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Quintessence and Supergravity

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Abstract

In the context of quintessence, the concept of tracking solutions allows to address the fine-tuning and coincidence problems. When the field is on tracks today, one has $Q \approx m_{\rm Pl}$ demonstrating that, generically, any realistic model of quintessence must be based on supergravity. We construct the most simple model for which the scalar potential is positive. The scalar potential deduced from the supergravity model has the form $V(Q) = \frac{\Lambda^{4+\alpha}}{Q^\alpha} e^{\frac{\kappa}{2}Q^2}$. We show that despite the appearence of positive powers of the field, the coincidence problem is still solved. If $\alpha \geq 11$, the fine-tuning problem can be overcome. Moreover, due to the presence of the exponential term, the value of the equation of state, ω_Q, is pushed towards the value -1 in contrast to the usual case for which it is difficult to go beyond $\omega_Q \approx -0.7$. For $\Omega_{\rm m} \approx 0.3$, the model presented here predicts $\omega_Q \approx -0.82$. Finally, we establish the $\Omega_{\rm m} - \omega_Q$ relation for this model.

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Recent measurements of the relation between the luminous distance and the redshift using type Ia supernovae seem to suggest that our present Universe undergoes an accelerated expansion [1]. If confirmed, this means that our Universe is dominated by a type of matter with unusual properties. This matter would contribute by 70% to the total energy of the Universe, the remaining 30% being essentially Cold Dark Matter ensuring that the Universe is spatially flat, $\Omega_0 = 1$, in agreement with the standard inflationary scenario. The unusual features of this fluid dominating the total energy of the Universe reveal themselves in the equation of state. One usually assumes that it takes the form $p = \omega_Q \rho$ leading to a negative ω_Q. The cosmological constant ($\omega_Q = -1$) is a possible candidate but one has to face the task of explaining an energy scale of $\approx 5.7 h^2 \times 10^{-47}\mathrm{GeV}^4$, i.e. a value far from the natural scales of Particle Physics. Quintessence [2] is an alternative scenario with a homogeneous scalar field Q whose equation of state is such that $-1 \le \omega_Q \le 0$. It has been shown [2] that, using the concept of "tracking fields", the coincidence and the fine-tuning problems can be solved. The inverse power law potential $V(Q) = \Lambda^{\alpha+4}/Q^{\alpha}$ is the prototype of such models which possess remarkable properties.

We assume that the matter content of the Universe is composed of five different fluids: baryons, cold dark matter, photons, neutrinos and the quintessential field Q. The energy density of baryons and cold dark matter evolves as $\rho_{\mathrm{m}} = \rho_{\mathrm{c}} \Omega_{\mathrm{m}} (1+z)^3$ where z is the redshift and $\rho_{\mathrm{c}} = 8.1 h^2 \times 10^{-47}\mathrm{GeV}^4$ the present value of the critical energy density. Observations indicate that $\Omega_{\mathrm{m}} = \Omega_{\mathrm{b}} + \Omega_{\mathrm{cdm}} \approx 0.3$ [3, 4]. Photons and neutrinos have an energy density given by $\rho_{\mathrm{r}} = \rho_{\mathrm{c}} \Omega_{\mathrm{r}} (1+z)^4$. The contribution of radiation is negligible today since $\Omega_{\mathrm{r}} = \Omega_\gamma + \Omega_\nu \approx 10^{-4}$. Finally, the fifth component is the scalar field Q. Its equation of state is characterized by $\omega_Q = [\frac{1}{2}\dot{Q}^2 - V(Q)]/[\frac{1}{2}\dot{Q}^2 + V(Q)]$ where a dot represents a derivative with respect to cosmic time. A priori, ω_Q is not a constant and is such that $-1 \le \omega_Q \le 1$. Since the Universe is supposed to be spatially flat, we always have $\Omega_{\mathrm{m}} + \Omega_{\mathrm{r}} + \Omega_Q = 1$ which leads to $\Omega_Q \approx 0.7$. The inverse power law potential was first studied in Ref. [5]. If one requires that, during the radiation dominated era, the energy density of the scalar field is subdominant, i.e. $\rho_Q \ll \rho_{\mathrm{r}}$, and redshifts as $\rho_Q \propto a^{-4\alpha/(\alpha+2)}$ then one is automatically led to the inverse power law potential. This was the original motivation of Ref. [5] for considering this potential. It is possible to find an exact solution to the Klein Gordon equation for which $Q \propto a^{4/(\alpha+2)}$. One can show that this solution is an attractor [5]. Then, if one follows the behaviour of the scalar field during the matter dominated era with the same potential, one can show [5] that $Q \propto a^{3/(\alpha+2)}$ is an exact solution which is still an attractor. For this solution, one has $\rho_Q \propto a^{-3\alpha/(\alpha+2)}$. The previous results are equivalent to say that the attractor is given by:

