

Ex 6

fredag 23. september 2016 10.47

$$dU = - \frac{G m_{\text{shell}} m_{\text{int}}}{r}$$

$$U = - G \int \frac{(4\pi r^2 \rho) \left(\frac{4\pi}{3} r^3 \rho\right)}{r} dr$$

$$= - G \frac{16\pi}{3} \rho^2 \int_0^R dr r^4$$

$$= - G \frac{16\pi}{3} \rho^2 \frac{R^5}{5}$$

$$= - G \frac{16\pi}{15} \rho^2 R^5$$

$$\rho = \frac{M}{\frac{4\pi}{3} R^3}$$

$$\Rightarrow U = - G \frac{16\pi}{3} \left(\frac{M}{\frac{4\pi}{3} R^3}\right)^2 \frac{R^5}{5}$$

$$U = - \frac{3 G M^2}{5 R}$$

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