Exercise 3.7

Date ______ 20 ____

(3) Sum is S and product is maximum.

let no 1 $= x$

no 2 $= y$

$S = x + y \Rightarrow y = S - x$

$P = xy$ (max:)

& $P = x(S - x)$

$P = Sx - x^2$

$P' = S - 2x$

$0 = S - 2x$

$\boxed{x = \frac{S}{2}}$ Since $P'' = -2 < 0$ always max:

Now

$y = S - \frac{S}{2}$

$\boxed{y = \frac{S}{2}}$

(4) Product is 192 and sum is minimum.

$P = xy = 192 \longrightarrow (i)$

$S = x + y$ (min)

from (i)

$y = \frac{192}{x}$

So

$S = x + \frac{192}{x}$

$S' = 1 - \frac{192}{x^2}$

$0 = 1 - \frac{192}{x^2}$

$x^2 = 192$

$\boxed{x = \pm\sqrt{192}}$ critical points

Now

$S'' = 0 + \frac{192}{x^3}$

put $x = \sqrt{192}$

$S'' = \frac{192}{(\sqrt{192})^3} > 0$ $\boxed{\text{maximum for } x = \sqrt{192}}$

$S'' = \frac{192}{(-\sqrt{192})^3} < 0$

minimum for $x = -\sqrt{192}$

So

$y = \frac{192}{+\sqrt{192}}$

$\boxed{y = \sqrt{192}}$

(5) Product is 192 sum of 1st plus three times the 2nd is minimum.

$P = xy = 192 \rightarrow (i)$

$S = x + 3y$ (min:) $\rightarrow$ (ii)

from (i)

$\boxed{y = \frac{192}{x}} \rightarrow$ (iii)

So $S = x + 3\left(\frac{192}{x}\right)$

$S = x + \frac{576}{x}$

$S' = 1 - \frac{576}{x^2}$

$0 = 1 - \frac{576}{x^2}$

$\boxed{x = \pm\sqrt{576}}$

critical number

Now $S'' = 0 + \frac{576}{x^3}$

$S'' = \frac{576}{x^3}$

for $x = \sqrt{576}$

$$S'' = \frac{576}{(\sqrt{576})^3} > 0$$

minimum for $x = \sqrt{576}$

for $x = -\sqrt{576}$

$$S'' = \frac{576}{(-\sqrt{576})^3} < 0$$

maximum for $x = -\sqrt{576}$

Since sum is minimum

So $\boxed{x = \sqrt{576}}$

and $\boxed{y = \frac{192}{\sqrt{576}}}$

⑦ Sum of the 1st number and twice the second is 100 and product is max:

$$S = x + 2y = 100 \rightarrow (1)$$

$$P = xy \quad (\text{max:})$$

from (1)

$$y = \frac{100 - x}{2}$$

$$P = x\left(\frac{100 - x}{2}\right)$$

$$P' = \frac{100 - 2x}{2}$$

$$0 = \frac{100 - 2x}{2}$$

$$\boxed{x = 50}$$

$$P'' = -2 < 0$$

always maximum nice

$$y = \frac{100 - 50}{2}$$

$$\boxed{y = 25}$$

$$\boxed{x = 50}$$

⑧ The sum is the first number squared and the 2nd is 27 and product is max:

$$S = x^2 + y = 27$$

$$y = 27 - x^2$$

and

$$P = xy \quad (\text{max}).$$

$$P = x(27 - x^2)$$

$$P = 27x - x^3$$

$$P' = 27 - 3x^2$$

$$0 = 27 - 3x^2$$

$$x^2 = \frac{27}{3}$$

$$x = \pm 3$$

$$P'' = 0 - 6x$$

$$P'' = -6x$$

for $x = 3$

$P'' = -18 < 0$ max

for $x = -3$

$P'' = 18 > 0$ min

we need : max

So $\boxed{x = 3}$

$$y = 27 - (3)^2$$

$$\boxed{y = 18}$$

Find the point on the graph of the function that is closest to the given point.

Date ______ 20 ____

(13) $f(x) = \sqrt{x}$

Point $= (4, 0)$

Sol:

Here $P_1 = (4, 0)$

So $P_2 = (x, \sqrt{x})$

$|P_1 P_2| = \sqrt{(x-4)^2 + (\sqrt{x} - 0)^2}$

$= \sqrt{x^2 - 8x + 16 - x}$

$= \sqrt{x^2 - 9x + 16}$

Here

$f(x) = x^2 - 9x + 16$

$f'(x) = 2x - 9$

$0 = 2x - 9$

$x = 9/2$

$f''(x) = 2 > 0$

minimum or closesel for $x = 9/2$

So

$y = \sqrt{x}$

$y = \sqrt{9/2}$

Therefore ~~min~~ closest point ~~the~~ will be

$\left(\frac{9}{2}, \sqrt{\frac{9}{2}}\right)$ Ans.

