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The SUGRA Quintessence Model Coupled to the MSSM

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Abstract: We study the cosmological evolution of the universe when quintessence is modeled within supergravity, supersymmetry is broken in a hidden sector, and we also include observable matter in a third independent sector. We find that the presence of hidden sector supersymmetry breaking leads to modifications of the quintessence potential. We focus on the coupling of the SUGRA quintessence model to the MSSM and investigate two possibilities. First one can preserve the form of the SUGRA potential provided the hidden sector dynamics is tuned. The currently available limits on the violations of the equivalence principle imply a universal bound on the vacuum expectation value of the quintessence field now, $\kappa^{1/2}Q \ll 1$. On the other hand, the hidden sector fields may be stabilised leading to a minimum of the quintessence potential where the quintessence field acquires a mass of the order of the gravitino mass, large enough to circumvent possible gravitational problems. However, the cosmological evolution of the quintessence field is affected by the presence of the minimum of the potential. The quintessence field settles down at the bottom of the potential very early in the history of the universe. Both at the background and the perturbation levels, the subsequent effect of the quintessence field is undistinguishable from a pure cosmological constant.

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1. Introduction

There is a host of observational evidence in favor of the existence of a non-zero vacuum energy density of the Universe driving the acceleration of the expansion of the Universe [1–3]. The simplest explanation for this new era in the history of the Universe is the presence of a cosmological constant of extraordinarily small value, some 120 orders of magnitude lower than the Planck scale. Such a small value is particularly difficult to accommodate when dynamical effects such as the Quantum Chromo-Dynamics (QCD) and electroweak phase transitions or even Grand Unified Theory (GUT) scale physics are taken into account. This has prompted the possibility of using extra dimensional models such as self–tuning scenarios [4] or brane induced gravity models [5]. Unfortunately these alternatives have drawbacks such as hidden fine–tunings [6]. Within four dimensional physics, there is an experimental way of discovering whether the vacuum energy is a true constant of nature or the result of more complicated dynamical effects. Indeed very active experimental programs are dedicated to the analysis of the so-called equation of state of the dark energy sector (the ratio between the pressure and the energy density). If the equation of state differs from -1 (and is greater than -1, otherwise see for instance Ref. [7]), then a plausible candidate for dark energy is quintessence [8–10], *i.e.* the dynamics of a scalar field rolling down a runaway potential. Of course, quintessence only accounts for the small and non-vanishing vacuum energy, it has nothing to say about the large cancellation of the overall cosmological constant.

One of the most stringent requirements imposed on quintessence models is the existence of attractors, *i.e.* long time stable solutions of the equations of motion [9]. Indeed the presence of an attractor implies an insensitivity to initial conditions of the quintessence field for the vacuum energy now. For a large class of potentials, attractors leading to vacuum energy dominance exist provided their large field behavior is of the inverse power law type. Such potentials are known under the name of Ratra–Peebles potentials [8]. In these cases, the quintessence field reaches an attractor, only to leave it when dominating the energy content of the universe. This happens when the field is of the order of the Planck scale.

This has drastic consequences on quintessence model building. Indeed, it requires a natural framework within which Planck scale physics is taking into account. Supergravity is a promising field theoretical arena where both particle physics and Planck scale physics can be described [11]. Models of quintessence in supergravity have been constructed [12–15] leading to interesting phenomenological consequences such as low values of the equation of state. In particular, the simplest model of quintessence in supergravity, often dubbed the SUGRA model in the literature, leads to the following potential

$$V_{\rm quint}(Q) = {\rm e}^{\kappa Q^2/2+\kappa\xi^2}\,\frac{M^{4+\alpha}}{Q^\alpha}\,, \tag{1.1}$$

with $\kappa \equiv 8\pi/m_{_{\rm Pl}}^2$ and $M^{4+\alpha} = \lambda^2\xi^4 m_{\rm c}^\alpha 2^{\alpha/2}$ and where, in this equation, Q is canonically normalized. The quantity α is a free positive index and λ is a dimensionless coupling constant and, in order to avoid any fine-tuning, we will always consider that $\lambda \sim 1$. $m_{\rm c}$ is the cut-off scale of the effective theory used in order to derive the SUGRA potential. Typically $m_{\rm c}$ can be thought as the GUT scale but we will see that the Planck scale is also possible (and, sometimes, necessary). Finally, ξ is a vacuum expectation value (vev) of some other field present in the quintessence sector, see below for more details. As a specific example, ξ can be realized as a Fayet-Iloupoulous term arising from the Green–Schwarz anomaly cancellation mechanism [12, 13]. The main feature of the above potential is that supergravity corrections have been exponentiated and appear in the prefactor. Phenomenologically, this potential has the nice feature that the equation of state $\omega \equiv p_Q/\rho_Q$ can be closer to -1 than with the Ratra–Peebles potential when the field approaches its present value $\kappa^{1/2}Q_{\rm now} \approx 1$. Moreover, a small value for M can be avoided. Indeed, since the vev of the quintessence field is now of the order of the Planck mass, requiring that the quintessence energy density be of the order of the critical energy density $\rho_{\rm cri} \sim 10^{-122}m_{_{\rm Pl}}^4$ implies that

$$\frac{M}{m_{_{\rm Pl}}} \sim 10^{-122/(4+\alpha)}\,, \tag{1.2}$$

and, therefore, M can be a large scale (by particle physics standard) for very reasonable values of the index α. For instance, it is above the TeV scale for $\alpha \gtrsim 4$. This mechanism is reminiscent of a "see-saw" mechanism where a very small scale (the cosmological constant scale) is explained in terms of a large one (the scale M) and a very large one (the Planck scale $m_{\rm Pl}$). Moreover, the scale ξ can have acceptable values. From the expression of the scale M, one obtains

