Fys 4170/9170, 2013

A short-version solution:

1a) $c_{\vec{p}\sigma}$ = destruction of particle

$d^{+}_{\vec{p}\sigma}$ = creation of anti-particle

$\psi^{(\pm)}_{\vec{p}\sigma}(x)$ = plane wave solutions of Dirac eq. ($\vec{p}$ = three (space) momentum, σ = spin quantum number) for positive (+) or negative (−) energy solutions; – corresponding to particles and antiparticles respectively

Hamiltonian*) $H = \sum_{\vec{p}\sigma} E_p \{ N^c_{\vec{p}\sigma} + N^d_{\vec{p}\sigma} \}$

where $E_p = \sqrt{(\vec{p})^2 + m^2}$ is the energy (m = mass of part.)

$$(N^c_{\vec{p}\sigma})^2 = (c_{\vec{p}\sigma})^{+} c_{p\sigma}\, c^{+}_{\vec{p}\sigma} c_{p\sigma} = c^{+}_{\vec{p}\sigma}(1 - c^{+}_{p\sigma} c_{p\sigma}) c_{p\sigma}$$

$$= N_{\vec{p}\sigma} \quad \text{because} \quad c_{p\sigma} c_{p\sigma} = 0 \; (c^{+}_{p\sigma} c^{+}_{p\sigma} = 0)$$

This shows that the number operator $N^c_{\vec{p}\sigma}$ (and sim. $N^d_{\vec{p}\sigma}$) has eigenvalues 0 ~~and~~ 1 for given $\vec{p}$ and σ.

---

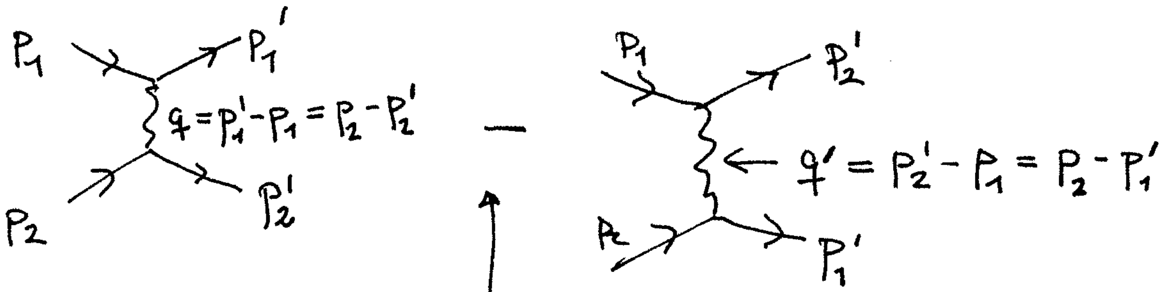
*) The answer $H = \int d^3\vec{x}\; \psi^{+}(\vec{\alpha}\cdot\vec{p} + \beta m)\psi$ is also acceptable

Here $\vec{p} = \frac{\hbar}{i}\vec{\nabla} \to -i\vec{\nabla}$

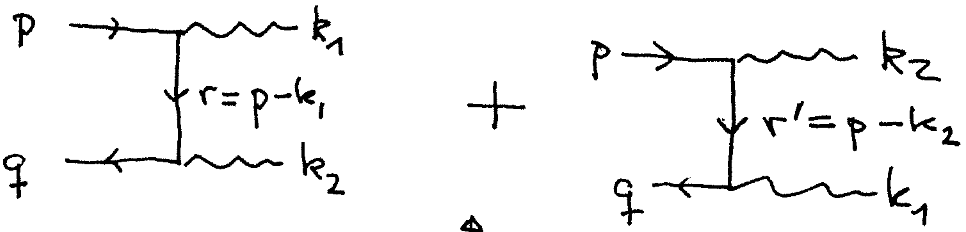
1b) $|e^-(\vec{p}_1\sigma_1)\, e^-(p_2\sigma_2)\rangle = c^+_{\vec{p}_1\sigma_1} c^+_{\vec{p}_2\sigma_2}|0\rangle$

