

# Dark Energy and the MSSM

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**ABSTRACT:** We consider the coupling of quintessence to observable matter in supergravity and study the dynamics of both supersymmetry breaking and quintessence in this context. We investigate how the quintessence potential is modified by supersymmetry breaking and analyse the structure of the soft supersymmetry breaking terms. We pay attention to their dependence on the quintessence field and to the electroweak symmetry breaking, *i.e.* the pattern of fermion masses at low energy within the Minimal Supersymmetric Standard Model (MSSM) coupled to quintessence. In particular, we compute explicitly how the fermion masses generated through the Higgs mechanism depend on the quintessence field for a general model of quintessence. Fifth force and equivalence principle violations are potentially present as the vacuum expectation values of the Higgs bosons become quintessence field dependent. We emphasize that equivalence principle violations are a generic consequence of the fact that, in the MSSM, the fermions couple differently to the two Higgs doublets. Finally, we also discuss how the scaling of the cold dark and baryonic matter energy density is modified and comment on the possible variation of the gauge coupling constants, among which is the fine structure constant, and of the proton-electron mass ratio.

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## 1. Introduction

Cosmological observations combining the large scale structures of the universe [1], the Hubble diagram of type Ia supernovae [2] and the anisotropies of the cosmic microwave background [3] all lead to the existence of an accelerated phase of the expansion of the universe. Such an acceleration is interpreted within Einsteinian relativity (see also Ref. [4]) as being driven by a vacuum energy of minute magnitude, some hundred and twenty orders below the Planck scale. Most scenarios suggest that such a very low value for the cosmological constant is hardly compatible with known physics such as the electroweak phase transition and the large radiative correction which are generated there. This has prompted the possibility that the acceleration of the universe expansion could have an extra dimensional origin. It could result from a self-tuning mechanism where the energy density on a brane-world curves a fifth dimension leaving an (almost) flat brane [5]. It could also be that our universe undergoes a phase of self-acceleration such as the ones existing in brane induced gravity models [6–8]. Another possibility is that the vacuum energy has an anthropic origin coming from the landscape of string vacua [9].

In the following we will use a more traditional approach and utilize four-dimensional field theory to tackle the problem of the acceleration of the universe expansion. More precisely we will study quintessence models [10–14]. As usual we consider that the cosmological constant problem is in fact two-pronged. The cosmological constant *per se* has to do with the exact cancellation of large energy densities coming from Quantum Chromodynamics (QCD), the electroweak phase transition, Grand Unified Theory (GUT) scale physics and other high energy phenomena. Quintessence models have nothing to say about this major problem. On the contrary we concentrate on modeling the small vacuum energy responsible for the acceleration. To do so, we assume that scalar field energy densities are responsible for it. We also require that quintessence models possess attractors [10], i.e. long time solutions implying that no sensitivity to initial conditions exists. Within such a class of models, exemplified by the Ratra–Peebles model, the quintessence field tends to have a large value now, very close to the Planck scale. As a consequence we embed quintessence models within supergravity (SUGRA) [13, 15–17], the best field theoretic candidate capturing physics close to the Planck scale.

In this paper, we are particularly interested in the coupling between the quintessence field and the observable sector of particle physics such as the Minimal Supersymmetric Standard Model (MSSM) or the mSUGRA model [18]. Such a model contains soft supersymmetry breaking terms whose origin is best described using spontaneously broken supersymmetry originating in a supersymmetry breaking sector. As gravitational experiments give stringent bounds on fifth forces and equivalence principle violations [19] mediated by light scalar fields such as the quintessence field, we impose that the quintessence field lives in a separate sector from the supersymmetry breaking and observable sectors. Hence we consider models with three sectors. Nevertheless gravitational couplings between these sectors are present and we study their consequences on the pattern of supersymmetry breaking, the generation of soft terms [20] and the electroweak symmetry breaking. The soft terms and the fermion masses after electroweak symmetry breaking [21] all become dependent on the quintessence field, i.e. we have Yukawa-like interaction of the form

$$m^2 \left( \frac{Q}{m_{\text{Pl}}} \right) \bar{\Psi} \Psi, \quad (1.1)$$

where  $\Psi$  represents a fermionic field and  $Q$  is the quintessence field. The main purpose of the paper is to calculate the function  $m(Q/m_{\text{Pl}})$  for a general model of quintessence. We also study how the quintessence potential itself is modified by the supersymmetry breaking and give the general expression of its new shape. In particular, the value of the quintessence field now can be drastically affected by the presence of the supersymmetry breaking sector and can lead to an extremely low value of  $Q$  compared to the Planck scale [22].

An interaction of the form (1.1) is such that the model reduces to a scalar–tensor–like theory [23] where matter couples both to gravity and the quintessence field. In particular, we find that matter couples to a different metric depending on which of the two MSSM Higgs fields gives its mass to a particle. This may lead to large gravitational problems such as strong violations of the weak equivalence principle [24], the presence of a fifth force or the variation of the proton–electron mass ratio. Another consequence of the interaction (1.1) is

that it implies the presence of an interaction between dark energy and dark matter [25,26]. As a consequence, the scaling of the cold dark and baryonic energy densities is modified and we briefly mention the observational consequences of this fact.

The paper is arranged as follows. In the next section we discuss the general coupling between quintessence, supersymmetry breaking and observable matter and calculate the resulting soft terms. In section 3, we apply these results to the electroweak symmetry breaking. In section 4, we analyze different scenarios and discuss the various physical consequences of the coupling between the quintessence field and the fields of the standard model of particles physics. The discussion is kept as general as possible while concrete examples and applications of the calculations developed in this article are studied in Ref. [22]. Finally, in Section 5, we present our conclusions.

## 2. Coupling Quintessence to Matter

### 2.1 General Setting

In the following we will deal with the coupling of a quintessence sector to an observable sector and a supersymmetry breaking sector. The supersymmetry breaking sector is assumed to be only gravitationally coupled to the observable sector as is standard in SUGRA [18]. Similarly we assume that the quintessence sector is separated from the observable sector and couples to ordinary matter only gravitationally. This is to prevent strong direct couplings between matter and quintessence which may lead to the existence of a fifth force or violations of the equivalence principle. In some sense, this is also the minimal and most conservative assumption since, in this case, the coupling between matter and quintessence sectors is completely fixed by SUGRA and no other physical input is necessary. Otherwise, it would have been necessary to choose an explicit coupling and it is unclear which form it should take. Hence, we assume that there are three different sectors in the theory: the observable, hidden and quintessence sectors. As a consequence, the Kähler and super potentials are given by the following expressions

$$K = K_{\text{quint}} + K_{\text{hid}} + K_{\text{obs}}, \quad W = W_{\text{quint}} + W_{\text{hid}} + W_{\text{obs}}. \quad (2.1)$$

Let us notice that the above choice, although very natural, is probably not unique. For instance, another possibility inspired by the physics of branes would be to consider the sequestered form [27] where one adds the exponential of the Kähler potentials, *i.e.*  $g = -3/\kappa + g_{\text{quint}} + g_{\text{hid}} + g_{\text{obs}}$  with  $g \equiv -(3/\kappa)e^{-\kappa K/3}$ ,  $\kappa$  being defined by  $\kappa \equiv 8\pi/m_{\text{Pl}}^2$  rather than the Kähler potentials themselves. Therefore, in this type of models,  $K$  is given by [27]:  $K = -(3/\kappa) \ln(1 - \kappa g_{\text{quint}}/3 - \kappa g_{\text{hid}}/3 - \kappa g_{\text{obs}}/3)$ . The analysis of the phenomenology of these sequestered models is left for future work.

Notice that we restrict our considerations to  $N = 1$  SUGRA in its usual formulation limited to second order in the derivatives. Higher order derivatives might give extra contributions such as  $R^4$  where  $R$  is the Ricci scalar, see Ref. [28]. However, these terms are Planck mass suppressed and since one considers an effective theory valid up to a cutoff small in comparison to the Planck mass (typically the GUT scale), one expects these terms

to be small. Otherwise, these corrections could lead to K-essence models that have been studied in Ref. [29] but are different from what is investigated in this paper.

In this article, the quintessence sector is left unspecified and no assumption will be made about  $K_{\text{quint}}$  and  $W_{\text{quint}}$ . In this “dark energy” sector, we collectively denote the fields by  $d_\alpha$  (if it contains more than just one field) among which is of course the quintessence field itself,  $Q$ . We denote the fields in the hidden sector by  $z_i$ . For simplicity, we take a flat Kähler potential and we do not specify the superpotential for the moment,

$$K_{\text{hid}} = \sum_i z_i z_i^\dagger + \dots, \quad W_{\text{hid}} = W_{\text{hid}}(z_i). \quad (2.2)$$

Finally, the fields in the matter sector are written  $\phi_a$ . This sector is supposed to contain all the (super) fields that are observable. As a consequence, we take this sector to be the MSSM or the mSUGRA model [18], namely

$$K_{\text{obs}} = \sum_a \phi_a \phi_a^\dagger + \dots, \quad W_{\text{obs}} = \frac{1}{3} \sum_{abc} \lambda_{abc} \phi_a \phi_b \phi_c + \frac{1}{2} \sum_{ab} \mu_{ab} \phi_a \phi_b + \dots, \quad (2.3)$$

with a supersymmetric mass matrix  $\mu_{ab}$  and Yukawa couplings  $\lambda_{abc}$ . The dots denote possible extra terms suppressed by the cutoff scale of the theory that will not be considered here. This is compatible with the superpotential of the mSUGRA model. In the following, when we treat the electroweak transition, we will be even more specific about the form of the superpotential.

In order to completely specify the observable sector, it is also necessary to choose the supergravity gauge coupling functions  $f_G$  appearing in the action as  $\int d^2\theta f_G W_G^2$  where  $W_G$  is the superfield strength of the gauge group  $G$ . If the function  $f_G$  depends on the quintessence field, this leads to variations of the gauge coupling constants such as the fine structure constant. In this article, we have chosen to keep the more general dependence and assume that all the  $f_G$ ’s to be  $z_i$  and  $d_\alpha$ -dependent (hence, a priori,  $Q$ -dependent).

