· Sum

$(f(x) + g(x))$

$(f+g)(x)$

$$\frac{d}{dx}\left(f(x)+g(x)\right) = \frac{df(x)}{dx} + \frac{dg(x)}{dx}$$

Exp: $h(x) = x^2 + 2x + \cos x$

$h'(x) = 2x + 2 - \sin x$.

Difference :-

$$\frac{d}{dx}\left(f(x) - g(x)\right) = f'(x) - g'(x)$$

$h(x) = x^3 - \sin x$

$h'(x) = 3x^2 - \cos x$ ✓.

$((f+g)(x))' = f'(x) + g'(x)$

✓ $((f-g)(x))' = f'(x) - g'(x)$

$H(x) = f(x) - g(x)$

$H(x+h) = f(x+h) - g(x+h)$

$$H'(x) = \lim_{h\to 0}\left[\frac{H(x+h) - H(x)}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{f(x+h) - g(x+h) - f(x) + g(x)}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{f(x+h) - f(x)}{h} - \frac{g(x+h) - g(x)}{h}\right]$$

$$= f'(x) - g'(x)$$

$$H'(x) = f'(x) - g'(x)$$

$$\frac{d}{dx}\left(f(x) - g(x)\right) = f'(x) - g'(x)$$

$$\frac{d}{dx}\left(k f(x)\right) = k \frac{df(x)}{dx}$$

$$h(x) = 3 \sin x$$

$$h'(x) = 3 \cos x.$$

$$H(x) = k f(x)$$

$$H(x+h) = k f(x+h)$$

Product Rule :-

$$\frac{d}{dx}\left(f(x) g(x)\right) = f'(x) g(x) + f(x) g'(x)$$

$$G(x) = f(x) g(x)$$

$$G(x+h) = f(x+h) g(x+h)$$

$$\frac{d G(x)}{dx} = \lim_{h \to 0} \left[\frac{G(x+h) - G(x)}{h}\right]$$

$$= \lim_{h \to 0} \left[\frac{f(x+h) g(x+h) - f(x) g(x)}{h}\right]$$

$$= \lim_{h \to 0} \left[\frac{f(x+h) g(x+h) - f(x) g(x+h) + f(x) g(x+h) - f(x) g(x)}{h}\right]$$

$$= \lim_{h \to 0} \left[\frac{g(x+h)\left(f(x+h) - f(x)\right) + f(x)\left(g(x+h) - g(x)\right)}{h}\right]$$

$$= \lim_{h \to 0} g(x+h) \cdot \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} + f(x) \lim_{h \to 0} \frac{\left(g(x+h) - g(x)\right)}{h}$$

$= g(x)\, f'(x) + f(x)\, g'(x)$

$$\frac{d}{dx} G(x) = f'(x)\, g(x) + f(x)\, g'(x)$$

$$\frac{d}{dx}\left(f(x) g(x)\right) = f' g + f g'$$

$$(fg)' = f'g + fg'$$

$$(\underline{f'g - fg'})' = f''g - fg''$$

<u>Prove</u>:

$$\frac{d}{dx}\left(f'g - fg'\right)$$

$$= \frac{d}{dx}(f'g) - \frac{d}{dx}(fg')$$

$$= (f''g + f'g') - (f'g' + fg'')$$

$$= f''g + \cancel{f'g'} - \cancel{f'g'} - fg''$$

$$= f''g - fg''$$

$$f(x) = \underline{x}^2 \sin x$$

$$f'(x) = (2x)(\sin x) + (x^2)(\cos x)$$

$$= 2x \sin x + x^2 \cos x$$

$(\sin x)' = \cos x,$ $\qquad$ $(\cos x)' = -\sin x$

$(\tan x)'$ $\qquad$ $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

$$\tan x = \frac{\sin x}{\cos x}$$

$$(\tan x)' = \left(\frac{\sin x}{\cos x}\right)' = \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} = \sec^2 x$$

$$(\tan x)' = \sec^2 x$$

$$(\sec x)' = \left(\frac{1}{\cos x}\right)' = \frac{(0)(\cos x) - (1)(-\sin x)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$$

$$= \tan x \sec x$$

$$(\sec x)' = \sec x \tan x$$

$$(\cot x)' = \left(\frac{\cos x}{\sin x}\right)' = -\operatorname{cosec}^2 x$$

$$(\operatorname{cosec} x)' = \left(\frac{1}{\sin x}\right)' = -\operatorname{cosec}(x)\cot(x)$$