$$\frac{\mathrm{d}^2 V(Q)}{\mathrm{d}Q^2} = \frac{9}{2}\frac{\alpha+1}{\alpha}(1-\omega_Q^2)H^2, \tag{1}$$

during both the radiation and matter dominated epochs. We can re-write the parameter ω_Q as $\omega_Q = (\alpha\omega_{\rm B} - 2)/(\alpha + 2)$ where $\omega_{\rm B}$ is either 1/3 or 0. Since ρ_Q redshifts slower than radiation or matter energy densities, the scalar field contribution becomes dominant at some stage of the evolution.

As shown in Refs. [2], this scenario possesses important advantages. Firstly, one can hope to avoid any fine-tuning. Indeed if the scalar field is on tracks today and begins to dominate and if, in addition, we require $\Omega_Q \approx 0.7$ then Eq. (1) says that $Q \approx m_{\rm Pl}$ which implies that $\Lambda \approx (\Omega_Q \rho_c m_{\rm Pl}^{\alpha})^{1/(4+\alpha)} \approx 10^{10}$GeV for $\alpha = 11$, a very reasonable scale from the High Energy Physics point of view (we take $h = 0.5$). Secondly, the solution will be on tracks today for a huge range of initial conditions including the equipartition for which $\Omega_{Qi} \approx 10^{-4}$. If one fixes the initial conditions at the end of inflation, i.e. $z \approx 10^{28}$, the allowed initial values for the energy density are such that $10^{-37}{\rm GeV}^4 \lesssim \rho_Q \lesssim 10^{61}{\rm GeV}^4$ where $10^{-37}{\rm GeV}^4$ is approximatively the background energy density at equality whereas $10^{61}{\rm GeV}^4$ represents the background energy density at the initial redshift. If the scalar field starts at rest, this means that $1.8 \times 10^{-10} m_{\rm Pl} \lesssim Q_{\rm i} \lesssim 0.16 m_{\rm Pl}$ initially. Thirdly, the value of ω_Q is automatically such that $-1 \le \omega_Q \le 0$ today. For example, if $\alpha = 11$ then one has $\omega_Q \approx -0.29$. This illustrates the fact that with inverse power law potentials, it is difficult to obtain values of ω_Q close to $\omega_Q = -1$. This shortcoming can be partially removed if one considers smaller values of α or more general potentials of the form $V(Q) = \sum_k c_k Q^{-k}, k > 0$[2]. However, it is not possible to reach a value lower than $\omega_Q \approx -0.7$. This seems in disagreement with recent estimates in Ref. [6]. Finally, there exists a relation $\Omega_{\rm m} - \omega_Q$ which only depends on the functional form of the potential. This relation could also be used as an observational test of quintessence.

This scenario raises the issue of the physical origin of the quintessential field. It is clear that this question should be adressed by the means of High Energy Physics [7, 8, 9, 10, 11]. Recently, an interesting supersymmetric (SUSY) model based upon the superpotential $W(Q) = \Lambda^{3+a}/Q^a$, where $\Lambda \ll m_{\rm Pl}$, leading to an inverse power law scalar potential has been proposed [12]. This model suffers from serious problems. Firstly, it seems difficult to understand how, in this model, one can have $Q > \Lambda$ which is mandatory at the end of the evolution as Λ is the UV cutoff at the gaugino condensation scale. Secondly, when the field is on tracks, one necessarily has $Q \approx m_{\rm Pl}$. One should take supergravity (SUGRA) into account. Let us emphasize that this is a generic property which comes from the very definition of a tracking solution. Therefore any model of quintessential tracking field coming from High Energy Physics must be based on SUGRA. In the context of the previous model, assuming that the Kähler potential is flat, $K(Q, Q^*) = Q^*Q$, one finds for the scalar potential:

$$V = e^{\frac{\kappa}{2}Q^2} \frac{\Lambda^{4+\alpha}}{Q^\alpha}\Big(\frac{(\alpha - 2)^2}{4} - (\alpha + 1)\frac{\kappa}{2}Q^2 + \frac{\kappa^2}{4}Q^4\Big), \tag{2}$$

where $\kappa \equiv 8\pi G/c^4$ and $\alpha = 2a + 2$. This example is typical of the difficulties that one encounters in more general situations. A serious problem arises due to the existence of negative contributions to the potential, a general property of any SUGRA model. These contributions entail that the energy density (and therefore the potential) becomes negative for $Q \approx m_{\rm Pl}$. We have studied numerically the case $\alpha = 11$ in more details. The appearence of negative contributions depends on the value of Λ. For example, if $\Lambda \approx 8.7 \times 10^{10}$GeV, this occurs at $z \approx 2.24$. It is possible to avoid this problem by changing the value of Λ. Indeed, there exist values of Λ such that the negative contributions do not show up. However, it is not possible to find a value such that $\Omega_Q \approx 0.7$. We have found that the best case is for $\Lambda \approx 2.1 \times 10^{10}$GeV for which $\Omega_Q \approx 0.055$ and $\omega_Q \approx -0.09$. This problem also occurs for other values of α.

The appearance of dangerous negative contributions to the potential is not accidental. The supergravity Lagrangian depends on two functions: the Kähler potential $K(\phi_i, \phi_i^*)$ governing the kinetic terms of the boson fields and the superpotential $W(\phi_i)$. The bosonic part of the Lagrangian can be derived from the following potential $G = \kappa K + \ln(\kappa^3 |W|^2)$. The kinetic terms are simply given by $K_{i\bar{\rm j}} \partial^\mu \phi^i \partial_\mu \bar{\phi}^{\bar{\rm j}}$ where $K_{i\bar{\rm j}} = \frac{\partial}{\partial \phi^{\rm i}} \frac{\partial}{\partial \bar{\phi}^{\bar{\rm j}}} K$. The scalar potential is obtained as:

$$V \equiv \frac{1}{\kappa^2} e^G (G^i G_i - 3) + V_{\rm D}, \tag{3}$$

where $V_{\rm D} \geq 0$ is a term coming from the gauge sector. The negative contribution stems from the -3 term in the potential. The most natural way out is to impose that the superpotential vanishes and that the scalar potential is entirely due to a non-flat Kähler potential. In this letter, we present a model where this can be achieved.

Let us consider a supergravity model where there are two types of fields, the quintessence field Q and charged matter fields (X, Y^i) under the gauge group. We assume that the gauge group of the model is broken along a flat direction of the D terms such that $X \neq 0, Y^i = 0$ where $V_{\rm D} = 0$. As already mentioned we impose that the scalar potential is positive to prevent any negative contribution to the energy density. This is achieved by considering that $\langle W \rangle = 0$ when evaluated along the flat direction. Moreover we assume that one of the gradients of the superpotential W_Y does not vanish. The scalar potential is given by Eq. (3) evaluated along the flat direction and becomes

$$V = e^{\kappa K} K^{Y\bar{Y}} |W_Y|^2. \tag{4}$$

As expected the scalar potential is positive and becomes a function of the quintessence field Q only.