Then, inserting the Kähler and the super potentials into the expression of the scalar potential (the  $F$ -term), one gets

$$\begin{aligned} V &= e^{\kappa K} (K^{-1})^{d_\alpha^\dagger d_\beta} \left( \kappa W \frac{\partial K_{\text{quint}}}{\partial d_\beta} + \frac{\partial W_{\text{quint}}}{\partial d_\beta} \right) \left( \kappa W^\dagger \frac{\partial K_{\text{quint}}}{\partial d_\alpha^\dagger} + \frac{\partial W_{\text{quint}}^\dagger}{\partial d_\alpha^\dagger} \right) \\ &\quad + e^{\kappa K} (K^{-1})^{z_i^\dagger z_j} \left( \kappa W \frac{\partial K_{\text{hid}}}{\partial z_j} + \frac{\partial W_{\text{hid}}}{\partial z_j} \right) \left( \kappa W^\dagger \frac{\partial K_{\text{hid}}}{\partial z_i^\dagger} + \frac{\partial W_{\text{hid}}^\dagger}{\partial z_i^\dagger} \right) \\ &\quad + e^{\kappa K} (K^{-1})^{\phi_a^\dagger \phi_b} \left( \kappa W \frac{\partial K_{\text{obs}}}{\partial \phi_b} + \frac{\partial W_{\text{obs}}}{\partial \phi_b} \right) \left( \kappa W^\dagger \frac{\partial K_{\text{obs}}}{\partial \phi_a^\dagger} + \frac{\partial W_{\text{obs}}^\dagger}{\partial \phi_a^\dagger} \right) \\ &\quad - 3\kappa e^{\kappa K} W W^\dagger \\ &\equiv V_1 + V_2 + V_3 - 3\kappa e^{\kappa K} W W^\dagger. \end{aligned} \quad (2.4)$$

In the above expression, the inverse Kählerian matrix is not the total inverse matrix but the inverse matrix in each sector, i.e., for instance,  $(K^{-1})^{d_\alpha^\dagger d_\beta} = (K_{\text{quint}}^{-1})^{d_\alpha^\dagger d_\beta}$ . This is

because these matrices involve derivatives of the Kähler potential and, therefore, kill all the contributions which are not in the sector considered. For convenience, we can separate this potential into several parts, namely  $V_1$ ,  $V_2$  and  $V_3$  as defined above. The first term gives the potential

$$\begin{aligned}
V_1 = & e^{\kappa K} (K^{-1})^{d_\alpha^\dagger d_\beta} \left( \kappa W_{\text{quint}} \frac{\partial K_{\text{quint}}}{\partial d_\beta} + \frac{\partial W_{\text{quint}}}{\partial d_\beta} \right) \left( \kappa W_{\text{quint}}^\dagger \frac{\partial K_{\text{quint}}}{\partial d_\alpha^\dagger} + \frac{\partial W_{\text{quint}}^\dagger}{\partial d_\alpha^\dagger} \right) \\
& + e^{\kappa K} (K^{-1})^{d_\alpha^\dagger d_\beta} \kappa^2 \frac{\partial K_{\text{quint}}}{\partial d_\beta} \frac{\partial K_{\text{quint}}}{\partial d_\alpha^\dagger} \left[ (W_{\text{hid}} + W_{\text{obs}}) (W_{\text{hid}}^\dagger + W_{\text{obs}}^\dagger) \right. \\
& + W_{\text{quint}} (W_{\text{hid}}^\dagger + W_{\text{obs}}^\dagger) + W_{\text{quint}}^\dagger (W_{\text{hid}} + W_{\text{obs}}) \left. \right] + e^{\kappa K} (K^{-1})^{d_\alpha^\dagger d_\beta} \kappa \\
& \times \left[ \frac{\partial K_{\text{quint}}}{\partial d_\beta} \frac{\partial W_{\text{quint}}^\dagger}{\partial d_\alpha^\dagger} (W_{\text{hid}} + W_{\text{obs}}) + \frac{\partial K_{\text{quint}}}{\partial d_\alpha^\dagger} \frac{\partial W_{\text{quint}}}{\partial d_\beta} (W_{\text{hid}}^\dagger + W_{\text{obs}}^\dagger) \right]. \quad (2.5)
\end{aligned}$$

In the above formula, the first term usually gives rise to a supergravity quintessence potential while the other terms represent the coupling of the quintessence field to the hidden and observable sectors.

Let us now consider the term  $V_2$  associated with the hidden sector. Using the fact that the Kähler potential is flat, it can be written as

$$V_2 = e^{\kappa K} \left( \kappa^2 W W^\dagger z_i z_i^\dagger + \kappa W \frac{\partial W_{\text{hid}}^\dagger}{\partial z_i^\dagger} z_i^\dagger + \kappa W^\dagger \frac{\partial W_{\text{hid}}}{\partial z_i} z_i + \left| \frac{\partial W_{\text{hid}}}{\partial z_i} \right|^2 \right). \quad (2.6)$$

Finally, there is the observable matter part. Again, since the Kähler potential is taken to be flat, the matrix  $(K^{-1})^{\phi_a^\dagger \phi_b}$  is the unit matrix. Therefore, one obtains

$$V_3 = e^{\kappa K} \left( \kappa^2 W W^\dagger \phi_a \phi_a^\dagger + \kappa W \frac{\partial W_{\text{obs}}^\dagger}{\partial \phi_a^\dagger} \phi_a^\dagger + \kappa W^\dagger \frac{\partial W_{\text{obs}}}{\partial \phi_a} \phi_a + \left| \frac{\partial W_{\text{obs}}}{\partial \phi_a} \right|^2 \right). \quad (2.7)$$

So far, all these expressions are exact and only depend on the assumption that we have three separate sectors in the theory. In the following we will discuss the physical consequences of these potentials and the coupling between the three sectors.

## 2.2 Breaking Supersymmetry

In the hidden sector, the supersymmetry breaking fields take a vacuum expectation value (vev) obtained by solving the following equation

$$\frac{\partial V(z_j, Q, \phi_a)}{\partial z_i^\dagger} = 0. \quad (2.8)$$

Solving the above expression for  $\langle z_i \rangle$  leads to

$$\kappa^{1/2} \langle z_i \rangle_{\min} = \kappa^{1/2} \langle z_i \rangle_{\min} (\langle Q \rangle, \langle \phi_a \rangle). \quad (2.9)$$

The previous considerations are valid at very high energies, well above the electroweak transition. In this case, one has  $\langle \phi_a \rangle = 0$ . The presence of the quintessence sector implies

that the dynamics of the supersymmetry breaking sector is perturbed. This cannot be neglected as the vev of the quintessence field is, *a priori*, not negligible and leads to  $Q$  dependent vev's, namely  $\kappa^{1/2} \langle z_i \rangle_{\min} = \kappa^{1/2} \langle z_i \rangle_{\min}(Q, \langle \phi_a \rangle = 0)$ . As a consequence, if we parameterize the hidden sector supersymmetry breaking in a model independent way, we have

$$\kappa^{1/2} \langle z_i \rangle_{\min} \sim a_i(Q), \quad \kappa \langle W_{\text{hid}} \rangle_{\min} \sim M_s(Q), \quad \kappa^{1/2} \left\langle \frac{\partial W_{\text{hid}}}{\partial z_i} \right\rangle_{\min} \sim c_i(Q) M_s(Q), \quad (2.10)$$

where  $a_i$  and  $c_i$  are coefficients of order one which depend on the detailed structure of the hidden sector. In the following, we focus on the direction in field space where the hidden supersymmetry breaking fields are located at their minimum satisfying (2.8). It is clear that, from the cosmological point of view, having time varying fields  $z_i$ 's with vev's of the order of the Planck mass can have drastic consequences. We will return to this question below.

The standard way to see whether supersymmetry is broken is to calculate the  $F$ -terms of the theory that are defined, for the fields in the hidden sector (similar expression would obviously hold in the other sectors), by

$$F_{z_i} \equiv \left\langle e^{\kappa K/2} \left( \kappa W \frac{\partial K}{\partial z_i} + \frac{\partial W}{\partial z_i} \right) \right\rangle. \quad (2.11)$$

Explicit calculations lead to the following expression

$$F_{z_i} = e^{\kappa K_{\text{quint}}/2 + \sum_i |a_i|^2/2} \frac{1}{\kappa^{1/2}} \left[ (M_s + \kappa \langle W_{\text{quint}} \rangle) a_i + M_s c_i \right]. \quad (2.12)$$

In the observable sector, the corresponding result is obviously zero but, in the quintessence sector, one generically obtains non vanishing results.

### 2.3 Computing the Soft Terms

In this section, we still consider the theory at high energy, say at the GUT scale. Our aim is then to calculate the soft terms [20], *i.e.* the Lagrangian for the observable fields after supersymmetry breaking. This means that in all the corresponding expressions, we have to replace the hidden sectors terms by their values at the minimum of the potential according to our general parameterization given by Eqs. (2.10). Let us also introduce the gravitino mass. It is defined by

$$m_{3/2} \equiv \left\langle \kappa W e^{\kappa K/2} \right\rangle. \quad (2.13)$$

In the present context, the gravitino mass may depend on the quintessence field and, therefore, may be a time-dependent quantity. In the following, we write

$$m_{3/2} = e^{\kappa K_{\text{quint}}/2} m_{3/2}^0 = e^{\kappa K_{\text{quint}}/2 + \sum_i |a_i|^2/2} (M_s + \kappa \langle W_{\text{quint}} \rangle), \quad (2.14)$$

where  $m_{3/2}^0$  is the mass that the gravitino would have without the presence of the quintessence field (in this case, as already discussed above, the coefficients  $a_i$ ,  $c_i$  and the mass  $M_s$  are constant and the term  $W_{\text{quint}}$  does not exist). The soft terms are obtained by taking the

limit  $m_{\text{Pl}} \rightarrow +\infty$  in all the expressions while keeping the gravitino mass, hence the mass  $M_{\text{S}}$ , fixed. This gives for the observable part of the total potential

$$V_3 = e^{\kappa K_{\text{quint}} + \sum_i |a_i|^2} \left\{ \left[ M_{\text{S}}^2 + M_{\text{S}} \kappa (W_{\text{quint}} + W_{\text{quint}}^\dagger) + \kappa^2 W_{\text{quint}} W_{\text{quint}}^\dagger \right] \phi_a \phi_a^\dagger \right. \\ \left. + (M_{\text{S}} + \kappa W_{\text{quint}}) \frac{\partial W_{\text{obs}}^\dagger}{\partial \phi_a^\dagger} \phi_a^\dagger + (M_{\text{S}} + \kappa W_{\text{quint}}^\dagger) \frac{\partial W_{\text{obs}}}{\partial \phi_a} \phi_a + \left| \frac{\partial W_{\text{obs}}}{\partial \phi_a} \right|^2 \right\}. \quad (2.15)$$

Notice the terms  $\exp(\sum_i |a_i|^2)$  which remain in the final expression of  $V_3$ . *A priori*, they depend on the vev's of the quintessence field and introduce an additional dependence on the quintessence field.

