

Quintessence models in Supergravity

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Scalar field models of quintessence typically require that the expectation value of the field today is of order the Planck mass, if we want them to explain the observed acceleration of the Universe. This suggests that we should be considering models in the context of supergravity. We discuss a particular class of supergravity models and analyze their behavior under different choices of the Kähler metric.

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I. INTRODUCTION

Several cosmological observations appear to suggest that the present Universe is dominated by an unknown form of energy with negative pressure, acting as an effective cosmological constant Λ . CMB and cluster distribution data, when combined with the measurements of the redshift–luminosity relation using high redshift type Ia supernovae, single out a flat cosmological model with $\Omega_{\text{matter}} \sim 1/3$ and $\Omega_{\Lambda} \sim 2/3$ [1]. The contribution to the energy density which has led to the observed acceleration is named “quintessence” and could well be some more general component than a bare cosmological term Λ [2]. In particular, a scalar field Q rolling down a potential $V(Q)$ can act as an effective dynamical cosmological constant with equation of state

$$w_Q = \frac{p_Q}{\rho_Q} = \frac{\dot{Q}^2/2 - V(Q)}{\dot{Q}^2/2 + V(Q)}. \quad (1)$$

Dynamical models are appealing because they could potentially solve the smallness and coincidence problems which arise when postulating that there is today a cosmological constant energy density contribution, $\rho_{\Lambda} = \Lambda/8\pi G$, of the same order of magnitude as the critical energy density $\rho_c \simeq 10^{-47} \text{GeV}^4$. The advantage of dealing with a dynamical scalar field is that there exist attractor solutions in a cosmological setting for some classes of potentials [3]. If this is the case, the scalar field will join the attractor solution before the present epoch for a very wide range of initial conditions, thus relieving the fine-tuning issue.

For example, the potential $V(Q) = M^{4+\alpha}Q^{-\alpha}$, with $\alpha > 0$, leads to a ‘tracking’ attractor solution for which the scalar energy density scales as $\rho_Q/\rho_B \sim a^{3(w_B - w_Q)}$, where ρ_B and w_B are the background energy density and equation of state (the latter being $0, 1/3$ during matter and radiation domination respectively), and a is the scale factor of the Universe. The equation of state of the attractor is given by $w_Q = (w_B\alpha - 2)/(\alpha + 2)$ and is always negative during matter domination [4]. If Q has already reached the attractor solution today, it must be of order of the Planck mass M_{Pl} . This follows from the fact that

for the tracking solutions $V''(Q) \simeq H^2$ and moreover we require $V \simeq \rho_c$ today. The scale M in the potential is fixed by requiring that $V(Q \simeq M_{\text{Pl}}) \simeq \rho_c$ and depends on the exponent α . This class of potentials is then a suitable candidate for quintessence and has been shown to be compatible with observations [4].

If instead an exponential potential is considered, $V(Q) = M^4 \exp(\lambda Q)$, two attractors are found [5], depending on the value of λ . If $\lambda > 3(w_B + 1)$, the late time attractor is a ‘scaling’ one, characterized by $w_Q = w_B$ and $\Omega_Q = 3(w_B + 1)/\lambda^2$. Note that in this case the ratio of the scalar to critical energy density is independent of the mass scale M in the potential. Unfortunately the equation of state is non negative and so cannot drive the Universe to an accelerated expansion. If $\lambda < 3(w_B + 1)$, instead, then the late time attractor is the scalar field dominated solution with $\Omega_Q = 1$ and $w_Q = -1 + \lambda^2/3$. Now the equation of state is negative, but clearly cannot be the solution for all times, as there is the tight constraint on the allowed contribution of the scalar field at nucleosynthesis of $\Omega_Q(1\text{MeV}) < 0.13$. Besides, one must allow time for structure formation before the Universe starts accelerating. A scenario like this requires that the scalar field starts with roughly the same energy density as today, and so provides no improvement on solving the fine tuning problem. The exponential potentials, then, cannot be used individually for modeling quintessence, but interesting modifications have been proposed in order to fit the data [6,7].

