Implicit Differentiation :-

$y = x^2 + 3$

$y = \sin(x)$

$y = \sqrt{1 - x^2}$

$y = \tan\left(\frac{1}{x^2+1}\right)$

$\longrightarrow$ Explicit form of a function.

$$\frac{dy}{dx} = \frac{d}{dx}(x^2+3)$$
$$= 2x + 0 = 2x$$

Implicit

$xy = 2 \longrightarrow y = \frac{2}{x}$ — Explicit form.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{2}{x}\right) = \frac{-2}{x^2}$$

$x^2 + y^2 = 2 \longrightarrow y = \pm\sqrt{1-x^2}$

$y(x)$ relation

$x^2 - 2y^3 + 4y = 2 \longrightarrow y = ?$

$y = \sin(x^2 + y^2) \longrightarrow y = ?$

$\sqrt{xy} = x - 2y \longrightarrow y = ?$

Implicit Differentiation:

$x^2 + y^2 = 1$

$\frac{dy}{dx}$

$$\frac{x^2}{2} + \frac{y^2}{3} = 1$$

$$\frac{dy}{dx} = ?$$

General Power Rule

$$\frac{d}{dx}\left((f(x))^n\right) = n\,(f(x))^{n-1} f'(x)$$

$y \rightarrow$ function of x

$$(x^n)' = n\,x^{n-1}$$

$$\frac{d}{dx}\left(y^n\right) = n\,y^{n-1}\,y'$$

$$\boxed{\begin{aligned}\frac{d}{dx}\left((f(y))^n\right) &= n\,(f(y))^n \frac{d}{dx}\left(f(y)\right) \\ &= n\,(f(y))^n f'(y)\,y'\end{aligned}}$$

$\times$

$$\frac{d}{dx}\left(\sin(y)\right) = \cos(y)\,\frac{dy}{dx}$$

$$\frac{d}{dx}\left(\cos(y)\right) = -\sin(y)\,y'$$

$$\frac{d}{dx}\left(\tan(y)\right) = \sec^2(y)\,y'$$

$$\frac{d}{dx}\left(f(y)\right) = f'(y)\,y'$$

$$\frac{d}{dx}\left(y^2 + 3y + \sin(y) - 3x\right)$$

$$= \frac{d}{dx}\left(y^2\right) + 3\frac{d}{dx}(y) + \frac{d}{dx}\left(\sin y\right) - 3\frac{d}{dx}(x)$$

$$= 2y\frac{dy}{dx} + 3\frac{dy}{dx} + \cos(y)\frac{dy}{dx} - 3(1)$$

$$= 2y\,y' + 3\,y' + (\cos y)\,y' - 3.$$

Ex 2

$$\frac{d}{dx}\left(x y^2\right)$$

$$= \frac{d}{dx}(x)\, y^2 + x \frac{d}{dx}\left(y^2\right)$$

$$= y^2 + x\left(2y\, y'\right)$$

$$= y^2 + 2xyy'$$

$$y^3 + y^2 - 5y - x^2 = -4 \qquad \frac{dy}{dx} = ?$$

$$\frac{d}{dx}\left(y^3 + y^2 - 5y - x^2\right) = \frac{d}{dx}(-4)$$

$$\frac{d}{dx}\left(y^3\right) + \frac{d}{dx}\left(y^2\right) - 5\frac{dy}{dx} - \frac{d}{dx}\left(x^2\right) = 0$$

$$3y^2 y' + 2yy' - 5y' - 2x = 0$$

$$y'\left(3y^2 + 2y - 5\right) = 2x$$

$$y' = \frac{2x}{3y^2 + 2y - 5}$$

$$2\sin(x)\cos(y) = 1$$

$\frac{dy}{dx} = ?$

$y = ?$

$$\frac{d}{dx}\left(2\sin(x)\cos(y)\right) = \frac{d}{dx}(1)$$

$$2\frac{d}{dx}\left(\underbrace{\sin(x)}_{f}\underbrace{\cos(y)}_{g}\right) = 0$$

