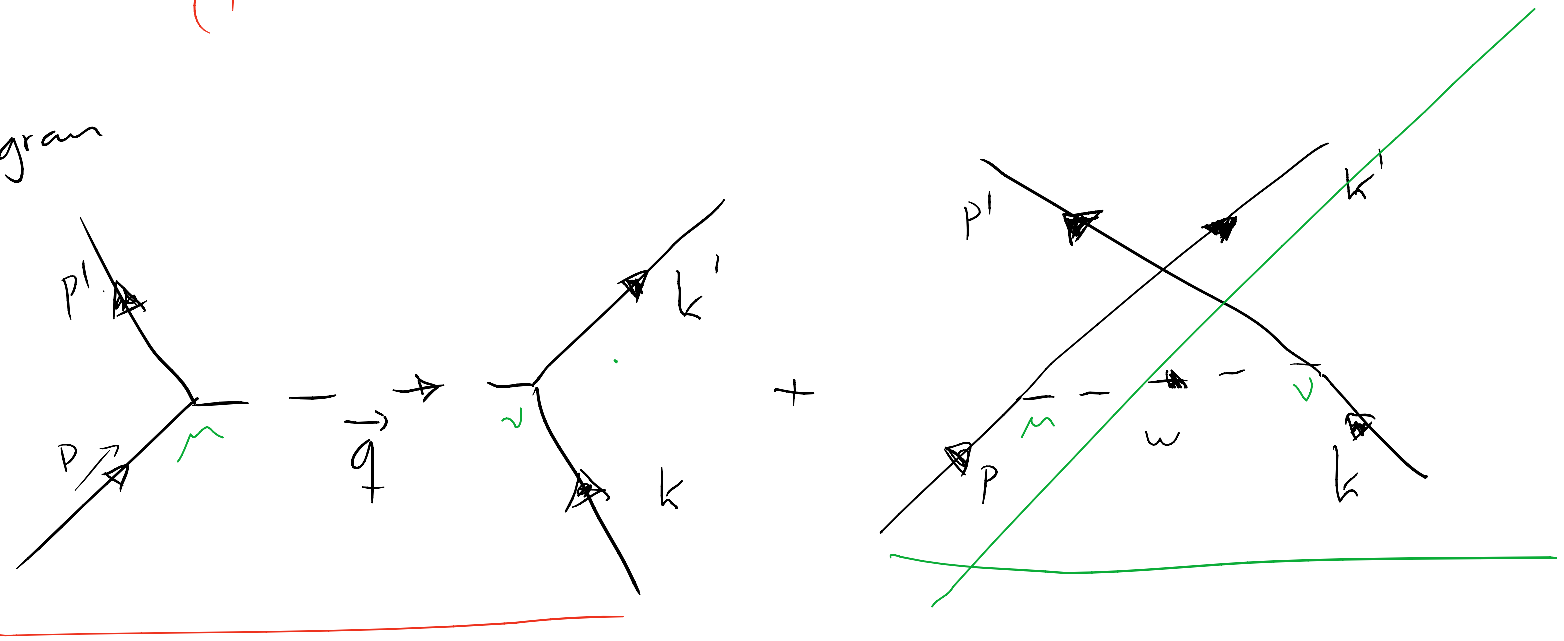
onsdag 26. oktober 2016 16.46

Problem 6

Yukawa theory: $\mathcal{L}_{int} = -g_e \bar{\Psi}\Psi\phi$, scattering

$(e^- \gamma \to e^- \gamma)$ → $(p + k \to p' + k')$

Feynman diagram

$iM =$ 

distinguishable particles

we get $(-ig)^2$ from the two vertexes, from the propagator q we get $\frac{i}{q^2 - m_\phi^2 [+i\epsilon]}$. pick up (-1) from fermi statistics. From external leg contractions we get $u^s(p')u^s(p)$ and $\bar{u}^s(k')u^s(k)$.

$m_\phi = m_e$

For the first diagram we then have:

$$-(-ig)^2 \bar{u}^s(p')u^s(p) \frac{i}{q^2 - m_\phi^2 [+i\epsilon]} \bar{u}^r(k') u^r(k)$$

$$\Rightarrow -ig^2\, \bar{u}^s(p')\, u^s(p)\, \frac{1}{q^2 - m_\Phi^2}\, \bar{u}^r(k')\, u^r(k) = -ig^2\, \bar{u}^s(p')\, u^s(p)\, \frac{1}{(p'-p)^2 - m_\Phi^2}\, \bar{u}^r(k')\, u^r(k)$$

Now

$$iM = -ig^2 \left(\bar{u}^{s'}(p')\, u^s(p)\, \frac{1}{(p'-p)^2 - m_\Phi^2}\, \bar{u}^{r'}(k')\, u^r(k) \right)$$