As a guiding principle we now present a model which illustrates the quintessential property in SUGRA. We use string-inspired models with an anomalous $U(1)_X$ gauge symmetry [13]. We consider the case of type I string theories[14]. The case of the usual compactification of the weakly coupled heterotic string[15] is phenomenologically disfavoured as the resulting scalar potential shows an exponential dependence on Q [16]. We suppose that the gauge group factorises as $G\times U(1)_X$ where G contains the standard model gauge group and $U(1)_X$ is an anomalous Abelian symmetry. The fields of the model split into three groups: the field X has a charge 1 under $U(1)_X$ and is neutral under G, the field Y is a matter field neutral under G and of charge -2 under $U(1)_X$ while the matter fields Y_i are charged under G and possess charges $q_i \neq -2$ under $U(1)_X$. The matter fields Y_i are spectators and will be discarded in the following. We also assume that there is a modular symmetry $SL(2,Z)$ stemming from the moduli space of the string compactification. We take into account a single modulus t such that the radius of the compact manifold is $R_{\rm c}\equiv (t+\bar{t})l_{\rm S}$ measured in units of the string scale $l_{\rm S}$. We assign different modular weights to the fields, i.e. X is neutral, Y has a weight $n_Y = n/p$ and Q has a weight $n_Q = 1-1/p$ where n and p are integers. Associated to $U(1)_X$ is the D term potential:

$$V_{\rm D} = \frac{g_X^2}{2}\left(K^X X - 2K^Y Y + \sum_i q_i K^{Y_i} Y_i - \xi^2\right)^2, \tag{5}$$

where g_X is the $U(1)_X$ gauge coupling and ξ is a Fayet-Iliopoulos term. The Kähler potential of the effective supergravity theory describing the string theory at low energy is a function of the different fields Q, X and Y as well as the UV cutoff $m_{\rm c} = 1/R_{\rm c}$. A compatible choice with the gauge and modular symmetries is

$$K = -\frac{1}{\kappa}\ln(t+\bar{t}) + XX^* + \frac{(QQ^*)^p}{m_{\rm c}^{2p-2}} + |Y|^2\frac{(QQ^*)^n}{m_{\rm c}^{2n}}. \tag{6}$$

The curvature along the flat direction possesses a delta function singularity at the origin. The D term potential vanishes altogether along a flat direction where the field X acquires a vacuum expectation value breaking the Abelian symmetry $U(1)_X$ at $\langle X\rangle = \xi$, while the other fields vanish altogether. Expanding the superpotential in terms of Yukawa couplings, we obtain $W = \lambda(t)X^2Y + \ldots$, where we have only taken into account the couplings such that $W_Y = \lambda(t)\langle X\rangle^2 \neq 0$ along the flat direction. The Yukawa coupling $\lambda(t)$ is a modular form of weight $-n/p-1$. Considering $Q = Q^*$, the scalar potential of this supergravity model is given by

$$V(Q) = \frac{\Lambda^{4+\alpha}}{Q^\alpha} e^{\frac{\kappa}{2}Q^2}. \tag{7}$$

In this equation, the field Q has been redefined such that the kinetic term takes the canonical form. The parameter Λ can be expressed in terms of $m_{\rm c}$ according to $\Lambda^{4+\alpha} = 2^{\frac{\alpha}{2}}\lambda^2(t+\bar{t})^{-1}\xi^4 m_{\rm c}^\alpha$ where $\alpha = 2n_Y$. In the type I string theories the Fayet-Iliopoulos term ξ is moduli dependent [17]. Using the relation $m_{\rm Pl}^2 \approx m_{\rm S}^8 m_{\rm c}^{-6}$ for a six-dimensional compact space [18] and demanding $\lambda \approx 1$ to avoid any fine-tuning of the coupling constant, we obtain $\xi \approx (m_{\rm Pl}/m_{\rm c})^{\alpha/4}(\Omega_Q\rho_{\rm c})^{1/4}$. Imposing that the UV cutoff is $m_{\rm c}\approx 10^{14}$GeV in order to be compatible with the fact that the evolution starts at the end of inflation and $\xi > 10^2$GeV as the extra $U(1)_X$

