



Calculus and Analytical Geometry

Quiz-1

Class: BS CS-II (H)

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Q1.

a. Show that $\lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{1 - \cos(3x)} = \frac{4}{9}$

b. Show that $\lim_{x \rightarrow 0} \frac{\sin^3 x}{\tan(x) - \sin(x)} = 2$. Hint: use $\tan(x) = \frac{\sin(x)}{\cos(x)}$ and $\sin^2(x) = 1 - \cos^2(x)$.

c. Evaluate $\lim_{x \rightarrow k} \frac{x^3 - k^3}{x - k}$

d. For what value(s) of k is following function continuous,

$$h(x) = \begin{cases} 48 & x = k \\ \frac{x^3 - k^3}{x - k} & x \neq k \end{cases}$$

at $x = k$?

e. Given that,

$$\frac{2a \sin^3(x)}{\tan(x) - \sin(x)} \leq a \frac{\sin(x)}{x} + b \leq \frac{1 - \cos(2x)}{1 - \cos(3x)} + 1$$

for all values of x near 0. Using Squeeze Theorem, determine the values of “a” and “b”.

Q2:

a. Differentiate, $f(x) = \sec(2x)(x^2 + 1)^3$

b. The ordering and transportation cost for the components used in manufacturing a product is

$$C(x) = 100\left(\frac{200}{x^2} + \frac{x}{x^2 + 30}\right), \quad x \geq 1$$

where $C(x)$ is measured in thousands of dollars and x is the order size in hundreds. Find the rate of change of $C(x)$ with respect to x when $x = 5$.

Q3:

- i. Prove that $\frac{d}{dx}(f'g - fg') = f''g - fg''$
- ii. Write down a polynomial function of your own choice whose derivative can never be zero.
- iii. The graph of a function is parallel to y-axis, what should be its derivative with respect to x?