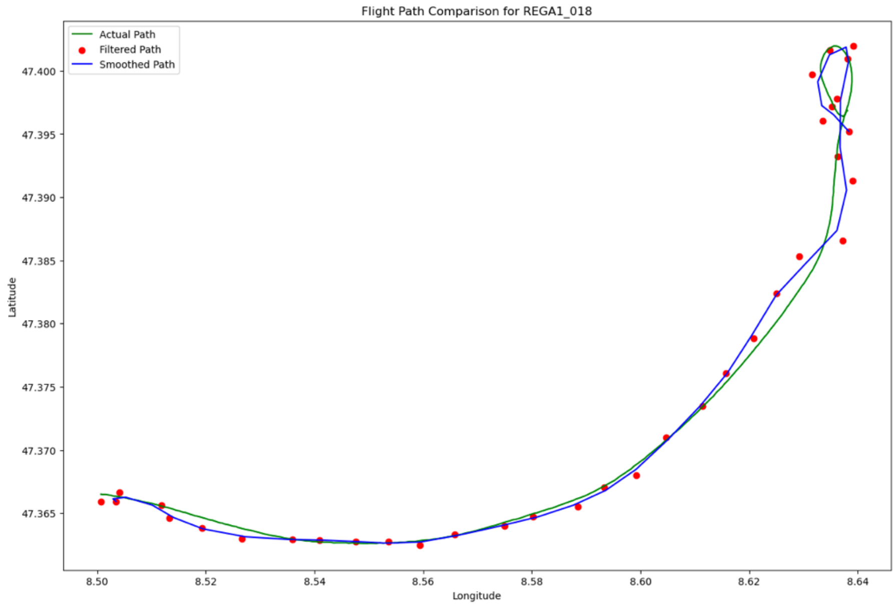


Figure 14: Plot of Latitude vs Longitude of actual path, filtered path and smoothed path for CALIBRA_021

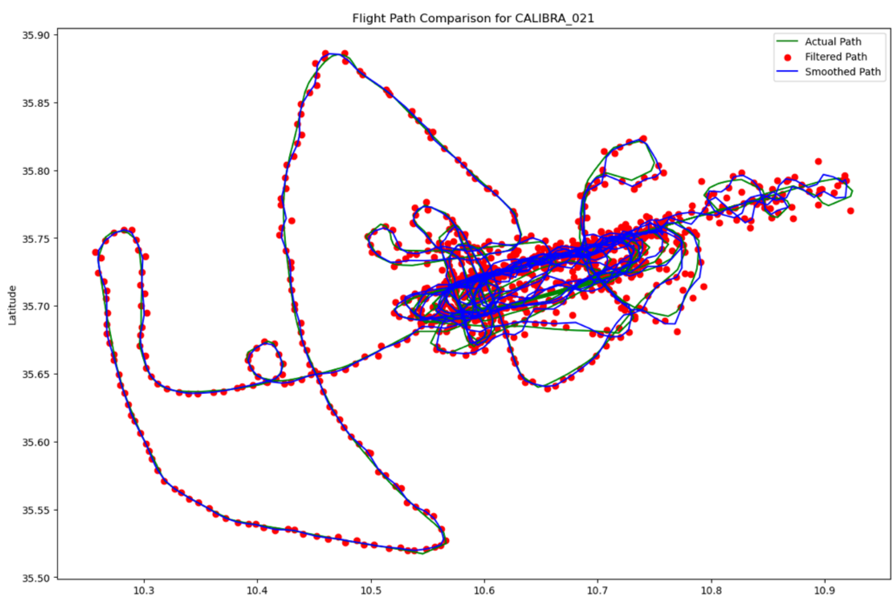


Figure 15: Plot of average error with variation in process noise and observation noise for flights ADA_025, CALIBRA_026, VOR05_034, REGA1_018, CALIBRA_021

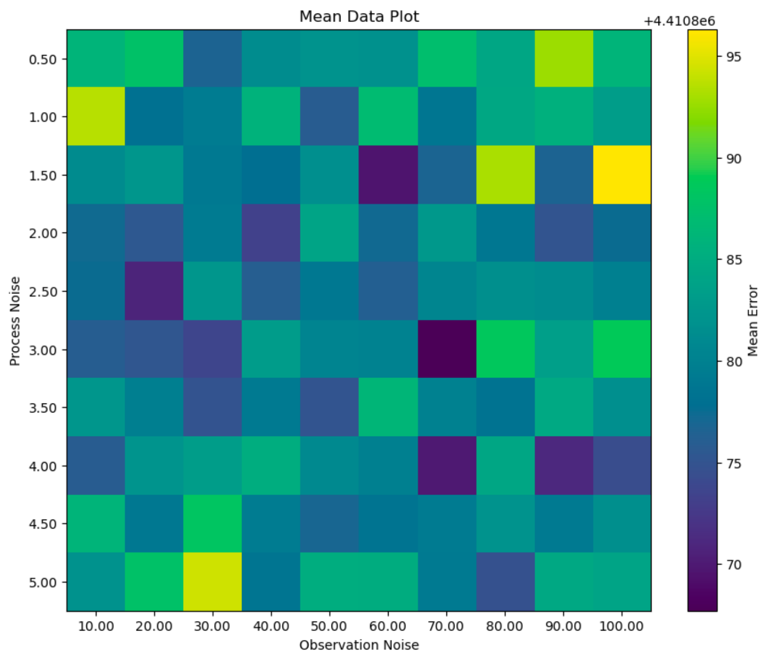


Figure 16: Plot of average maximum error with variation in process noise and observation noise for flights ADA_025, CALIBRA_026, VOR05_034, REGA1_018, CALIBRA_021