Chain Rule :-

$G(x) = f(g(x))$

$G'(x) = f'(g(x))\, g'(x)$

- $\frac{d}{dx}\left(f(g(x))\right) = \frac{df(g(x))}{dg(x)} \cdot \frac{dg(x)}{dx}$

- $\left(f(g(x))\right)' = f'(g(x))\, g'(x)$

$$\begin{cases} y = f(g(x)) \\ u = g(x) \quad\Big|\quad y = f(u) \\ \frac{du}{dx} = g'(x) \quad\Big|\quad \frac{dy}{du} = f'(u) \\ \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \end{cases}$$

$\frac{d}{dx}\left((x)^3\right) = 3x^2.$

CR $\frac{d}{dx}\left((x^2+2x)^3\right) =$

$\frac{d}{dx}(\sin x) = \cos x$

CR $\frac{d}{dx}\left(\sin(x^2+2x)\right) = ?$

$\sin^3(x)$

$(\sin x)^3$

$\cos^3(2x+1)$

General Power Rule :-

Simple Rule

$y = x^n$

$\frac{dy}{dx} = n x^{n-1}$

$y = x^9$

$y' = 9x^8$

General Power Rule

$y = (f(x))^n$

$\frac{dy}{dx} = n\,(f(x))^{n-1} f'(x)$

$y = (1 + x^2 + \sin x)^9$

$\frac{dy}{dx} = \left(9\,(1 + x^2 + \sin x)^8\right)(0 + 2x + \cos x)$

$= 9\,(1 + x^2 + \sin x)^8\,(2x + \cos x)$

$$f(x) = \sqrt{3x^2 - x + 1}$$
$$= (3x^2 - x + 1)^{1/2}$$
$$f'(x) = \left(\frac{1}{2}(3x^2 - x + 1)^{\frac{1}{2}-1}\right)(6x - 1 + 0)$$
$$= \frac{1}{2}(3x^2 - x + 1)^{-1/2}(6x - 1)$$
$$= \frac{6x - 1}{2\sqrt{3x^2 - x + 1}}$$

$$h(x) = \sqrt{g(x)}$$
$$h'(x) = \frac{d}{dx}\left((g(x))^{1/2}\right)$$
$$= \frac{1}{2}(g(x))^{-1/2}\, g'(x)$$
$$= \frac{1}{2}\,\frac{g'(x)}{\sqrt{g(x)}}$$

$$h(x) = \tan^3(x)$$
$$h(x) = (\tan(x))^3$$

$$\tan^3(x) = (\tan x)^3$$
$$\tan(x^3) \neq (\tan x)^3$$
$$\tan(2x) \neq 2\tan x$$

$$h(x) = \tan^3 x$$
$$h(x) = (\tan x)^3$$
$$h'(x) = \frac{d}{dx}\left((\tan x)^3\right)$$
$$= 3(\tan x)^{3-1}(\sec^2 x)$$
$$= 3(\tan x)^2 \sec^2 x$$

$= 3\tan^2 x \sec^2 x$

$\sin^2 x = (\sin x)^2$

$\cos^9(x) = (\cos x)^9$

$\tan^{10}(x) = (\tan x)^{10}$

Product + Chain Rule

$y = x\sqrt{1-x^2}$

$f \quad g$

$$\frac{dy}{dx} = (x)'\sqrt{1-x^2} + x\left(\sqrt{1-x^2}\right)'$$

$$= \sqrt{1-x^2} + x\frac{d}{dx}\left((1-x^2)^{\frac{1}{2}}\right)$$

$$= \sqrt{1-x^2} + x\left[\frac{1}{2}(1-x^2)^{\frac{1}{2}-1}(-2x)\right]$$

$$= \sqrt{1-x^2} - \frac{\cancel{2}x^2}{\cancel{2}\sqrt{1-x^2}}$$

Chain + Quotient Rule

$$g(x) = \frac{x}{\sqrt{x^2+1}}$$

$$\left(\frac{N}{D}\right)' = \frac{N'D - ND'}{D^2}$$

$$= \frac{(1)\sqrt{x^2+1} - x\frac{d}{dx}\left(\sqrt{x^2+1}\right)}{\left(\sqrt{x^2+1}\right)^2}$$