$$\frac{\xi}{m_{\rm Pl}} \sim \left(\frac{\rho_{\rm cri}}{m_{\rm Pl}}\right)^{1/4}\left(\frac{m_{\rm Pl}}{m_{\rm c}}\right)^{\alpha/4}, \tag{1.3}$$

and for $\alpha \gtrsim 11$ and a cut-off $m_{\rm c}$ of the order of the GUT scale, ξ is above the TeV scale [12]. However, considering $m_{\rm c} < m_{\rm Pl}$ can also be viewed as problematic since, as already mentioned, the vev of the quintessence field tends to be of the order of the Planck mass. In this case, it is difficult to control the shape of the Kähler potential, see Eq. (2.2). Facing this issue, a natural choice could be $m_{\rm c} = m_{\rm Pl}$. Then the above equation indicates that ξ needs to be fine-tuned as the same level as the cosmological constant even if the model remains different since the equation of state is redshift dependent. Moreover, in this situation, the choice of the Kähler potential strongly influences the shape of the scalar potential [13] since the SUGRA corrections are of order one. Hence, it is no longer possible to see the Kähler potential of Eq. (2.2) as a Taylor expansion but it should rather be considered as a specific choice. Another route is to argue that, in these circumstances, the quintessence energy density can easily remain less than $m_{\rm c}^4$, indicating that the theory can still be meaningful if the cut-off is interpreted as the cut-off to the energy density scales and not to the vev's of the fields. In any case, it is clear that there are some fine-tuning problems at this level even if it is arguably less acute than in the cosmological constant case.

However, it is clear that the quintessence sector cannot be considered as disconnected from the particle physics standard model (or its extensions) and should be embedded in a more general structure. In Ref. [16], we have investigated the coupling between the quintessence sector and a hidden sector where supersymmetry is broken. A general formalism to calculate the corresponding implications was presented. On very general grounds, it was shown that, as a consequence of this coupling, the shape of the quintessence potential is changed and that the particle masses become dependent on the quintessence field which implies the presence of a fifth force, a violation of the equivalence principle and possibly, depending on the complexity of the model, a variation of the gauge couplings and of the proton to electron mass ratio.

The main goal of the present article is to apply this general formalism to a concrete case, namely the SUGRA one, where the quintessence potential is described by Eq. (1.1). We find that the coupling between the quintessence, observable and hidden sectors has tremendous consequences on the dynamics of the quintessence field. The first one is that the shape of the quintessence potential is modified. When the hidden sector fields have been stabilized, the potential acquires a minimum located at a very small value of the field (in comparison with the Planck mass). As a result, the model becomes equivalent to a pure cosmological constant as the quintessence field settles at the minimum of a potential before Big Bang Nucleosynthesis (BBN). The mass of the field is also changed and becomes equal to the gravitino mass. The above conclusion is also true at the perturbative level as no growing mode is present despite the smallness of the Jeans length. On top of this, the energy scales in the quintessence sector have to be fine–tuned at a highest level than the cosmological constant itself. This makes the whole scenario very unappealing. On the other hand, preserving the shape of the SUGRA potential requires a fine-tuning of the hidden sector dynamics. If this is done, one may wonder whether the model is still phenomenologically acceptable. We show that, in this case, the scenario tends to be ruled out by local tests of gravity rather than by cosmological considerations.

Our results have been obtained using the SUGRA model [12] of quintessence. Effectively the only crucial ingredient is the fact that the potential in the quintessence sector reduces to the Ratra–Peebles form for small values of the quintessence field. Within this framework, *i.e.* a quintessence sector with a Ratra–Peebles potential at small values of the quintessence field coupled to a hidden sector gravitationally, our conclusions apply and are generic even if a complete scan of the parameter space $(m_{3/2}, m_{1/2}, \omega_Q)$ has not yet been performed.

The paper is arranged as follows. In the following section, *i.e.* section 2, we discuss the coupling of SUGRA quintessence to the hidden sector of supersymmetry breaking and the observable sector. In particular we find that the generic quintessence potential has a minimum with a mass for the quintessence field of the order of the gravitino mass. On the other hand one can fine–tune the hidden sector dynamics to preserve the form of the SUGRA potential. In section 3, we investigate the implications for gravity experiments of this fine–tuning of the SUGRA potential. In section 4, we discuss the cosmological evolution of the generic case where the quintessence potential develops a minimum and show that the field settles at the minimum of the potential before nucleosynthesis. We then examine the cosmological perturbations around the minimum of the potential and show that there are no growing modes for quintessence perturbations. We then conclude and mention possible way-outs in section 5.

2. Quintessence and Supersymmetry Breaking

2.1 The Framework

As described in the Introduction, the usual approach consists in taking into account the features of the quintessence sector only. However, the non-observation of supersymmetric partners to the standard model particles implies that SUSY must be broken at a scale of the order of a TeV. One must also take into account the existence of an observable sector, modeled as the Minimal Super Symmetric Model (MSSM) or mSUGRA model [11], where the standard model particles live. The calculations based on this more realistic description are performed in Ref. [16] where the model is separated in three sectors. There is a quintessence sector as already presented. It couples gravitationally to a visible sector where the standard model particles live. The supersymmetry breaking sector is also interacting gravitationally with the other two sectors. At the level of the Kähler K and super potentials

W, it is a simple sum of the contributions from each sector.

$$K = K_{\rm quint} + K_{\rm hid} + K_{\rm obs}\,, \quad W = W_{\rm quint} + W_{\rm hid} + W_{\rm obs}\,. \tag{2.1}$$

In the present article, we use the general results obtained in Ref. [16] and apply them to a specific class of quintessence model.

