onsdag 26. oktober 2016 11.28

Problem 3

QED Lagrangian:

$$\mathcal{L} = \mathcal{L}_0 - eA_\mu \bar{\psi}\gamma^\mu \psi$$

note that $\mathcal{L}_0$ is invariant under C, P and T, so we need to check the qed part $eA_\mu \bar{\psi}\gamma^\mu \psi$ (p. 71 P&S)

Charge conjugation: $C^T\psi(x)C = -i\gamma^2\psi^*(x)$

$$\begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \phi \\ A_x \\ A_y \\ A_z \end{pmatrix}^T \cdot \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} -iA_z \\ iA_y \\ iA_x \\ -i\phi \end{pmatrix}$$

$$= \begin{pmatrix} -\phi \\ -A_x \\ -A_y \\ -A_z \end{pmatrix} \Rightarrow C^T A_\mu C = -A_\mu$$

So the electromagnetic potential switches sign under charge conjogation. Further

$$C\bar\psi\gamma^\mu\psi C = -\bar\psi\gamma^\mu\psi \qquad \left(= \left[i\gamma^0\gamma^2\psi\right]^T\gamma^\mu\left[-i\bar\psi\gamma^0\gamma^2\right]^T = -1\right)$$

Then the QED addition is invariant under charge conjogation!

Parity:

A_μ changes sign under parity

$$P e A_\mu \bar\psi\gamma^\mu\psi P = e\, P A_\mu P\, P\bar\psi P P\gamma^\mu P P \psi P$$

$$= -e A_\mu \begin{cases} \bar\psi\gamma^\mu\psi(t,-\vec{x}) & \text{for } \mu = 0 \\ -\bar\psi\gamma^\mu\psi(t,-\vec{x}) & \text{for } \mu = 1,2,3 \end{cases}$$

then the QED addition is invariant for $\mu = 1,2,3$ and changes signfor $\mu=0$.

Time reversal