A. Particle physics models

While dealing with what is thought to be one of the fundamental components of the present universe, it is mandatory to ask if we can find any firmer motivation for introducing a cosmological scalar field. We actually know that scalar fields arise quite naturally in unified theories and are a necessity in supersymmetric theories where they play the role of ‘superpartners’ of the Standard model fermion fields. Scalar fields are also required by another crucial cosmological mechanism, inflation. Indeed, it has been recently shown [8] that the same unique

field could possibly dominate both the inflationary and quintessence phases of our universe. The issue of finding deeper roots for the quintessence scalar into ‘realistic’ particle physics models is then a pressing one.

It is well known that inverse power law scalar potentials arise in supersymmetry (SUSY) gauge theories due to non-perturbative effects [9]. They have been studied as candidates for quintessence [10] and shown to provide a viable phenomenology both in the one-scalar and multi-scalar cases. Exponential potentials are also common in high energy physics, since they arise as a consequence of Kaluza–Klein type compactifications of string theory. The attempt of building a particle physics motivated model for quintessence is then a well-posed problem, although many points remain to be clarified. For example, exponential-like models as those proposed in [6,7] still await a firmer high energy motivation. At the same time, inverse power law models based on SUSY QCD need to be revisited taking into account the possibility that the scalar potential receive corrections originating from quantum or supergravity effects.

This last issue was recently addressed by Brax and Martin [11]. In particular they showed that inverse power law models which lead to quintessence are stable against quantum corrections, both in the supersymmetric and non-supersymmetric cases. Furthermore, discussing curvature and Kählerian corrections in the globally supersymmetric case, they found that while curvature corrections are negligible, Kählerian ones might be important in the early evolution of Q .

The most interesting question, though, concerns supergravity (SUGRA) corrections [11]. Since today the vacuum expectation value (vev) of the scalar field is of order Planck mass, these corrections are unavoidable and may lead to problems for these power law quintessence models. In particular, if the scalar potential is dramatically altered for field values corresponding to the most recent epoch, then we might have a completely different phenomenology from the attractor case just described. In ref. [11], a possible way of avoiding the dangerous SUGRA corrections was outlined, which involved the restrictive condition $\langle W \rangle = 0$. The aim of this paper is to further develop this line of attack by proposing an alternative way of tackling SUGRA corrections. As an example, we will study the cosmology of a specific SUGRA-inspired quintessence model and show that it is compatible with observations for a wide range of initial conditions.

II. SUPERGRAVITY MODELS

A. Global supersymmetry

Globally supersymmetric QCD theories with N_c colors and $N_f < N_c$ flavors may give an explicit realization of a model of quintessence with an inverse power law

scalar potential [10]. The matter content of the theory is given by the chiral superfields Q_i and \bar{Q}_i ($i = 1 \dots N_f$) transforming according to the N_c and \bar{N}_c representations of $SU(N_c)$, respectively.

Supersymmetry and anomaly-free global symmetries constrain the superpotential to take the form

$$W = (N_c - N_f) \left(\frac{\Lambda^{(3N_c - N_f)}}{\det T} \right)^{\frac{1}{N_c - N_f}} \quad (2)$$

where the gauge-invariant matrix superfield $T_{ij} = Q_i \cdot \bar{Q}_j$ appears. Λ is the only mass scale of the theory. It is the supersymmetric analogue of Λ_{QCD} , the renormalization group invariant scale at which the gauge coupling of $SU(N_c)$ becomes non-perturbative.

If the Kähler metric is flat, the scalar potential is given by*

$$\begin{aligned} V(Q_i, \bar{Q}_i) &= V_F + V_D \\ &= \sum_{i=1}^{N_f} (|F_i|^2 + |F_{\bar{i}}|^2) + \frac{1}{2} D^a D^a \end{aligned} \quad (3)$$

where $F_i = \partial W / \partial Q_i$, $F_{\bar{i}} = \partial W / \partial \bar{Q}_i$, and