(14) $f(x) = \sqrt{x-8}$

$P_1 = (2, 0)$.

Sol:

$P_2 = (x, \sqrt{x-8})$

$|P_1 P_2| = \sqrt{(x-2)^2 + (\sqrt{x-8} - 0)^2}$

$= \sqrt{x^2 - 4x + 4 + x - 8}$

$= \sqrt{x^2 - 3x - 4}$

Here

$f(x) = x^2 - 3x - 4$

$f'(x) = 2x - 3$

$0 = 2x - 3$

$x = 3/2$

$f''(x) = 2 > 0$

always minimum or close for ~~all values of~~ x

$x = 3/2$

So

$y = \sqrt{x-8}$

$y = \sqrt{3/2 - 8}$

$y =$

Date ______ 20 ____

(15) $f(x) = x^2$

$P_1 = (2, \frac{1}{2})$

So

$P_2 = x, x^2$

$|P_1P_2| = \sqrt{(x-2)^2 + (x^2 - \frac{1}{2})^2}$

$= \cancel{(x^2 - 4x + 4) + (x^4 - x^2 + \frac{1}{4})}$

$= \sqrt{(x^2 - 4x + 4) + (x^4 - x^2 + \frac{1}{4})}$

$= \sqrt{x^2 - 4x + 4 + x^4 - x^2 + \frac{1}{4}}$

$= \sqrt{x^4 - 4x + \frac{17}{4}}$

Here $f(x) = x^4 - 4x + \frac{17}{4}$

$f'(x) = 4x^3 - 4$

$0 = 4x^3 - 4$

$x^3 = \frac{4}{4}$

$x^3 = 1$

$\boxed{x = 1}$

$f''(x) = 12x^2$

$= 12(1)^2 > 0$

minimum.

for $x = 1$

So $y = (1)^2$

$y = 1$

$\boxed{P_2 = (1, 1)}$ Ans

(16) $f(x) = (x+1)^2$

$P_1 = (5, 3)$.

So

$P_2 = (x, (x+1)^2)$

$|P_1P_2| = \sqrt{(x-5)^2 + ((x+1)^2 - 3)^2}$

$= \sqrt{x^2 - 10x + 25 + (x^2 + 2x + 1 - 3)^2}$

$= \sqrt{x^2 - 10x + 25 + (x^2 + 2x - 2)^2}$

Here

$f(x) = x^2 - 10x + 25 + (x^2 + 2x - 2)^2$

$f'(x) = 2x - 10 + 2(x^2 + 2x - 2)(2x+2)$

$f'(x) = 2x - 10 + (2x^2 + 4x - 4)(2x+2)$

$f'(x) = 2x - 10 + 4x^3 + 4x^2 + 8x^2 \cancel{+8x} \cancel{-8x} - 8$

$f'(x) = 4x^3 + 12x^2 + 2x - 18$

$0 = 4x^3 + 12x^2 + 2x - 18$

2 common.

$2x^3 + 6x^2 + x - 9 = 0$

factors of $9 = \pm 1, \pm 3, \pm 9$.

put $x = 1$

$2 + 6 + 1 - 9 = 0$

$9 = 0$

So

$(x-1)$ is factor

by dividing $x-1$ with $2x^3 + 6x^2 + x - 9$ ~~we will get~~

$f(x)$

$$x-1\overline{\big)\,2x^3+6x^2+x-9\,}\big(\,2x^2+8x+9$$
$$\underline{-2x^3 \mp 2x^2}$$
$$8x^2+x-9$$
$$\underline{-8x^2 \mp 8x}$$
$$9x-9$$
$$\underline{-9x \mp 9}$$

Therefore

$$2x^3+6x^2+x-9=0$$
$$(x-1)(2x^2+8x+9)=0$$

$x-1=0$ or $2x^2+8x+9=0$

$\boxed{x=1}$ No real solution for x.

$$f'(x)=2x^3+6x^2+x-9=0$$
$$f''(x)=6x^2+12x+1=0$$

for $x=1$

$$f''(1)=6+12+1>0$$

minimum for $x=1$

So

$$y=(x+1)^2$$
$$y=(1+1)^2$$
$$y=4$$

~~ans~~ closest point will be $P_2(1,4)$ to the $(5,3)$ Ans

(17) For what value of x the rate of chemical reaction will be greatest

$$\frac{dQ}{dx}=kx(Q_0-x).$$

That is 1st derivative
now

$$\frac{d''Q}{dx}=\frac{d}{dx}(kxQ_0-kx^2)$$
$$\frac{d''Q}{dx}=kQ_0-2kx$$
$$0=kQ_0-2kx$$
$$x=\frac{KQ_0}{2K}$$
$$x=\frac{Q_0}{2}$$
$$\frac{d'''Q}{dx}=0-2k$$
$$=-2k<0$$

maximum when

$$\boxed{x=\frac{Q_0}{2}}$$

Ans:

18) $$F = \frac{V}{22 + 0.02V^2}$$

what speed will maximize the flow of rate?