Let us now consider the term  $V_2$ . Then, as before, one takes the limit  $m_{\text{Pl}} \rightarrow +\infty$ , keeping the gravitino mass fixed and remembering that, *a priori*,  $\langle z_i \rangle$  is of order  $m_{\text{Pl}}$  (as it is the case for the quintessence field vev). This gives

$$V_2 = e^{\kappa K_{\text{quint}} + \sum_i |a_i|^2} \left\{ W_{\text{obs}} \left[ (M_{\text{S}} + \kappa W_{\text{quint}}^\dagger) \sum_i |a_i|^2 + M_{\text{S}} \sum_i a_i c_i \right] \right. \\ \left. + W_{\text{obs}}^\dagger \left[ (M_{\text{S}} + \kappa W_{\text{quint}}) \sum_i |a_i|^2 + M_{\text{S}} \sum_i a_i c_i \right] + \frac{1}{\kappa} (M_{\text{S}}^2 + M_{\text{S}} \kappa W_{\text{quint}} + M_{\text{S}} \kappa W_{\text{quint}}^\dagger \right. \right. \\ \left. \left. + \kappa^2 W_{\text{quint}} W_{\text{quint}}^\dagger) \sum_i |a_i|^2 + \frac{M_{\text{S}}}{\kappa} \left[ (2M_{\text{S}} + \kappa W_{\text{quint}} + \kappa W_{\text{quint}}^\dagger) \sum_i a_i c_i \right. \right. \right. \\ \left. \left. \left. + M_{\text{S}} \sum_i |c_i|^2 \right] \right\}. \quad (2.16)$$

The same expression can also be expressed as

$$V_2 = \sum_i |F_{z_i}|^2 + e^{\kappa K_{\text{quint}} + \sum_i |a_i|^2} \left\{ W_{\text{obs}} \left[ (M_{\text{S}} + \kappa W_{\text{quint}}^\dagger) \sum_i |a_i|^2 + M_{\text{S}} \sum_i a_i c_i \right] \right. \\ \left. + W_{\text{obs}}^\dagger \left[ (M_{\text{S}} + \kappa W_{\text{quint}}) \sum_i |a_i|^2 + M_{\text{S}} \sum_i a_i c_i \right] \right\}, \quad (2.17)$$

where we have used the expression of  $F_{z_i}$ , see also Eq. (2.12). It is important to notice that the above potential  $V_2$  is made out of terms which are proportional to  $m_{\text{Pl}}^0$  and to  $m_{\text{Pl}}^2$  (the terms proportional to negative powers of the Planck mass having been eliminated by the limit  $m_{\text{Pl}} \rightarrow +\infty$ ). On the contrary,  $V_{\text{obs}}$  is only made of terms proportional to  $m_{\text{Pl}}^0$ . Therefore, one could worry that, in  $V_{\text{hid}}$ , the limit  $m_{\text{Pl}} \rightarrow +\infty$  is ill-defined. However, all the terms proportional to  $m_{\text{Pl}}^2$  appear in the term  $|F_{z_i}|^2$ . The presence of this term is linked to the cosmological constant problem and  $|F_{z_i}|^2$  will be fine-tuned in order to avoid a large cosmological constant. For this reason, we do not need to worry about these terms. On the other hand, the terms that participate to the A and B terms in the soft SUSY breaking potential have a perfectly well-defined limit  $m_{\text{Pl}} \rightarrow +\infty$ .



Finally, remains the term  $V_1$ . In the same limit, and taking into account that the vev of  $Q$  is *a priori* of the order of the Planck mass, one obtains

$$\begin{aligned}
V_1 = & e^{\kappa K} (K^{-1})^{d_\alpha^\dagger d_\beta} \left( \kappa W_{\text{quint}} \frac{\partial K_{\text{quint}}}{\partial d_\beta} + \frac{\partial W_{\text{quint}}}{\partial d_\beta} \right) \left( \kappa W_{\text{quint}}^\dagger \frac{\partial K_{\text{quint}}}{\partial d_\alpha^\dagger} + \frac{\partial W_{\text{quint}}^\dagger}{\partial d_\alpha^\dagger} \right) \\
& + M_s^2 e^{\kappa K} (K^{-1})^{d_\alpha^\dagger d_\beta} \frac{\partial K_{\text{quint}}}{\partial d_\beta} \frac{\partial K_{\text{quint}}}{\partial d_\alpha^\dagger} + M_s e^{\kappa K} (K^{-1})^{d_\alpha^\dagger d_\beta} \left[ \frac{\partial K_{\text{quint}}}{\partial d_\beta} \frac{\partial K_{\text{quint}}}{\partial d_\alpha^\dagger} \right. \\
& \times \left( \kappa W_{\text{quint}} + \kappa W_{\text{quint}}^\dagger \right) + \left( \frac{\partial K_{\text{quint}}}{\partial d_\beta} \frac{\partial W_{\text{quint}}^\dagger}{\partial d_\alpha^\dagger} + \frac{\partial K_{\text{quint}}}{\partial d_\alpha^\dagger} \frac{\partial W_{\text{quint}}}{\partial d_\beta} \right) \left. \right] \\
& + W_{\text{obs}} e^{\kappa K} (K^{-1})^{d_\alpha^\dagger d_\beta} \left[ \left( M_s + \kappa W_{\text{quint}}^\dagger \right) \kappa \frac{\partial K_{\text{quint}}}{\partial d_\beta} \frac{\partial K_{\text{quint}}}{\partial d_\alpha^\dagger} + \kappa \frac{\partial K_{\text{quint}}}{\partial d_\alpha^\dagger} \frac{\partial W_{\text{quint}}}{\partial d_\beta} \right] \\
& + W_{\text{obs}}^\dagger e^{\kappa K} (K^{-1})^{d_\alpha^\dagger d_\beta} \left[ \left( M_s + \kappa W_{\text{quint}} \right) \kappa \frac{\partial K_{\text{quint}}}{\partial d_\beta} \frac{\partial K_{\text{quint}}}{\partial d_\alpha^\dagger} + \kappa \frac{\partial K_{\text{quint}}}{\partial d_\beta} \frac{\partial W_{\text{quint}}^\dagger}{\partial d_\alpha^\dagger} \right].
\end{aligned} \tag{2.18}$$

This part of the potential contains a coupling between the observable superpotential and the quintessence field. It is of dimension 5 in the fields but is not suppressed by the Planck mass as  $Q$  is of the order of  $m_{\text{Pl}}$ .

Putting everything together, in particular taking into account the term  $-3WW^\dagger$ , we find  $V = V_{\text{DE}} + V_{\text{mSUGRA}}$  with

$$\begin{aligned}
V_{\text{DE}} = & e^{\sum_i |a_i|^2} V_{\text{quint}} + M_s^2 e^{\kappa K_{\text{quint}} + \sum_i |a_i|^2} \left[ (K^{-1})^{d_\alpha^\dagger d_\beta} \frac{\partial K_{\text{quint}}}{\partial d_\beta} \frac{\partial K_{\text{quint}}}{\partial d_\alpha^\dagger} - \frac{3}{\kappa} \right] \\
& + M_s e^{\kappa K_{\text{quint}} + \sum_i |a_i|^2} \left\{ \left[ (K^{-1})^{d_\alpha^\dagger d_\beta} \frac{\partial K_{\text{quint}}}{\partial d_\beta} \frac{\partial K_{\text{quint}}}{\partial d_\alpha^\dagger} - \frac{3}{\kappa} \right] \left( \kappa W_{\text{quint}} + \kappa W_{\text{quint}}^\dagger \right) \right. \\
& \left. + (K^{-1})^{d_\alpha^\dagger d_\beta} \left( \frac{\partial K_{\text{quint}}}{\partial d_\beta} \frac{\partial W_{\text{quint}}^\dagger}{\partial d_\alpha^\dagger} + \frac{\partial K_{\text{quint}}}{\partial d_\alpha^\dagger} \frac{\partial W_{\text{quint}}}{\partial d_\beta} \right) \right\} + \sum_i |F_{z_i}|^2,
\end{aligned} \tag{2.19}$$

where  $V_{\text{quint}}$  is the potential that one would have obtained by considering the dark sector alone, namely

$$\begin{aligned}
V_{\text{quint}}(Q) = & e^{\kappa K_{\text{quint}}} \left[ \left( K_{\text{quint}}^{-1} \right)^{d_\alpha^\dagger d_\beta} \left( \kappa W_{\text{quint}} \frac{\partial K_{\text{quint}}}{\partial d_\beta} + \frac{\partial W_{\text{quint}}}{\partial d_\beta} \right) \left( \kappa W_{\text{quint}}^\dagger \frac{\partial K_{\text{quint}}}{\partial d_\alpha^\dagger} \right. \right. \\
& \left. \left. + \frac{\partial W_{\text{quint}}^\dagger}{\partial d_\alpha^\dagger} \right) - 3\kappa W_{\text{quint}} W_{\text{quint}}^\dagger \right].
\end{aligned}$$

Therefore, as expected, the breaking of supersymmetry has changed the shape of the quintessence potential. As will be discussed in the following, these corrections are important since they can in principle modify the cosmological evolution of  $Q$ . On the other

hand, in the observable sector, one has

$$\begin{aligned}
V_{\text{mSUGRA}} = & e^{\kappa K_{\text{quint}} + \sum_i |a_i|^2} \left| \frac{\partial W_{\text{obs}}}{\partial \phi_a} \right|^2 \\
& + e^{\kappa K_{\text{quint}} + \sum_i |a_i|^2} \left[ M_S^2 + M_S \left( \kappa W_{\text{quint}} + \kappa W_{\text{quint}}^\dagger \right) + \kappa^2 W_{\text{quint}} W_{\text{quint}}^\dagger \right] \phi_a \phi_a^\dagger \\
& + e^{\kappa K_{\text{quint}} + \sum_i |a_i|^2} \left[ (M_S + \kappa W_{\text{quint}}) \frac{\partial W_{\text{obs}}^\dagger}{\partial \phi_a^\dagger} \phi_a^\dagger + (M_S + \kappa W_{\text{quint}}^\dagger) \frac{\partial W_{\text{obs}}}{\partial \phi_a} \phi_a \right] \\
& + e^{\kappa K_{\text{quint}} + \sum_i |a_i|^2} W_{\text{obs}} \left\{ \left( M_S + \kappa W_{\text{quint}}^\dagger \right) \left[ \kappa (K^{-1})^{d_\alpha^\dagger d_\beta} \frac{\partial K_{\text{quint}}}{\partial d_\beta} \frac{\partial K_{\text{quint}}}{\partial d_\alpha^\dagger} \right. \right. \\
& \left. \left. + \sum_i |a_i|^2 - 3 \right] + \kappa (K^{-1})^{d_\alpha^\dagger d_\beta} \frac{\partial K_{\text{quint}}}{\partial d_\alpha^\dagger} \frac{\partial W_{\text{quint}}}{\partial d_\beta} + M_S \sum_i a_i c_i \right\} \\
& + e^{\kappa K_{\text{quint}} + \sum_i |a_i|^2} W_{\text{obs}}^\dagger \left\{ \left( M_S + \kappa W_{\text{quint}} \right) \left[ \kappa (K^{-1})^{d_\alpha^\dagger d_\beta} \frac{\partial K_{\text{quint}}}{\partial d_\beta} \frac{\partial K_{\text{quint}}}{\partial d_\alpha^\dagger} \right. \right. \\
& \left. \left. + \sum_i |a_i|^2 - 3 \right] + \kappa (K^{-1})^{d_\alpha^\dagger d_\beta} \frac{\partial K_{\text{quint}}}{\partial d_\beta} \frac{\partial W_{\text{quint}}^\dagger}{\partial d_\alpha^\dagger} + M_S \sum_i a_i c_i \right\}. \quad (2.20)
\end{aligned}$$

This is the effective potential for the observable fields after supersymmetry breaking. It contains a part involving the quintessence field. It also contains the soft supersymmetry breaking terms of the observable sector. Using the explicit form of the superpotential of the mSUGRA model, see Eq. (2.3), the soft terms can easily be estimated. The scalar potential in the observable sector takes the form which precisely defines the soft terms

$$\begin{aligned}
V_{\text{mSUGRA}} = & \cdots + e^{\kappa K} V_{\text{susy}} + A_{abc} \left( \phi_a \phi_b \phi_c + \phi_a^\dagger \phi_b^\dagger \phi_c^\dagger \right) + B_{ab} \left( \phi_a \phi_b + \phi_a^\dagger \phi_b^\dagger \right) \\
& + m_{a\bar{b}}^2 \phi_a \phi_b^\dagger. \quad (2.21)
\end{aligned}$$