$$D^a = Q_i^\dagger t^a Q_i - \bar{Q}_i t^a \bar{Q}_i^\dagger, \quad (4)$$

with the t^a ’s being the generators of the gauge group. Of interest to us are the dynamics of the field expectation values which take place along directions in field space in which the above D-term vanishes, *i.e.* the perturbatively flat directions $\langle Q_{i\alpha} \rangle = \langle \bar{Q}_{i\alpha}^\dagger \rangle$, where $\alpha = 1 \dots N_c$ is the gauge index. At the non-perturbative level these directions acquire a non vanishing potential from the F-terms in (3), which are zero to any order in perturbation theory. Gauge and flavor rotations can be used to diagonalize the $\langle Q_{i\alpha} \rangle$ and put them in the form

$$\langle Q_{i\alpha} \rangle = \langle \bar{Q}_{i\alpha}^\dagger \rangle = \begin{cases} Q_i \delta_{i\alpha} & 1 \leq \alpha \leq N_f \\ 0 & N_f \leq \alpha \leq N_c \end{cases}.$$

Along these directions, if the expectation values of all the N_f scalars are taken to be equal, $\langle Q_i \rangle = Q$, $i = 1, \dots, N_f$, the cosmological evolution of the scalar vev Q is given by

$$\begin{aligned} \ddot{Q} &= -3H\dot{Q} + \beta \frac{\Lambda^{4+2\beta}}{Q^{2\beta+1}}, \\ \beta &= \frac{N_c + N_f}{N_c - N_f}, \end{aligned} \quad (5)$$

thus reproducing exactly the case of a single scalar field Q in a potential $V = \frac{\Lambda^{4+2\beta}}{2} Q^{-2\beta}$. The ‘tracker’ solution is characterized [10] by an equation of state

*In the following, the same symbols will be used for the superfields Q_i , \bar{Q}_i , and their scalar components.

$$w_Q = \frac{N_c + N_f}{2N_c} w_B - \frac{N_c - N_f}{2N_c} \quad (6)$$

as a function of the parameters of the theory, with the scalar energy density growing with respect to the matter one as

$$\rho_Q/\rho_m = a^{3(N_c - N_f)/2N_c} \quad (7)$$

Requiring that the scalar Q both has reached the tracker today and is starting to dominate the energy density, we obtain that at the present epoch $Q \simeq M_{\text{Pl}}$ and that the mass scale in the potential must satisfy $\Lambda^{4+\beta} \simeq \rho_c M_{\text{Pl}}^\beta$, introducing a degree of fine tuning in the problem.

B. Supergravity corrections

The above discussion is valid as far as the global SUSY limit can be taken as a good approximation. This is correct for most of the cosmological evolution of the scalar Q . However, for the present epoch we are not allowed to neglect SUGRA corrections, since we enter the regime for which they might become important. In this case, the F-term in the scalar potential in general is

$$\begin{aligned} V(Q) &= F^2 - e^{\kappa^2 K} 3\kappa^2 |W|^2 \\ &= e^{\kappa^2 K} [(W_i + \kappa^2 W K_i) K^{j*i} (W_j + \kappa^2 W K_j)^* \\ &\quad - 3\kappa^2 |W|^2] \end{aligned} \quad (8)$$

where the subscript i indicates the derivative with respect to the i -th field, and $\kappa^2 = 8\pi G = 8\pi M_{\text{Pl}}^{-2}$.

Brax and Martin [11] discuss the case of a theory with superpotential $W = \Lambda^{3+\alpha} Q^{-\alpha}$ and a flat Kähler potential, $K = QQ^*$. It is straightforward to compute the resulting scalar potential:

$$V(Q) = e^{\frac{\kappa^2}{2} Q^2} \frac{\Lambda^{4+\beta}}{Q^\beta} \left(\frac{(\beta-2)^2}{4} - (\beta+1) \frac{\kappa^2}{2} Q^2 + \frac{\kappa^4}{4} Q^4 \right) \quad (9)$$

where $\beta = 2\alpha + 2$. The main effect of the supergravity corrections is that the scalar potential can now become negative due to the presence of the second term. This is a serious drawback for the model, which becomes ill defined for $Q \simeq M_{\text{Pl}}$. They go on to propose a possible solution by imposing the condition that the expectation value of the superpotential vanishes, $\langle W \rangle = 0$. We then see from equation (8) that the negative contribution to the scalar potential disappears, and it takes the form

$$V(Q) = \frac{\Lambda^{4+\alpha}}{Q^\alpha} e^{\frac{\kappa^2}{2} Q^2} \quad (10)$$

