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http://www.uio.no/studier/emner/matnat/fys/FYS4160/v17/

Lecture spring 2017:

General Relativity

Problem sheet 11

⇝ These problems are scheduled for discussion on **Thursday, 27 April 2017**.

Problem 36

In the weak field limit, and for a suitable coordinate choice, the metric of a mass M sitting at the origin is given by

$$ds^2 = -(1+2\phi)dt^2 + (1-2\phi)\delta_{ij}dx^i dx^j \,,$$

where $\phi = -GM/\sqrt{x^2+y^2+z^2} \ll 1$.

a) Instead of being at rest, the mass now moves in the x-direction with constant velocity v such that its position is given by $x = vt$. What is the metric in this case?

b) A photon falls freely in the y direction with offset b behind the x-directions, for $t \to -\infty$, i.e. its undeflected trajectory would be $\mathbf{x}_0 = -b\,\mathbf{e}_x + t\,\mathbf{e}_y$. By what angle is the actual photon path deflected w.r.t. to this trajectory?
Hint: There are (at least) two very different ways of addressing this problem. Explicit Lorentz transformations to the rest frame of the moving mass, and back, is much easier than using the geodesic equation directly in a brute force approach.

c) By how much changes the frequency of the deflected photon? How would you guess is this change in energy compensated?

Problem 37

In the lecture on Friday we will derive the linearized version of Einstein's equations,

$$G^{(0)}_{\mu\nu} = 8\pi G T_{\mu\nu} \,,$$

where $G^{(0)}_{\mu\nu}$ is given by Eq. (7.8) in the book.

a) By deriving the transformation properties of $h_{\mu\nu}$ from the way it was introduced, $h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$, show that this describes a Lorentz-invariant theory of a symmetric rank-2 tensor field (h) on flat spacetime.

b) Show that this theory follows from the Lagrangian given in Eq. (7.9) in the book, after adding a matter part $\mathcal{L}_M$!

Problem 38

Discuss in what sense the theory introduced in the previous problem is invariant under the replacement $h_{\mu\nu} \to h_{\mu\nu} + 2\partial_{(\mu}\xi_{\nu)}$, and relate this to the situation of gauge transformations in electrodynamics! Show explicitly that, for a metric decomposition as in (7.16, 7.17), the gauge transformations of linearized gravity are given by (7.33).