<u>Exercise Sheet 1:</u>

Mathematics Content-III   **Instructor: Abdul Fatah**

Note:

- **Please attempt each question.**
- **You can help one another (Understand don't just copy and paste because it will create trouble for you in Exams).**
- **You should visit MRC.**
- **You can take help from internet or any other resources but again don't just copy, understand what you are doing.**
- **You can discuss with me if you are having difficulty in solving the sheet.**

1. Let f be a real valued function then, what do we mean by the following expressions
   a. $\lim\limits_{x \to x_0} f(x) = L$
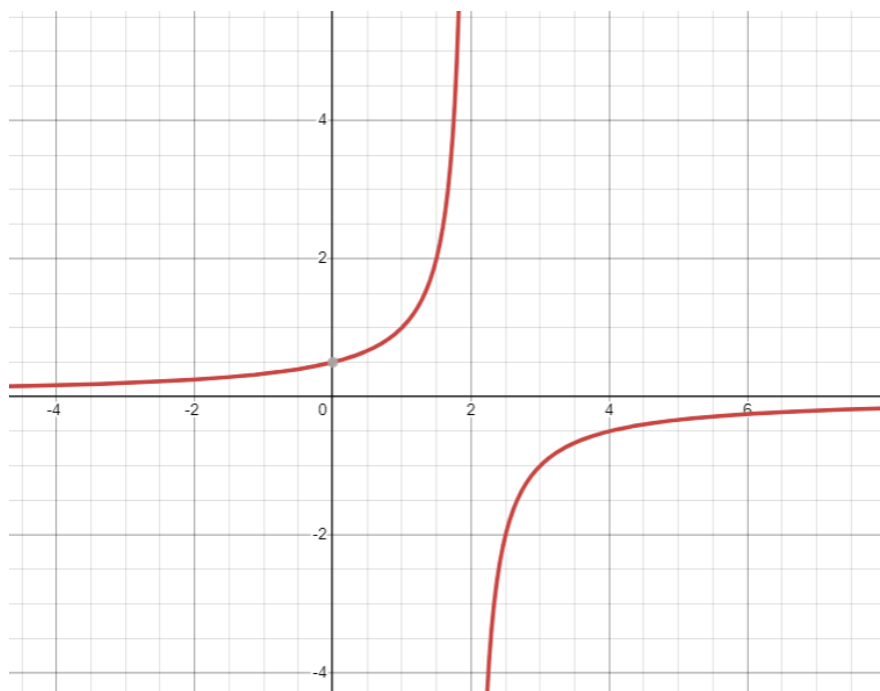   b. $\lim\limits_{x \to x_0^+} f(x) = L$
   c. $\lim\limits_{x \to x_0^-} f(x) = L$

2. It has been observed that $\lim\limits_{x \to x_0^+} f(x)$ and $\lim\limits_{x \to x_0^-} f(x)$ always exist. When does the limit $\lim\limits_{x \to x_0} f(x)$ is said to exist?