The quintessence sector is chosen such that it gives the SUGRA version of the Ratra-Peebles potential. It has been shown in Ref. [12] that this can be obtained using

$$K_{\rm quint} = QQ^\dagger + XX^\dagger + YY^\dagger \frac{(QQ^\dagger)^p}{m_{\rm c}^{2p}} + \sum_{\alpha=1}^{n}\left(X_\alpha X_\alpha^\dagger + Y_\alpha Y_\alpha^\dagger\right) + \cdots\,, \tag{2.2}$$

$$W_{\rm quint} = \lambda X^2 Y + \sum_{\alpha=1}^{n} \lambda_\alpha X_\alpha^2 Y_\alpha + \cdots\,, \tag{2.3}$$

where X_α and Y, Y_α are superfields satisfying $\langle X\rangle = \xi$, $\langle X_\alpha\rangle = \xi_\alpha$ and $\langle Y\rangle = \langle Y_\alpha\rangle = 0$ at the GUT scale where the model is defined (of course, in this context, the GUT scale is just an illustration), see however the discussion after Eq. (1.3). This implies that $\langle W_{\rm quint}\rangle = 0$ and guarantees the positivity of the potential in the quintessence sector. The quantities λ, λ_α are dimensionless coupling constants and p is a free index. As already mentioned, the scale $m_{\rm c}$ is the GUT scale below which the theory under consideration is valid. This means that the theory is valid only for vevs that are much less than $m_{\rm c}$ or energy densities less than $m_{\rm c}^4$ according to the interpretation given to the cut-off. The influence of the Kählerian corrections are a priori important and have been studied in Ref. [13]. In the above expression and in the rest of this paper the dots stand for the higher order terms suppressed by $m_{\rm c}$. In the "dark energy" sector, we collectively denote the fields by $d_\alpha = X, Y, X_\alpha, Y_\alpha, Q$. Let us notice that we have assumed that the Kähler potential in the quintessence sector is regular at the origin. This excludes the no-scale case which deserves a special treatment, see Ref. [17]. It is interesting to consider regular Kähler potentials since they naturally lead to inverse power law scalar potentials while no scale Kähler potentials tend to give exponential potentials with very different properties, see Ref. [17].

For the hidden sector, we follow Ref. [16]. We denote the fields in the hidden sector by z_i and assume that $K_{\rm hid}$ is regular for small values of the hidden fields and can be Taylor expanded. Without specifying the superpotential for the moment, this leads to

$$K_{\rm hid} = \sum_i z_i z_i^\dagger + \cdots\,, \quad W_{\rm hid} = W_{\rm hid}(z_i)\,, \tag{2.4}$$

where, as before, the dots denote the higher order terms that are not considered in this article.

Finally, the fields in the matter sector are written ϕ_a. This sector is supposed to contain all the (super) fields that are observable (including the dark matter). As a consequence, following again Ref. [16], we take this sector to be the MSSM or the mSUGRA model [11], that is to say

$$K_{\rm obs} = \sum_a \phi_a \phi_a^\dagger + \cdots\,, \quad W_{\rm obs} = \frac{1}{3}\sum_{abc}\lambda_{abc}\phi_a\phi_b\phi_c + \frac{1}{2}\sum_{ab}\mu_{ab}\phi_a\phi_b\,, \tag{2.5}$$

with a supersymmetric mass matrix μ_{ab} and Yukawa couplings λ_{abc}. In order to completely specify the observable sector, it is also necessary to choose the supergravity gauge coupling functions $f_{_G}$. *A priori*, all the $f_{_G}$'s are z_i and d_α–dependent. If, indeed, these functions are not constant, then this implies variations of the coupling constants. In particular, this leads to a variation of the fine structure constant, see Ref. [16].

Let us now discuss the breaking of supersymmetry in more details. In the hidden sector, the supersymmetry breaking fields z_i take a vev determined by

$$\partial_{z_i} V = 0 \tag{2.6}$$

where V is the total potential obtained from the previous model. The presence of the quintessence field affects the dynamics of the hidden sector and the vev's of the hidden sector fields become a priori Q dependent. They are parameterized in a model independent way by the coefficients $a_i(Q)$ and $c_i(Q)$ according to

$$\kappa^{1/2}\left\langle z_i\right\rangle_{\min} \sim a_i(Q)\,,\quad \kappa\left\langle W_{\rm hid}\right\rangle_{\min} \sim M_{_{\rm S}}(Q)\,,\quad \kappa^{1/2}\left\langle\frac{\partial W_{\rm hid}}{\partial z_i}\right\rangle_{\min} \sim c_i(Q) M_{_{\rm S}}(Q)\,, \tag{2.7}$$

where $M_{_{\rm S}}$ is the supersymmetry scale. Notice that, if the cut-off of the theory is much less than $m_{_{\rm Pl}}$, then we expect $a_i \ll 1$ since the vev's of $\langle z_i\rangle$ must be at most of the order of $m_{\rm c}$. Notice also that, a priori, nothing can be said about the function $c_i(Q)$. On the other hand, if the cut-off is close to the Planck mass, the dynamics of the hidden sector cannot be analyzed in a model independent way and we must rely on particular models for the hidden sector to go any further. This is not surprising (and is even to be expected) since the situation is in fact exactly similar to what happens in the standard case of the mSUGRA model where the hidden sector is often described by the Polonyi model and the hidden field is stabilized at a vev of the order of the Planck mass. However, in the standard case, our ignorance of the hidden sector is parameterized in terms of two numbers, a_i and c_i, or equivalently $m_{3/2}$ and $m_{1/2}$. In presence of dark energy, the situation is more complicated since our ignorance of the hidden sector is now described by two free functions $a_i(Q)$ and $c_i(Q)$.

