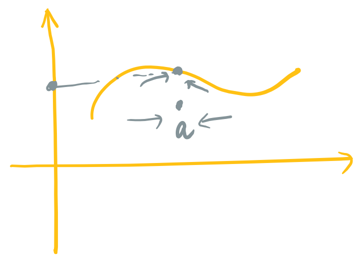
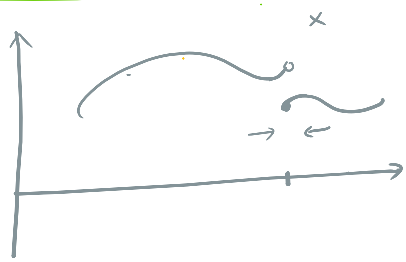
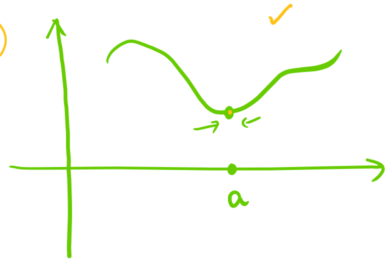
⊙ Continuity of a function at a point:





(1)

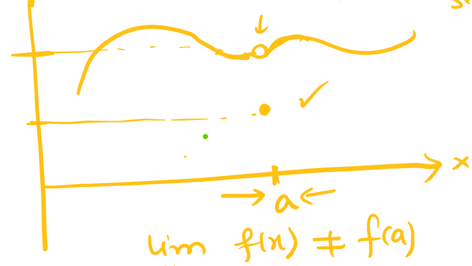


limit at $x=a$ ✓
exists

$f(a)$ ✓

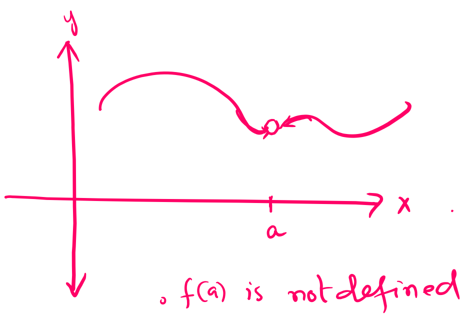
(2)

Limit exists
$f(a)$ makes sense



$\lim\limits_{x \mapsto a} f(x) \neq f(a)$

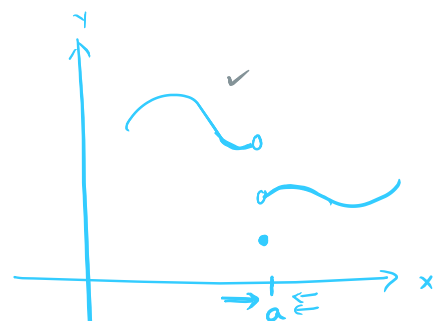
(3)



- $f(a)$ is not defined
- limit of $f(x)$ exists at $x=a$

$\lim\limits_{x \mapsto a} f(x)$ exists.

(4)

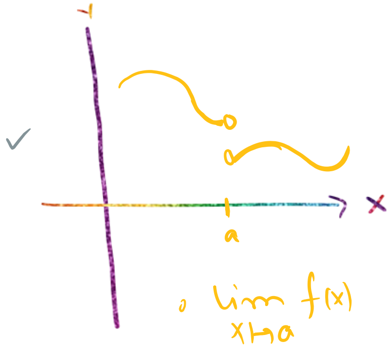


$\lim\limits_{x \mapsto a^+} f(x) \neq \lim\limits_{x \mapsto a^-} f(x)$

$\Rightarrow \lim\limits_{x \longmapsto a} f(x)$ DNE

but $f(a)$ is defined.

5



- $\lim_{x \to a} f(x)$ DNE
- $f(a)$ is not defined.

Definition:

A real valued function $f(x)$ is said to be continuous at $x = a$ if

- $f(x)$ is defined at $x = a$. "$f(a) \in \mathbb{R}$"
- $\lim_{x \to a} f(x)$ exists.
- $\lim_{x \to a} f(x) = f(a)$.

- A function is said to be continuous on open (a,b) if it is continuous at every value in (a,b).

Examples:

① $f(x) = \frac{1}{x-1}$, Is it continous at $x = 1$?

$f(1) = \frac{1}{1-1} = \frac{1}{0}$ ☹

No, $f(x)$ is not continuous at $x = 1$.

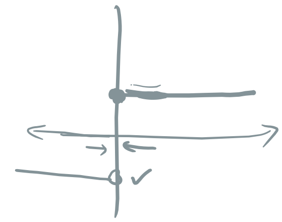
② $y = \frac{x^2-4}{x-2}$, $x = 2$?