$= -c^+_{\vec{p}_2\sigma_2} c^+_{\vec{p}_1\sigma_1}|0\rangle = -|e^-(\vec{p}_2\sigma_2)\, e^-(\vec{p}_1\sigma_1)\rangle$

i.e. antisymmetry – in accordance with the Pauli principle



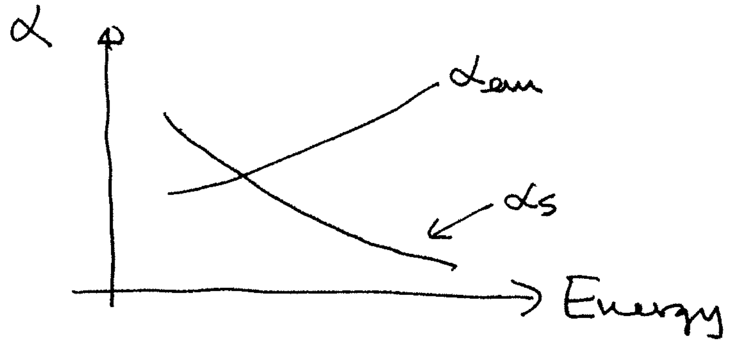
relative minus-sign between the two amplitudes



relative + sign between ampl.

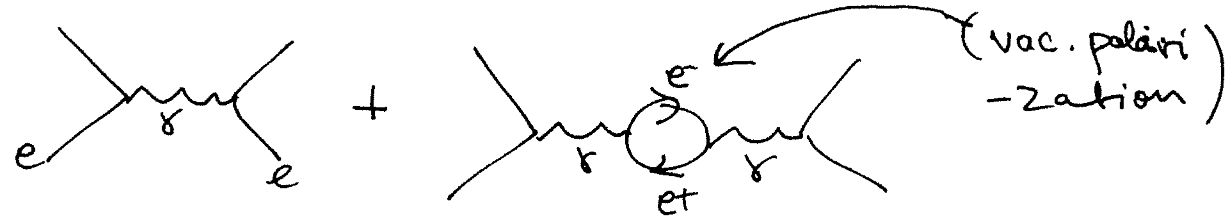
1c) Qualitative behaviour of elmagn structure "constant" $\alpha_{em} \equiv \frac{e^2}{4\pi} \equiv \frac{(g_\gamma)^2}{4\pi}$

and strong (quark-gluon) structure constant $\alpha_s \equiv \frac{g_s^2}{4\pi}$ with increasing energy:

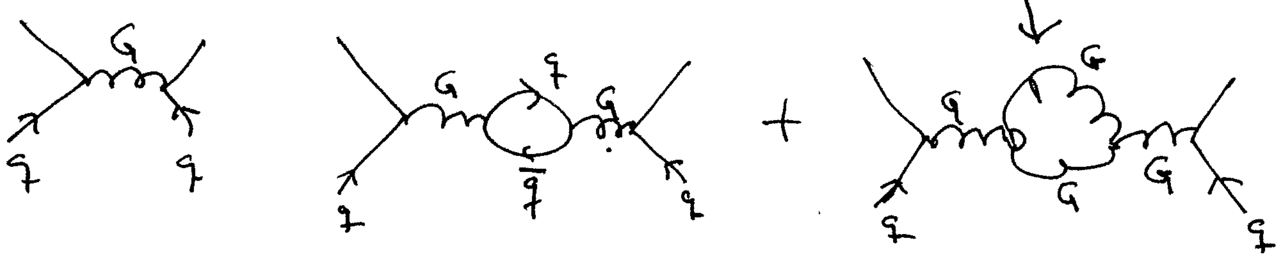


(Qualitative Figure !)

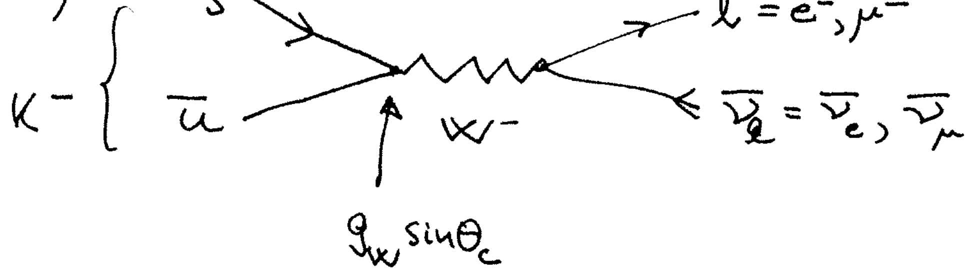
For elmagn case "screening" of charge



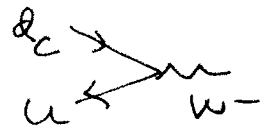
For strong case screening – but also (dominating!) anti-screening due to "gluonic vac. pol."