Sol.

$$F' = \frac{(1)(22+0.02V^2) - V(0+0.04V)}{(22+0.02V^2)^2}$$

$$F' = \frac{22 + 0.02V^2 - 0.04V^2}{(22+0.02V^2)^2}$$

$$F' = \frac{22 - 0.02V^2}{(22+0.02V^2)^2}$$

$$0 = \frac{22 - 0.02V^2}{(22+0.02V^2)^2}$$

$$V^2 = \frac{22}{0.02}$$

$$V^2 = \frac{2200}{2}$$

$$V = \pm\sqrt{1100}$$

By 1st derivative test V is maximized at

$$\boxed{V = \sqrt{1100}}$$

19) A farmer plans to fence a rectangular pasture adjacent to a river. ....

Sol: Given Data:

$A = 180,000\ m^2$

dimensions = ?

↳ $l = ?$

$b = ?$

no fencing needed along the river.

$$A = lb = 180,000\ m^2$$

$$lb = 180,000$$

$$b = \frac{180,000}{l} \rightarrow (i)$$

$$P = 2l + b$$

(Because one side there is river).

$$P = 2l + \frac{180,000}{l}$$

$$P' = 2 - \frac{180,000}{l^2}$$

$$0 = 2 - \frac{180,000}{l^2}$$

$$l^2 = \frac{180,000}{2}$$

$$l^2 = 90000$$

$$l = \pm\sqrt{90000}$$

length is always +ve

$$\boxed{l = 300}$$

$$b = \frac{180,000}{300} \rightarrow \boxed{b = 600}$$

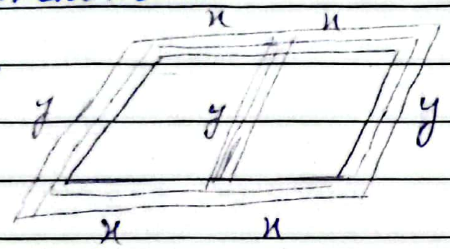
nice

Teacher's Sign...

Date ______ 20 ____

(20) A rancher has 200 feet of fencing with which to enclose two adjacent, rectangular corrals. What dimentions should be used so that the enclosed area will be maximum.

Sol:



$$\text{Perimeter} = 4x + 3y$$

$$200 = 4x + 3y$$

$$y = \frac{200 - 4x}{3} \rightarrow (i)$$

$$\text{Area} = xy + xy$$

$$A = 2xy$$

$$A = 2x\left(\frac{200 - 4x}{3}\right)$$

$$A = \frac{400x - 8x^2}{3}$$

$$A' = \frac{400 - 16x}{3}$$

$$0 = \frac{400 - 16x}{3}$$

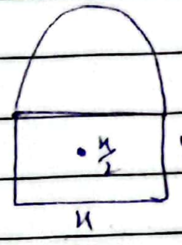
$$x = \frac{400}{16} \Rightarrow \boxed{x = 25}$$ Ans

$$y = \frac{200 - 4(25)}{3}$$

$$\boxed{y = \frac{100}{3}}$$ Ans.

(23) A norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window. Find the dimentions of a Norman window of maximum area if Perimeter is 16 feet.

Sol:



Given

$P = 16$

$A(\max) = ?$

$$P = \text{Perimeter of semicircle} + \text{Perimeter of rectangle}$$

$$16 = \pi r + 2y + x$$

Here $r = \frac{x}{2}$

$$16 = \frac{\pi x}{2} + 2y + x$$

$$y = \left(16 - \frac{\pi x}{2} - x\right)\frac{1}{2}$$

$$y = \frac{32 - \pi x - 2x}{4} \rightarrow (i)$$

$$\text{Aerea} = \text{Area of semicircle} + \text{area of rectangle}$$

$$A = \frac{\pi r^2}{2} + xy$$

Here $r = \frac{x}{2}$

put value of y from (i).