$$= \frac{\sqrt{x^2+1} - x\left((x^2+1)^{\frac{1}{2}}\right)'}{(x^2+1)}$$

$$= \frac{\sqrt{x^2+1} - x\left(\frac{1}{\cancel{2}}(x^2+1)^{-\frac{1}{2}}(\cancel{2}x)\right)}{x^2+1}$$

$$= \left(\frac{\sqrt{x^2+1}}{1} - \frac{x^2}{\sqrt{x^2+1}}\right) \div (x^2+1)$$

$$= \left(\frac{\cancel{x^2} + 1 - \cancel{x^2}}{\sqrt{x^2+1}}\right) \times \frac{1}{x^2+1}$$

Chain + Quotient

Quotient Rule inside Chain.

$$y = \left(1 + \frac{t^2}{t^3+2}\right)^2$$

$$\frac{dy}{dt} = \frac{d}{dt}\left(\left(1+\frac{t^2}{t^3+2}\right)^2\right)$$

$$= \left(2\left(1+\frac{t^2}{t^3+2}\right)^{2-1}\right)\left(\frac{d}{dt}\left(1+\frac{t^2}{t^3+2}\right)\right)$$

$$= 2\left(1+\frac{t^2}{t^3+2}\right)\left(0+\frac{d}{dt}\left(\frac{t^2}{t^3+2}\right)\right)$$

$$= 2\left(\frac{1}{1}+\frac{t^2}{t^3+2}\right)\left(\frac{(2t)(t^3+2) - t^2(3t^2)}{(t^3+2)^2}\right)$$

$$= 2\left(\frac{t^3+2+t^2}{t^3+2}\right)\left(\frac{2t^4+4t-3t^4}{(t^3+2)^2}\right)$$

$$= \frac{2(t^3+t^2+2)}{(t^3+2)} \cdot \frac{(4t-t^4)}{(t^3+2)^2}$$

$$= \frac{2(t^3+t^2+2)(4t-t^4)}{(t^3+2)^3}$$

$$y = \sqrt{x\sin x}$$

$$y = (x\underline{\sin x})^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}(x\sin x)^{-1/2}\frac{d}{dx}(x\sin x)$$

$$= \frac{1}{2\sqrt{x\sin x}} \cdot ((1)\sin x + x\cos x)$$

$$= \frac{\sin x + x\cos x}{2\sqrt{x\sin x}} \checkmark$$

$$y = \sin(f(x))$$

$$y' = \cos(f(x)) \cdot f'(x),$$

$y = \underline{\sin}^9 (\underline{x^2+1})$

$y' = \left(9 \sin^8(x^2+1)\right)\left(\cos(x^2+1)\right)\left((2x)\right)$

$= 18x \sin^8(x^2+1) \cos(x^2+1)$ ✓

$y = \underline{\cos}^8(\underline{\tan x})$

$y' = 8 \cos^7(\tan x)\left(-\sin(\tan x)\right) \sec^2 x$

$= -8 \cos^7(\tan x) \sin(\tan x) \sec^2 x.$

✓

$f(x) = \operatorname{cosec}^{10}(\underline{3x^2+2x+1}) = \left(\underline{\operatorname{cosec}(3x^2+2x+1)}\right)^{10}$

$f'(x) = 10 \operatorname{cosec}^9(3x^2+2x+1)\left(-\operatorname{cosec}(3x^2+2x+1)\cot(3x^2+2x+1)\right)$

$(6x+2)$