The condition $\langle W \rangle = 0$ is possible to realize, for example, in a model in which we allow matter fields to be present in addition to the quintessence scalar field [11]. Then, if at least one of the gradients of the superpotential with

respect to the matter fields is non-zero, the scalar potential will always be positive.

This is not the only possibility, though. There are two obvious problems with the potential (9): one, as already stated, is the negative term in the general expression (8) and the second is the choice of the Kähler metric which makes this term grow with the field's vev, relative to the other terms in the potential. As mentioned above, setting $\langle W \rangle = 0$ is a tight restriction, so we will address the issue by relaxing this constraint but allow for more general forms of the Kähler metric.

Such an approach was recently adopted in [12,13] as a method of obtaining a minimum for the dilaton field in string theory. It had the advantage of relying on only one gaugino condensate and provided an alternative approach to the phenomenology associated with ‘racetrack’ models [14]. In this scenario, the Kähler potential acquires string inspired non-perturbative corrections. A further nice feature of these models is that it is possible to have a minimum with zero or small positive cosmological constant [13,15], and moreover it is possible to establish that the dilaton can be stabilized in such a minimum in a cosmological setting [16].

In general, for different choices of the Kähler metric, the negative term in (8) does not always lead to the disaster of a negative minimum in the scalar potential. For a general Kähler, we do not know *a priori* the shape of the potential or the location of the minimum. In fact, in what follows we will show through explicit examples that the scalar potential might always remain positive through a suitable choice of the Kähler metric. Moreover, with this approach there is no need to introduce additional fields in the model.

Let us now go on to study SUGRA corrections to inverse power law quintessence models by choosing more general Kähler potentials. Consider, for example, a theory with superpotential $W = \Lambda^{3+\alpha} \tilde{Q}^{-\alpha}$ and a Kähler $K = -\ln(\kappa \tilde{Q} + \kappa \tilde{Q}^*)/\kappa^2$, the type of term which is present at tree level for both the dilaton and moduli fields in string theory. In this case, the resulting scalar potential, expressed in terms of the canonically normalized field $Q = (\ln \kappa \tilde{Q})/\sqrt{2}\kappa$, is

$$V(Q) = M^4 e^{-\sqrt{2}\beta\kappa Q} \quad (11)$$

where $M^4 = \Lambda^{5+\beta} \kappa^{1+\beta} (\beta^2 - 3)/2$ with $\beta = 2\alpha + 1 > \sqrt{3}$ to allow for positivity of the potential. This corresponds to the ‘scaling’ solution discussed in the introduction and so cannot lead to a negative equation of state for the field in a matter dominated regime.

Another example follows as a natural extension of the one just described and leads to potentials with more than one exponential. For a superpotential of the form $W = \Lambda^{3+\alpha} \tilde{Q}^{-\alpha} + \Lambda^{3+\beta} \tilde{Q}^{-\beta}$ and the same Kähler metric as above, then in terms of the same canonically normalized field $Q = (\ln \kappa \tilde{Q})/\sqrt{2}\kappa$, the scalar potential becomes

$$V(Q) = (M_1)^4 e^{-\sqrt{2}\gamma\kappa Q} + (M_2)^4 e^{-\sqrt{2}\delta\kappa Q} + (M_3)^4 e^{-\frac{\gamma+\delta}{\sqrt{2}}\kappa Q}, \quad (12)$$

where $\gamma = 2\alpha + 1$, $\delta = 2\beta + 1$ and

$$\begin{aligned} (M_1)^4 &= \Lambda^{5+\gamma} \kappa^{1+\gamma} (\gamma^2 - 3)/2, \\ (M_2)^4 &= \Lambda^{5+\delta} \kappa^{1+\delta} (\delta^2 - 3)/2, \\ (M_3)^4 &= \Lambda^{5+\frac{\gamma+\delta}{2}} \kappa^{1+\frac{\gamma+\delta}{2}} (\gamma\delta - 3). \end{aligned} \quad (13)$$

At first sight this appears to be of the form required in [6] in that it involves multiple exponential terms. However, closer analysis indicates that the slopes of the exponentials are not adequate to satisfy the bounds arising from nucleosynthesis constraints, whilst also providing a positive cosmological constant type contribution today.