$$A = \frac{\pi\left(\frac{x}{2}\right)^2}{2} + x\left(\frac{32 - \pi x - 2x}{4}\right)$$

$$A = \frac{\pi x^2}{8} + \frac{32x - \pi x^2 - 2x^2}{4}$$

$$A = \frac{\pi x^2 + 64x - 2\pi x^2 - 4x^2}{8}$$

$$A = \frac{-\pi x^2 - 4x^2 + 64x}{8}$$

$$A' = \frac{-2\pi x - 8x + 64}{8}$$

$$0 = \frac{-2\pi x - 8x + 64}{8}$$

$$-x(2\pi + 8) = -64$$

$$x = \frac{64}{2(\pi+4)}$$

$$\boxed{x = \frac{32}{\pi+4}}$$

$$A'' = \frac{-2\pi - 8}{8} < 0$$

So area is maximum

when $\boxed{x = \frac{32}{\pi+4}}$ Ans: of x

Now put x in (i)

$$y = \frac{32 - \pi\left(\frac{32}{\pi+4}\right) - 2\left(\frac{32}{\pi+4}\right)}{4}$$

$$y = \frac{32(\pi+4) - 32\pi - 64}{4(\pi+4)}$$

$$y = \frac{\cancel{32\pi} + 128 - \cancel{32\pi} - 64}{4(\pi+4)}$$

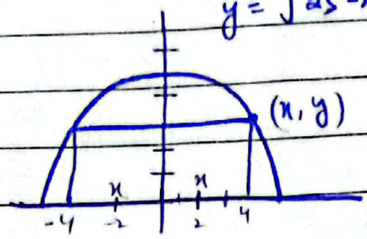
$$y = \frac{64}{4(\pi+4)}$$

$$\boxed{y = \frac{16}{\pi+4}}$$

Ans: of y.

(27)

$y = \sqrt{25 - x^2}$



Area of rectangle $= (2x)(y)$

$$A = 2x\left(\sqrt{25 - x^2}\right)$$

$$A' = 2\sqrt{25-x^2} + \cancel{2}x \times \frac{1}{\cancel{2}\sqrt{25-x^2}}(-2x)$$

$$A' = 2\sqrt{25-x^2} - \frac{2x^2}{\sqrt{25-x^2}}$$

$$A' = \frac{2(25-x^2) - 2x^2}{\sqrt{25-x^2}}$$

$$A' = \frac{-2x^2 + 50 - 2x^2}{\sqrt{25-x^2}}$$

$$A' = \frac{-4x^2 + 50}{\sqrt{25-x^2}}$$

$$0 = \frac{-4x^2 + 50}{\sqrt{25-x^2}}$$

$$4x^2 = 50$$

$$x^2 = \frac{25}{2}$$

$$x = \frac{5}{\sqrt{2}} \Rightarrow \boxed{x = \frac{5\sqrt{2}}{2}}$$

$$y = \sqrt{25 - \left(\frac{5\sqrt{2}}{2}\right)^2}$$

$$y = \sqrt{25 - \left(\frac{25 \times 2}{4}\right)} \Rightarrow y = \frac{5}{\sqrt{2}}$$

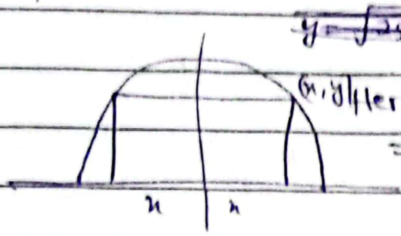
or $\boxed{y = \frac{5\sqrt{2}}{2}}$

(28) Find the dimentions of the largest rectangle that can be inscribed in a semicircle of radius r.

Sol. ~~From question (27)~~

~~$y = \sqrt{25 - x^2}$~~

~~$y = \sqrt{25 - x^2}$~~



Here radius = r.

Since equation of circle is

$y^2 + x^2 = r^2$

$y = \sqrt{r^2 - x^2}$

Area $= 2xy$

$= 2x\sqrt{r^2 - x^2}$

$A' = \cancel{2}x \times \frac{1}{\cancel{2}}(r^2 - x^2)^{-1/2}(-2x) + 2\sqrt{r^2 - x^2}$

$A' = \frac{-2x}{\sqrt{r^2 - x^2}} + 2\sqrt{r^2 - x^2}$

$A' = \frac{-2x + 2(r^2 - x^2)}{\sqrt{r^2 - x^2}}$

$0 = \frac{-2x + 2r^2 - 2x^2}{\sqrt{r^2 - x^2}}$

$+4x^2 = 2r^2$

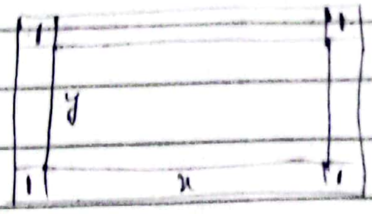
$x^2 = \frac{\cancel{2}r^2}{\cancel{4}\,2}$

$x = \frac{r}{\sqrt{2}} \Rightarrow \boxed{x = \frac{\sqrt{2}\, r}{2}}$

So $\boxed{y = \frac{\sqrt{2}\, r}{2}}$

(29) A rectangular page is to contain 30 square ~~inches~~ of print. the margins of each side are 1 inch. Find the dimentions of the page such that at least amount of paper is used.

