SUKKUR IBA UNIVERSITY
Worksheet-1

Subject: Calculus
Class: CS-(E&F)
Topics: Limits
Instructor: Abdul Fatah Rajri

Q1: By using Numerical Techniques of Limit of a functions. Calculate the following limits if they exist.

a. $\lim_{x\to 0} \frac{e^x-1}{x}$
b. $\lim_{x\to 0} \frac{|x+1|-|x-1|}{x}$

Q2: Find the limit of the following functions by directly putting the values of x.

a. $\lim_{x\to 3}(x^3 - 125x + 125)$
b. $\lim_{x\to a}(2022)$
c. $\lim_{N\to b} r - a(N-b)^2$
d. $\lim_{x\to a}(a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \cdots a_nx^n$
e. $\lim_{x\to 0} \frac{3x^3-2x^2+8}{4x^2+2}$

Q3: By using Factoring Techniques of Limit of a functions. Calculate the following limits if they exist.

a. $\lim_{x\to 0}[\frac{(x-a)}{x^n-a^n}]$ {Where n is the last digit of your CMS ID}
b.

$$\lim_{x\to 4} \frac{x^2 - 5x + 4}{x^2 - 2x - 8}$$

Q4: By using Rationalization Techniques of Limit of a functions. Calculate the following limits if they exist.

a. $\lim_{x\to 0} \frac{\sqrt{x+n}-\sqrt{n}}{x}$ {Where n is the last digit of your CMS ID}
b.

$$\lim_{x\to 3} \frac{\sqrt{x+1} - 2}{x - 3}$$

Q5: By simplifying. Calculate the following limits if they exist.

a. $\lim_{y\to 0} \frac{\frac{1}{y+n}-\frac{1}{n}}{y}$ {Where n is the last digit of your CMS ID}
b.

, $\lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x}$

Q6: Determine by the following trigonometric limits:

a. $\lim_{x \to 0} \frac{\sin(nx)}{\sin(nx+10x)}$ {Where n is the last digit of your CMS ID}

b. $\lim_{x \to \pi/4} \frac{1 - \tan x}{\sin x - \cos x}$

c. $\lim_{y \to 0} y\, cosec(y)$

d. $\lim_{x \to 0} \frac{cosx-1}{2x^2}$

e. $\lim_{x \to 0} \frac{1-cos(x^n)}{x}$ {Where n is the last digit of your CMS ID}

Q7: Answer the following conceptual Questions:

a. Find condition on k such that $\lim_{x \to 0} \frac{\sin(x^k)}{x} = 0$.

b. Determine $\lim_{x \to \frac{\pi}{2}} \frac{\sin(\cos(x))}{x - \frac{\pi}{2}}$ if it exists {hint: let y=cosx}

c. Why following limits does not exit?

i. $\lim_{x \to 0} \frac{x}{|x|}$

ii. $\lim_{x \to 0} \frac{1}{x^3}$

iii. $\lim_{x \to a} f(x)$ where $f(x) = \begin{cases} x^2 + a, & x < a \\ x^2 - a, & x \geq a \end{cases}$

iv. $\lim_{x \to 0} f(x)$ where $f(x) = \begin{cases} \frac{a^x - 1}{x}, & x \neq 0 \\ \text{a}, & x = 0 \end{cases}$

d. Decide whether followings are true or false

i. If $\lim_{x \to a^+} f(x) = L$ then $\lim_{x \to a^{-1}} f(x) = L$

ii. If $\lim_{x \to a^-} f(x) = L$ then $\lim_{x \to a^{+1}} f(x) = L$

iii. If $\lim_{x \to a^+} f(x) = L$ then $\lim_{x \to a} f(x) = L$

iv. If $\lim_{x \to a^-} f(x) = L$ then $\lim_{x \to a} f(x) = L$

v. If $\lim_{x \to a} f(x) = L$ then $\lim_{x \to a^{+1}} f(x) = L$

vi. If $\lim_{x \to a} f(x) = L$ then $\lim_{x \to a^{-1}} f(x) = L$

e. If $\lim_{x \to 0} \frac{(10^x - 1)}{x} = \ln(10)$ then what will be the value of $\lim_{x \to 0^+} \frac{(10^x - 1)}{x}$ and $\lim_{x \to 0} \frac{(10^x - 1)}{x}$

Q8: Suppose $g(x) \leq f(x) - 1 \leq h(x)$ for all real values of x and $lim_{x\to a} g(x) = lim_{x\to a} h(x) = -1$ then find $lim_{x\to a} f(x)$.

Q9: Solve following questions

In Exercises 87 and 88, use the Squeeze Theorem to find $\lim_{x\to c} f(x)$.

87. $c = 0$

$4 - x^2 \leq f(x) \leq 4 + x^2$

88. $c = a$

$b - |x - a| \leq f(x) \leq b + |x - a|$

In Exercises 89–94, use a graphing utility to graph the given function and the equations $y = |x|$ and $y = -|x|$ in the same viewing window. Using the graphs to observe the Squeeze Theorem visually, find $\lim_{x\to 0} f(x)$.

89. $f(x) = x \cos x$

90. $f(x) = |x \sin x|$

91. $f(x) = |x| \sin x$

92. $f(x) = |x| \cos x$

93. $f(x) = x \sin \frac{1}{x}$

94. $h(x) = x \cos \frac{1}{x}$

Q10: Decide true or false for the following questions

113. $\lim_{x\to 0} \frac{|x|}{x} = 1$

114. $\lim_{x\to a} \frac{\sin x}{x} - 1$

115. If $f(x) = g(x)$ for all real numbers other than $x = 0$, and

$\lim_{x\to 0} f(x) = L$, then $\lim_{x\to 0} g(x) = L$.

116. If $\lim_{x\to c} f(x) = L$, then $f(c) = L$.

117. $\lim_{x\to 2} f(x) = 3$, where $f(x) = \begin{cases} 3, & x \leq 2 \\ 0, & x > 2 \end{cases}$

118. If $f(x) < g(x)$ for all $x \neq a$, then

$\lim_{x\to a} f(x) < \lim_{x\to a} g(x)$.

Q11: The function used in the logistic Regression is given by

$$f(x) = \frac{e^{ax+b}}{1 + e^{ax+b}}$$

Find $\lim_{x\to 0} f(x)$.

Q12: The function used in the Linear Regression is given by

$$\hat{y} = a\hat{x} + b$$

Find b such that $\lim_{x \to -\frac{1}{a}} \hat{y} = 1$.

Q13: For a Revenue function $R(x)$ the Marginal Revenue when $x = a$, can be defined by

$\lim_{x \to a} \frac{R(x) - R(a)}{x - a}$.

Suppose a company is selling laptops and its revenue can be modeled by following function

$$\boldsymbol{f(x) = -x^2 + 50x + 60}$$

Where x represents number of laptops sold and f(x) represents revenue in thousands of dollars.

a. Find the Marginal revenue when x=15.

Best of Luck!