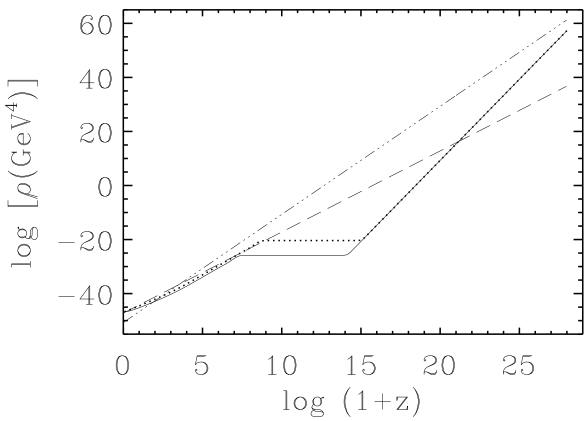

Figure 1: Evolution of the different energy densities. The dashed-dotted line represents the energy density of radiation whereas the dashed line represents the energy density of matter. The solid line is the energy density of quintessence in the SUGRA model with $\alpha = 11$. The dotted line is the energy density of quintessence for the potential $V(Q) = \Lambda^{4+\alpha}Q^{-\alpha}$ with the same α. The initial conditions are such that equipartition, i.e. $\Omega_{Qi} \approx 10^{-4}$, is realized just after inflation.

symmetry has to be broken above the weak interaction scale, we deduce that $\alpha \geq 11$ leading to $\Lambda \gtrsim 10^{10}$GeV. The string scale is given by $m_{\rm S} \gtrsim 10^{15}$GeV. The fine tuning problem can be avoided by allowing a sufficiently large value of α.

Let us investigate the properties of the potential given by Eq. (7) for $\alpha = 11$. First of all, the energy scale is now $\Lambda \approx 3.8 \times 10^{10}$GeV. Secondly, the presence of the exponential factor implies that this potential possesses arbitrary positive powers of Q. This could destroy the nice properties of the tracking solutions [2]. However, it is clear that this factor will play a role only at small redshifts since, initially, the value of the field is very small in comparison with the Planck mass. One expects a modification only at the very end of the evolution. We have checked numerically that the insensitivity to the initial conditions is totally preserved for the SUGRA potential. The initial value of ρ_Q can change by 100 orders of magnitude, the final result is always the same as in the usual case. The corresponding values for the quintessence field are now such that $5.8\times 10^{-11}m_{\rm Pl} \lesssim Q_{\rm i} \lesssim 0.05 m_{\rm Pl}$. The evolution is displayed in Fig. 1 and is very similar to the evolution already found in Refs. [2]. Let us now study the evolution of the equation of state. The presence of the exponential factor is crucial. It has the effect of reinforcing the potential energy in comparison to the kinetic one and

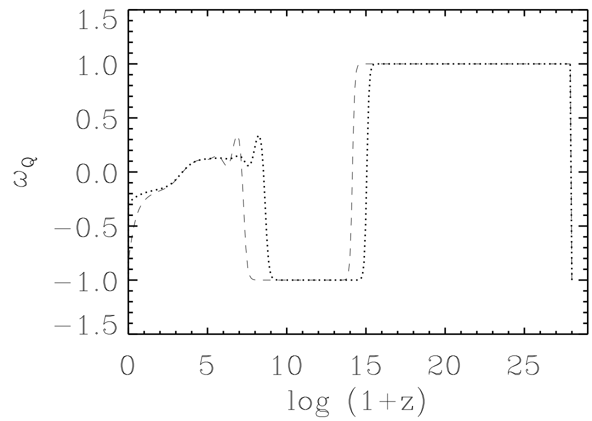

Figure 2: The dotted line represents the evolution of ω_Q for the potential $V(Q) = \Lambda^{4+\alpha}Q^{-\alpha}$ with $\alpha = 11$. The dashed line represents the evolution of ω_Q in the SUGRA model for the same value of α. In this case $\omega_Q \approx -0.82$ today.