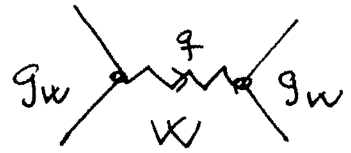
2a)



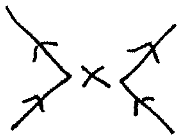
$\sin\theta_c$ enters because of Cabibbo-mixing (4 quark case). The decaying quark "formally" is $d_c = d\cos\theta_c + s\sin\theta_c$



2b)

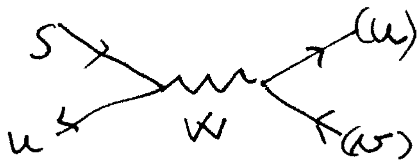


$\xrightarrow[\text{Fermi limit}]{} \left(\frac{1}{\sqrt{2}}G_F\right)$



$$\frac{G_F}{\sqrt{2}} = \left(\frac{\frac{1}{2}g_W}{M_W}\right)^2 = \lim_{q\to 0}\left(\frac{-\left(\frac{1}{2}g_W\right)^2}{q^2 - M_W^2}\right)$$

The weak interaction is weak because the W-boson is heavy! (g_W of same order as $g_\gamma = |e|$ = elmagn coupling)



The weak (leptonic) current is

$$j_\mu(\ell^- \nu_\ell) = \overline{u(p_\ell)}\,\gamma_\mu L\, v(p_\nu)$$

$$L = \tfrac{1}{2}(1-\gamma_5)$$

2c) $\mathcal{M}(K^- \to \ell^- \bar{\nu}_\ell) = G\left(-\frac{i}{\sqrt{2}} f_K\right) \cdot q^\mu j_\mu(\ell^- \nu_\ell)$

Using the Dirac equation

$$\underline{q^\mu j_\mu(\ell^- \bar{\nu}_\ell)} = (p_\ell^\mu - p_\nu^\mu)\, \bar{u}(p_\ell)\,\underbrace{\gamma_\mu L}_{=R\gamma_\mu}\, v(p_\nu)$$

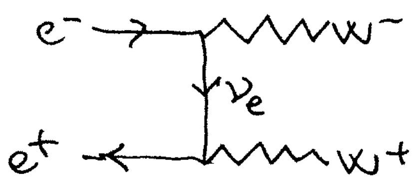
$$= [\underbrace{\bar{u}(p_\ell)\,\gamma\cdot p_\ell}_{=m_\ell \bar{u}(p_\ell)}]L\, v(p_\nu) - \bar{u}(p_\ell)\cdot R[\underbrace{\gamma\cdot p_\nu\, v(p_\nu)}_{\simeq 0}]$$

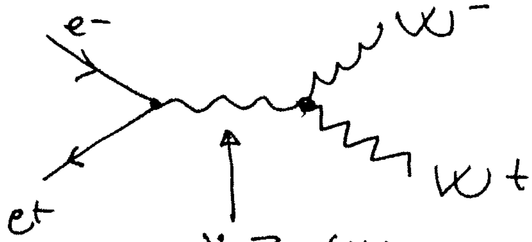
$$\simeq \underline{m_\ell\, \bar{u}(p_\ell) L\, v(p_\nu)}$$

Thus the amplitude for $\ell^- = \mu^-$ is bigger than the one for $\ell^- = e^-$ and decay to μ^- has bigger probability

3a)

$e^+e^- \to W^+W^-$



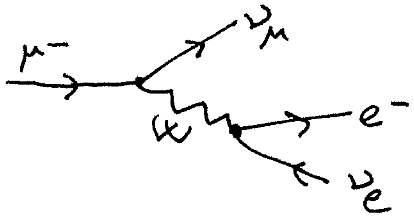


$\gamma, Z, (H)$

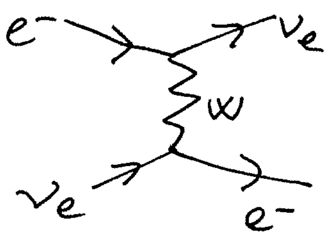
↖ small coupling to electron

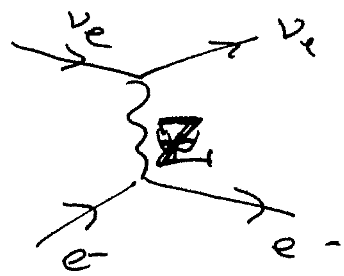
$g_H \sim \frac{m_e}{M_W} g_W$

μ-decay

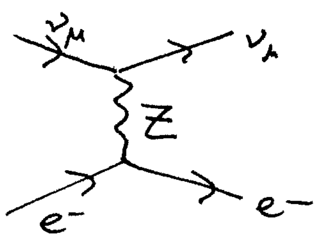


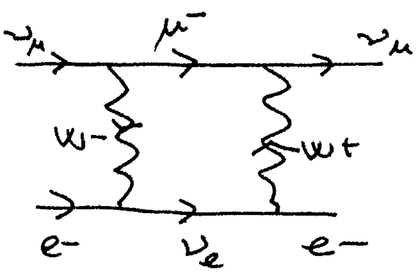
$e^- \nu_e \to e^- \nu_e$:





$e^- \nu_\mu \to e^- \nu_\mu$





Double W-exchange (if no Z) ⤴

(NB: Neutrino-mixing change this!)
(small)

↑ analogous to quark mixing)

3b) The b-quark can decay to a c- or u-quark due to <u>quark mixing</u>

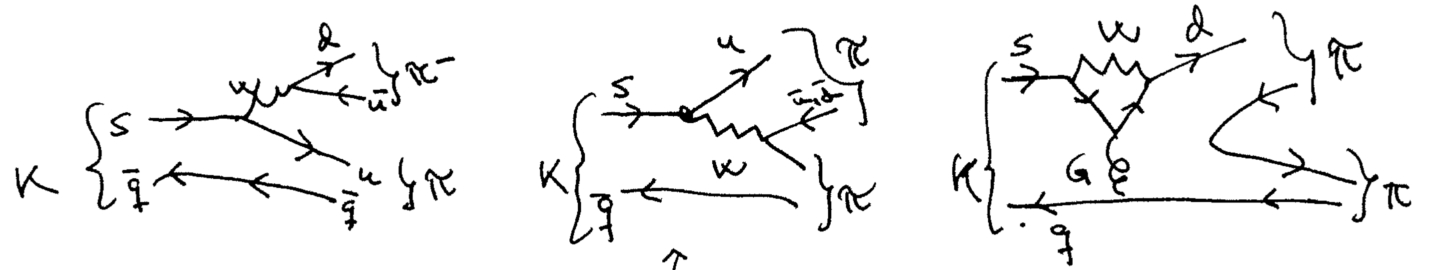
Formally the b-quark "decays" to

$$t_{CKM} = t\cdot V_{tb} + c\,V_{cb} + u\,V_{ub} \qquad \text{(CKM)}$$

where the V_{qb}'s are complex numbers

The mixing occurs because quark masses are different (For details; - see the repetition pages (rep13.pdf, pages 42-44))

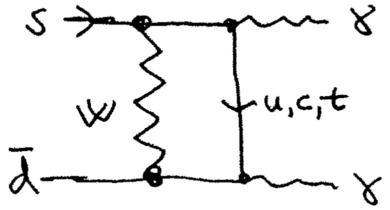
Typical $K \to \pi\pi$ diagrams



"Color suppressed"

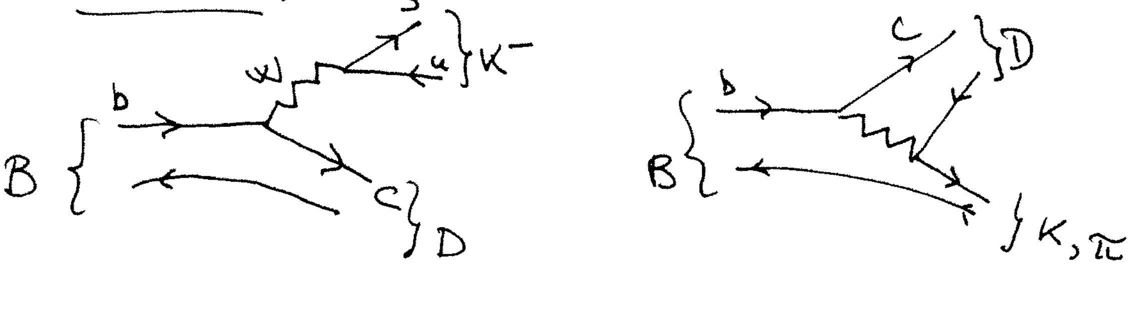
"Penguin diagram"

<u>$K^0 \to 2\gamma$</u>



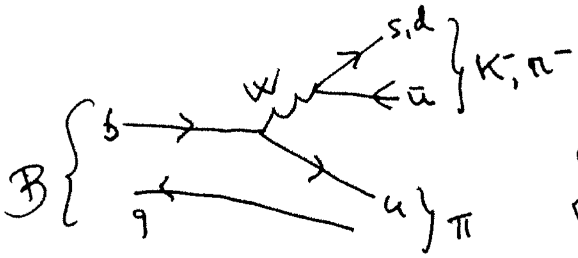
(+ others...)