$y = \frac{2^2-4}{2-2} = \frac{0}{0}$ ✗ No, it is not contin.

③ $f(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$ at $x = 0$?

Greatest integer $f(x) = [x]$

① $f(0) = 1$ (Yes, defined)



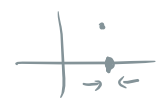
② $\lim_{x \to 0^-} f(x) = -1$, $\lim_{x \to 0^+} f(x) = +1$

left limit ≠ right limit

⇒ ~~f(0)~~ $\lim_{x \to 0} f(x)$ DNE

⇒ $f(x)$ is not contum at $x = 0$.

④ $f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & x \neq 2 \\ 5 & x = 2 \end{cases}$



at $x = 2$?

① $f(2) = 5$ Yes, it is defined.

② $\lim_{x \to 2} \dfrac{x^2-4}{x-2} = \lim_{x \to 2} \dfrac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = 4$

$f(2) = 5$, $\lim_{x \to 2} f(x) = 4$

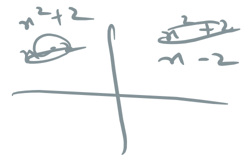
$\lim_{x \to 2} f(x) \neq 5$

⑤ $f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & x \neq 2 \\ 4 & x = 2 \end{cases}$

$f(2) = 4$. $\lim_{x \to 2} f(x) = 4$

contnous at $x = 2$.

⑥ $h(x) = \begin{cases} x^2+2 & x \le 0 \\ x-2 & x > 0 \end{cases}$ at $x=0$?



① $h(0) = 0^2+2 = 2$.

② $\left.\begin{aligned} \lim_{x \to 0^-} x^2+2 &= 2 \\ \lim_{x \to 0^+} x-2 &= -2 \end{aligned}\right\} \Rightarrow \lim_{x \to 0} h(x) = DNE$

③ $h(x)$ is not continuous at $x=0$.

- If discontinuity is because of non-existence of limit then it not <u>removable</u>.
- If ~~doesn~~ limit exists but $f(x)$ is discontinuous then this discontinuity is removable.

$f(x) = \dfrac{x^2-9}{x-3}$ $\quad x=3$?

① $f(3) = \dfrac{0}{0}$

② $\lim_{x \to 3} \dfrac{x^2-9}{x-3} = 6$

$f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & x \neq 3 \\ 6 & x=3 \end{cases}$

$$g(x) = \begin{cases} \dfrac{\sin x}{x} & x \neq 0 \\ 2 & x = 0 \end{cases}, \quad \text{at } x = 0?$$

(i) $g(0) = 2$, (ii) $\lim_{x \to 0} \dfrac{\sin x}{x} = 1$, (iii) $\lim_{x \to 0} g(x) \neq g(0)$

discontin at $x = 0$.

$$g(x) = \begin{cases} \dfrac{\sin x}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$$

$h(x) = \dfrac{1}{x-1}$ at $x = 1$

$h(1)$, $\lim_{x \to 1} \dfrac{1}{x-1}$ DNE

- All Polynomial functions are continuous at every real number.

  $y = x$, $f(x) = x^3 - x^2 + 2x + 3$.

- ~~All those~~ $R(x) = \dfrac{N(x)}{D(x)}$ $D(x) \neq 0$

  $g(x) = \dfrac{x^2 - 4}{x - 2}$, $x \neq 2$,

  $R(x) = \dfrac{x^2 + 3x + 5}{x^2 + 9}$

- $f(x) = \sin x$, $x \in \mathbb{R}$

  $g(x) = \cos x$, $x \in \mathbb{R}$

  $\boxed{h(x) = \tan x = \dfrac{\sin x}{\cos x}}$, $\cos x$ is zero at all the odd multiples of $\frac{\pi}{2}$.

  is continuous at all real numbers except odd multiples of $\frac{\pi}{2}$.

  $\ldots -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

(4) $\sec x = \dfrac{1}{\cos x}$

(5) $\operatorname{cosec} x = \dfrac{1}{\sin x}$

$\sin x = 0$, when x is multiple of π.

$\boxed{\ldots, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, \ldots}$

These are the values where cosec x is discont.

(6) $\cot x = \dfrac{\cos x}{\sin x}$

(0) $Y = \begin{cases} \dfrac{\cos x - 1}{x} & x \neq 0 \\ 3 \checkmark & x = 0 \end{cases}$

(1) $f(0) = 3 \checkmark$

(2) $\lim_{x \to 0} \dfrac{\cos x - 1}{x} = 0 \checkmark$

Discontn at $x = 0$.

- Exponential fun
- log: