onsdag 26. oktober 2016 15.57

Problem 5

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}(\partial_\mu \Phi)^2 - \frac{1}{2} m_\Phi^2 \Phi^2 - \frac{\mu}{2} \Phi \phi^2$$

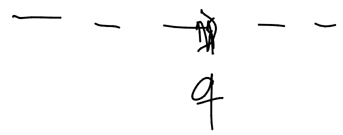
a)

Mass dimension of $\mathcal{L} = 4$ then

$$\left[\frac{\mu}{2} \Phi \phi^2\right] = 4 \Rightarrow [\mu] = \underline{\underline{1}}$$

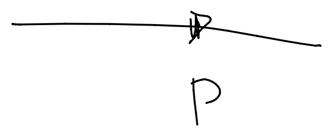
Feynman rules:

New Propagators $\quad \contraction{}{\phi}{}{\phi}\phi\phi$



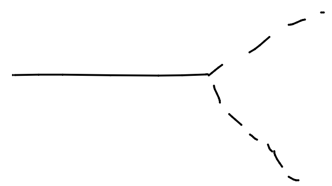
$$= \frac{i}{q^2 + i\varepsilon}$$

$\contraction{}{\Phi}{}{\Phi}\Phi\Phi$



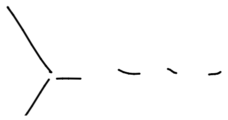
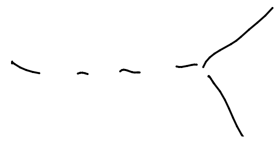
$$= \frac{i}{p^2 - m_\Phi^2 + i\varepsilon}$$

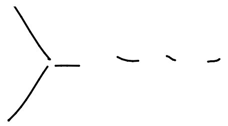
New vertex:



$$= \underline{-\frac{\mu}{2}}$$

External leg contractions

$\contraction{}{\phi}{|}{q}\phi|q\rangle =$  $= 1$, $\contraction{\langle}{q}{|}{\phi}\langle q|\phi =$  $= 1$

$\phi | q\rangle =$  $= 1$, $\langle q | \phi =$  $= 1$

b) decay rate

$$\Gamma = \frac{1}{2m_\Phi} d\Pi_2 \quad |\mathcal{M}(\Phi \to 2\phi)|^2$$

$$d\Pi_2 = \frac{d^3p_1}{(2\pi)^3}\frac{1}{2E_1}\frac{d^3p_2}{(2\pi)^3}\frac{1}{2E_2}(2\pi)^4 \underbrace{\delta^{(4)}(p_\Phi - p_1 - p_2)}_{\delta(E_{cm} - E_1 - E_2)\,\delta^{(3)}(p_\Phi - p_1 - p_2)}$$

$$= \frac{d^3p_1}{(2\pi)^2}\frac{1}{4E_1E_2}$$

delta functions impose $p_1 = -p_2$

the invariant matrix element becomes

$$i\mathcal{M} = -i\left(\frac{\mu}{2} + \frac{\mu}{2}\right) = -i\mu$$

$$\Rightarrow |\mathcal{M}|^2 = \mu^2$$

$$\Rightarrow |M|^2 = \mu^2$$

$$\Rightarrow \Gamma = \frac{1}{2m_\Phi} \frac{d^3 p_1}{(2\pi)^2} \frac{\mu^2}{4E_1 E_2}$$

$E_1 = |p_1| = |p_2| = E_2$

$E_1 + E_2 = m_\Phi$

$$\Gamma = \frac{\mu^2}{16\pi^2 m_\Phi} d\Omega \frac{p_1^2}{|p_1^2|}$$

$$= \frac{\mu^2}{16\pi^2 m_\Phi} d\Omega$$

$$= \frac{\mu^2}{16\pi^2 m_\Phi} \int_0^{\pi} \int_0^{2\pi} d\phi \, d\theta$$

$$= \frac{\mu^2}{8 m_\Phi}$$