to push ω_Q towards the value -1 (recall that if the kinetic energy vanishes then $\omega_Q = -1$). This behaviour is illustrated in Fig. 2. For $\alpha = 11$, the final value of ω_Q is -0.82 whereas it is $\omega_Q \approx -0.29$ for the potential $V(Q) = \Lambda^{4+\alpha}Q^{-\alpha}$. In this last case, the value of ω_Q changes from $\omega_Q \approx -0.63$ to $\omega_Q \approx -0.29$ when α goes from 2 to 11 at fixed $\Omega_Q = 0.7$. In the SUGRA model, ω_Q changes from $\omega_Q \approx -0.89$ to $\omega_Q \approx -0.82$ in the same conditions; ω_Q does not strongly depend on α. These properties render the SUGRA potential more attractive than the tracking solutions of Refs. [2] for which w_Q cannot be less than ≈ -0.7 (see Fig. 7 of that reference). In Ref. [4], the constraint $-1 \le \omega_Q \le -0.6$ is given whereas in Refs. [6, 19] a value between -1 and -0.8 is favoured. According to Ref. [6], $\omega_Q \approx -0.82$ is less than 1σ from the likelihood value. If the latter turns out to be confirmed the SUGRA model presented here could account for this. Finally let us study the $\Omega_{\mathrm{m}} - \omega_Q$ relation which is displayed in Fig. 3. The parameter ω_Q now varies between -0.22 and -0.995. If $\Omega_{\mathrm{m}} \approx 0.25$ instead of 0.3 then $\omega_Q \approx -0.86$. The curve $\Omega_{\mathrm{m}} - \omega_Q$ has almost no dependence on α. This is due to the fact that the value of ω_Q is mainly determined by the exponential factor which is α independent. In order to illustrate this property, the curve $\omega_Q - \alpha$ is displayed in Fig. 4. In this sense, the curve $\Omega_{\mathrm{m}} - \omega_Q$ presented here should be typical of any model based on SUGRA.

In conclusion, we would like to emphasize the generic character of the results found in this letter. Nice properties arise (possibility to avoid the fine tuning problem, insensitivity to the initial conditions) if the quintessence field is on tracks today. This means that $Q \approx m_{\mathrm{Pl}}$ now and SUGRA must be taken into

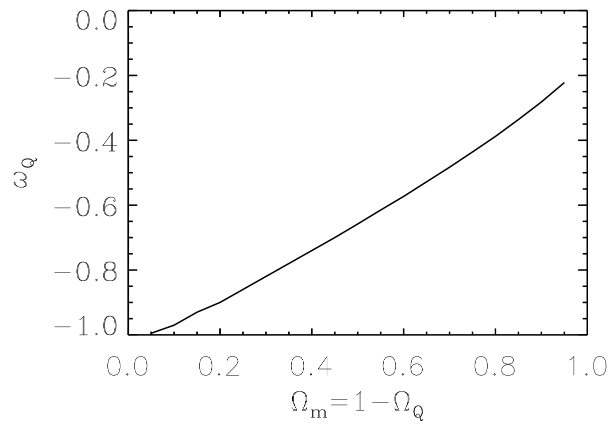

Figure 3: $\Omega_{\rm m} - \omega_Q$ relation for the SUGRA potential given by $V(Q) = \Lambda^{4+\alpha}Q^{-\alpha}e^{\kappa Q^2/2}$ with $\alpha = 11$.

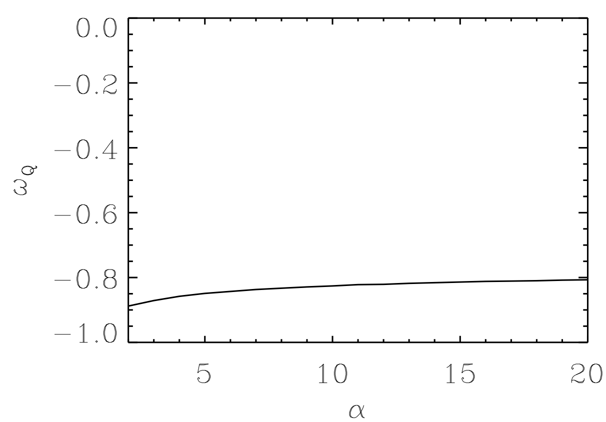

Figure 4: $\omega_Q - \alpha$ relation for the SUGRA potential.

account if one wishes to construct a realistic model. As a consequence, the scalar potential possesses an exponential factor which pushes ω_Q towards the value -1. We have constructed explicitly a natural model in this context. If $\alpha \geq 11$ then the fine tuning problem can be overcome. In this simple case, we find that $\omega_Q \approx -0.82$ if $\Omega_{\rm m} \approx 0.3$.

Finally let us comment on the supersymmetry breaking issue. When SUGRA is broken by the dilatonic F term the model is not modified whereas in the case where it is broken by the moduli F term the coupling between Q and t induces an inverse power law potential. More studies are required in the moduli breaking scenario. This question will be adressed elsewhere [20].

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