As we mentioned earlier, it is possible to have more general Kähler potentials, and with that in mind we now consider the original model $W = \Lambda^{3+\alpha} \tilde{Q}^{-\alpha}$, but with a Kähler potential which depends on a parameter γ

$$K = \frac{1}{\kappa^2} \left[\ln(\kappa\tilde{Q} + \kappa\tilde{Q}^*) \right]^\gamma, \quad \gamma > 1. \quad (14)$$

In this case, the second derivative of the Kähler is

$$\begin{aligned} K_{\tilde{Q}\tilde{Q}^*} &= \frac{\gamma(\gamma-1)}{\kappa^2 (\tilde{Q} + \tilde{Q}^*)^2} [\ln(\kappa\tilde{Q} + \kappa\tilde{Q}^*)]^{\gamma-2} \\ &\times \left(1 - \frac{\ln(\kappa\tilde{Q} + \kappa\tilde{Q}^*)}{\gamma-1} \right) \end{aligned} \quad (15)$$

and the canonically normalized field Q can be obtained as a function of \tilde{Q} by integrating the following expression

$$dQ = \sqrt{2K_{\tilde{Q}\tilde{Q}^*}} d\tilde{Q}. \quad (16)$$

In order to avoid the singularity at $\tilde{Q} + \tilde{Q}^* = 1/\kappa^2$, when $\ln(\kappa\tilde{Q} + \kappa\tilde{Q}^*)$ passes through zero (see equation (16)), the only possible choice is $\gamma = 2$. We then obtain:

$$K_{\tilde{Q}\tilde{Q}^*} = \frac{2[1 - \ln(\kappa\tilde{Q} + \kappa\tilde{Q}^*)]}{\kappa^2(\tilde{Q} + \tilde{Q}^*)^2} \quad (17)$$

and as a consequence

$$Q = -\frac{2}{3\kappa} [1 - \ln(2\kappa\tilde{Q})]^{3/2}. \quad (18)$$

Implying that the theory is well defined for

$$-\infty < \ln(2\kappa\tilde{Q}) < 1$$

which corresponds to $0 < \tilde{Q} < e/2\kappa$.

The scalar potential in terms of the canonically normalized field Q reads

$$\begin{aligned} V &= M^4 \left[2x^2 + (4\alpha - 7)x + 2(\alpha - 1)^2 \right] \frac{1}{x} \\ &\times \exp[(1-x)^2 - 2\alpha(1-x)] \end{aligned} \quad (19)$$

where for notational convenience we have defined the quantities

$$x \equiv \left(-\frac{3}{2} \kappa Q \right)^{2/3} = 1 - \ln(2\kappa\tilde{Q}), \quad (20)$$

$$M^4 = \Lambda^{6+2\alpha} \kappa^{2+2\alpha} 2^{2\alpha}. \quad (21)$$

Note that the canonically normalized field Q has a range $-\infty < Q < 0$.

We can see from equations (19)–(20) that the scalar potential behaves like an exponential for $|Q| \gg 1$ and like an inverse power law for $|Q| \ll 1$, and thus develops a minimum at a finite value Q_m . Note that the potential is always positive for any $\alpha > 1.25$. Thus, we have found that in this case the supergravity corrections induce a finite minimum in the potential but do not spoil its positivity. Note also that the field's value in the minimum is exactly in the region where we expect the supergravity corrections to become relevant. For example, with $\alpha = 5$ we obtain $Q_m \simeq -0.02$ (in $8\pi G = 1$ units), which corresponds to $\tilde{Q} \simeq 1.2$. Imposing that the minimum of the potential equals the critical energy density today, we can also estimate the mass scale Λ , depending on α . In the case $\alpha = 5$ we have that $V(Q = Q_m) \simeq 10^{-47} \text{GeV}^4$ which corresponds to $\Lambda \simeq 6 \cdot 10^{10} \text{GeV}$.