Another quantity of interest is of course the gravitino mass. It is defined by the following expression

$$m_{3/2} \equiv \left\langle \kappa W {\rm e}^{\kappa K/2}\right\rangle\,, \tag{2.8}$$

where K and W are the total Kähler and super potentials (*i.e.* taking into account the three sectors). In the present context, the gravitino mass may depend on the vev of the quintessence field. Therefore, it is natural to write

$$m_{3/2} = {\rm e}^{\kappa K_{\rm quint}/2+\sum_i |a_i|^2/2} M_{_{\rm S}} \equiv {\rm e}^{\kappa K_{\rm quint}/2} m_{3/2}^0\,, \tag{2.9}$$

where $m_{3/2}^0$ is the mass that the gravitino would have without the presence of the quintessence field. This quantity explicitly appears in the expression of the scalar potential. Let us notice that $W_{\rm quint}$ does no appear in the previous formula because one has $\langle W_{\rm quint}\rangle = 0$ for the particular case of the SUGRA model considered here.

2.2 The Dark Energy Sector

In this subsection, we study how the shape of the SUGRA model is changed by supersymmetry breaking. As discussed in Ref. [16], see Eq. (2.18) of that article, the new potential in the dark sector is given by

$$\begin{aligned} V_{\rm DE} &= {\rm e}^{\sum_i \vert a_i\vert^2} V_{\rm quint} + M_{\rm S}^2 {\rm e}^{\kappa K_{\rm quint} + \sum_i \vert a_i\vert^2}\left[\left(K^{-1}\right)^{d_\alpha^\dagger d_\beta}\frac{\partial K_{\rm quint}}{\partial d_\beta}\frac{\partial K_{\rm quint}}{\partial d_\alpha^\dagger} - \frac{3}{\kappa}\right] \\ &+ M_{\rm S} {\rm e}^{\kappa K_{\rm quint} + \sum_i \vert a_i\vert^2}\left\{\left[\left(K^{-1}\right)^{d_\alpha^\dagger d_\beta}\frac{\partial K_{\rm quint}}{\partial d_\beta}\frac{\partial K_{\rm quint}}{\partial d_\alpha^\dagger} - \frac{3}{\kappa}\right]\left(\kappa W_{\rm quint} + \kappa W_{\rm quint}^\dagger\right)\right. \\ &+ \left.\left(K^{-1}\right)^{d_\alpha^\dagger d_\beta}\left(\frac{\partial K_{\rm quint}}{\partial d_\beta}\frac{\partial W_{\rm quint}^\dagger}{\partial d_\alpha^\dagger} + \frac{\partial K_{\rm quint}}{\partial d_\alpha^\dagger}\frac{\partial W_{\rm quint}}{\partial d_\beta}\right)\right\} + \sum_i \vert F_{z_i}\vert^2 \,, \end{aligned} \tag{2.10}$$

where $V_{\rm quint}$ is the potential that one would have obtained by considering the dark energy sector alone and F_{z_i} the F–term in the hidden sector which takes the form

$$F_{z_i} = {\rm e}^{\kappa K_{\rm quint}/2 + \sum_i \vert a_i\vert^2/2}\frac{1}{\kappa^{1/2}}\left[\left(M_{\rm S} + \kappa\left\langle W_{\rm quint}\right\rangle\right) a_i + M_{\rm S} c_i\right] . \tag{2.11}$$

In the case of the model described by Eqs. (2.2), the above expressions simplify a lot, as $\langle W_{\rm quint}\rangle = 0$. Instead of the usual shape of the SUGRA potential, $V(Q) = {\rm e}^{\kappa\left(Q^2+\xi^2+\sum_\alpha \xi_\alpha^2\right)} \times M^{4+2p}Q^{-2p}$, see Eq. (1.1) (in the minimal approach, the fields X_α are not present which explains the absence of the term $\sum_\alpha \xi_\alpha^2$ in the above mentioned formula), we now have

$$V_{\rm DE}(Q) = {\rm e}^{\kappa\left(Q^2+\xi^2+\sum_\alpha \xi_\alpha^2\right)}\left[\frac{M^{4+2p}}{Q^{2p}} + \left(m_{3/2}^0\right)^2 Q^2 - \Upsilon^4(Q)\right] , \tag{2.12}$$

where the function $\Upsilon(Q)$ encodes our ignorance of the hidden sector and is given by

$$\begin{aligned} -\Upsilon^4(Q) &= \left(m_{3/2}^0\right)^2\left(\xi^2 + \sum_{\alpha=1}^n \xi_\alpha^2\right) + {\rm e}^{\sum_i \vert a_i\vert^2}\sum_{\alpha=1}^n \lambda_\alpha^2\xi_\alpha^4 \\ &+ \frac{1}{\kappa}\left(m_{3/2}^0\right)^2\left\{\sum_i\left[a_i(Q) + c_i(Q)\right]\right\}^2 - \frac{3}{\kappa}\left(m_{3/2}^0\right)^2 , \end{aligned} \tag{2.13}$$

and $M^{4+2p} = {\rm e}^{\sum_i \vert a_i\vert^2}\lambda^2\xi^4 m_{\rm c}^{2p}$. In fact the quintessence field is not correctly normalized in this expression. To obtain a correctly normalized field, it is sufficient to replace Q with $Q/\sqrt{2}$. The new shape of the potential is still not fixed in the above expression and is only known when the functions $a_i(Q)$ and $c_i(Q)$ are specified, *i.e.* when the hidden sector is known explicitly.

We can envisage two different situations. They are distinguished by the equation of state w_Q of the quintessence sector when Q takes its present value in the history of the universe. First of all, let us assume that the equation of state is $w_Q \neq -1$. This can only be achieved when the potential is of runaway type with an effective mass for the quintessence field $m_Q \sim H_0$ the present Hubble rate. This situation can only be achieved when the functions $a_i(Q)$ and $c_i(Q)$ are not constant and such that, despite the new terms which modify the shape of $V_{\rm DE}$, the runaway shape of the potential is preserved. This means that the susy breaking fields z_i are not stabilized or, more precisely, that the fields follow a trajectory in the field space (z_i, Q). In this case, as we discuss in the following, one should precisely evaluate how serious the gravitational problems are. Obviously, this cannot be done in detail unless the functions $a_i(Q)$ and $c_i(Q)$ are known exactly. In the following we will emphasize the case where $a_i \ll 1$ and c_i is tuned to obtain a runaway potential.

