fredag 23. september 2016 09.23

Householder = Phase 1

$T = S^T A S$

$$S_1^T A S_1 = \begin{bmatrix} a_{11} & e_1 & 0 & \cdots & 0 \\ e_1 & & & & \\ 0 & & & & \\ \vdots & & \tilde{A}_1 & & \\ 0 & & & & \end{bmatrix}$$

$$S_2^T S_1^T A S_1 S_2 = \begin{bmatrix} a_{11} & e_1 & 0 & \cdots & 0 \\ e_1 & a_{22} & e_2 & 0 \cdots & 0 \\ 0 & e_2 & & & \\ \vdots & 0 & & \tilde{\tilde{A}} & \\ \vdots & \vdots & & & \\ 0 & 0 & & & \end{bmatrix}$$

Continue with this $S_{n-2}^T S_{n-3}^T \cdots S_1^T A S_1 \cdots S_{n-2}$

$$= \begin{bmatrix} a_{11} & e_1 & & & & \\ e_1 & \tilde{a}_{22} & e_2 & & O & \\ & e_2 & \ddots & \ddots & & \\ & & \ddots & \ddots & e_{n-1} & \\ & O & & e_{n-1} & \tilde{a}_{nn} & \end{bmatrix}$$

$$S_1^T A S_1 = \begin{bmatrix} a_{11} & e_1 & 0 & \cdots & 0 \\ e_1 & \tilde{a}_{22} & \tilde{a}_{23} & \cdots & \\ 0 & \tilde{a}_{23} & \ddots & & \\ \vdots & \vdots & & \ddots & \\ 0 & \vdots & & & \tilde{a}_{nn} \end{bmatrix}$$

$$S_1 = \begin{bmatrix} \mathbb{1} & 0^T \\ 0 & P \end{bmatrix}$$

$0^T = [0, 0, \cdots 0]$

$n-1$ vector

$P = n-1 \times m-1$ matrix

$$P = P^T \Rightarrow P^2 = \mathbb{1} = P^T P$$

2×2

Jacobi: $\begin{bmatrix} c & s \\ -s & c \end{bmatrix}$ (rotation)

Householder $S = \begin{bmatrix} c & s \\ s & -c \end{bmatrix}$ (reflection along a line)

$$S^T S = c^2 + s^2 = \mathbb{1}$$

$$P = \mathbb{1} - 2\underbrace{u u^T}_{\text{unknown}}$$

$$P^2 = \mathbb{1} - 4(u u^T)(u u^T) + 4(u u^T)(u u^T) = \mathbb{1}$$

$$P_{ij} = \delta_{ij} - 2 u_i u_j$$

$$P_{ij} = \delta_{ij} - 2u_i u_j$$

$$\vec{v}^T = (a_{12} \quad a_{12} \quad \cdots \quad a_{1n}]$$

$$S_1^T A S_1 = \begin{bmatrix} a_{11} & (P\cdot\vec{v})^T \\ (P\cdot\vec{v}) & \tilde{A} \end{bmatrix}$$

$$(P\cdot\vec{v})^T = [e_1, 0, \ldots 0]$$

$$P\cdot\vec{v} = \vec{v} - 2\vec{u}(\vec{u}^T\vec{v}) = e_1\vec{e}$$

$$\vec{e} = (1, 0, \cdots 0)$$

$$\dim(\vec{e}) = n-1$$

$$(P\vec{v})^T(P\vec{v}) = \vec{v}^T P^2 \vec{v} = v^2 = e_1^2$$

$$e_1^2 = \sum_{i=2}^{n} a_{1i}^2$$