

FYS4160-2017

Problem sheet 8

NOTE: These notes are complementary to the discussion in the exercise sessions and are thus very condensed. If you want to discuss parts in more detail, please use our platform PIAZZA. If you spot some typos or mistakes, please email to *magdalena.kersting@fys.uio.no*.

Problem 28: Lagrangian for a scalar field

Let us define

$$\hat{\mathcal{L}} = \mathcal{L}/\sqrt{-g} = -\frac{1}{2}(\nabla^\mu\phi)(\nabla_\mu\phi) - V(\phi). \quad (1)$$

To derive the stress-energy tensor, we follow equations 4.75 - 4.79 in the book and get:

$$T^{\mu\nu} = \nabla^\mu\phi\nabla^\nu\phi + g^{\mu\nu}\hat{\mathcal{L}} \quad (2)$$

For proving it to be covariantly conserved we need the following:

- $\nabla^\mu\phi = \partial^\mu\phi$ and $\nabla^\mu V(\phi) = \partial^\mu V(\phi)$, because they are scalar quantities.
- $\nabla^\mu\nabla_\mu\phi = \frac{dV(\phi)}{d\phi}$, is the equation of motion for the scalar field (see eqn 4.53)
- $\nabla_\rho g^{\mu\nu} = 0$, because of metric compatibility

Using this, we see that

$$\begin{aligned} \nabla_\mu T^{\mu\nu} &= \nabla_\mu(\nabla^\mu\phi\nabla^\nu\phi) - g^{\mu\nu}\nabla_\mu\left(\frac{1}{2}(\nabla^\rho\phi)(\nabla_\rho\phi) + V(\phi)\right) \\ &= \partial^\mu\phi\nabla_\mu\partial^\nu\phi + \partial^\nu\phi\nabla_\mu\partial^\mu\phi - \nabla^\nu\left(\frac{1}{2}(\nabla^\rho\phi)(\nabla_\rho\phi) + V(\phi)\right) \\ &= \partial^\mu\phi\nabla_\mu\partial^\nu\phi + (\partial^\nu\phi)\frac{dV(\phi)}{d\phi} - \frac{1}{2}\partial^\rho\phi\nabla^\nu\partial_\rho\phi - \frac{1}{2}\partial_\rho\phi\nabla^\nu\partial^\rho\phi - \partial^\nu V(\phi) \\ &= 0 \end{aligned} \quad (3)$$

In the third line we have used the equation of motion and renaming the indices you can see that the first term cancels with the third and fourth and the second term cancels the last.

Problem 29: Kepler problem

This is a standard problem in classical mechanics, compare for example to Arnold's Mathematical Methods of Classical Mechanics or have a look at PIAZZA where we will collect useful online resources.