Another case of particular interest is when a satisfactory model of the hidden sector has been found and the z_i's are correctly stabilized, *i.e.* $a_i(Q)$ and $c_i(Q)$ are independent of Q. In addition, if we assume that the fields in the hidden sector are stabilized at vev's compatible with the cut-off $m_{\rm c}$ of the theory, *i.e.* we assume that $\langle z_i\rangle \ll m_{\rm c}$, and if $m_{\rm c} \ll m_{\rm Pl}$, then the coefficients a_i are very small (since $m_{\rm c} \ll m_{\rm Pl}$) but this does not imply anything about the coefficients c_i. On the other hand, if $m_{\rm c}$ is not small in comparison to $m_{\rm Pl}$, then nothing can be said about a_i.

Let us now focus on the simplest model of supersymmetry breaking where there is only one field z with a flat Kähler potential and a constant superpotential, $W_{\rm hid} = m^3$ [18]. Then, one can even justify the previous choice. Indeed, first of all, this immediately implies that $M_{\rm S} = \kappa m^3$ and $c = 0$. Then, the total potential (for simplicity, here, we assume $\lambda_\alpha = 0$) reads

$$V = {\rm e}^{\kappa K_{\rm quint}}{\rm e}^{\kappa z z^\dagger}\kappa m^6\left[\kappa z z^\dagger - 3 + \kappa\left(\xi^2 + \sum_{\alpha=1}^{n}\xi_\alpha^2 + QQ^\dagger\right) + \frac{\lambda^2\xi^4 m_{\rm c}^{2p}}{\kappa m^6 (QQ^\dagger)^p} + \cdots\right], \tag{2.14}$$

where the dots now indicate the part containing the observable sector terms. These terms do not play a role at high energy and therefore can be ignored in the present context. At the minimum, we have $\partial V/\partial z^\dagger = 0$, that is to say

$${\rm e}^{\kappa K_{\rm quint}}{\rm e}^{\kappa z z^\dagger}\kappa^2 m^6 z\left[\kappa z z^\dagger - 2 + \kappa\left(\xi^2 + \sum_{\alpha=1}^{n}\xi_\alpha^2 + QQ^\dagger\right) + \frac{\lambda^2\xi^4 m_{\rm c}^{2p}}{\kappa m^6 (QQ^\dagger)^p}\right] = 0\,. \tag{2.15}$$

The constraint coming from the smallness of the vacuum energy implies (this is just another manifestation of the fact that quintessence has nothing to say about the cosmological constant problem)

$$\xi^2 + \sum_{\alpha=1}^{n}\xi_\alpha^2 \sim \frac{3}{\kappa}\,, \tag{2.16}$$

and, therefore, the quantity in the squared bracket is positive. As a result, the only solution is

$$\langle z\rangle_{\rm min} = 0\,, \tag{2.17}$$

i.e. $a = 0$. Hence, we find that this simple model satisfies $a_i = c_i = 0$ and, moreover, gives a SUSY breaking scale that does not depend on the quintessence field. In this case the function $\Upsilon(Q)$ becomes a constant and the potential in Q admits a minimum. At this minimum, it is clear that the mass of the quintessence field is of the order of the gravitino mass, $m_Q \sim m_{3/2} = O(1)$TeV. In more complex settings, one can envisage a case where one of the previous results is relaxed, for instance where $c(Q)$ becomes Q-dependent while the other functions describing the hidden sector remain constant. This is what will be done in the following.

We have thus reached one important conclusion, namely that when the quintessence sector corresponds to the SUGRA potential and when the hidden sector is correctly stabilized then the quintessence field acquires a mass of the order of the gravitino mass. If the hidden sector is more complicated, the runaway shape can be preserved and it is interesting to see whether local gravity tests can constrain this type of models. The cosmological implications of the new potential (2.12) are worked out in detail in Sec. 4.

2.3 The Observable Sector

After having described how the dark energy sector looks like for the SUGRA model, we now study the observable sector. On very general grounds, it was shown in Ref. [16] that the interaction between the dark energy sector and the observable sector implies a Yukawa like interaction of the form

$$m^2\left(\frac{Q}{m_{_{\rm Pl}}}\right)\bar{\Psi}\Psi\,, \tag{2.18}$$

where Ψ is a fermionic field. Therefore, the above expression implies that the fermion masses become quintessence field dependent quantities. As we discuss in the following, this implies a series of interesting effects as the presence of a fifth force and/or a violation of the weak equivalence principle. Our goal in this subsection is to use the general results obtained in Ref. [16] and to apply them to the SUGRA model in order to compute explicitly the functions $m(Q/m_{_{\rm Pl}})$.

Computing the masses of the fermions first requires to compute the soft terms. This was done in Ref. [16], see Eqs. (2.21)–(2.23) of that article, and the general result reads

$$\begin{aligned}
A_{abc} &= \lambda_{abc}{\rm e}^{\kappa K_{\rm quint}+\sum_i|a_i|^2}\Biggl\{\left(M_{_{\rm S}}+\kappa W_{\rm quint}^\dagger\right)+\frac{1}{3}\left(M_{_{\rm S}}+\kappa W_{\rm quint}^\dagger\right)\Biggl[\kappa\left(K^{-1}\right)^{d_\alpha^\dagger d_\beta}\\
&\quad\times\frac{\partial K_{\rm quint}}{\partial d_\beta}\frac{\partial K_{\rm quint}}{\partial d_\alpha^\dagger}+\sum_i|a_i|^2-3\Biggr]+\frac{1}{3}\kappa\left(K^{-1}\right)^{d_\alpha^\dagger d_\beta}\frac{\partial K_{\rm quint}}{\partial d_\alpha^\dagger}\frac{\partial W_{\rm quint}}{\partial d_\beta}\\
&\quad+\frac{1}{3}M_{_{\rm S}}\sum_i a_i c_i\Biggr\}\,, \qquad (2.19)\\
B_{ab} &= \mu_{ab}{\rm e}^{\kappa K_{\rm quint}+\sum_i|a_i|^2}\Biggl\{\left(M_{_{\rm S}}+\kappa W_{\rm quint}^\dagger\right)+\frac{1}{2}\left(M_{_{\rm S}}+\kappa W_{\rm quint}^\dagger\right)\Biggl[\kappa\left(K^{-1}\right)^{d_\alpha^\dagger d_\beta}\\
&\quad\times\frac{\partial K_{\rm quint}}{\partial d_\beta}\frac{\partial K_{\rm quint}}{\partial d_\alpha^\dagger}+\sum_i|a_i|^2-3\Biggr]+\frac{1}{2}\kappa\left(K^{-1}\right)^{d_\alpha^\dagger d_\beta}\frac{\partial K_{\rm quint}}{\partial d_\alpha^\dagger}\frac{\partial W_{\rm quint}}{\partial d_\beta}\\
&\quad+\frac{1}{2}M_{_{\rm S}}\sum_i a_i c_i\Biggr\}\,, \qquad (2.20)\\
m_{a\bar b}^2 &= {\rm e}^{\kappa K_{\rm quint}+\sum_i|a_i|^2}\left[M_{_{\rm S}}^2+M_{_{\rm S}}\left(\kappa W_{\rm quint}+\kappa W_{\rm quint}^\dagger\right)+\kappa^2W_{\rm quint}W_{\rm quint}^\dagger\right]\delta_{a\bar b}\,. \qquad (2.21)
\end{aligned}$$

This is the general form of the soft terms, calculated at the GUT scale. Then, we have to specialize the above formulas to the dark sector described by Eqs. (2.2). Again the fact that $\langle W_{\rm quint}\rangle = 0$ considerably simplifies the calculations. Straightforward manipulations lead to

$$A_{abc} = \lambda_{abc} m_{3/2}^0 {\rm e}^{\kappa K_{\rm quint}} {\rm e}^{\sum_i |a_i|^2/2} \left[1 + \frac{1}{3}\sum_i |a_i|^2 + \frac{1}{3}\sum_i a_i c_i + \frac{1}{3}\left(\kappa Q^2 + \kappa \xi^2 + \kappa \sum_{\alpha=1}^{n} \xi_\alpha^2 - 3\right)\right], \tag{2.22}$$

$$B_{ab} = \mu_{ab} m_{3/2}^0 {\rm e}^{\kappa K_{\rm quint}} {\rm e}^{\sum_i |a_i|^2/2} \left[1 + \frac{1}{2}\sum_i |a_i|^2 + \frac{1}{2}\sum_i a_i c_i + \frac{1}{2}\left(\kappa Q^2 + \kappa \xi^2 + \kappa \sum_{\alpha=1}^{n} \xi_\alpha^2 - 3\right)\right], \tag{2.23}$$

$$m_{a\bar{b}} = m_{3/2}^0 {\rm e}^{\kappa K_{\rm quint}/2} \delta_{a\bar{b}}\,, \tag{2.24}$$

where, again for simplicity, we have assumed $\lambda_\alpha = 0$. In the following, one will neglect ξ in comparison with the vev of Q (as ξ has to be extremely small in order for the quintessence energy density to be of the order of the critical energy density today, see above) and one will also consider that $\sum_{\alpha=1}^{n}\xi_\alpha^2 = 3/\kappa$. This last relation appears in the model of the hidden sector discussed before, see Eq. (2.16), but is also natural in the present context when one treats the case where the potential keeps its runaway shape despite the appearance of the new terms coming from the hidden sector. This means that $V_{\rm DE}$ should vanish at infinity and, therefore, that the constant terms cancel in the term $\Upsilon^4(Q)$. Again, this issue is linked to the cosmological constant problem.

In order to obtain the masses of the fermions, one should use the following procedure. In the MSSM, they are two Higgs doublets $H_{\rm u}$ and $H_{\rm d}$ and the fermions either couple to the Higgs doublet "u" or "d". These couplings are different and, therefore, through the Higgs mechanism, the masses of the fermions either depend on the vev of the Higgs "u" or of the Higgs "d". Explicitly, one has [16]

$$\begin{aligned} m_{\rm u} &= \lambda_{\rm u}^{\rm F} {\rm e}^{K_{\rm quint}/2 + \sum_i |a_i|^2/2} \frac{v \tan\beta}{\sqrt{1+\tan^2\beta}} \\ &= \lambda_{\rm u}^{\rm F} {\rm e}^{K_{\rm quint}/2 + \sum_i |a_i|^2/2} \left[v(Q) + \mathcal{O}\left(\frac{1}{\tan^2\beta}\right)\right], \end{aligned} \tag{2.25}$$

$$\begin{aligned} m_{\rm d} &= \lambda_{\rm d}^{\rm F} {\rm e}^{K_{\rm quint}/2 + \sum_i |a_i|^2/2} \frac{v}{\sqrt{1+\tan^2\beta}} \\ &= \lambda_{\rm d}^{\rm F} {\rm e}^{K_{\rm quint}/2 + \sum_i |a_i|^2/2} \left[\frac{v(Q)}{\tan\beta} + \mathcal{O}\left(\frac{1}{\tan^2\beta}\right)\right], \end{aligned} \tag{2.26}$$

