

Exercise 1

$$1) \quad \vec{v} = H \vec{r} \quad \left(H = \frac{\dot{a}}{a} \right)$$

$$\frac{d\rho}{dt} = -\rho \nabla \cdot \vec{v}$$

$$\frac{\partial \rho}{\partial t} + \vec{v} \cdot \nabla \rho = -\rho \nabla \cdot \vec{v}$$

$$\frac{\partial \rho}{\partial t} + H \vec{r} \cdot \nabla \rho = -\rho \nabla \cdot H \vec{r}$$

$$\dot{\rho} = \rho \nabla \cdot (H \vec{r}) - H \vec{r} \cdot \nabla \rho$$

$$= \rho \nabla \cdot \frac{\dot{a}}{a} \vec{r} - \frac{\dot{a}}{a} \vec{r} \cdot (\nabla \rho) = 0?$$

$$= - \nabla \rho \cdot \frac{\dot{a}}{a} \vec{r}$$

$$= - \frac{\dot{a}}{a} \nabla \cdot \rho \vec{r}$$

$$= - \frac{\dot{a}}{a} \nabla \cdot \rho \vec{r}$$

$$d\rho = - \frac{da}{a} \nabla \cdot \rho \vec{r} = - \frac{da}{a} (\rho \nabla \cdot \vec{r} + \vec{r} \cdot \nabla \rho)$$

$$= - \frac{da}{a} \rho 3$$

$$\ln \rho \Big|_{t=t_0}^t = -3 \ln a$$

$$\ln \frac{\rho(t)}{\rho(t_0)} = -3 \ln a$$

$$\frac{\rho(t)}{\rho(t_0)} = a^{-3}$$

$$\boxed{\rho(t) = \rho(t_0) a^{-3}}$$

if $\nabla \rho = 0$

this is valid, but

I don't see why it should...