As a consequence, they read

$$\begin{aligned}
A_{abc} = & \lambda_{abc} e^{\kappa K_{\text{quint}} + \sum_i |a_i|^2} \left\{ \left( M_S + \kappa W_{\text{quint}}^\dagger \right) + \frac{1}{3} \left( M_S + \kappa W_{\text{quint}}^\dagger \right) \left[ \kappa (K^{-1})^{d_\alpha^\dagger d_\beta} \right. \right. \\
& \times \frac{\partial K_{\text{quint}}}{\partial d_\beta} \frac{\partial K_{\text{quint}}}{\partial d_\alpha^\dagger} + \sum_i |a_i|^2 - 3 \left. \right] + \frac{1}{3} \kappa (K^{-1})^{d_\alpha^\dagger d_\beta} \frac{\partial K_{\text{quint}}}{\partial d_\alpha^\dagger} \frac{\partial W_{\text{quint}}}{\partial d_\beta} \\
& \left. + \frac{1}{3} M_S \sum_i a_i c_i \right\}, \quad (2.22)
\end{aligned}$$

$$\begin{aligned}
B_{ab} = & \mu_{ab} e^{\kappa K_{\text{quint}} + \sum_i |a_i|^2} \left\{ \left( M_S + \kappa W_{\text{quint}}^\dagger \right) + \frac{1}{2} \left( M_S + \kappa W_{\text{quint}}^\dagger \right) \left[ \kappa (K^{-1})^{d_\alpha^\dagger d_\beta} \right. \right. \\
& \times \frac{\partial K_{\text{quint}}}{\partial d_\beta} \frac{\partial K_{\text{quint}}}{\partial d_\alpha^\dagger} + \sum_i |a_i|^2 - 3 \left. \right] + \frac{1}{2} \kappa (K^{-1})^{d_\alpha^\dagger d_\beta} \frac{\partial K_{\text{quint}}}{\partial d_\alpha^\dagger} \frac{\partial W_{\text{quint}}}{\partial d_\beta} \\
& \left. + \frac{1}{2} M_S \sum_i a_i c_i \right\}, \quad (2.23)
\end{aligned}$$

$$m_{a\bar{b}}^2 = e^{\kappa K_{\text{quint}} + \sum_i |a_i|^2} \left[ M_S^2 + M_S \left( \kappa W_{\text{quint}} + \kappa W_{\text{quint}}^\dagger \right) + \kappa^2 W_{\text{quint}} W_{\text{quint}}^\dagger \right] \delta_{a\bar{b}}. \quad (2.24)$$

This is the general form of the soft terms, calculated at the GUT scale. Notice that as we have chosen the hidden and observable fields to be canonically normalized, we find that the soft terms are universal as expected. Here the soft terms are explicitly  $Q$  dependent. This has severe consequences on the particle masses as we find in the following section. Indeed, the previous result allows to compute the  $Q$ -dependence of the fermions mass in the mSUGRA model. As we discuss below, it is necessary to use the renormalization group equations to be able to use the previous equations not only at GUT scale but also at the electroweak transition.

Finally, another soft term can be evaluated, the gaugino masses. As discussed before, we assume that the gauge kinetic functions are given by  $f_G = f_G(d_\alpha, z_i)$ . Then, we find that the gaugino masses at GUT scale can be expressed as

$$(m_{1/2})_G = \frac{1}{f_G + f_G^\dagger} \left( \sum_\alpha F^{d_\alpha} \frac{\partial f_G}{\partial d_\alpha} + \sum_i F^{z_i} \frac{\partial f_G}{\partial z_i} \right) \equiv e^{\kappa K_{\text{quint}}/2} (m_{1/2}^0)_G, \quad (2.25)$$

where the  $F_{z_i}$  term has already been evaluated in Eq. (2.12). Notice that it depends on an arbitrary function  $f_G(d_\alpha, z_i)$  for each gauge group  $G$ .

### 3. Electroweak Symmetry Breaking in Presence of Dark Energy

#### 3.1 The Higgs Potential in Presence of Quintessence

We now consider the application of the previous results to the electroweak symmetry breaking since this is the way fermions in the standard model are given a mass. We consider that the soft terms have been run down from the GUT scale to the weak scale. The next step consists in computing the potential in the Higgs sector which belongs to the observable sector. In the MSSM, there are two  $SU(2)_L$  Higgs doublets

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \quad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}, \quad (3.1)$$

that have opposite hypercharges, *i.e.*  $Y_u = 1$  and  $Y_d = -1$ . The only term which is relevant in the superpotential is  $W_{\text{obs}} = \mu H_u \cdot H_d + \dots = \mu(H_u^+ H_d^- - H_u^0 H_d^0) + \dots$  such that the superpotential remains gauge invariant. For indices  $a, b$  running in the Higgs sector, we have  $\mu_{ab} = \mu \epsilon_{ab}$ , where  $\epsilon_{11} = \epsilon_{22} = 0$  and  $\epsilon_{12} = -\epsilon_{21} = 1$ . This term gives contribution to the globally susy term  $V_{\text{susy}}$ , namely

$$V_{\text{susy}} = |\mu|^2 \left( |H_u^+|^2 + |H_u^0|^2 + |H_d^0|^2 + |H_d^-|^2 \right). \quad (3.2)$$

Then, we have the contribution coming from the soft susy-breaking terms. As can be seen in Eq. (2.23), the coefficient  $B_{ab}$  is proportional to  $\mu_{ab}$  and we choose to write it as

$B_{ab} \equiv \mu B e^{\kappa K_{\text{quint}}} \epsilon_{ab}$ , where  $B = B(Q)$  is now a function of the quintessence field, namely

$$B(Q) = e^{\sum_i |a_i|^2} \left\{ \left( M_s + \kappa W_{\text{quint}}^\dagger \right) + \frac{1}{2} \left( M_s + \kappa W_{\text{quint}}^\dagger \right) \left[ \kappa (K^{-1})^{d_\alpha^\dagger d_\beta} \right. \right. \\ \times \frac{\partial K_{\text{quint}}}{\partial d_\beta} \frac{\partial K_{\text{quint}}}{\partial d_\alpha^\dagger} + \sum_i |a_i|^2 - 3 \left. \right] + \frac{1}{2} \kappa (K^{-1})^{d_\alpha^\dagger d_\beta} \frac{\partial K_{\text{quint}}}{\partial d_\alpha^\dagger} \frac{\partial W_{\text{quint}}}{\partial d_\beta} \\ \left. + \frac{1}{2} M_s \sum_i a_i c_i \right\}. \quad (3.3)$$

Then the B-soft susy-breaking term in the scalar potential can be expressed as

$$V_{\text{B-soft}} = \mu B(Q) e^{\kappa K_{\text{quint}}} \left[ H_u^+ H_d^- - H_u^0 H_d^0 + (H_u^+)^\dagger (H_d^-)^\dagger - (H_u^0)^\dagger (H_d^0)^\dagger \right]. \quad (3.4)$$

Finally, the soft masses remain to be evaluated from Eq. (2.24). For this purpose, one writes  $m_{11}^2 = m_{H_u}^2 e^{\kappa K_{\text{quint}}}$ , and  $m_{22}^2 = m_{H_d}^2 e^{\kappa K_{\text{quint}}}$ , where according to Eq. (2.14) and (2.24),  $m_{H_u} = m_{H_d} = m_{3/2}^0$  at the GUT scale. This degeneracy is lifted by the renormalization group evolution as necessary to obtain the radiative breaking of the electroweak symmetry [21], as explained below. One obtains

$$V_{\text{m-soft}} = m_{H_u}^2 e^{\kappa K_{\text{quint}}} \left( |H_u^+|^2 + |H_u^0|^2 \right) + m_{H_d}^2 e^{\kappa K_{\text{quint}}} \left( |H_d^0|^2 + |H_d^-|^2 \right), \quad (3.5)$$

where according to the previous considerations, there is no reason to assume that  $m_{H_u} = m_{H_d}$ , nor that  $m_{H_u}^2 > 0$  at the electroweak scale. As is well-known the renormalization group equations drive the initial value of  $m_{H_u}^2 = \left( m_{3/2}^0 \right)^2$  towards negative values which is necessary in order to have the electroweak transition. This important property is not modified by the presence of the quintessence field. Let us notice that, in the above expressions, there are *a priori* contributions coming from the Kähler potential in the Higgs sector of the form  $e^{\kappa K_{\text{Higgs}}}$ . We ignore these contributions since the vevs of the Higgs is very small in comparison with  $m_{\text{Pl}}$ .

This is not yet the final expression of the potential because the D-term remain to be calculated. They are given by

$$V_{\text{D}}^{\text{Higgs}} = \sum_G \frac{g_G^2}{2} \sum_\alpha \left( G^a T_G^\alpha \phi_a \right) \left( G^b T_G^\alpha \phi_b \right), \quad (3.6)$$

where, in the summation, we have two gauge groups, namely  $\text{SU}(2)_L$  (coupling constant  $g$ ),  $\text{U}(1)_Y$  (coupling constant  $g'$ ) and  $T_\alpha$  are the generators of the gauge groups under consideration. We recall that  $G_a \equiv K_a + \partial_a W/W$ . Thanks to gauge invariance implying that  $\delta_G W = \partial_a W i T_G^\alpha \phi_a = 0$ , only the part involving the Kähler potential gives a non vanishing contribution. Then, a standard calculation leads to

$$V_{\text{D}}^{\text{Higgs}} = \frac{1}{8} (g^2 + g'^2) \left( |H_u^+|^2 + |H_u^0|^2 - |H_d^0|^2 - |H_d^-|^2 \right)^2 + \frac{1}{2} g^2 \left| H_u^+ H_d^{0\dagger} + H_u^0 H_d^{-\dagger} \right|^2. \quad (3.7)$$

Notice that the D-term potential does depend on the quintessence field due to the dependence of the GUT scale gauge coupling constants  $g$  and  $g'$  on the fields in the quintessence and hidden sector.

The total Higgs potential is the sum of the F-term potential and of the D-term. The explicit and complete expression reads

$$V^{\text{Higgs}} = e^{\kappa K_{\text{quint}}} \left\{ \left( |\mu|^2 e^{\sum_i |a_i|^2} + m_{H_u}^2 \right) \left( |H_u^+|^2 + |H_u^0|^2 \right) + \left( |\mu|^2 e^{\sum_i |a_i|^2} + m_{H_d}^2 \right) \right. \\ \times \left( |H_d^0|^2 + |H_d^-|^2 \right) + \mu B(Q) \left[ H_u^+ H_d^- - H_u^0 H_d^0 + (H_u^+)^{\dagger} (H_d^-)^{\dagger} - (H_u^0)^{\dagger} (H_d^0)^{\dagger} \right] \Big\} \\ + \frac{1}{8} (g^2 + g'^2) \left( |H_u^+|^2 + |H_u^0|^2 - |H_d^0|^2 - |H_d^-|^2 \right)^2 + \frac{1}{2} g^2 \left| H_u^+ H_d^{0\dagger} + H_u^0 H_d^{-\dagger} \right|^2 \quad (3.8)$$

As mentioned above, it contains an explicit  $Q$  dependence from the F-term potential but also from the D-term if the gauge functions  $f_G$  are not trivial.