where $\lambda_{\rm u,d}^{\rm F}$ are the Yukawa coupling which are, in the minimal setting considered here, independent of the quintessence field. In the previous equations, v is defined by $\sqrt{v_{\rm u}^2 + v_{\rm d}^2}$, where $v_{\rm u} \equiv v\sin\beta$ and $v_{\rm d} \equiv v\cos\beta$ are the vevs of the Higgs $H_{\rm u}$ and $H_{\rm d}$ respectively. We have expanded the two previous expressions in terms of $1/\tan\beta$. The explicit expression of $\tan\beta$ reads [16]

$$\tan\beta(Q) = \frac{2|\mu|^2 \mathrm{e}^{\sum_i |a_i|^2} + m^2_{H_\mathrm{u}}(Q) + m^2_{H_\mathrm{d}}(Q)}{2\mu B(Q)} \times\left(1 \pm \sqrt{1 - 4\mu^2 B^2(Q)\left[2|\mu|^2 \mathrm{e}^{\sum_i |a_i|^2} + m^2_{H_\mathrm{u}}(Q) + m^2_{H_\mathrm{d}}(Q)\right]^{-2}}\right). \quad (2.27)$$

Notice that there are two possibilities according to the sign in the above expression. In the following, we work in the limit of large $\tan\beta$ and, therefore, take the largest value. In Eq. (2.27), $m^2_{H_\mathrm{u}}(Q)$ and $m^2_{H_\mathrm{d}}(Q)$ are the two loops renormalized Higgs masses given by [16,19]

$$m^2_{H_\mathrm{u}}(Q) = m^2_{H_\mathrm{d}}(Q) - 0.36\left(1 + \frac{1}{\tan^2\beta}\right)\left\{\left(m^0_{3/2}\right)^2\left(1 - \frac{1}{2\pi}\right) + 8\left(m^0_{1/2}\right)^2 + \left(0.28 - \frac{0.72}{\tan^2\beta}\right)\left[A(Q) + 2m^0_{1/2}\right]^2\right\}, \quad (2.28)$$

$$m^2_{H_\mathrm{d}}(Q) = \left(m^0_{3/2}\right)^2\left(1 - \frac{0.15}{4\pi}\right) + \frac{1}{2}\left(m^0_{1/2}\right)^2. \quad (2.29)$$

We see from the two above expressions that the quintessence field dependence of the fermions masses is controlled by two functions, $A(Q)$ and $B(Q)$. The explicit expressions of these soft terms, see Eqs. (2.22)–(2.24), allow us to extract the functions $A(Q)$ and $B(Q)$ by $A_{abc} \equiv \mathrm{e}^{\kappa K_\mathrm{quint}} A(Q)\lambda_{abc}$ and $B_{ab} \equiv \mathrm{e}^{\kappa K_\mathrm{quint}} \mu B(Q)\epsilon_{ab}$. If we restrict our considerations to $a_i = 0$, which is a case of interest since this is at the same time compatible with $\langle z_i\rangle \ll m_\mathrm{C}$ and with a possible runaway potential (using a non trivial dependence of the coefficient c_i, see the discussions before), one arrives at

$$A(Q) = M_\mathrm{S}\left(1 + \frac{\kappa Q^2}{3}\right), \quad B(Q) = M_\mathrm{S}\left(1 + \frac{\kappa Q^2}{2}\right). \quad (2.30)$$

Notice that A and B follow the same universal relationship as in the mSUGRA model despite the presence of the quintessence field. Then, one deduces that $\tan\beta(Q)$ in the SUGRA model can be expressed as

$$\tan\beta(Q) \simeq \frac{\delta_1 + \delta_2\kappa Q^2 + \delta_3\kappa^2 Q^4}{\delta_4 + \delta_5\kappa Q^2}\left[1 + \sqrt{1 + \frac{(\delta_4 + \delta_5\kappa Q^2)^2}{(\delta_1 + \delta_2\kappa Q^2 + \delta_3\kappa^2 Q^4)^2}}\right], \quad (2.31)$$

where the coefficients δ_1, δ_2, δ_3, δ_4 and δ_5 can easily be evaluated in terms of the physical parameters characterizing the model from the previous equations, *i.e.* μ, $m^0_{3/2}$ and $m^0_{1/2}$ given at the GUT scale. As already mentioned, this result is valid both in the case where the hidden sector fields are stabilized with $a_i = c_i = 0$ and when the hidden sector dynamics is tuned to reach $a_i = 0$ and $c_i \neq 0$ possibly leading to a runaway potential. In the latter the role of the gravitational tests is crucial in discriminating models.

The expression of the scale $v(Q)$ can also be obtained from the minimization of the Higgs potential along the lines described in Ref. [16]. One obtains

$$v(Q) = \frac{2}{\sqrt{g^2 + g'^2}} \mathrm{e}^{\kappa K_{\mathrm{quint}}/2} \sqrt{\left| |\mu|^2 + m_{H_\mathrm{u}}^2 \right|} + \mathcal{O}\left(\frac{1}{\tan\beta}\right) , \tag{2.32}$$

where as before, we have used $a_i = 0$. The value of v today is known and is $v \sim 174\,\mathrm{GeV}$. Therefore, recalling that $m_{Z^0}^2 = (g^2 + g'^2)v^2/2$ with $m_{Z^0} \sim 91.6\,\mathrm{GeV}$ and expressing Eq. (2.32) at vanishing redshift allows us to determine the μ parameter. Explicitly, one has

$$|\mu| = \sqrt{\frac{1}{2} m_{Z^0}^2 \mathrm{e}^{-\kappa\left(Q_{\mathrm{now}}^2 + \xi^2 + \sum_\alpha \xi_\alpha^2\right)} - m_{H_\mathrm{u}}^2} \, . \tag{2.33}$$

In the following, as already explained, we neglect ξ and use $\sum_{\alpha=1}^{n} \xi_\alpha^2 = 3/\kappa$. The above formula also depends on the value of the vev of the quintessence field today, Q_{now}. This one should be determined by the cosmological evolution of the field in the potential V_{DE} and is therefore fixed once the parameters (*i.e.* $m_{3/2}^0$, $m_{1/2}^0$ etc $\dots$) have been chosen. However, in the runaway case, it is clear that one always have $\kappa^{1/2} Q_{\mathrm{now}} \sim 1$ and, for definiteness, we will take $\kappa^{1/2} Q_{\mathrm{now}} = 1$ in the following, the final results being (almost) independent of the precise value of $\kappa^{1/2} Q_{\mathrm{now}}$.