C. Supersymmetry breaking

If supersymmetry is to be realized in nature, it must be broken at a mass scale M_S such that $M_S^2 \sim \langle F \rangle \gtrsim (10^{10} \text{GeV})^2$ or $M_S^2 \sim \langle F \rangle \gtrsim (10^4 \text{GeV})^2$ (for gravity and gauge mediated cases respectively), in order to lift the supersymmetric scalar particle masses above 10^2GeV .

This then requires the superpotential in (8) to be $W \sim \langle F \rangle \kappa^{-1} \sim m_{3/2} \kappa^{-2}$ in order to cancel the F-term contribution and consequently to give a negligible total vacuum energy density ($m_{3/2}$ is the gravitino mass).

From the discussion in the last section[†], it is clear that the dynamical cosmological constant provided by the quintessence potential cannot be the dominant source of SUSY breaking in the Universe as $W \simeq \Lambda^{3+\alpha} \kappa^{-\alpha} \sim (10^{-3} \text{eV})^2 \kappa^{-1} \ll \langle F \rangle \kappa^{-1}$. Therefore, we need some additional source of SUSY breaking. If we consider now the superpotential,

$$W = \Lambda^{3+\alpha} Q^{-\alpha} + m_{3/2} \kappa^{-2}, \quad (22)$$

then one gains a correction to the scalar potential in (19) of,

$$\delta V \sim \Lambda^{3+\alpha} m_{3/2} \kappa^{-\alpha} + m_{3/2}^2 \kappa^{-2}, \quad (23)$$

[†]We thank K. Choi and D. Lyth for discussions on this point.

for $Q \sim M_{\text{Pl}}$ today [18].

The first term can in principle be controlled for sufficiently large α , however, the constant second term unavoidably leads to a disruption of the quintessence potential. This is a very serious problem of all supergravity models in quintessence [‡].

The situation gets worse, since, as pointed out in [17], it appears that even if we imagine that the amount of SUSY breaking that we observe in the universe today comes from a hidden sector other than the quintessence one, there will still be gravitational couplings between the two sectors that rekindle the original problem.

However, some recent proposals point in a slightly different direction for solving this problem. The basic idea is that the traditional approach to SUSY breaking might not be the best way to explain the world we live in. For example, the mass difference between the superpartners could arise in a 4D world with *unbroken* SUSY through some higher dimensional effects [19]. If this is the case, we would not need to break supersymmetry, and the quintessence potential would be preserved. Another possibility [20] is that the relation between the SUSY breaking scale M_S and the cosmological constant $\langle F \rangle^2$ is not what is usually considered. If $M_S = \kappa^{-1}[\langle F \rangle^2 \kappa^4]^\beta$ and $\beta = 1/8$, instead of the usual $1/4$, then the observed cosmological constant would provide just the right amount of SUSY breaking. In this case we wouldn't have any dangerous F-term of order $\sim \kappa^{-2} M_S^2$ which spoils the quintessence potential.

III. COSMOLOGY OF THE MODEL

We can now study in further detail the model we presented in the previous section. It can be easily checked that for $Q \rightarrow 0$ the potential (19) goes as $V \sim Q^{-2/3}$, while for $Q \rightarrow -\infty$ it is $V \sim Q^{2/3} e^{Q^{4/3}}$. This behavior is *independent* of the parameter α in our theory, which plays no role in the asymptotic form of the potential. For all the values of α we then find the same qualitative behavior for the scalar Q . For a very wide range of the initial conditions, indeed, we obtain scalar field dominance and negative equation of state at the present epoch.

In Figure 1 and Figure 2 we plot the evolution of the scalar energy density and equation of state for $\alpha = 5$.