Sol.



Area with~~out~~ margins

$A = (x+2)(y+2)$ ~~$= 30$~~

$A = xy + 2x + 2y + 4$ ~~$= 30$~~ $\rightarrow$ (i)

~~$y(x+2) = 30 - 4 - 2x$~~

~~$y = \frac{26 - 2x}{x+2}$~~ $\rightarrow$ (i)

~~Area without margins~~

~~$A = xy = 30$~~

~~$= x\left(\frac{26 - 2x}{x+2}\right)$~~

~~$A' = \frac{26x - 2x^2}{x+2}$~~

Area without margins

$A = xy = 30$

$y = \frac{30}{x} \rightarrow$ (ii)

put in eq (i)

$A = x\left(\frac{30}{x}\right) + 2x + 2\left(\frac{30}{x}\right) + 4$

$A = 30 + 2x + \frac{60}{x} + 4$

$\frac{dA}{dx} = 0 + 2 - \frac{60}{x^2} + 0$

$A' = 2 - \frac{60}{x^2}$

put $A' = 0$

$2 - \frac{60}{x^2} = 0$

$\frac{60}{x^2} = 2$

$x^2 = \frac{60}{2}$

$x = \pm\sqrt{30}$

x (length) can't be negative

$\boxed{x = \sqrt{30}}$

So $y = \frac{30}{\sqrt{30}}$

$y = \frac{\cancel{30}\sqrt{30}}{\cancel{30}}$

$\boxed{y = \sqrt{30}}$

Now dimensions of page are

$\boxed{x + 2 = \sqrt{30} + 2}$
$\boxed{y + 2 = \sqrt{30} + 2}$

Ans:

(30) A rectangular page contains 36 sq inches of print. The margins on each side are to be $1\frac{1}{2}$ inches. Find the dimensions of page such that least amount of paper is used.

Sol: $A = 36$

margin $= 1\frac{1}{2} = \frac{3}{2}$

Now

Area without margin

$A = xy = 36$

$y = \frac{36}{x} \rightarrow (i)$

Area with margin

$A = (x + 1\frac{1}{2} + 1\frac{1}{2})(y + 1\frac{1}{2} + 1\frac{1}{2})$

$A = (x+3)(y+3)$

$\because y = \frac{36}{x}$

$A = (x+3)(\frac{36}{x}+3)$

$A = 36 + 3x + \frac{108}{x} + 9$

$A = 3x + \frac{108}{x} + 39$

$A' = 3 - \frac{108}{x^2}$

$0 = 3 - \frac{108}{x^2}$

$\frac{108}{x^2} = 3$

$x^2 = \frac{\cancel{108}^{36}}{\cancel{3}}$

$x = \sqrt{36} \Rightarrow x = 6$

So

$y = \frac{36}{6}$

$y = 6$

Now dimentions.

$\boxed{y + 3 = 6 + 3 = 9}$
$\boxed{x + 3 = 6 + 3 = 9}$

Ans.

Exercise 4.1 (I verified by taking integration of L.H.S)

Date ______ 20 ____

(1) $\int -\frac{9}{x^4}\,dx = \frac{3}{x^3} + c$

taking L.H.S

$\int \frac{-9}{x^4}\,dx = \int -9x^{-4}\,dx$

$\cancel{\int \frac{-9x^{-4+1}}{-4+1}}$

$= \frac{-9x^{-4+1}}{-4+1} + c$

$= \frac{3\cancel{9}x^{-3}}{\cancel{3}} + c$

$= \frac{3}{x^3} + c = RHS$

(3) $\int (x-2)(x+2)\,dx = \frac{1}{3}x^3 - 4x + c$

L.H.S

$\int [(x)^2 - (2)^2]\,dx$

$\int (x^2 - 4)\,dx$

$\int x^2\,dx - \int 4\,dx$

$\frac{x^3}{3} - 4x + c = RHS$

(4) $\int \frac{x^2-1}{x^{3/2}}\,dx = \frac{2(x^2+3)}{3\sqrt{x}} + c$

$\int \left(\frac{x^2}{x^{3/2}} - \frac{1}{x^{3/2}}\right) dx$

$\int x^{2-3/2}\,dx - \int x^{-3/2}\,dx$

$\int x^{1/2}\,dx - \int x^{-3/2}\,dx$

$\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + c$

$\frac{x^{3/2}}{3/2} - \frac{x^{-1/2}}{-1/2} + c$

$\frac{2}{3}x^{3/2} + \frac{2}{\sqrt{x}} + c$

$\frac{2(x^{\frac{3}{2}+\frac{1}{2}}) + 2(3)}{3\sqrt{x}} + c$

$\frac{2(x^2) + 6}{3\sqrt{x}}$

$\frac{2(x^2+3)}{3\sqrt{x}} + c = R.H.S$

Find general solution.