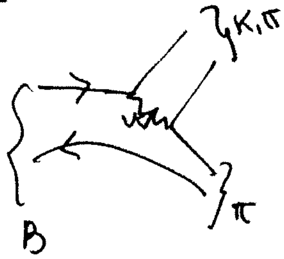
<u>$B \to DK$:</u>

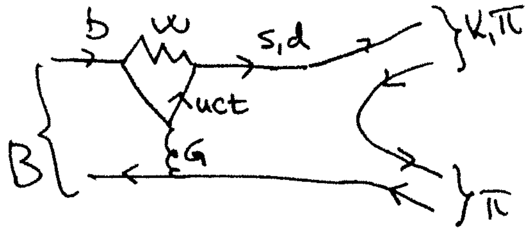


(+ Penguin diagram for $b \to sG$)

$B \to K\pi, \pi\pi$

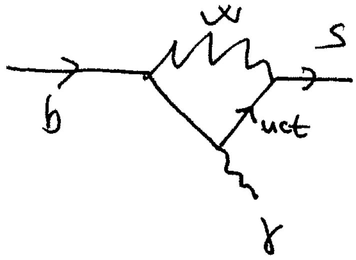




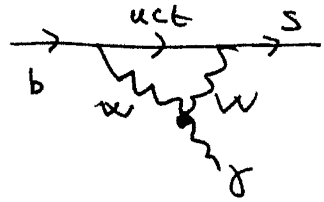


"Penguin"

$\underline{b \to \gamma s}$



Also



4a) $SU(2)_L \times U(1)_Y$ transf: $U = U_2 \cdot e^{iY\cdot\alpha}$,

Y = hypercharge, $U_2 \in SU(2)_L$ (α = arbitrary real)

$\chi_L \to \chi_L' = U\chi_L$, $\phi \to \phi' = U\phi$

$\overline{\chi_L} \to \overline{\chi_L'} = \overline{\chi_L}\, U^+$, $\phi^+ \to \phi'^+ = \phi^+ U^+$

Thus $(\overline{\chi_L}\phi)$ and $\phi^+\chi_L$ are invariant under $SU(2)_L \times U(1)_Y$ trans. | $e_R \to e_R' = e^{iY_R\alpha} e_R$ for $U(1)_Y$ transf.

In total $\mathcal{L}_{e\phi}$ is invariant provided

$$Y_R + Y_\phi - Y_\chi = 0$$

(See also rep13.pdf)

Special case (in Higgs-mechanism)

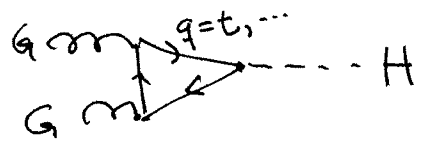
$$\phi \to \phi' = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ (v+H) \end{pmatrix}$$ gives (see rep13.pdf)

$$\mathcal{L}'_{e\phi} = -\frac{G_e}{\sqrt{2}} \bar{e}' e' (v+H)$$

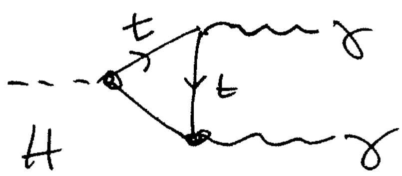
giving $\underline{m_e = \frac{G_e}{\sqrt{2}} \cdot v}$.

($e' = \psi'_e$ = physical electron field)

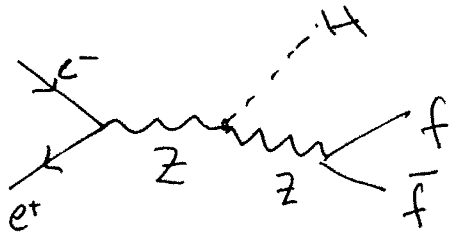
4b) The Higgs-particle H doesnt couple directly to gluons such that we might have

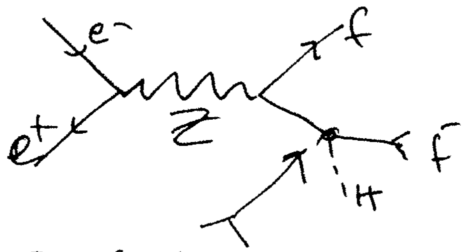


$H \to 2\gamma$



$\underline{e^- e^+ \to f \bar{f} H}$





Big if f = top quark