### 3.2 Minimizing the Higgs potential

We now investigate the minimum of this potential. Using a  $SU(2)_L$  transformation, one can always require  $H_u^+ = 0$ . Then, writing  $\partial V^{\text{Higgs}} / \partial H_u^+ = 0$ , implies  $H_d^- = 0$  as usual since the presence of the quintessence dependent coefficient does not modify the structure of the potential in terms of the Higgs fields. Moreover,  $H_u^0$  and  $H_d^0$  can be taken real since they have opposite hypercharges. Finally, the potential reads

$$V^{\text{Higgs}} = e^{\kappa K_{\text{quint}}} \left[ \left( |\mu|^2 e^{\sum_i |a_i|^2} + m_{H_u}^2 \right) |H_u^0|^2 + \left( |\mu|^2 e^{\sum_i |a_i|^2} + m_{H_d}^2 \right) |H_d^0|^2 \right. \\ \left. - 2\mu B(Q) |H_u^0| |H_d^0| \right] + \frac{1}{8} (g^2 + g'^2) \left( |H_u^0|^2 - |H_d^0|^2 \right)^2. \quad (3.9)$$

Given this potential, there are conditions for the existence of a stable minimum for non vanishing values of the Higgs vevs. First, in order to avoid that the potential be unbounded from below along the direction  $H_u^0 = H_d^0$ , we have to require that

$$2|\mu|^2 e^{\sum_i |a_i|^2} + m_{H_u}^2 + m_{H_d}^2 - 2\mu B(Q) > 0. \quad (3.10)$$

In addition, one must require that the origin be a saddle point which leads to

$$\left( |\mu|^2 e^{\sum_i |a_i|^2} + m_{H_u}^2 \right) \left( |\mu|^2 e^{\sum_i |a_i|^2} + m_{H_d}^2 \right) < \mu^2 B^2(Q). \quad (3.11)$$

If  $|\mu|^2 e^{\sum_i |a_i|^2} + m_{H_u}^2$  is negative, then the last condition is automatically satisfied. As mentioned before, this is natural since the renormalization group equations pushes towards negative values of  $m_{H_u}^2 < m_{H_d}^2$ . We now discuss this point in more detail. Let us define the Higgs vevs as  $\langle H_u^0 \rangle \equiv v_u$  and  $\langle H_d^0 \rangle \equiv v_d$ . The link between these vevs and the mass of the gauge bosons,  $W^{\pm}$ ,  $Z^0$  is not modified by the presence of dark energy. Technically, this is due to the fact that the kinetic terms of the Higgs are standard because the corresponding Kähler potentials are flat. The standard calculation leads to

$$m_{W^{\pm}}^2 = \frac{g^2}{2} (v_u^2 + v_d^2) \equiv \frac{g^2}{2} v^2, \quad m_{Z^0}^2 = \frac{1}{2} (g^2 + g'^2) (v_u^2 + v_d^2). \quad (3.12)$$

where the energy scale  $v = v(Q)$  is equal to  $\sim 174\text{GeV}$  today, *i.e.* at vanishing redshift. The gauge boson masses pick up a  $Q$ -dependence from the gauge coupling constants and from  $v$ . For the same reason as above, the cancellation of the photon mass leads to  $g' \cos \theta = g \sin \theta$ , where  $\theta$  is a  $Q$ -dependent Weinberg angle via

$$\tan \theta(Q) = \frac{g'(Q)}{g(Q)}, \quad (3.13)$$

if the gauge functions  $f_G$  are non trivial. Despite the  $Q$  dependence of  $\theta$ , we are guaranteed that the photon remains massless. Finally, we define  $\tan \beta \equiv v_u/v_d$ , or  $v_u = v \sin \beta$  and  $v_d = v \cos \beta$ . Then, the two loop expression for the renormalized Higgs masses gives [30]

$$m_{H_u}^2(Q) = m_{H_d}^2(Q) - 0.36 \left(1 + \frac{1}{\tan^2 \beta}\right) \left\{ \left[m_{3/2}^0(Q)\right]^2 \left(1 - \frac{1}{2\pi}\right) + 8 \left[m_{1/2}^0(Q)\right]^2 + \left(0.28 - \frac{0.72}{\tan^2 \beta}\right) \left[A(Q) + 2m_{1/2}^0\right]^2 \right\}, \quad (3.14)$$

$$m_{H_d}^2(Q) = \left[m_{3/2}^0(Q)\right]^2 \left(1 - \frac{0.15}{4\pi}\right) + \frac{1}{2} \left[m_{1/2}^0(Q)\right]^2, \quad (3.15)$$

where  $m_{1/2}^0$  is the gaugino mass at GUT scale, see Eq. (2.25). Let us notice that the gaugino mass is also a  $Q$ -dependent quantity. In the previous expression, we have identified  $A_{abc} = e^{\kappa K_{\text{quint}}} A \lambda_{abc}$ , see Eq. (2.22), which leads to

$$A(Q) = e^{\sum_i |a_i|^2} \left\{ \left(M_s + \kappa W_{\text{quint}}^\dagger\right) + \frac{1}{3} \left(M_s + \kappa W_{\text{quint}}^\dagger\right) \left[ \kappa (K^{-1})^{d_\alpha^\dagger d_\beta} \times \frac{\partial K_{\text{quint}}}{\partial d_\beta} \frac{\partial K_{\text{quint}}}{\partial d_\alpha^\dagger} + \sum_i |a_i|^2 - 3 \right] + \frac{1}{3} \kappa (K^{-1})^{d_\alpha^\dagger d_\beta} \frac{\partial K_{\text{quint}}}{\partial d_\alpha^\dagger} \frac{\partial W_{\text{quint}}}{\partial d_\beta} + \frac{1}{3} M_s \sum_i a_i c_i \right\}. \quad (3.16)$$

The above expressions of the  $Q$ -dependent Higgs mass are obtained by assuming than one starts at GUT scale with equal masses ( $= m_{3/2}^0$ ), as indicated by the previous calculation of the soft terms, see Eq. (2.24). They are then evaluated at the electroweak scale. After the electroweak breaking, the masses are stabilized to the previous values. The choice of the parameters should be made such that, in absence of any dark energy, the Higgs potential has a minimum, *i.e.* such that the conditions given by Eqs. (3.10) and (3.11) are satisfied. Then, for non-vanishing values of the quintessence field, it could very well turn out that these conditions no longer hold.

The next step is to perform the minimization of the Higgs potential given by Eq. (3.9). Straightforward calculations give

$$e^{\kappa K_{\text{quint}}} \left( |\mu|^2 e^{\sum_i |a_i|^2} + m_{H_u}^2 \right) = \mu B(Q) \frac{e^{\kappa K_{\text{quint}}}}{\tan \beta} + \frac{m_{Z^0}^2}{2} \cos(2\beta), \quad (3.17)$$

$$e^{\kappa K_{\text{quint}}} \left( |\mu|^2 e^{\sum_i |a_i|^2} + m_{H_d}^2 \right) = \mu B(Q) e^{\kappa K_{\text{quint}}} \tan \beta - \frac{m_{Z^0}^2}{2} \cos(2\beta). \quad (3.18)$$

Adding the two equations for the minimum, we obtain a quadratic equation determining  $\tan \beta$ . The solution can easily be found and reads

$$\tan \beta(Q) = \frac{2|\mu|^2 e^{\sum_i |a_i|^2} + m_{H_u}^2(Q) + m_{H_d}^2(Q)}{2\mu B(Q)} \times \left( 1 \pm \sqrt{1 - 4\mu^2 B^2(Q) \left[ 2|\mu|^2 e^{\sum_i |a_i|^2} + m_{H_u}^2(Q) + m_{H_d}^2(Q) \right]^{-2}} \right). \quad (3.19)$$

*A priori*, this equation is a transcendental equation determining  $\tan \beta$  as  $\tan \beta$  also appears in the right-hand-side of the above formula, more precisely in the Higgs masses, see Eqs. (3.14) and (3.15). However, if one performs an expansion in  $1/\tan \beta$ , that is to say if  $\beta$  is not too far from  $\pi/2$  then the factor  $\tan \beta$  in Eqs. (3.14) and (3.15) can be forgotten and Eq. (3.19) becomes an algebraic expression giving the tangent of  $\beta$ . Another remark is that we have in fact two solutions for  $\tan \beta$ , one corresponding to the plus sign, the other corresponding to the minus sign.

From the equations (3.17) and (3.18), one can also deduce how the scale  $v$  depends on the quintessence field. This leads to

$$v(Q) = \frac{2e^{\kappa K_{\text{quint}}/2}}{\sqrt{g^2 + g'^2}} \sqrt{|\mu|^2 e^{\sum_i |a_i|^2} + m_{H_u}^2} + \mathcal{O}\left(\frac{1}{\tan \beta}\right). \quad (3.20)$$

Notice the absolute value in the square root that insures the positivity of the argument which is necessary since, as already mentioned, the renormalization group equations pushes  $m_{H_u}^2$  towards negative values. Through Eqs. (3.12), the above equation can be used to calculate the gauge bosons mass explicitly.

Then, finally, one has for the vevs of the two Higgs

$$v_u(Q) = \frac{v(Q) \tan \beta(Q)}{\sqrt{1 + \tan^2 \beta(Q)}} = v(Q) + \mathcal{O}\left(\frac{1}{\tan^2 \beta}\right), \quad (3.21)$$

$$v_d(Q) = \frac{v(Q)}{\sqrt{1 + \tan^2 \beta(Q)}} = \frac{v(Q)}{\tan \beta} + \mathcal{O}\left(\frac{1}{\tan^2 \beta}\right), \quad (3.22)$$

at leading order in  $1/\tan^2 \beta$  (but if we insert the expression of  $v$ , then  $v_u$  and  $v_d$  are only determined at first order in  $1/\tan \beta$ ). It is necessary to perform the expansion in  $1/\tan^2 \beta$  in the above equations as it would be inconsistent to keep higher order corrections in those formulae while neglecting them in Eq. (3.19). The two expressions (3.21) and (3.22) constitute the main result of this article. It implies that fermion masses become  $Q$  dependent. Moreover there are two kinds of masses, depending on whether the fermions couple to  $H_u$  or  $H_d$

$$m_{u,a}^F(Q) = \lambda_{u,a}^F e^{\kappa K_{\text{quint}}/2 + \sum_i |a_i|^2/2} v_u(Q), \quad m_{d,a}^F(Q) = \lambda_{d,a}^F e^{\kappa K_{\text{quint}}/2 + \sum_i |a_i|^2/2} v_d(Q), \quad (3.23)$$

where  $\lambda_{u,a}^F$  and  $\lambda_{d,a}^F$  are the Yukawa coupling of the particle  $\phi_a$  coupling either to  $H_u$  or  $H_d$ . Clearly, in order to go further and compute explicitly the  $Q$ -dependence of the fermions mass, it is necessary to specify the hidden and quintessence sectors.