$$\cos(x)\cos(y) + \sin(x)\left(-\sin(y)\right)\frac{dy}{dx} = 0$$

$$\cos(x)\cos(y) - \left(\sin(x)\sin(y)\right)y' = 0$$

$$\cos x\cos y = \underline{\sin x}\sin y\ y'$$

$$y' = \frac{\cos(x)\cos(y)}{\sin(x)\sin(y)} = \cot(x)\cot(y)$$

$$y' = \cot(x)\cot(y)$$

$$\frac{dy}{dx} = \cot(x)\cot(y)$$

$$\sqrt{xy} = x - 2y$$

$$\Rightarrow (xy)^{\frac{1}{2}} = x - 2y$$

$$\Rightarrow \frac{d}{dx}\left((xy)^{1/2}\right) = \frac{d}{dx}(x-2y)$$

$$\Rightarrow \frac{1}{2}(xy)^{\frac{1}{2}-1}\frac{d}{dx}(xy) = 1 - 2y'$$

$$\Rightarrow \frac{1}{2(xy)^{1/2}}\left[\frac{d}{dx}(xy)\right] = 1 - 2y'$$

$$\Rightarrow \frac{1}{2\sqrt{xy}}\left[(1)y + xy'\right] = 1 - 2y'$$

$$\Rightarrow \frac{y + xy'}{2\sqrt{xy}} = 1 - 2y'$$

$\Rightarrow \quad y + xy' = (1-2y')(2\sqrt{xy})$

$\Rightarrow \quad y + xy' = 2\sqrt{xy} - 2y'\sqrt{xy}$

$\Rightarrow \quad xy' + 2y'\sqrt{xy} = 2\sqrt{xy} - y$

$\Rightarrow \quad y'(x + 2\sqrt{xy}) = 2\sqrt{xy} - y$

$\Rightarrow \quad y' = \dfrac{2\sqrt{xy} - y}{x + 2\sqrt{xy}}$ ✓

$3(x^2$

⊙ $3(x^2+y^2)^2 = 100xy$ ✓ $\qquad (3,1)$

Equation of tangent to – at $(3,1)$

$\dfrac{d}{dx}\left[3(x^2+y^2)^2\right] = \dfrac{d}{dx}(100xy)$

$\Rightarrow \quad 3\dfrac{d}{dx}\left((x^2+y^2)^2\right) = 100\dfrac{d}{dx}(xy)$

$\Rightarrow \quad 3\left[2(x^2+y^2)'\dfrac{d}{dx}(x^2+y^2)\right] = 100\left[(1)y + xy'\right]$

$\Rightarrow \quad 3\left[2(x^2+y^2)(2x+2yy')\right] = 100(y+xy') \qquad (3,1)$

$\Rightarrow \quad 3\left[2(9+1)(6+2y')\right] = 100(1+3y')$

$\Rightarrow \quad 60(6+2y') = 100 + 300y'$

$\Rightarrow \quad 360 + 120y' = 100 + 300y'$

$\Rightarrow \quad 260 = 180y'$

$\Rightarrow \quad y' = \dfrac{13}{9}$

$m = \dfrac{13}{9}$ at $(3,1)$ to $(\ )$

$(3,1)$ $\quad m = \frac{13}{9}$

$$y-1 = \frac{13}{9}(x-3)$$

$$9y - 9 = 13x - 39$$

$$9y - 13x = -30$$

$\odot$ $\quad \frac{dy}{dx} = ?$

$y = \tan(x^2+y^2)$ $\quad \frac{dy}{dx} = ?$ $\quad x=0,\ y=0$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\tan(\underline{x^2+y^2})\right)$$

$$y' = \left(\sec^2(x^2+y^2)\right)\frac{d}{dx}(x^2+y^2)$$

$$y' = \left(\sec^2(x^2+y^2)\right)(2x+2yy')$$

$$y' = \left(\sec^2(0)\right)(0+0)$$

$$y' = 0$$

$m = 0$

$m=0,$ $\quad x=0,$ $\quad y=0$

$(0,0)$

$$y-0 = 0(x-0)$$

$$y = 0$$

$(x^2 + y^2 - 1)$

⑥ $(x^2+y^2-1)^3 = x^2y^3$

$\frac{dy}{dx} =$

$x=1, y=0$

$\Rightarrow \frac{d}{dx}\left((x^2+y^2-1)^3\right) = \frac{d}{dx}(x^2y^3)$

$\Rightarrow 3(x^2+y^2-1)^2 \frac{d}{dx}(x^2+y^2-1) = (2x)y^3 + x^2(3y^2y')$

$\Rightarrow 3(x^2+y^2-1)^2(2x+2yy'-0) = 2xy^3 + 3x^2y^2y'$

$\Rightarrow 3(1+0-1)(2\cancel{(x)}+2(0)y') = 2(1)(0)^3$

$(x^2+y^2-1)^3 = x^2y^3$

$\Rightarrow \frac{d}{dx}\left((x^2+y^2-1)^3\right) = \frac{d}{dx}(x^2y^3)$

$\Rightarrow 3(x^2+y^2-1)^2 \frac{d}{dx}(x^2+y^2-1) = (2x)y^3 + x^2(3y^2y')$

$\Rightarrow 3(x^2+y^2-1)^2(2x+2yy') = 2xy^3 + 3x^2y^2y'$ ✓

$\Rightarrow 6x(x^2+y^2-1)^2 + 6yy'(x^2+y^2-1)^2 = 2xy^3 + 3x^2y^2y'$

$\Rightarrow 6yy'(x^2+y^2-1)^2 - 3x^2y^2y' = 2xy^3 - 6x(x^2+y^2-1)^2$

$\Rightarrow y'\left(6y(x^2+y^2-1)^2 - 3x^2y^2\right) = 2xy^3 - 6(x^2+y^2-1)^2$

$\Rightarrow \frac{dy}{dx} = \boxed{\frac{2xy^3 - 6(x^2+y^2-1)^2}{6y(x^2+y^2-1)^2 - 3x^2y^2}}$