If the scalar field is rolling down the potential towards the minimum from the side $Q_{\text{in}}/Q_{\text{m}} \ll 1$, then it will exhibit a 'tracking' behavior as in the general case $V \sim Q^{-\beta}$, with $\beta = 2/3$. This is characterized by an equation of state $w_Q = (w_B \beta - 2)/(\beta + 2) = -2/3, -3/4$ during radiation and matter domination respectively. When the

field Q approaches the minimum, it will depart from the attractor solution and enter a regime of damped oscillations. The non-zero vacuum energy will rapidly take over the kinetic term and the equation of state be driven towards $w_Q = -1$ (see Figure 2).

If, instead, Q rolls down from the side $Q_{\text{in}}/Q_{\text{m}} \gg 1$ the exponential in (19) is then important. As it will be shown below, this leads to an attractor with $w_Q \simeq w_B$ up to a scale factor dependent logarithmic correction. Eventually, the field will settle down in the minimum with $w_Q = -1$, after a stage of small oscillations about the minimum as before. The earlier requirement made on α , namely $\alpha > 1.25$ is also sufficient to respect the nucleosynthesis bound of $\Omega_Q(1\text{MeV}) < 0.13$.

In the second reference of [4] a general formula for the scalar equation of state in the case of 'tracking' potentials was derived:

$$w_Q \simeq \frac{w_B - 2(\Gamma - 1)}{1 + 2(\Gamma - 1)}, \quad (24)$$

where Γ is defined as $\Gamma \equiv V''V/(V')^2$, with a prime denoting the derivative with respect to the scalar Q .

In deriving eq. (24), it is assumed that Γ is nearly constant. A tracker solution with $w_Q < w_B$ is reached if $\Gamma > 1$. For a potential of the form

$$V = M^4 e^{\alpha(\kappa Q)^\beta}, \quad (25)$$

we obtain

$$\Gamma = 1 + \frac{\beta - 1}{\alpha\beta} \frac{1}{(\kappa Q)^\beta}, \quad (26)$$

which is larger than 1 and slowly varying as required. Actually the dependence of Γ on the scale factor $a(t)$

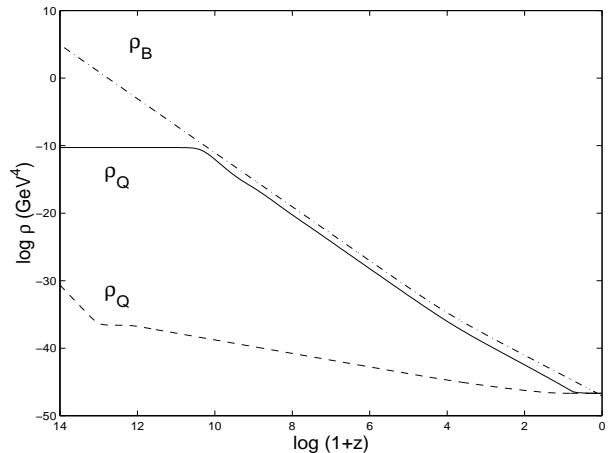


FIG. 1. The evolution of the energy densities ρ of different cosmological components is given as a function of redshift, for the case $\alpha = 5$ and with $\Omega_Q(\text{today}) \simeq 0.7$. The dot-dashed line represents the background energy density $\rho_B = \rho_{\text{radiation}} + \rho_{\text{matter}}$. The solid line is the evolution of the scalar field energy density when the field Q starts from an initial value $Q_{\text{in}}/Q_{\text{m}} \gg 1$, while the dashed line corresponds to starting with $Q_{\text{in}}/Q_{\text{m}} \ll 1$.

[‡]In order to avoid the SUGRA corrections problem, Choi proposes a Goldstone-type quintessence model inspired in heterotic M-Theory [18].

