



Calculus and Analytical Geometry

Quiz-2

Class: BS CS-II (H)

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Q1: [6 Marks]

c. In an autocatalytic chemical reaction, the product formed is a catalyst for the reaction. If $2Q$ is the amount of the original substance and “ x ” is the amount of catalyst formed, the rate of chemical reaction is

$$R(x) = kx(2Q - x^2)$$

For what value of x will the rate of chemical reaction be greatest?

Q3:[4+3 Marks] The velocity V of the flow of blood at a distance r from the central axis of an artery of radius R is

given by

$$V(r) = k(R^2 - r^2)$$

Where k is the constant of proportionality. Find the average rate of flow of blood along a radius of the artery. (Use 0 and R as the limits of integration.)

Q4[3 Marks]: Buffon's Needle Experiment: A horizontal plane is ruled with parallel lines 2 inches apart. A two-inch needle is tossed randomly onto the plane. The probability that the needle will touch a line is,

$$P = \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \sin(\theta) d\theta$$

Where θ is the acute angle between the needle and any one of the parallel lines. **Find this probability.**

Q5[Marks 12]:

- a. Given that $\int_{-2}^2 f(x)dx = 30$ and $\int_2^0 f(x)dx = -9$. Find $\int_{-2}^0 f(x)dx$.
- b. Recall that $\log_a x = \frac{\ln(x)}{\ln(a)}$. Show that $\frac{d}{dx} \log_a x = \frac{1}{x \ln(a)}$. Also show that $\frac{d}{dx} \log(x) = \frac{1}{x \ln(10)}$.
- c. Find a trigonometric function $f(x)$, such that $\int_0^\pi f(x)dx = 1$.
- d. Given that if $f(x)$ is odd function, then $\int_{-a}^a f(x)dx = 0$. Using this idea, evaluate
- $$\int_{-5}^5 [\sin(x^3) + 1]dx$$
- e. Given that $\mathbf{g}(x) = (x)(\int_0^x f(t)dt)$, Show that $\mathbf{g}'(x) = \int_0^x f(t)dt + xf(x)$.
- f. Show that $y = 5\cos(2t + 3)$ satisfies the differential equation $\frac{d^2y}{dt^2} + 4y = 0$.