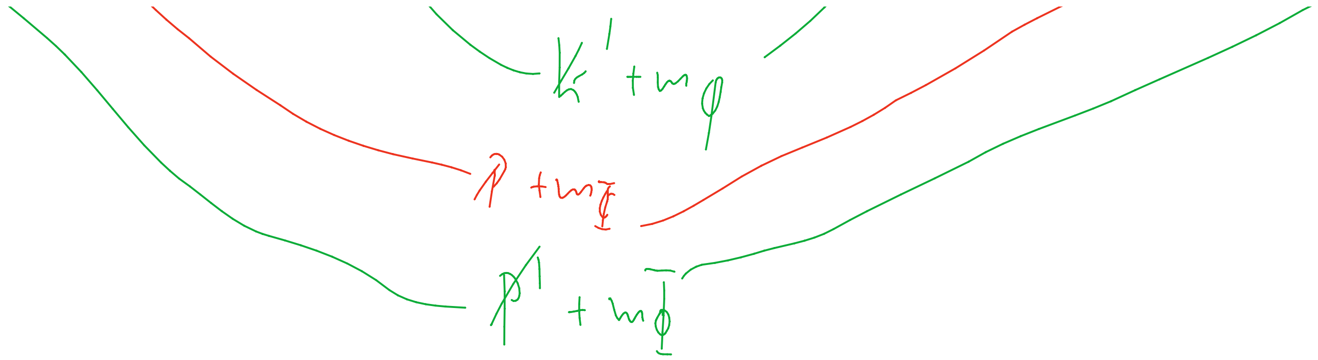
Unpolarized:

$$|\bar{M}|^2 = \frac{1}{\underbrace{2}_{e^-} \cdot \underbrace{2}_{\text{scalar particle}}} \sum_{\text{spins}} |M|^2$$

$$|\bar{M}|^2 = \frac{g^4}{4} \sum_{r,s,r',s'} \left[\bar{u}^{s'}(p')\, u^s(p)\, \frac{1}{(p'-p)^2 - m_\Phi^2}\, \bar{u}^{r'}(k')\, u^r(k) \right] \left[\bar{u}^r(k)\, u^{r'}(k')\, \frac{1}{(p'-p)^2 - m_\Phi^2}\, \bar{u}^s(p)\, u^{s'}(p') \right].$$

$\not{k} + m_\phi$

$\not{k}' + m_\phi$



$$= \frac{1}{4} \left(\frac{g^4}{(p'-p)^2 - m_\Phi^2} \right)^2 \underbrace{\mathrm{Tr}\left[(\not{p}' + m_\Phi)(\not{p} + m_\Phi) \right] \mathrm{Tr}\left[(\not{k}' + m_\varphi)(\not{k} + m_\varphi) \right]}$$

$$\cdot\ \mathrm{Tr}\left[p' \cdot p\, \gamma^\mu \gamma^\nu + p' m_\Phi \gamma^\mu + p\, m_\Phi \gamma^\nu + m_\Phi^2 \right]$$

$$= p' \cdot p\, \underbrace{\mathrm{Tr}\left[\gamma^\mu \gamma^\nu \right]}_{= 4 g^{\mu\nu}} - 4 m_\Phi^2 g^{\mu\nu}$$

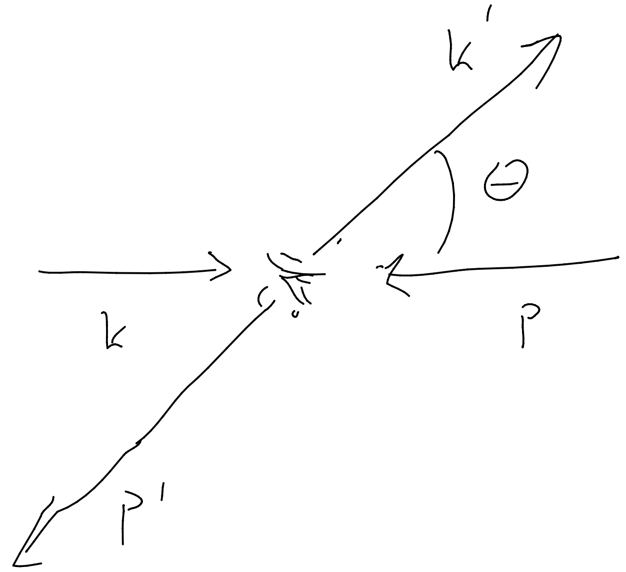
$$\mathrm{Tr}(\gamma^\mu) = 0$$

$$= 4 (p' \cdot p - m_\Phi^2)\, g^{\mu\nu}$$

$$m_\Phi \gg m_\varphi$$

$$\Rightarrow |\bar{\mathcal{M}}|^2 \simeq \frac{1}{4} \frac{g^4}{\left[(p'-p)^2 - m_\Phi^2 \right]^2} \cdot 4 (p' \cdot p - m_\Phi^2) \cdot 4\, k' \cdot k$$

$$= \frac{4g^4}{\left[(p'-p)^2 - m_\Phi^2\right]^2}\left(p'\cdot p - m_\Phi^2\right) k'\cdot k$$



