Ex 2

fredag 23. september 2016 11.25

1) i) $\Omega_m = 1$

$$\frac{\dot a}{a} = H_0 a^{-3/2}$$

$$\sqrt{a}\,\dot a = H_0$$

$$\sqrt{a}\,da = H_0\,dt$$

$$\frac{2}{3}a^{3/2} = H_0 t$$

$$a = \left(\frac{3H_0 t}{2}\right)^{2/3}$$

$$\dot a = \left(\frac{3H_0}{2}\right)^{2/3}\frac{d}{dt}\left(t^{2/3}\right)$$

$$= \left(\frac{3}{2}H_0\right)^{2/3}\frac{2}{3}t^{-1/3}$$

$$\frac{\dot a}{a} = \left[\left(\frac{3}{2}\cancel{H_0}\right)^{2/3}\frac{2}{3}t^{-1/3}\right] \Big/ \left[\frac{3\cancel{H_0}t}{2}\right]^{2/3}$$

$$\boxed{\frac{\dot a}{a} = \frac{2}{3}t^{-1}}$$

$$H = H_0\sqrt{\Omega_m a^{-3} + \Omega_\Lambda + \Omega_r a^{-4}}$$

2) $$\frac{d^2\delta}{dt^2} + \frac{\dot a}{a}\frac{d\delta}{dt} = \delta\alpha \qquad \alpha = 4\pi G\rho_0 - k^2 c_s^2$$

$x = \ln a$, $y = \ln\delta$ $\qquad a = e^x$

$x = \ln a \quad , \quad y = \ln$ $\quad a = e$

$$\frac{d\delta}{dt} = \frac{d\delta}{dx}\frac{dx}{da}\frac{da}{dt}$$

$$= \frac{d\delta}{dx}\frac{1}{a}\dot{a}$$

$$= H\delta'$$

$$\frac{d}{dt}H\delta' = \delta'\frac{dH}{dt} + H\frac{d\delta'}{dt}$$

$$= \delta'\frac{d}{dt}\left(\frac{\dot{a}}{a}\right) + H\frac{d}{dt}\frac{d\delta}{dx}$$

$$= \delta'\frac{\ddot{a}a - \dot{a}^2}{a^2} + H\underbrace{\frac{d^2\delta}{dt\,dx}}$$

$$= \frac{d}{dx}\frac{d\delta}{dt}$$

$$= \frac{d}{dx}H\delta'$$

$$= \frac{d}{dx}\frac{\dot{a}}{a}\delta'$$

$$= \frac{d}{dx}\frac{\dot{a}}{e^x}\delta'$$

$$= \delta'\frac{d}{dx}\frac{\dot{a}}{e^x} + \frac{\dot{a}}{a}\delta''$$

$$= \delta'\underbrace{\frac{d}{dx}\left(\dot{a}e^{-x}\right)} + H\delta''$$

$$\frac{d}{dx}\left(\frac{da}{dt}e^{-x}\right) = a\frac{d}{dx}\dot{a} + \dot{a}\frac{da}{dx}$$

$d \quad 1 \quad 2\dot{a}) \; a\frac{da}{dt} + \dot{a}\frac{da}{dx}$

$$= \frac{d}{dt}\frac{1}{a^2} = -\frac{2\dot a}{a^3}\Big) \quad a\frac{da}{dx\,dt} + \dot a\frac{da}{dx}$$

$$= a\dot a + H$$

$$\boxed{\frac{d^2\delta}{dt^2} = -\delta'\left(\frac{\ddot a}{a} - (H)^2\right) - \delta'(a\dot a + H) + H^2\delta''}$$

$$\boxed{\frac{d\delta}{dt} = H\delta'}$$

$$\ddot a = \frac{d}{dt}\left(H_0 a\sqrt{\Omega_m a^{-3} + \Omega_\Lambda}\right)$$

$$= H_0\frac{d}{dt}\sqrt{\Omega_m a^{-1} + a^2\Omega_\Lambda} = H_0\frac{d}{da}\frac{da}{dt}\sqrt{\dots}$$

$$= H_0\,\dot a\,\frac{d}{da}\sqrt{\Omega_m a^{-1} + a^2\Omega_\Lambda}$$

$$= H_0\cancel{\dot a}\, a^{-1}\frac{1}{2}\left(-\Omega_m a^{-2} + 2a\Omega_\Lambda\right)$$

$$\ddot a = H_0 a\left(\Omega_\Lambda - \frac{1}{2}\Omega_m a^{-3}\right)$$

$$\frac{d^2\delta}{dt^2} = -\delta'\left(H_0\left(\Omega_\Lambda - \frac{1}{2}\Omega\quad a^{-3}\right) - H_0^2\left(\Omega_m a^{-3} + \Omega_\Lambda\right)\right) - \delta'(a\dot a + H) + H^2\delta''$$

$$H^2\delta'' - \delta'\left[H_0\left(\Omega_\Lambda - \frac{1}{2}\Omega_m a^{-3}\right) - H_0^2\left(\Omega_m a^{-3} + \Omega_\Lambda\right)\right] - \delta'(a\dot a + H) + 2H^2\delta' = \delta\alpha$$

$$H^2\delta'' - \delta'\left(H_0\left(\Omega_\Lambda - \frac{1}{2}\Omega_m a\right) - H_0\left(\Omega_m a + \Omega_\Lambda\right)\right) \ldots$$

$$H^2\delta'' - \delta'\left(H_0\left(\Omega_\Lambda - \frac{1}{2}\Omega_m a^{-3}\right) - 3H^2 + a^2 H + H\right) = \delta\alpha$$

$$\delta'' - \delta'\left(\frac{H_0}{H^2}\left(\Omega_\Lambda - \frac{1}{2}\Omega_m a^{-3}\right) - 3 + \frac{a^2}{H} + H^{-1}\right) = \delta\alpha$$

We did not manage the computational part due to bad time management, therefore we could not finish exercise 2.3.