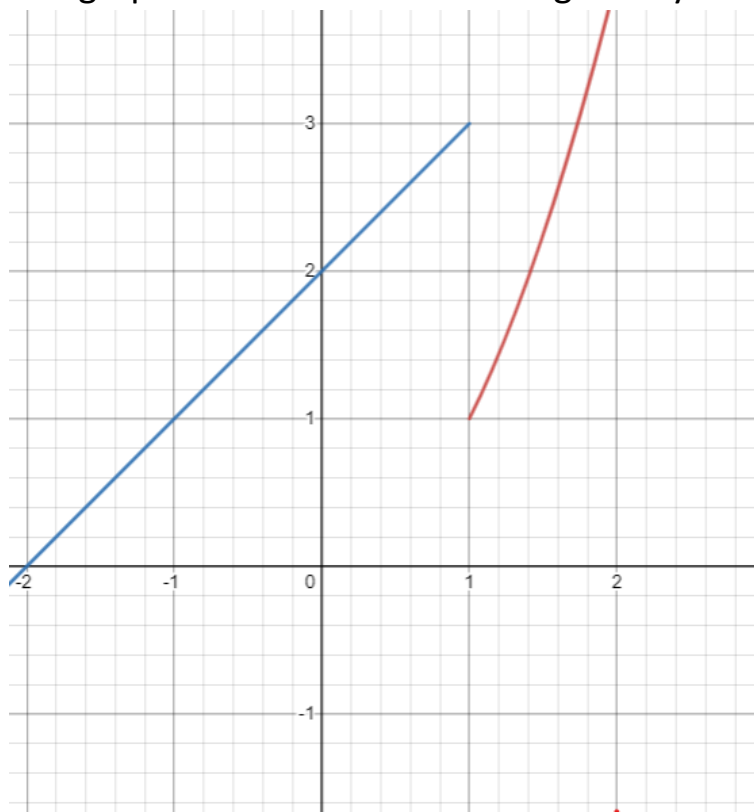3. For a certain function $f(x)$, $\lim\limits_{x \to x_0^+} f(x) = 2$ and $\lim\limits_{x \to x_0^+} f(x) = -2$ what can you say about $\lim\limits_{x \to x_0} f(x)$?

4. The graph of a certain function is given by
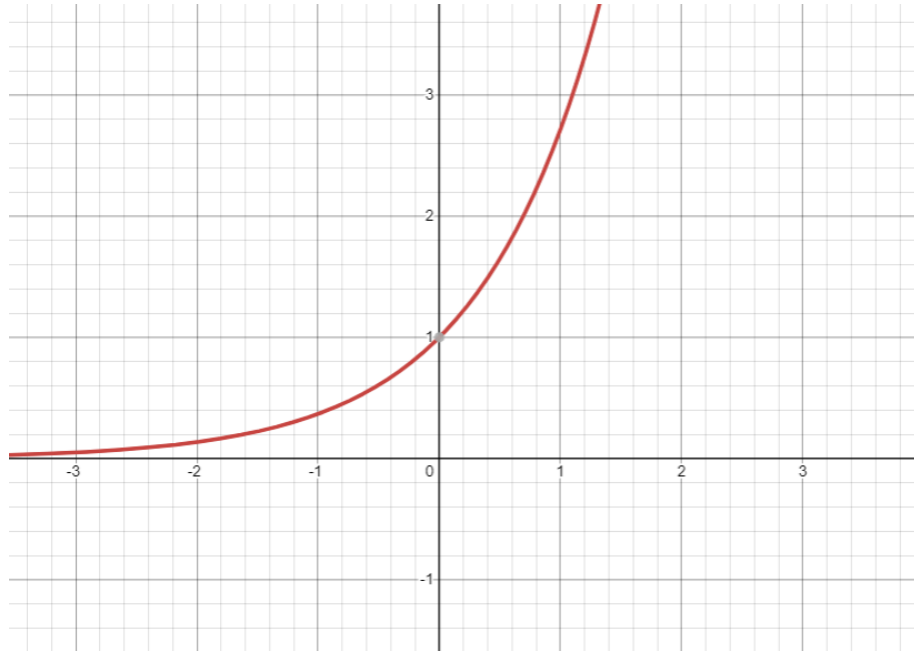
What can you say about the $\lim_{x\to 2^+} f(x)$, $\lim_{x\to 2^-} f(x)$ and $\lim_{x\to 2} f(x)$?

Is there any other value limit of f does not exist?

5. The graph of a certain function is given by

What can you say about the $\lim\limits_{x\to 1^+} f(x)$, $\lim\limits_{x\to 1^-} f(x)$ and $\lim\limits_{x\to 1} f(x)$?

Is there any other value limit of f does not exist?

6. The graph of a certain function is given by

What can you say about the $\lim\limits_{x\to 0^+} f(x)$, $\lim\limits_{x\to 0^-} f(x)$ and $\lim\limits_{x\to 0} f(x)$?