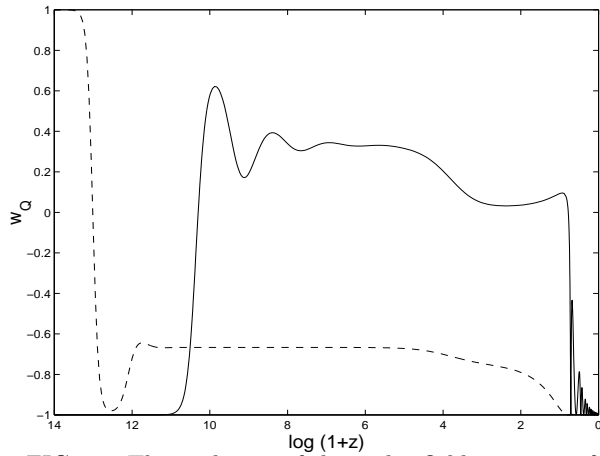


FIG. 2. The evolution of the scalar field equation of state as a function of redshift, corresponding to the two cases of FIG 1.

through $Q(a)$ will give the logarithmic variation of the equation of state mentioned earlier.

By substituting the expression (26) for Γ in eq. (24), we obtain

$$w_B = w_Q + (w_Q + 1) \frac{2(\beta - 1)}{\beta} \frac{1}{(\kappa Q)^\beta}, \quad (27)$$

with

$$(\kappa Q)^\beta = \ln \left(\frac{V_0}{M^4} \right) - 3(w_Q + 1) \ln \left(\frac{a}{a_0} \right), \quad (28)$$

where V_0 is the value of the scalar field potential at some time $a = a_0$, when the attractor has already been reached, and we have assumed the following dependence of the potential on the scale factor

$$V(a) = V_0 \left(\frac{a}{a_0} \right)^{-3(w_Q + 1)} \quad (29)$$

which is exact if w_Q is constant and can be taken as a good approximation when w_Q is slowly varying.

Note that, when $\beta = 1$, then V reduces to a simple exponential and from (27) we recover the ‘scaling’ behaviour with $w_Q = w_B$.

IV. CONCLUSIONS

In this paper we have studied supergravity corrections to quintessence models with a superpotential $W = \Lambda^{\alpha+3} Q^{-\alpha}$. The motivation for this is simple. Scalar field models of quintessence typically require that the expectation value of the field today is of order the Planck mass, if we want it to explain the observed acceleration of the Universe. This suggests that we should be considering models in the context of supergravity.

We have proposed a new line of attack to a serious problem that these models share, *i.e.* the negativity of

the minimum in the resulting scalar potential. Allowing for nonlinear modifications to the Kähler potential this problem can effectively be cured. Such modifications are, as far as we are aware, perfectly acceptable and lead to some interesting features.

In particular we discussed in detail a model with Kähler potential $K = [\ln(\kappa Q + \kappa Q^*)]^2 / \kappa^2$ and superpotential $W = \Lambda^{\alpha+3} Q^{-\alpha}$. In this case, the minimum of the resulting scalar potential is always positive for any $\alpha > 1.25$ and is located close to the point where the SUGRA corrections start to be important, *i.e.* at $Q \sim M_{\text{Pl}}$. We found that the resulting scalar potential yields two possible attractor solutions, both of which lead the scalar field towards the minimum without dominating the Universe dynamics before today. After reaching the minimum, the scalar field will mimic a cosmological constant with $w_Q \simeq -1$.

We also noted that the dynamical cosmological constant described above cannot be the dominant source of SUSY breaking in the universe, since the F-term is not large enough. This means that we need some additional source of SUSY breaking, which in turn implies we must ensure that the SUSY breaking sector does not interact with the quintessence sector, in order not to spoil the behaviour described above. Unfortunately, this particular problem still remains to be resolved [17], without invoking some extra symmetry principle in the action [18]. A possible, but still speculative resolution has been proposed by Witten [19], who has suggested that we may actually live in a vacuum with unbroken supersymmetry. As an alternative, Banks [20] has proposed a mechanism through which the measured cosmological constant might well be able to provide the observed amount of supersymmetry breaking.

In closing we note that an effective equation of state $w_Q \simeq -1$ for the present time, as the one we find, is favored by the available data [21]. Unfortunately this makes it even harder to observationally distinguish the scalar field contribution from the pure cosmological constant case.

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