Before discussing the above results, let us make the following remark. Denoting by  $v_{u,d}(0)$  the vev in the absence of the quintessence field we can introduce the coupling constants

$$A_{u,d}(Q) \equiv e^{\kappa K_{\text{quint}}/2 + \sum_i |a_i|^2/2} \frac{v_{u,d}(Q)}{v_{u,d}(0)}. \quad (3.24)$$

Using this coupling constant, the theory below the electroweak scale becomes a scalar tensor theory

$$S = -\frac{1}{2\kappa} \int d^4x \sqrt{-g} \left[ R + \frac{1}{2} g^{\mu\nu} \partial_\mu Q \partial_\nu Q + V_{\text{DE}}(Q) \right] + S_{\text{mat}} [\phi_{u,a}, A_u^2(Q) g_{\mu\nu}] \\ + S_{\text{mat}} [\phi_{d,a}, A_d^2(Q) g_{\mu\nu}]. \quad (3.25)$$

The quintessence field has not decoupled and corresponds to the scalar sector of the theory. The action is written in the Einstein frame with metric  $g_{\mu\nu}$  while the matter action is split in two parts corresponding to the matter fields  $\phi_{u,v}^a$  having coupled either to  $H_u$  or  $H_d$ . Notice that the presence of the coupling to the metrics  $A_{u,v}^2(Q) g_{\mu\nu}$  implies that a fifth force due to the scalar exchange of  $Q$  is generated. Moreover, matter of type  $\phi_u$  couples differently to the metric than  $\phi_d$ . This is a violation of the weak equivalence principle.

In addition, the presence of two types of matter with two couplings  $A_{u,d}$  prevents one from defining a Jordan frame where the gravitational constant depends on the quintessence field and matter couples to the metric uniquely. This is very different from scalar-tensor theories and string theory with a universal dilaton. Even when string loops are taken into account, it has been claimed in Ref. [31] that a least coupling principle is at play whereby the gravitational couplings to matter are proportional to a single function of the dilaton. A cosmological attractor implies that the dilaton runs towards the minimum of this function where gravitational constraints are evaded. Here we have two intrinsically different couplings. This is a feature of the MSSM coupled to quintessence and comes from the necessity of having two Higgs fields to cancel the gauge anomalies. Hence violations of the weak equivalence principle are intrinsic when coupling quintessence to the MSSM and/or mSUGRA.

## 4. Discussion

The results obtained in the previous sections have three main consequences: firstly, the shape of the quintessence potential is modified as compared to what is obtained if one just considers a separate dark energy sector, see Eq. (2.19). Secondly, the mass of the observable sector particles becomes  $Q$ -dependent quantities. This is of course valid for the spin-1/2 fermions and also for the gravitino and the gauge bosons. Thirdly, the gauge couplings, among which is the fine structure constant, can also become a function of the quintessence field depending on the choice of the function  $f_G$ . At this point, the following remark is in order. The fact that the mass of the particles becomes  $Q$ -dependent has already been noticed and some of the corresponding consequences have been worked out, see for instance Ref. [32]. However, what is usually done is to keep the quintessence potential unchanged and to postulate some dependence for the fermions masses (while, here, we calculate this



dependence from first principle). We see from the previous considerations that this is not really consistent. In the context of supergravity (and recall that the use of this framework is mandatory since the vacuum expectation value of the quintessence is a priori of the order of the Planck mass), particles acquire a mass through the Higgs mechanism which, in turns, is possible only if supersymmetry is broken, see the calculation of the soft terms in the last section. But, as we have also seen, if supersymmetry is broken then the quintessence potential is modified. This modification can of course change the cosmological evolution of the quintessence field and the determination of the free parameters characterizing the shape of the potential (as, for instance, the typical mass scale). Therefore, when we take into account the fact that the fermions masses are  $Q$ -dependent, it is also necessary to pay attention to the corresponding modifications in the quintessence potential.

#### 4.1 Modifications of the Quintessence Potential

Let us now discuss in more detail the three main consequences described before. The new shape of the quintessence potential is only known when the functions  $a_i(Q)$  and  $c_i(Q)$  are specified. From Eq. (2.19), we see that even if  $a_i(Q)$  and  $c_i(Q)$  vanish or are just constant, that is to say if the vacuum expectation value of the susy breaking fields  $z_i$  are very small in comparison with the Planck mass or if the  $z_i$  fields are stabilized, the potential is still not the original one. We can then envisage two different situations. They are differentiated by the equation of state  $w_Q$  of the quintessence sector when  $Q$  takes its present value in the history of the universe. First of all, let us assume that the equation of state is  $w_Q \neq -1$ . This case can only be achieved when the potential is of runaway type with an effective mass for the quintessence field  $m_Q \sim H_0$  the present Hubble rate. In this case, this means that, despite the appearance of new terms in  $V_{DE}$ , the original shape is stable and still presents a runaway behavior. Another possibility is that the new potential has a minimum. In this situation, the field will have a completely different mass in general and, from Eq. (2.19), a typical expectation is  $m_Q \sim M_s \sim m_{3/2}^0$  although it should be possible to avoid this conclusion in some particular cases. If the field is stabilized at this minimum early in the history of the Universe, then the effective equation of state is  $w_Q = -1$  and the model is equivalent to a cosmological constant. In this case, the quintessence hypothesis clearly becomes useless.

#### 4.2 Fifth Force

Let us now discuss the consequences of having  $Q$ -dependent masses. As already mentioned before, this can lead to strong constraints coming from gravitational experiments. Two situations can be envisaged depending on the mass of the quintessence field. If the mass of the quintessence field is larger than  $10^{-3}\text{eV}$ , the gravitational constraints are always satisfied as the range of the force mediated by  $Q$  is less than one millimeter. We see that this typically occurs in the case where the quintessence potential has developed a minimum, see above, first because, in such a situation, the mass is expected to be  $\sim m_{3/2}^0 \gg 10^{-3}\text{eV}$  and second because, if the field is stabilized at the minimum, then there is no variation at all. Therefore, the case where there is a minimum appears to be free from gravitational problems. On the other hand, we have seen before that this is not a satisfactory model of

dark energy. If the mass is less than  $10^{-3}\text{eV}$ , the range of the quintessence field is large and generically there will be violations of the weak equivalence principle (as discussed in the next subsection) and a large fifth force. In order not to be in contradiction with fifth force experiments such as the recent Cassini spacecraft experiment, one must require that the Eddington (post-Newtonian) parameter  $|\gamma - 1| \leq 5 \times 10^{-5}$ , see Ref. [19]. If one defines the parameter  $\alpha_{\text{u,d}}$  by

$$\begin{aligned}\alpha_{\text{u,d}}(Q) &\equiv \left| \frac{1}{\kappa^{1/2}} \frac{d \ln m_{\text{u,d}}^{\text{F}}(Q)}{dQ} \right| \\ &= \left| \frac{1}{\kappa^{1/2}} \frac{d \ln \lambda_{\text{u,d}}^{\text{F}}(Q)}{dQ} + \frac{1}{\kappa^{1/2}} \frac{d \ln \left[ e^{\kappa K_{\text{quint}}/2 + \sum_i |a_i|^2/2} v_{\text{u,d}}(Q) \right]}{dQ} \right|, \quad (4.1)\end{aligned}$$

then the difficulties are avoided by imposing that  $\alpha_{\text{u,d}}^2 \leq 10^{-5}$  since one has  $\gamma = 1 + \alpha_{\text{u,d}}^2$ . This result is valid for a gedanken experiment involving the gravitational effects on elementary particles. For macroscopic bodies, the effects are more subtle and will be discussed later. In this case, in principle, one should precisely compute the  $Q$  dependence of  $\alpha_{\text{u,d}}$  to evaluate how serious the gravitational problems are. Obviously, this cannot be done in detail unless the functions  $a_i(Q)$  and  $c_i(Q)$  and the quintessence sector are known exactly. However, one can give some generic arguments. Complying with the Cassini bound probably requires either a fine-tuning of the functions  $a_i(Q)$  and  $c_i(Q)$  such that the global  $Q$ -dependence of the masses is almost canceled out. One could also use a non-minimal setting whereby the Yukawa couplings  $\lambda_{\text{u,d}}^{\text{F}}$  become  $Q$ -dependent. In this last case, it could turn out that, in Eq. (4.1), this dependence cancels exactly the dependence of the Higgs vevs  $v_{\text{u,d}}(Q)$ . This is a functional fine-tuning which seems difficult to justify. If the Yukawa couplings are not  $Q$ -dependent, then it is interesting to calculate the expressions of the gravitational couplings  $\alpha_{\text{u,d}}$  using the results obtained in the last section. One obtains

$$\alpha_{\text{u}} = \frac{\kappa^{1/2}}{2} \partial_Q K_{\text{quint}} + \frac{\kappa^{-1/2}}{2} \sum_i \partial_Q |a_i|^2 + \frac{\kappa^{-1/2}}{\tan \beta (1 + \tan^2 \beta)} \frac{d \tan \beta}{dQ} + \frac{\kappa^{-1/2}}{v} \frac{dv}{dQ}, \quad (4.2)$$

$$\alpha_{\text{d}} = \frac{\kappa^{1/2}}{2} \partial_Q K_{\text{quint}} + \frac{\kappa^{-1/2}}{2} \sum_i \partial_Q |a_i|^2 - \frac{\kappa^{-1/2} \tan \beta}{1 + \tan^2 \beta} \frac{d \tan \beta}{dQ} + \frac{\kappa^{-1/2}}{v} \frac{dv}{dQ}, \quad (4.3)$$

where the derivative of the function  $\tan \beta(Q)$  can be expressed as

$$\begin{aligned}\frac{d \tan \beta}{dQ} &= \left( \frac{dm_{\text{H}_u}^2}{dQ} + \frac{dm_{\text{H}_d}^2}{dQ} \right) \left( 2|\mu|^2 e^{\sum_i |a_i|^2} + m_{\text{H}_u}^2 + m_{\text{H}_d}^2 \right)^{-1} \tan \beta - \frac{1}{B(Q)} \frac{dB(Q)}{dQ} \tan \beta \\ &\quad \pm 2\mu \left( 2|\mu|^2 e^{\sum_i |a_i|^2} + m_{\text{H}_u}^2 + m_{\text{H}_d}^2 \right)^{-1} \left[ 1 - 4\mu^2 B^2(Q) \left( 2|\mu|^2 e^{\sum_i |a_i|^2} + m_{\text{H}_u}^2 \right. \right. \\ &\quad \left. \left. + m_{\text{H}_d}^2 \right)^{-2} \right]^{-1/2} \times \left[ -\frac{dB(Q)}{dQ} + B(Q) \left( \frac{dm_{\text{H}_u}^2}{dQ} + \frac{dm_{\text{H}_d}^2}{dQ} \right) \left( 2|\mu|^2 e^{\sum_i |a_i|^2} + m_{\text{H}_u}^2 \right. \right. \\ &\quad \left. \left. + m_{\text{H}_d}^2 \right)^{-1} \right], \quad (4.4)\end{aligned}$$

assuming in this last equation, for simplicity, that the susy breaking fields are stabilized at a vev small in comparison with the Planck mass, that is to say  $a_i = 0$  but not necessarily  $c_i = 0$ . The derivatives of the soft terms  $A$  and  $B$  are known once the quintessence sector is specified and one has from the renormalization group equations (3.14) and (3.15)

$$\begin{aligned} \frac{dm_{\text{H}_u}^2}{dQ} &\simeq \frac{dm_{\text{H}_d}^2}{dQ} - 0.72 \left(1 - \frac{1}{2\pi}\right) m_{3/2}^0 \frac{dm_{3/2}^0}{dQ} - 0.72 \times 8m_{1/2}^0 \frac{dm_{1/2}^0}{dQ} \\ &\quad - 0.72 \times 0.28 \left[ \frac{dA(Q)}{dQ} + 2 \frac{dm_{1/2}^0}{dQ} \right] \times \left[ A(Q) + 2m_{1/2}^0 \right], \end{aligned} \quad (4.5)$$

$$\frac{dm_{\text{H}_d}^2}{dQ} \simeq 2 \left(1 - \frac{0.15}{4\pi}\right) m_{3/2}^0 \frac{dm_{3/2}^0}{dQ} + m_{1/2}^0 \frac{dm_{1/2}^0}{dQ}, \quad (4.6)$$

the symbol “approximate” in the two last equations meaning that we have used the fact that the terms in  $1/\tan^2 \beta$  have been neglected in the expression of the Higgs masses. In this case, one should keep the leading order only in Eq. (4.4) which amounts to

$$\begin{aligned} \frac{d \tan \beta}{dQ} &\simeq \left( \frac{dm_{\text{H}_u}^2}{dQ} + \frac{dm_{\text{H}_d}^2}{dQ} \right) \left( 2|\mu|^2 e^{\sum_i |a_i|^2} + m_{\text{H}_u}^2 + m_{\text{H}_d}^2 \right)^{-1} \tan \beta \\ &\quad - \frac{1}{B(Q)} \frac{dB(Q)}{dQ} \tan \beta. \end{aligned} \quad (4.7)$$