7. Calculate following limits numerically,

a. $\lim\limits_{x\to -5} \dfrac{x^3+125}{x+5}$

b. $\lim\limits_{x\to 0} \dfrac{sinx}{x}$

c. $\lim\limits_{x\to 0} \dfrac{1-cosx}{x}$

8. Evaluate the following limits of the polynomial functions:

a. $\lim\limits_{x\to 1} 4x(1-x)$

b. $\lim\limits_{x\to 3}(x^3 - 125x + 125)$

c. $\lim\limits_{x\to a}(2022)$

d. $\lim\limits_{N\to b} r - a(N-b)^2$

e. $\lim\limits_{x\to a}(a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \cdots a_nx^n)$

9. Suppose $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \cdots a_nx^n$ then show that $\lim_{x \to a} p(x) = p(a)$. [Hint: Use part c of question 8]

10. Evaluate the following limits of the Rational functions:

a. $\lim_{x \to 0} \dfrac{3x^3 - 2x^2 + 8}{4x^2 + 2}$

b. $\lim_{x \to 2} \dfrac{x^2 - 4}{x - 4}$

c. $\lim_{x \to 3} \dfrac{x^2 - 9}{x^2 + 3}$

d. $\lim_{x \to 0} \dfrac{3x^3 - 2x^2 + x}{4x^2 + 2x}$

e. $\lim_{x \to 2} \dfrac{x^2 - 5x + 6}{x^2 - 7x + 10}$

f. $\lim_{t \to 5} \dfrac{t^2 + 3t - 10}{2t^2 + 13t - 10}$

g. $\lim_{x \to a} \dfrac{x^n - a^n}{x - a}$ [Hint: Use the formula of $x^n - a^n$=]

h. https://www.physicsforums.com/threads/expanding-x-n-a-n-without-binomial-theroem.413309/

$$x^n - a^n = (x-a)\,(x^{n-1} + x^{n-2}\,a + x^{n-3}\,a^2 + \ldots + x\,a^{n-2} + a^{n-1})$$

i. $\lim_{x \to a} \dfrac{x^m - a^m}{x^n - a^n}$

j. $\lim_{x \to 1} \dfrac{x^n - 1}{x - 1}$ [Hint: Be smart, You can use part g with $a = 1$]

k. $\lim_{h \to 0} \dfrac{h}{h}$

l. $\lim_{y \to 0} \dfrac{y^3 - 216}{y - 6}$

m. $\mathrm{Lim}_{y \to 0} \dfrac{6y^3 - 3y^2 + 3}{7 + y}$

n. $\lim_{h \to 0} \dfrac{(h+1)^2 - (h-1)^2}{h}$

o. $\lim_{u \to 0} \dfrac{u^2 - u + 12}{u + 3}$

p. $\lim_{x \to 5} \dfrac{6}{x - 5}$ [You might need to calculate left and right limit in order decide the answer]

q. $\lim\limits_{x\to 5} \dfrac{6}{(x-5)^2}$ [You might need to calculate left and right limit in order decide the answer]

11. If $f(x) = x^3$ then evaluate the following

a. $f(x + h)$

b. $f(x + h) - f(x)$

c. $\dfrac{f(x+h)-f(x)}{h}$

d. $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

If $\dfrac{df}{dx} = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$ then what is $\dfrac{df}{dx}$ for above given f?

12. Evaluate following limits either by numerical ways or otherwise:

a. $\lim\limits_{x\to 0} sinx$

b. $\lim\limits_{x\to 0} cosx$

c. $\lim\limits_{x\to 0} tanx$

d. $\lim\limits_{x\to \pi} sinx$

e. $\lim\limits_{x\to \pi} cosx$

f. $\lim\limits_{x\to \frac{\pi}{2}} sinx$

g. $\lim\limits_{x\to \frac{\pi}{2}} cosecx$

Good Luck