onsdag 21. september 2016 18.23

Ex 4

Parametrisation:

$$R = A(1-\cos\theta)$$

$$t = B(\theta - \sin\theta)$$

$$A^3 = GMB^2$$

Show that this satisfy

$$\ddot{R} = -\frac{GM}{R^2}$$

$$\frac{dt}{d\theta} = B(1-\cos\theta) \Rightarrow \left(\frac{1}{d\theta} = \frac{B(1-\cos\theta)}{dt}\right)^2$$

$$\frac{d^2t}{d\theta^2} = -B\sin\theta \qquad \neq \qquad B(1-\cos\theta)^2$$

$$= B(1+\cos^2\theta - 2\cos\theta)$$

$$\frac{1}{d^2t} = -\frac{1}{B\sin\theta\, d\theta^2}$$

$$\frac{d^2R}{dt^2} = -\frac{1}{B\sin\theta}\frac{d^2R}{d\theta^2}$$

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$$= -\frac{1}{B\sin\theta}\frac{d}{d\theta}(A\sin\theta)$$

$$= -\frac{A}{B\tan\theta}$$

nah

try bruteforcing it using $\frac{1}{dt} = \frac{1}{B(1-\cos\theta)\,d\theta}$

$$\frac{d}{dt}\left(\frac{1}{B(1-\cos\theta)}A\sin\theta\right)$$

$$= \frac{A}{B^2(1-\cos\theta)}\frac{d}{d\theta}\left[\frac{\sin\theta}{(1-\cos\theta)}\right]$$

$$= \frac{A}{B^2(1-\cos\theta)}\frac{1}{(\cos\theta-1)}$$

$$= \frac{A}{B^2(1-\cos\theta)}\left(-\frac{1}{1-\cos\theta}\right)$$

$$= -\frac{A}{B^2(1-\cos\theta)^2}$$

$$= \frac{A}{B^2(1-\cos\theta)}\left(-\frac{1}{1-\cos\theta}\right)$$

$$= -\frac{A}{B^2(1-\cos\theta)^2}$$

$$= -\frac{A}{\frac{A^3}{GM}(1-\cos\theta)^2}$$

$$= -\frac{GM}{\left(A(1-\cos\theta)\right)^2}$$

$$= -\frac{GM}{R^2} \quad \square \quad \checkmark$$

$$A = \frac{R}{(1-\cos\theta)}$$

$$\Rightarrow \quad \ddot{R} = -\frac{GM}{R^2}\left(1+\frac{\sin\theta}{1-\cos\theta}\right)$$