fredag 23. september 2016 09.41

$$e_1 = \pm v \qquad v = \sqrt{\sum_{i=2}^{n} a_{1i}^2}$$

$$\vec{v} = e_1 \vec{e} = 2\vec{u}(\vec{u}^T \vec{v})$$

$$4(\vec{u}^T \vec{u})(\vec{u}^T \vec{v})^2 = 2(v^2 - e_1 a_{12})$$

$$2(u^T v)^2 = (v^2 \pm a_{12} v)$$

$$\Rightarrow \quad \vec{u} = \frac{\vec{v} - e_1 \vec{e}}{2(u^T v)}$$

can we get the eigenvalues from an LU decomp?

(Hint: Triangular Matrices)

(Matrices $A \in \mathbb{R}^{n \times n}$ |

Lanczos' algo: (Matrices $A \in \mathbb{R}^{n \times n}$ with $n > 10^5$)

<u>Iterative algo:</u>

Basis $\qquad T = S^T A S$

Solved iteratively with a start guess

$$S \cdot \vec{e}_1 = \vec{s}_1 \qquad S = [\vec{s}_1, \vec{s}_2, \dots \vec{s}_n]$$

$$S^T S = \mathbb{1} \Rightarrow s_k^T s_l = \delta_{kl}$$

$$T = \begin{bmatrix} \alpha_1 & \beta_1 & & \\ \beta_1 & \alpha_2 & \beta_2 & \\ & \beta & \alpha & \ddots \end{bmatrix}$$

$$
\begin{bmatrix}
\beta_1 & \ddots & & & \\
& \beta_2 & \alpha_3 & \ddots & \\
& & \ddots & \ddots & \ddots \\
& & \beta_{n-2} & \alpha_{n-1} & \beta_{n-1} \\
& & & \beta_{n-1} & \alpha_n
\end{bmatrix}
$$

$$S S^{\dagger} = \mathbb{1}$$

$$T = S^{\dagger} A S \Rightarrow S T = A S$$

equate collumns;

$$A [s_1 \quad s_2 \quad \cdots \quad s_n]$$

$$A \vec{S}_k = \beta_{k-1} \vec{S}_{k-1} + \alpha_k \vec{S}_k + \beta_k \vec{S}_{k+1}$$

$$\Rightarrow \quad s_k^T = s_k^+ A s_k = \alpha_k$$

Def.

$$\vec{r}_k = (A - \alpha_k I)\, \vec{s}_k - \beta_{k-1} \vec{s}_{k-1}$$

$$s_{k+1} = \frac{\vec{r}_k}{\beta_k} \qquad \beta_k = \pm \| a_k \|_2$$

Algo

$\vec{r}_0 = \vec{s}_1 \qquad \beta_0 = 0 \qquad \vec{s}_0 = 0 \qquad k = 0$

while $(\beta_k \neq 0)$

$$s_{k+1} = \frac{\vec{r}_k}{\beta_k} \qquad k \mathrel{+}= 1$$

$$\alpha_k = s_k^T A s_k$$

$$\vec{r}_k = (A - \alpha_k I)\vec{s}_k - \beta_{k-1}\vec{s}_{k-1}$$

$$\beta_k = \|r_k\|_2$$

end