(5) $\frac{dy}{dt} = 3t^2$

$y = \int 3t^2\,dx$

$y = \frac{3t^3}{3} + c$

$y = t^3 + c$ Ans.

(6) $\frac{dr}{d\theta} = \pi$

$r = \int \pi\,d\theta$

$r = \int \pi\theta^0\,d\theta$

$r = \frac{\pi\theta^{0+1}}{1} + c$

$r = \pi\theta + c$ Ans.

others same

Find the integral (verify yourself by taking derivative of answer). Date ______ 20 ____

(27) $\int \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right) dx$

$\int x^{1/2}\, dx + \int \frac{1}{2x^{1/2}}\, dx$

$\int x^{1/2}\, dx + \frac{1}{2}\int x^{-1/2}\, dx$

$\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{1}{2}\left(\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right) + c$

$\frac{x^{3/2}}{\frac{3}{2}} + \frac{1}{2}\left(\frac{x^{+1/2}}{+1/2}\right) + c$

$\frac{2x^{3/2}}{3} + \frac{1}{\cancel{2}}\left(\sqrt{x}\, \cancel{2}\right) + c$

$\frac{2x^{3/2}}{3} + \sqrt{x} + c$ Ans.

(37) $\int \frac{x^2+x+1}{\sqrt{x}}\, dx$

$\int \left(\frac{x^2}{x^{1/2}} + \frac{x}{x^{1/2}} + \frac{1}{x^{1/2}}\right) dx$

$\int x^{2-\frac{1}{2}}\, dx + \int x^{1-\frac{1}{2}}\, dx + \int x^{-1/2}\, dx$

$\int x^{3/2}\, dx + \int x^{1/2}\, dx + \int x^{-1/2}\, dx$

$\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c$

$\frac{x^{5/2}}{5/2} + \frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + c$

$\frac{2x^{5/2}}{5} + \frac{2x^{3/2}}{3} + 2x^{1/2} + c$

Ans.

(29) $\int (x+1)(3x-2)\, dx$

$\int (3x^2 - 2x + 3x - 2)\, dx$

$\int (3x^2 + x - 2)\, dx$

D.Y.S.

(30) $\int (2t^2-1)^2\, dt$

$= \int (4t^4 - 4t^2 + 1)\, dt$

$= \int 4t^4\, dt - \int 4t^2\, dt + \int dt$

D.Y.S.

(31) $\int y^2\sqrt{y}\, dy.$

$\int y^2 \cdot y^{1/2}\, dy$

$\int y^{2+\frac{1}{2}}\, dy$

$\int y^{5/2}\, dy$

$\frac{y^{\frac{5}{2}+1}}{\frac{5}{2}+1} + c$

$\frac{2y^{7/2}}{7} + c$ Ans.

(33) $\int dx$

$= \int x^0\, dx$

$= \frac{x^{0+1}}{0+1} = x + C$

Ans.

(34) $\int 3\, dt$

$= \int 3t^0\, dt$

$= 3t + c$ Ans.

(35) $\int (2\sin x + 3\cos x)\, dx.$

$\int 2\sin x\, dx + \int 3\cos x\, dx$

$2\int \sin x\, dx + 3\int \cos x\, dx$

$2(-\cos x) + 3(\sin x) + c$

$3\sin x - 2\cos x + c$ Ans.