One should also evaluate the derivative of the scale  $v(Q)$ . Straightforward manipulations lead to

$$\begin{aligned} \frac{1}{v} \frac{dv}{dQ} &\simeq \frac{\kappa}{2} \partial_Q K_{\text{quint}} - \frac{1}{g^2 + g'^2} \left( g \frac{dg}{dQ} + g' \frac{dg'}{dQ} \right) + \frac{1}{2} \left( |\mu|^2 e^{\sum_i |a_i|^2} + m_{\text{H}_u}^2 \right)^{-1} \\ &\quad \times \left( |\mu|^2 e^{\sum_i |a_i|^2} \sum_i \partial_Q |a_i|^2 + \frac{dm_{\text{H}_u}^2}{dQ} \right) + \mathcal{O} \left( \frac{1}{\tan \beta} \right). \end{aligned} \quad (4.8)$$

In fact, in order to be fully consistent at leading order, Eqs. (4.2) and (4.3) should be written as

$$\begin{aligned} \alpha_u &= \frac{\kappa^{1/2}}{2} \partial_Q K_{\text{quint}} + \frac{\kappa^{-1/2}}{2} \sum_i \partial_Q |a_i|^2 + \frac{\kappa^{-1/2}}{v} \frac{dv}{dQ} + \mathcal{O} \left( \frac{1}{\tan^2 \beta} \right), \\ \alpha_d &= \frac{\kappa^{1/2}}{2} \partial_Q K_{\text{quint}} + \frac{\kappa^{-1/2}}{2} \sum_i \partial_Q |a_i|^2 - \kappa^{-1/2} \left( \frac{dm_{\text{H}_u}^2}{dQ} + \frac{dm_{\text{H}_d}^2}{dQ} \right) \\ &\quad \times \left( 2|\mu|^2 e^{\sum_i |a_i|^2} + m_{\text{H}_u}^2 + m_{\text{H}_d}^2 \right)^{-1} + \kappa^{-1/2} \frac{d \ln B(Q)}{dQ} + \frac{\kappa^{-1/2}}{v} \frac{dv}{dQ} \\ &\quad + \mathcal{O} \left( \frac{1}{\tan^2 \beta} \right). \end{aligned} \quad (4.10)$$

As already noticed above, if we use the explicit expression of the derivative of  $v(Q)$  evaluated before, then the above formulas giving  $\alpha_u$  and  $\alpha_d$  are accurate to the order  $1/\tan \beta$  only. Let us also notice that, in principle, for a given model, the transcendental equation (3.19) can be solved numerically and all the subsequent quantities evaluated by this method if necessary.

### 4.3 Violation of the Weak Equivalence Principle

As already mentioned, we also have violations of the weak equivalence principle. This is due to the fact that, in the MSSM/mSUGRA, the fermions couple differently to the two Higgs doublets  $H_u$  and  $H_d$ . Technically, this can be seen as the consequence of having two different scalar tensor theories with two different conformal factor  $A_u(Q)$  and  $A_d(Q)$ . Violations of the weak equivalence principle are quantified in terms of the  $\eta_{AB}$  parameter defined by [24, 31, 33]

$$\eta_{AB} \equiv \left( \frac{\Delta a}{a} \right)_{AB} = 2 \frac{a_A - a_B}{a_A + a_B}, \quad (4.11)$$

for two test bodies A and B in the gravitational background of a third one E. Current limits [34] indicate that  $\eta_{AB} = (+0.1 \pm 2.7 \pm 1.7) \times 10^{-13}$ .

The parameter  $\eta_{AB}$  can be calculated in the following way. The interaction between two bodies can be written as  $-G(1 + \alpha_A \alpha_B) m_A m_B / r_{AB}$ , where the dimensionless coefficient  $\alpha_A$  is defined according to Eq. (4.1), namely  $\alpha_A = \kappa^{-1/2} d \ln m_A / dQ$ . Evaluating the parameter  $\eta_{AB}$  for  $r_{AE} = r_{BE}$  leads to

$$\eta_{AB} \sim \frac{1}{2} \alpha_E (\alpha_A - \alpha_B). \quad (4.12)$$

For our gedanken experiment involving two elementary particles of types u and d, this implies a strong violation of the equivalence principle as  $\eta_{ud}$  can be large.

For macroscopic bodies as the ones involved in the Cassini experiment or the solar system tests, the gravitational effects involve the inner structure of the tested bodies. In Ref. [31], it was shown that the mass of an atom can be written as

$$m_{\text{Atom}}(Q) \sim \Lambda_{\text{QCD}} M + \sigma'(N + Z) + \delta'(N - Z) + a_3 \alpha_{\text{QED}} E \Lambda_{\text{QCD}}, \quad (4.13)$$

where  $N$  is the number of neutrons and  $Z$  the number of protons. A priori,  $\Lambda_{\text{QCD}}$  and  $\alpha_{\text{QED}}$  are  $Q$ -dependent quantities when the functions  $f_G$  are non trivial. Their evolution is given by the running of the gauge group couplings, namely

$$\frac{1}{\alpha_i(m)} = 4\pi f_i - \frac{b_i}{2\pi} \ln \left( \frac{m_{\text{GUT}}}{m} \right), \quad (4.14)$$

where  $i = 1$  corresponds to the group  $G = \text{U}(1)_Y$ ,  $i = 2$  to  $G = \text{SU}(2)$  and  $i = 3$  to  $G = \text{SU}(3)$ . In the above expression, the gauge couplings are evaluated at a scale  $m$  starting from  $m_{\text{GUT}}$ , a high energy scale at which our effective theory breaks down, chosen here to be the GUT scale. In the MSSM, the coefficients  $b_i$  are given by (for  $i = 1, \dots, 3$ )  $b_i = (-33/5, -1, 3)$ . We study the consequences of the  $Q$ -dependence of the gauge couplings in the next subsection. Here, we just need to recall that  $\Lambda_{\text{QCD}}$  is the QCD scale  $m$  at which  $\alpha_{\text{QCD}}$  blows up. Using Eq. (4.14), it can be expressed as

$$\Lambda_{\text{QCD}} = m_{\text{GUT}} e^{-8\pi^2 f_3 / b_3}. \quad (4.15)$$

In the previous expression, threshold corrections have been neglected. They have been examined in more detail in Refs. [35–37]. On the other hand,  $\alpha_{\text{QED}}$  is the fine structure constant and is linked to  $\alpha_1$  and  $\alpha_2$  by

$$\alpha_{\text{QED}}(Q) = \frac{\alpha_2^2}{\alpha_1 + \alpha_2}, \quad (4.16)$$

where the evolution of  $\alpha_1$  and  $\alpha_2$  can be obtained from Eq. (4.14). Let us continue the description of the quantities appearing in Eq. (4.13).  $M$  can be written as  $M = (N + Z) + E_{\text{QCD}}/\Lambda_{\text{QCD}}$ , where  $E_{\text{QCD}}$  is the strong interaction contribution to the binding energy of the nucleus. The quantity  $a_3\alpha_{\text{QED}}\Lambda_{\text{QCD}}E = E_{\text{QED}}$ ,  $E_{\text{QED}}$  being the Coulomb interaction of the nucleus.  $E$  is given by  $E = Z(Z - 1)/(N + Z)^{1/3}$ . Finally, the coefficients  $\sigma'$  and  $\delta'$  are given by

$$\sigma' = \frac{1}{2}(m_u + m_d)(b_u + b_d) + \frac{\alpha_{\text{QED}}}{2}(C_n + C_p) + \frac{1}{2}m_e, \quad (4.17)$$

$$\delta' = -\frac{1}{2}(m_u - m_d)(b_u - b_d) + \frac{\alpha_{\text{QED}}}{2}(C_n - C_p) - \frac{1}{2}m_e, \quad (4.18)$$

where  $b_u, b_d, C_n/\Lambda_{\text{QCD}}$  and  $C_p/\Lambda_{\text{QCD}}$  are dimensionless coefficients. The masses  $m_{u,d}$  have already been defined before and  $m_e$  is the mass of the electron. From the above formulas, one sees that the  $Q$  dependence arises from the  $Q$  dependences of  $m_{u,d}$ ,  $m_e$  and the three functions  $f_i$ .

