Calculus and Analytical Geometry

Quiz-2

Class: BS CS-II (H) Instructor: Abdul Fatah Rajri

Q1: **[7 Marks]**

a. Find all the relative extrema of $\boldsymbol{f(x) = x + \frac{121}{x}}$.

b. Given that $\boldsymbol{f'(t) < 0}$, for every value of t in the interval $(\boldsymbol{-10, 10})$.

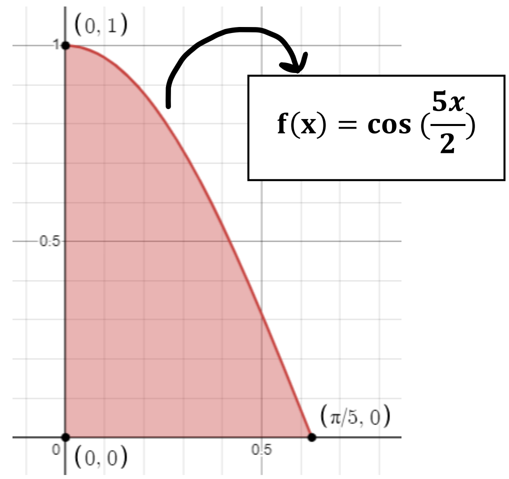
Explain Why $\boldsymbol{f(0) > f(10)}$?

Q3:[3 Marks]

a. Find the area of under the graph of

$$\boldsymbol{y = \cos\left(\frac{5x}{2}\right)}$$

Between the ordinates as shown in figure.



Q4[3 Marks]: Buffon's Needle Experiment: A horizontal plane is ruled with parallel lines 2 inches apart. A two-inch needle is tossed randomly onto the plane. The probability that the needle will touch a line is,

$$\boldsymbol{P = \frac{\pi}{2}\int_0^{\frac{\pi}{2}} \sin(\theta)\, d\theta}$$

Where $\boldsymbol{\theta}$ is the acute angle between the needle and any one of the parallel lines. **Find this probability**.

Q5:**[12 Marks]**

a. **Shifted Unit Step Function** is defined by,

$$U(t) = \begin{cases} 1 & t < a \\ 0 & t > a \end{cases}$$

Show that $\int_a^b U(t)dt = b - a$ where $b > a$.`

b. Show that if is continuous on the entire real number line, then,

$$\int_{x_1}^{x_2} f(x+h)dx = \int_{x_1+h}^{x_2+h} f(u)du.$$

c. Recall, if $g(x) \le f(x) \le h(x)$ then $\int_a^b g(x)dx \le \int_a^b f(x)dx \le \int_a^b h(x)dx$. Using this fact show that,

$$a - b \le \int_a^b \sin(x)dx \le b - a.$$

d. Find a function that satisfy,

$$\int_0^1 x^2 f(x)dx = a^2$$

e. Given that if $f(x)$ is even, then $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx.$ Show that average value of an even function $E(x)$ on $[0, 2]$ is given by $\int_0^2 E(x)dx.$

f. Given that $\frac{d}{dx}\left(tan^{-1}\left(\frac{x}{a}\right)\right) = \frac{a}{a^2+x^2}$. Evaluate

$$\int \frac{3}{9+x^2}dx$$