UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam in: Relativistic quantum field theory (FYS4170)

Day of exam: December 2, 2015

Exam hours: 4 hours

This examination paper consists of 4 pages. (including the title page)

Appendices: none

Permitted materials: 3 A4 pages (two-sided) with own notes.

Make sure that your copy of this examination paper is complete before answering.

Final exam

Lecture autumn 2015: Relativistic quantum field theory (FYS4170)

$\rightsquigarrow$ *Carefully read* all *questions before you start to answer them! Note that you don't have to answer the questions in the order presented here, so try to answer those first that you feel most sure about (even for a very good grade, you will not be required to have attempted all questions!). Keep your descriptions as short and concise as possible! Answers given in English are preferred – but feel free to write in Scandinavian if you struggle with formulations! Maximal number of available points: 60.*

Good luck!

Problem 1 *(5 points)*
Simplify the following expressions involving Dirac gamma matrices (assuming that the momentum p is on shell, i.e. that it belongs to a physical particle with mass m):

a) $\not{p}\not{p}$

b) $\mathrm{Tr}[(\not{p}+m)P_R(\not{p}-m)P_L]\,,$

c) $\mathrm{Tr}[(\not{p}+m)P_R(\not{p}-m)P_R]\,,$

d) $\gamma^\mu\gamma^\rho\gamma^\sigma\gamma_\mu$

where $P_{R,L} = (1 \pm \gamma^5)/2$.

Problem 2
Consider the following Lagrangian describing two real scalar fields ϕ and Φ:

$$\begin{aligned}\mathcal{L} \;=\;& \frac{1}{2}\left(\partial_\mu\phi\right)^2 - \frac{1}{2}m^2\phi^2 + \frac{1}{2}\left(\partial_\mu\Phi\right)^2 - \frac{1}{2}M^2\Phi^2 \\ & -\lambda M\Phi\phi^2 - \frac{2\lambda}{M}\phi\left(\partial_\mu\phi\right)\left(\partial^\mu\Phi\right) \\ & -\frac{1}{2}\lambda^2\phi^4 - \frac{2\lambda^2}{M^2}\phi^2\left(\partial_\mu\phi\right)\left(\partial^\mu\phi\right)\end{aligned}$$

a) Write down the Feynman rules for propagators and vertices in this theory, using dotted lines for the ϕ fields and dashed lines for the Φ fields. *(6 points)*

b) Draw all Feynman diagrams that contribute at lowest order in λ to the process $\phi\phi \to \phi\phi$, and write down the corresponding amplitudes. Compute the differential cross section $d\sigma/d\Omega$, in the center-of-mass frame, in terms of Mandelstam variables. *(8 points)*

c) Assuming $M > 2m$ (why?), compute the decay rate of Φ at tree-level. *(3 points)*

d) Identify a discrete symmetry in $\mathcal{L}$. What does this imply for the decay rate of ϕ (assuming this time that $m > 2M$)? *(3 points)*

Problem 3

Consider the following Lagrangian:

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + i\overline{\psi}\not{D}P_R\psi + |D_\mu\phi|^2 - V(|\phi|),$$

where $F_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu$ and ϕ is a *complex* scalar field with potential V. The scalar ϕ and the fermion ψ may have different charges, such that the covariant derivative is given by $D_\mu = \partial_\mu + igQ_{\psi,\phi}B_\mu$. (For the definition of P_R, see problem 1).

a) State the field transformations under which this Lagrangian is symmetric! Which of those symmetries forbids a mass term for the 'photon' B_μ, and which for the fermion ψ? *(5 points)*

b) What are the Feynman rules for *i)* the coupling between the 'photon' field B and the scalar field ϕ and *ii)* the coupling between B and the fermion field ψ? *(6 points)*

c) Now assume that $V(|\phi|)$ is chosen such that the scalar field acquires a non-vanishing vacuum expectation value $v \equiv \langle\phi\rangle \neq 0$. How does the field content of the theory change, compared to a situation with $V(|\phi|) = m^2|\phi|^2$, and what is this mechanism referred to? Why is there no Goldstone boson? Show explicitly which mass the 'photon' acquires in this way! Can the fermion also acquire a mass (possibly by extending the above Lagrangian) ? *(5 points)*

d) For the situation described in c), compute the unpolarized transition amplitude $\overline{|M|}^2$ for the process $\overline{\psi}\psi \to \phi\phi$! *(4 points)*

Problem 4

The *Yang-Mills* Lagrangian for a general gauge group G is given by

$$\mathcal{L}_{\rm YM} = -\frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \overline{\psi}\left(i\not{D}\right)\psi - m\overline{\psi}\psi\,.$$

a) What kind of fields does this Lagrangian describe? How do the involved fields transform under infinitesimal gauge transformations? *(2 points)*

b) How are the quantities $F^a_{\mu\nu}$ and D_μ defined that appear above? *(2 points)*

c) Which of the terms in $\mathcal{L}_{\mathrm{YM}}$ does not appear in the standard model? Why? *(3 points)*

d) Briefly describe the main steps that led to the construction of the Yang-Mills Lagrangian! *(4 points)*

e) The *Faddeev-Popov* Lagrangian results from a particular way of fixing the gauge and is given by $\mathcal{L}_{\mathrm{FP}} = \mathcal{L}_{\mathrm{YM}} - \frac{1}{2\xi}\left(\partial^\mu A^a_\mu\right)^2 - \bar{c}\,(\partial^\mu D_\mu)\,c$. What is the problem with the Yang-Mills Lagrangian that prompted Faddeev and Popov to carry out the procedure leading to these additional two terms? What is the physical significance of ξ and c appearing here? Is D_μ given by the same expression as in the Yang-Mills Lagrangian (and why?)? *(4 points)*