It is then straightforward to calculate the coefficient  $\alpha$  for the atom A. It reads

$$\begin{aligned} \kappa^{1/2}\alpha_A \sim & -\frac{8\pi^2}{b_3}\frac{\partial f_3}{\partial Q} + \frac{N_A + Z_A}{M_A}\frac{\partial}{\partial Q}\left(\frac{\sigma'}{\Lambda_{\text{QCD}}}\right) + \frac{N_A - Z_A}{M_A}\frac{\partial}{\partial Q}\left(\frac{\delta'}{\Lambda_{\text{QCD}}}\right) \\ & + a_3\frac{E_A}{M_A}\frac{\partial\alpha_{\text{QED}}}{\partial Q}, \end{aligned} \quad (4.19)$$

where

$$\begin{aligned} \frac{\partial}{\partial Q}\left(\frac{\sigma'}{\Lambda_{\text{QCD}}}\right) = & \frac{1}{2}\frac{\kappa^{1/2}}{\Lambda_{\text{QCD}}}(b_u + b_d)(\alpha_u m_u + \alpha_d m_d) + \frac{1}{2}\frac{\kappa^{1/2}}{\Lambda_{\text{QCD}}}\alpha_d m_e + \frac{8\pi^2}{b_3}\frac{\sigma'}{\Lambda_{\text{QCD}}}\frac{\partial f_3}{\partial Q} \\ & + \frac{C_n + C_p}{2\Lambda_{\text{QCD}}}\frac{\partial\alpha_{\text{QED}}}{\partial Q}, \end{aligned} \quad (4.20)$$

$$\begin{aligned} \frac{\partial}{\partial Q}\left(\frac{\delta'}{\Lambda_{\text{QCD}}}\right) = & -\frac{1}{2}\frac{\kappa^{1/2}}{\Lambda_{\text{QCD}}}(b_u - b_d)(\alpha_u m_u - \alpha_d m_d) - \frac{1}{2}\frac{\kappa^{1/2}}{\Lambda_{\text{QCD}}}\alpha_d m_e + \frac{8\pi^2}{b_3}\frac{\sigma'}{\Lambda_{\text{QCD}}}\frac{\partial f_3}{\partial Q} \\ & + \frac{C_n - C_p}{2\Lambda_{\text{QCD}}}\frac{\partial\alpha_{\text{QED}}}{\partial Q}, \end{aligned} \quad (4.21)$$

and

$$\frac{\partial\alpha_{\text{QED}}}{\partial Q} = -4\pi\frac{(2\alpha_1 + \alpha_2)\alpha_2^3}{(\alpha_1 + \alpha_2)^2}\frac{\partial f_2}{\partial Q} + 4\pi\frac{\alpha_1^2\alpha_2^2}{(\alpha_1 + \alpha_2)^2}\frac{\partial f_1}{\partial Q}. \quad (4.22)$$

Let us notice that the coefficient  $\alpha_d$  appears in front of the mass of the electron  $m_e$  because, in the MSSM, the electron behaves as a “d” particle. For numerical estimates of the previous expressions, one can use  $\Lambda_{\text{QCD}} \sim 180\text{MeV}$ ,  $b_u + b_d \sim 6$ ,  $b_u - b_d \sim 0.5$ ,  $C_p\alpha_{\text{QED}} \sim 0.63\text{MeV}$ ,  $C_n\alpha_{\text{QED}} \sim -0.13\text{MeV}$ ,  $\sigma'/\Lambda_{\text{QCD}} \sim 3.8 \times 10^{-2}$ ,  $\delta'/\Lambda_{\text{QCD}} \sim 4.2 \times 10^{-4}$ ,  $a_3\alpha_{\text{QED}} \sim 0.7710^{-3}$ ,  $m_u \sim 5\text{MeV}$ ,  $m_d \sim 10\text{MeV}$  and  $m_e \sim 0.5\text{MeV}$ .

As discussed before, a conservative way of complying with the Cassini results is to impose that  $|\alpha_A| \leq 10^{-3}$  in order to satisfy the constraint on the Eddington parameter. Knowing the gravitational coupling we can express the violations of the weak equivalence

principle

$$\eta_{AB} = \frac{1}{2}\kappa^{-1/2}\alpha_E \left[ \frac{\partial}{\partial Q} \left( \frac{\sigma'}{\Lambda_{\text{QCD}}} \right) \left( \frac{N_A + Z_A}{M_A} - \frac{N_B + Z_B}{M_B} \right) + \frac{\partial}{\partial Q} \left( \frac{\delta'}{\Lambda_{\text{QCD}}} \right) \left( \frac{N_A - Z_A}{M_A} - \frac{N_B - Z_B}{M_B} \right) + a_3 \frac{\partial \alpha_{\text{QED}}}{\partial Q} \left( \frac{E_A}{M_A} - \frac{E_B}{M_B} \right) \right]. \quad (4.23)$$

In particular for two pairs of test bodies, the ratio  $\eta_{AB}/\eta_{BC}$  is independent of the background object E.

Now although it is a tortuous route, the calculations of the fifth force and equivalence principle violations are directly related to the supergravity Lagrangian and can be computed from first principle.

#### 4.4 Variations of Constants

Another consequence of the interaction between dark energy and the observable sector is the variations of the gauge couplings. As described before, this is linked to the choice of the functions  $f_G(Q, z_i)$ . A non-vanishing function  $f_i$  implies, for example, a variation of the gauge coupling constants,

$$\frac{\Delta f_i}{f_i} = -\frac{\Delta \alpha_i}{\alpha_i}, \quad (4.24)$$

evaluated at the grand unification scale  $m_{\text{GUT}}$ . To obtain the low energy variations one must use the running of the fine structure constant between the GUT scale and the weak scale of the  $\text{SU}(2) \times \text{U}(1)_Y$  coupling constants using Eq. (4.14).

Experimentally, the Webb *et al.* result [38] suggests that  $\Delta \alpha_{\text{QED}}/\alpha_{\text{QED}} \sim (-0.76 \pm 0.28) \times 10^{-5}$  since a redshift  $z = 3$ . If confirmed, this is a strong constraint on the  $Q$  dependence of  $f$ . However, in contrast, other groups [39] reported a null result from observations on the Southern Hemisphere. One must also consider the Oklo bound  $|\Delta \alpha_{\text{QED}}/\alpha_{\text{QED}}| \leq 10^{-7}$  between  $z = 0.5$  and now [33].

Another important constraint comes from the variation of the proton to electron mass ratio  $r \equiv m_p/m_e$  where

$$m_p = C_{\text{QCD}} \Lambda_{\text{QCD}} + b_u m_u + b_d m_d + C_p \alpha_{\text{QED}}, \quad (4.25)$$

where  $C_{\text{QCD}} \sim 5.2$  is a constant. Using the results obtained before, namely  $m_e = A_d(Q)m_e^0$ ,  $m_{u,d} = A_{u,d}(Q)m_{u,d}^0$ , we get

$$\frac{\Delta r}{r} \sim -\frac{8\pi^2}{b_3} \Delta f_3 + b_u \frac{m_u}{m_p} \Delta \alpha_u + \left( b_d \frac{m_d}{m_p} - 1 \right) \Delta \alpha_d + \frac{C_p \alpha_{\text{QED}}}{m_p} \frac{\Delta \alpha_{\text{QED}}}{\alpha_{\text{QED}}}. \quad (4.26)$$

Experimentally this ratio was measured to be  $\Delta r/r \sim (5.02 \pm 1.81) \times 10^{-5}$  at  $z = 3$  [40] and, very recently, there is also an indication for a possible variation,  $\Delta r/r \sim (2.0 \pm 0.6) \times 10^{-5}$  [41]. If these results are confirmed experimentally, this gives bounds on  $\alpha_{u,d}$  and the variations of  $f_i$ .

It should be noticed that, in the present framework, a variation of the gauge couplings is not necessarily linked to a variation of the particle masses. Variation of particle masses

seems to be unavoidable while variation of the gauge couplings depends on the choice of the functions  $f_i$ . In particular, in a minimal setting, one could choose a constant  $f_i$  in which no variation of  $\alpha_{\text{QED}}$  would be present while the fermion masses would still be quintessence vev dependent quantities. However, a dependence on the hidden sector is still necessary in order to generate gaugino masses. This may lead to variations of the fine structure constant if the hidden sector fields have  $Q$ -dependent vevs.

#### 4.5 Cold Dark and Baryonic Energy Densities

Another consequence of having  $Q$ -dependent masses is that, *a priori*, the energy density of cold dark and baryonic matters no longer scales as  $1/a^3$  but as

$$\rho \sim \frac{1}{a^3} \sum_a n_a m_{\text{u,d}}^{\text{F}} \left( \frac{Q}{m_{\text{Pl}}} \right), \quad (4.27)$$

where  $n_a$  is the number of non-relativistic particles. The above claim is justified by the fact that dark matter is usually considered as belonging to the observable sector (for instance, the lightest supersymmetric particle is often presented as the most popular candidate for dark matter). It has been shown in Ref. [32] that this type of interaction between dark matter and dark energy can result in an effective dark energy equation of state less than  $-1$ . It is also interesting to notice that, even if the mass of the quintessence field is larger than  $10^{-3}\text{eV}$  in which case, as explained before, all the gravitational tests are satisfied, there still exists a signature, of cosmological nature, of the interaction between dark matter and dark energy, at least as long as the quintessence field is not stabilized at its minimum (if this one exists). The coupling between matter and quintessence induces a modification of the quintessence potential

$$V_{\text{eff}}(Q) = V_{\text{DE}}(Q) + A_{\text{CDM}}(Q) \frac{\rho_{\text{CDM}}^0}{a^3}, \quad (4.28)$$

where  $A_{\text{CDM}}(Q) \equiv m_{\text{CDM}}(Q)/m_{\text{CDM}}(0)$ . In the present context,  $m_{\text{CDM}}$  is the mass of the dark matter particle, typically the lightest supersymmetric particle. In this case, it is necessary to diagonalize the neutralino mass matrix and, as a result, the  $Q$ -dependence of  $m_{\text{CDM}}$  can be non trivial. When the effective potential admits a time-dependent minimum, the model is known as a chameleon model [42].

Another important constraint comes from the "kicks" received by the quintessence field when particle species become non-relativistic [31, 42]. This may induce large variations of the quintessence field. This is particularly dangerous for the electron where the transition occurs during nucleosynthesis and may lead to large mass variations threatening nucleosynthesis.

## 5. Conclusions

We have studied the coupling of quintessence to both observable and hidden matter in supergravity. We have shown that the resulting dynamics are modified by the breaking of supersymmetry in the hidden sector. In particular, the shape of the quintessence potential

is changed and, consequently, its mass is no longer necessarily small (*i.e.* of the order of the Hubble parameter today). We have argued that, if the potential acquires a minimum, then the mass becomes of the order of the gravitino mass.

We have also paid attention to the electroweak physics and the influence of quintessence on the particle masses after electroweak symmetry breaking. We have found that the particle masses become quintessence dependent in a way which is parametrized by the dynamics of the hidden sector and we have provided a general framework to compute how the fermion masses depend on the quintessence field vev. The resulting theory is equivalent to a scalar-tensor theory. If the quintessence mass is greater than  $10^{-3}\text{eV}$ , which is clearly the case when the mass of the quintessence field is of the order of the gravitino mass, then the corresponding model is free from gravitational problems. On the contrary, if  $m_Q$  is smaller than  $10^{-3}\text{eV}$  then the range of the force is large and, therefore, subject to the constraints existing on the presence of a large fifth force and/or of a strong violation of the equivalence principle.

The main goal of the paper was to develop a general formalism which allows us to compute  $m(Q/m_{\text{Pl}})$  in Eq. (1.1) in the most general case, *i.e.* without having to specify the details of the underlying quintessence model. In a companion paper [22], we apply this formalism to concrete cases which illustrate the results obtained here and investigate how the cosmological evolution of the quintessence field is changed.

For instance, the SUGRA model coupled to the observable and supersymmetry breaking sectors receives drastic modifications [13, 22]. Two alternatives are possible. In the first one corresponding to stabilized ( $Q$ -independent) hidden sector vev's, the quintessence potential develops a minimum at low values of  $Q$  where the mass of the quintessence field is of the order of the gravitino mass. In this case gravitational problems are evaded. Yet the cosmological evolution of the quintessence field is such that it settles down at the minimum of the potential before Big Bang Nucleosynthesis, implying that the model is indistinguishable from a pure cosmological constant. On the other hand, when the hidden sector vev's are not all stable, the potential can remain runaway. In this case, constraints on the non-observation of violations of the equivalence principle imply that  $\kappa^{1/2}Q_{\text{now}} \ll 1$ . Scanning over the various models, as necessary to get concrete predictions, requires a long discussion which is performed in detail in Ref. [22].

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