(36) $\int (t^2 - \sin t)\, dt.$

$\int t^2\, dt - \int \sin t\, dt$

$\frac{t^3}{3} - (-\cos(t)) + c$

$\frac{t^3}{3} + \cos(t) + c$ Ans

(37) $\int (1 - \csc t \cot t)\, dt$

$\int dt - \int \csc t \cot t\, dt$

$t + \csc(x) + c$ A

(38) $\int (\theta^2 + \sec^2\theta)\, d\theta$

$\int \theta^2\, d\theta + \int \sec^2\theta\, d\theta$

$\frac{\theta^3}{3} + \tan\theta + c$ A

(39) $\int (\sec^2\theta - \sin\theta)\, d\theta$

$\int \sec^2\theta\, d\theta - \int \sin\theta\, d\theta$

$\tan\theta + \cos\theta + c$ A

(40) $\int \sec y (\tan y - \sec y)\, dy$

$\int (\sec y \tan y - \sec^2 y)\, dy$

$\sec y - \tan y + c.$ A

(41) $\int (\tan^2 y + 1)\, dy$

$= \int (\sec^2 y - 1 + 1)\, dy$

$= \int \sec^2 y\, dy = \tan y + c$ Ans

(42) $\int \frac{\cos x}{1 - \cos^2 x}\, dx.$

$\int \frac{\cos x}{\sin^2 x}\, dx$

$\int \frac{\cos x}{\sin x} \times \frac{1}{\sin x}\, dx$

$\int \cot x \csc x\, dx$

$= -\csc x + c$ Ans

Solve the differential equation. Date ______ 20 ____

(55) $f'(x) = 4x, \quad f(0) = 6$

Sol

$f(x) = \int f'(x)\,dx$

$= \int 4x$

$f(x) = \frac{4x^2}{2} + c$

$f(0) = \frac{4(0)^2}{2} + c$

$\boxed{6 = c}$

$f(x) = \frac{\cancel{4}^2 x^2}{\cancel{2}} + 6$

$\boxed{f(x) = 2x^2 + 6}$

(56) $g'(x) = 6x^2, \quad g(0) = -1$

$g(x) = \int g'(x)\,dx$

$= \int 6x^2$

$= \frac{\cancel{6}^2 x^3}{\cancel{3}} + c$

$g(x) = 2x^3 + c$

$g(0) = 2(0)^3 + c$

$c = -1$

$\boxed{g(x) = 2x^3 - 1}$

(57) $h'(t) = 8t^3 + 5, \quad h(1) = -4$

$h(t) = \int h'(t)\,dt$

$= \int (8t^3 + 5)\,dt$

$= \frac{\cancel{8}^2 t^4}{\cancel{4}} + 5x + c$

$h(t) = 2t^4 + 5x + c$

$h(1) = 2 + 5 + c$

$-4 = c + 7$

$\boxed{c = -11}$

So, $\boxed{h(t) = 2t^4 + 5x - 11}$

(59) $f''(x) = 2,$

$f'(0) = 6$

$f(0) = 3.$

Sol

$f'(x) = \int f''(x)\,dx$

$= \int 2\,dx$

$f'(x) = 2x + c$

$f'(0) = 2(0) + c$

$\boxed{6 = c}$

$f'(x) = 2x + 6$

$f(x) = \int f'(x)\,dx$

$= \int (2x + 6)\,dx$

$= \frac{\cancel{2}x^2}{\cancel{2}} + 6x + c$

$f(x) = x^2 + 6x + c$

$f(0) = 0 + 0 + c$

$\boxed{c = 3}$

$\boxed{f(x) = x^2 + 6x + 3}$

Teacher's Signature ______

Date ______ 20 ____

(62) $f''(x) = \sin(x)$

$f'(0) = 1$

$f(0) = 6$

Sol:

$f'(x) = \int f''(x)\,dx$

$= \int \sin x\,dx$

$f'(x) = -\cos x + c$

$f'(0) = -\cos(0) + c$

$1 = -1 + c$

$\boxed{c = 2}$

$f'(x) = -\cos x + 2$

$f(x) = \int f'(x)\,dx$

$= \int (-\cos(x) + 2)\,dx$

$= -\int \cos x + 2\int dx$

$f(x) = -\sin x + 2x + c$

$f(0) = -\sin(0) + 2(0) + c$

$6 = 0 + 0 + c$

$\boxed{c = 6}$

$\boxed{f(x) = -\sin x + 2x + 6}$

(63) **Tree Growth:**

(a) ----

$\frac{dh}{dt} = 1.5t + 5$

$h(0) = 12$

$h(t) = \int \frac{dh}{dt}\,dt$

$= \int (1.5t + 5)\,dt$

$h(t) = \frac{1.5t^2}{2} + 5t + c$

$h(0) = 0 + 0 + c$

$c = 12$

$h(t) = \frac{15t^2}{20} + 5t + 12$

$\boxed{h(t) = \frac{3t^2}{4} + 5t + 12}$

Height after t years **Ans:**

(b) How tall are the shrubs when are sold.

Sol: They were sold after 6 years.

$t = 6$

$h(6) = \frac{3(6)^2}{4} + 5(6) + 12$

$= \frac{3 \times 36}{4} + 30 + 12$

$= 27 + 30 + 12$

$h(6) = 69$ cm **Ans:**

(64) Same as 63.

Data: $P(t) = \int \frac{dp}{dt}\,dt$

$P(t) = \int k\sqrt{t}\,dt$

$P(0) = 500$

$P(1) = 600$

$P(7) = ?$

(67) A ball is thrown vertically upward from a height of 6 feet with an initial velocity of 60 feet per second. How high will the ball go?