Frame

$p = (E, -E\hat{z})$ $\qquad p' = (E, -\vec{p}')$

$k = (E, E\hat{z})$ $\qquad k' = (E, \vec{p}')$

$$\Rightarrow k'\cdot k = E^2 - E|\vec{p}'|\cos\theta$$

$$(E, E\hat{z})\cdot(E, \vec{p}') = E^2 + E\hat{z}\cdot\vec{p}'$$
$$= E^2 + E|\vec{p}'|\cos\theta$$

$\Rightarrow$

$$p'\cdot p = E^2 - E|\vec{p}'|\cos\theta$$

$$(p'-p)^2 = p'p' - p'p - pp' + pp$$

$$= \cancel{E^2} - |\vec{p}'|^2 - \cancel{E^2} + E|\vec{p}'|\cos\theta - E^2 + E|\vec{p}'|\cos\theta + \cancel{E^2} - \cancel{E^2}$$

$$= 2E|\vec{p}'|\cos\theta + |\vec{p}'|^2 - E^2$$

$4g^4$ ... $(E^2 - |\vec{p}'|$ ...

$$\Rightarrow |\bar{M}|^2 = \frac{4g^4}{\left[2E|\vec{p}'|\cos\theta - \cancel{|\vec{p}|^2 + E^2 - m_\Phi^2}\right]^2} \underbrace{\left(E^2 - E|\vec{p}'|\cos\theta - m_\Phi^2\right)\left(E^2 - E|\vec{p}'|\cos\theta\right)}$$

$$E^2 = p^2 + m^2$$

$$E^2\left(E^2 - E|\vec{p}'|\cos\theta - m_\Phi^2\right) - E|\vec{p}'|\cos\theta\left(E^2 - E|\vec{p}'|\cos\theta - m_\Phi^2\right)$$

$$E^2\left(E^2 - 2E|\vec{p}'|\cos\theta - m_\Phi^2 + |\vec{p}'|^2\cos^2\theta\right) + E|\vec{p}'|\cos\theta\, m_\Phi^2$$

$$E^2\left(-2E|\vec{p}'|\cos\theta - |\vec{p}'|^2\sin^2\theta\right) + E|\vec{p}'|\cos\theta\, m_\Phi^2$$

$$\Rightarrow |\bar{M}|^2 = \frac{\cancel{4}g^4}{\cancel{4}\cancel{E^2}|\vec{p}'|^2} \frac{\cancel{E^2}}{\cos^2\theta}\left[\frac{|\vec{p}'|\cos\theta\, m_\Phi^2}{E} - 2E|\vec{p}'|\cos\theta - |\vec{p}'|^2\sin^2\theta\right]$$

$$= g^4\left[\frac{m_\Phi^2}{|\vec{p}'|^2\cos\theta\, E} - \frac{2E}{|\vec{p}'|^2\cos\theta} - \tan^2\theta\right]$$

Now we can use eq. 4.84 from P&S for a CM system.

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{1}{2E_A 2E_B |v_A - v_B|} \frac{|\vec{p}'|}{(2\pi)^2 4E_{cm}} |\bar{M}|^2$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \frac{}{2E_\Phi\, 2E_\phi\, |v_\Phi - v_\phi|} \qquad \frac{}{(2\pi)^2\, 4 E_{cm}}$$

but $E_\Phi = E_\phi = \frac{1}{2} E_{cm}$ and $v_\Phi = \frac{p_\Phi}{E_\Phi} = 1 = -v_\phi \Rightarrow |v_\Phi - v_\phi| = 2$

$$= \frac{1}{2E_{cm}^2} \frac{|\vec{p}\,'|}{16\pi^2 E_{cm}} g^4 \left[\frac{m_\Phi^2}{|\vec{p}\,'|^2 \cos\theta\, E} - \frac{2E}{|\vec{p}\,'|^2 \cos\theta} - \tan^2\theta \right]$$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{g^4}{32\pi^2 E_{cm}^3} \left[\frac{m_\Phi^2}{|\vec{p}\,'| \cos\theta\, E} - \frac{2E}{|\vec{p}\,'| \cos\theta} - |\vec{p}\,'| \tan^2\theta \right]}$$

Klein-Nishima formula:

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{m^2} \left(\frac{\omega'}{\omega}\right) \left[\frac{\omega'}{\omega} + \frac{\omega}{\omega'} - \sin^2\theta \right]$$

I probably messed up with the mass terms somewhere, note

I probably messed up with the mass terms somewhere, note that m_Φ should it be m_e. Nevertheless the Yukawa scattering cross section bears resemblance to the Klein-Nishima formula in its form. Obvioosly $\tan^2\theta$ and $\sin^2\theta$ does not behave the same way and the constant in front looks abit different due to the different vertices in the two theories. The Yukawa scattering could probably be simplified further (also fixing the mass terms may change some things), but there is no time.