Exercise 1

fredag 23. september 2016 11.02

2) We have $\rho = \rho_0 + \delta\rho$, $p = p_0 + \delta p$

$\phi = \phi_0 + \delta\phi$, $\vec{v} = \vec{v}_0 + \delta\vec{v}$

Inserting to Euler eq.:

$$\frac{d(\vec{v}_0 + \delta\vec{v})}{dt} = -\frac{1}{\rho_0+\delta\rho}\left(\nabla(p_0+\delta p)\right) - \nabla(\phi_0+\delta\phi)$$

$$\frac{\partial}{\partial t}(\vec{v}_0 + \delta\vec{v}) + (\vec{v}_0 + \delta\vec{v})\nabla(\vec{v}_0 + \delta\vec{v}) = -\frac{1}{\rho_0+\delta\rho}(\nabla p_0 + \nabla\delta p) - \nabla\phi_0 - \nabla\delta\phi$$

$$\underline{\frac{\partial}{\partial t}\vec{v}_0 + \vec{v}_0\nabla\vec{v}_0} + \frac{\partial}{\partial t}\delta\vec{v} + \vec{v}_0\nabla\cdot\delta\vec{v} + \delta\vec{v}\,\nabla\cdot\vec{v} = -\frac{1}{\rho_0+\delta\rho}(\underline{\nabla p_0} + \nabla\delta p) - \underline{\nabla\phi_0} - \nabla\delta\phi$$

Expanding $\frac{1}{\rho_0+\delta\rho}$ in $\delta\rho$, only including first order terms give $\frac{1}{\rho_0+\delta\rho} \approx \frac{1}{\rho_0}$. Also $\nabla\cdot\delta\vec{v} = 0$, since there is no prefered direction. Then the blue underlines indicate the unperturbed Euler eq. and equal zero.

Thus

$$\boxed{\frac{\partial}{\partial t}\delta\vec{v} + \delta\vec{v}\,\nabla\cdot\vec{v}_0 = -\frac{1}{\rho_0}\nabla\delta p - \nabla\delta\phi}$$

Now we insert into the Poisson eq.:

$$\nabla^2(\phi_0 + \delta\phi) = 4\pi G(\rho_0 + \delta\rho)$$

$$\underline{\nabla^2\phi_0} + \nabla^2\delta\phi = 4\pi G(\underline{\rho_0} + \delta\rho)$$

Again the blue underline indicets the unperturbed equation and equals zero, giving:

$$\boxed{\nabla^2\delta\phi = 4\pi G\,\delta\rho}$$