Sol. Data

acceleration $a(t) = -32 \ f/s^2$

velocity $v(t) = \int a(t)\, dt$

$v(0) = 60$

$h(0) = 6$

$h(t) = ?$

Now

$v(t) = \int (-32)\, dt$

$v(t) = -32t + c$

$v(0) = 0 + c$

$c = 60$

So

$v(t) = -32t + 60 \longrightarrow (i)$

at maximum height $v(t) = 0$

$0 = -32t + 60$

$t = \frac{60}{32}$

$\boxed{t = \frac{15}{8}}$

$h(t) = \int v(t)\, dt$

$= \int (-32t + 60)\, dt$

$= \frac{-32t^2}{2} + 60t + c$

$h(t) = -16t^2 + 60t + c$

$h(0) = 0 + 0 + c$

$c = 6$

Therefore

$h(t) = -16t^2 + 60t + 6$

$t = \frac{15}{8}$ when $h(t)$ is max

$h\left(\frac{15}{8}\right) = -16\left(\frac{15}{8}\right)^2 + 60\left(\frac{15}{8}\right) + 6$

$= \frac{-16 \times 225}{8 \times 8} + \frac{225}{2} + 6$

$= -\frac{450}{8} + \frac{225}{2} + 6$

$= \frac{-450 + 900 + 48}{8}$

$\boxed{h\left(\frac{15}{8}\right) = \frac{498}{8}}$ Ans.

(68) Show that height above the ground of an object thrown upward from a point S_0 feet above the ground with an initial velocity of V_0 feet per second is given by function.

$f(t) = -16t^2 + V_0 t + S_0$

Data:

$a(t) = -32 \ f/s^2$

$V(0) = V_0$

$h(0) = S_0$

first we find $V(t)$.

$V(t) = \int -32\, dt$

$V(t) = -32t + c$

$V(0) = 0 + c$

$c = V_0$

nice

Teacher's Signature ______

Date ________ 20 ____

$V(t) = -32t + V_0$

Now

$h(t) = \int (-32t + V_0)\, dt$

$h(t) = \frac{-32t^2}{2} + V_0 t + c$

$h(t) = -16t^2 + V_0 t + c$

$h(0) = 0 + 0 + c$

$c = S_0$

Therefore

$h(t) = -16t^2 + V_0 t + S_0$

or $\boxed{f(t) = -16t^2 + V_0 t + S_0}$

Hence shown.

(69) With what initial velocity must an object be thrown upward (from ground level) to reach the top of the washington Monument (approx: 550 feet)

Sol Since we found that

$V(t) = -32t + V_0$

and $h(t) = -16t^2 + V_0 t + S_0$

at max: height velocity $V(t)$ becomes 0. So

$0 = -32t + V_0$

$\boxed{t = \frac{V_0}{+32}}$

Now we find equation max: height at $t = \frac{V_0}{+32}$

$h\left(\frac{V_0}{+32}\right) = -16\left(\frac{V_0}{+32}\right)^2 + V_0\left(\frac{V_0}{+32}\right) + S_0$

$h\left(\frac{V_0}{32}\right) = -16\left(\frac{V_0^2}{32 \times 32}\right) + \frac{V_0^2}{32} + S_0$

Since max: height = 550 at $h(0)$ So S_0 will be 0

$550 = \frac{-V_0^2}{64} + \frac{V_0^2}{32} + 0$

$550 = \frac{-V_0^2 + 2V_0^2}{64}$

$V_0^2 = 550 \times 64$

$V_0^2 = 35200$

$\boxed{V_0 = 187.61}$ Ans.

(70) data: $V(t) = 16\, f/s$

$S_0 = 64\, f$

(a) $t = ?$ when $h(t) = 0$.

$h(t) = -16t^2 + V_0 t + S_0$

$0 = -16t^2 + 16t + 64$

$0 = -16(t^2 - t + 4)$

$t^2 - t + 4 = 0$

$t = \frac{1 \pm \sqrt{17}}{2}$

t can't be -ve

$\boxed{t = \frac{1+\sqrt{17}}{2}}$ when $h(t)$ is 0 and $V = 16$

(b) $V(t) = ?$ when $h(t)$ is 0.

Sol Since $h(t)$ is 0 when $t = \frac{1+\sqrt{17}}{2}$

So $V(t) = -32t + V_0$

$V\left(\frac{1+\sqrt{17}}{2}\right) = -32\left(\frac{1+\sqrt{17}}{2}\right) + 16$

$= -16(1+\sqrt{17}) + 16$

(others are related to these).