Let us also remark that the above formula giving $\tan\beta$ and v are only approximated formulas and, as announced above, valid only when terms like $1/\tan^2\beta$ are negligible in Eqs. (2.28) and (2.29). Otherwise one would have to deal with a transcendental equation. If necessary, this equation can always be solved numerically, but, in this article, we always consider the approximation where the various quantities of interest are expanded in $1/\tan\beta$. The corresponding evolution of $\tan\beta$ as given by Eq. (2.31) is represented in Fig. 1. We see that $\tan\beta$ does not change sign, which would have been problematic once the sign of μ is fixed. In Fig. 1, one check that the electroweak symmetry breaking conditions are indeed satisfied in the whole range for which $\tan\beta$ is plotted. We mentioned before that this plot has been obtained by neglecting the terms $1/\tan^2\beta$ in Eqs. (2.28) and (2.29). One can verify that, for our choice of parameters, this is a good approximation since $\tan\beta \gtrsim 5$. In Fig. 2, we have also represented the scale $v(Q)$ given by Eq. (2.32) for the same values of the parameters as before.

If the expressions of $\tan\beta$ and $v(Q)$ are used in Eqs. (2.25) and (2.26), this gives $m_{\mathrm{u,d}}(Q)$. To our knowledge, this is the first time that the Q dependence of the fermions masses is calculated in a precise model from first principles. The vev $v_\mathrm{u}(Q)$ is equal to $v(Q)$ at leading order and the vev $v_\mathrm{d}(Q)$ is represented in Fig. 3.

Finally, let us remark that, very often in the literature, the function $m_{\mathrm{u,d}}(Q)$ is just postulated see, for instance, Ref. [20] where $m_{\mathrm{u,d}}(Q) \propto \exp\left(\beta\kappa^{1/2}Q/2\right)$, β being a free parameter. We see that, even in our oversimplified model, the dependence can be much more complicated.

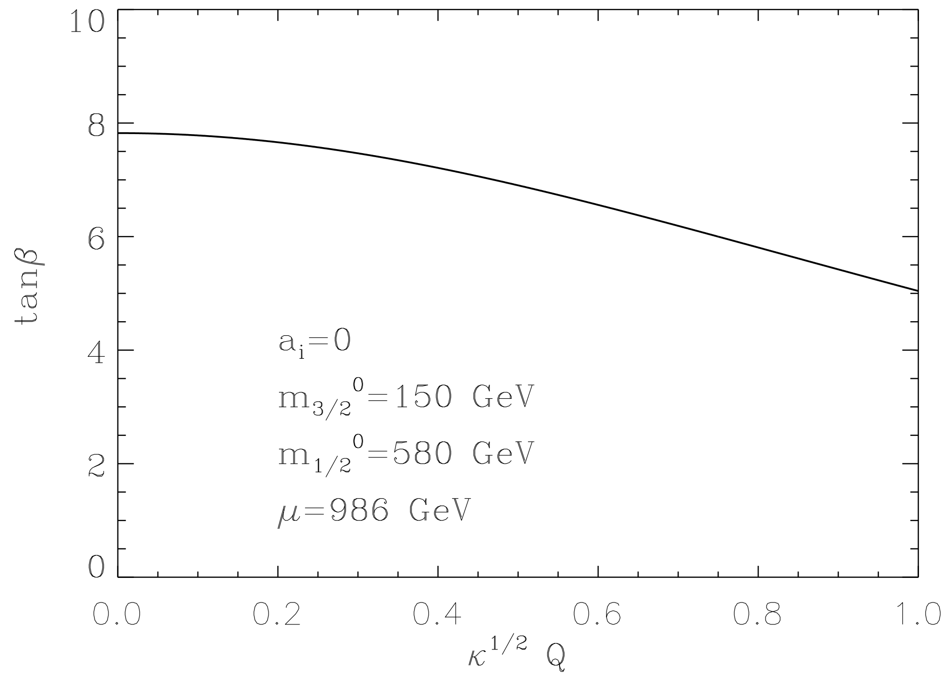

Figure 1: Evolution of $\tan\beta$ versus the vev of the quintessence field according to Eq. (2.27) or Eq. (2.31).

3. Implications for Gravity Experiments

3.1 Fifth Force Constraints

In this section, we study the consequences of the fact that the fermions masses are now Q-dependent quantities. It is known that this can cause serious problems coming from gravitational experiments since this implies the presence of a fifth force and a violation of the weak equivalence principle, see the next subsection. In fact, this crucially depends on the mass of the quintessence field. If the mass of the quintessence field is larger than 10^{-3}eV, then the gravitational constraints are always satisfied as the range of the force mediated by Q is less than one millimeter. We see that this occurs when the functions $a_i(Q)$ and $c_i(Q)$ vanish or are constant. Indeed, we have shown that, in this case, the potential has a minimum and acquires a mass $\sim m_{3/2}^0 \gg 10^{-3}$eV. Therefore, we reach the conclusion that the SUGRA model, with a hidden sector such that the fields are correctly stabilized, is free from gravitational problems. As discussed in the next section, the problems rather originate from cosmological considerations.

On the other hand, if the mass of the quintessence field is less than 10^{-3}eV, the range of the quintessence field is large and generically there will be violations of the weak equivalence principle and a large fifth force. In the case of the SUGRA model, this requires non trivial functions $a_i(Q)$ and $c_i(Q)$. In this situation, in order to avoid fifth force experiments such

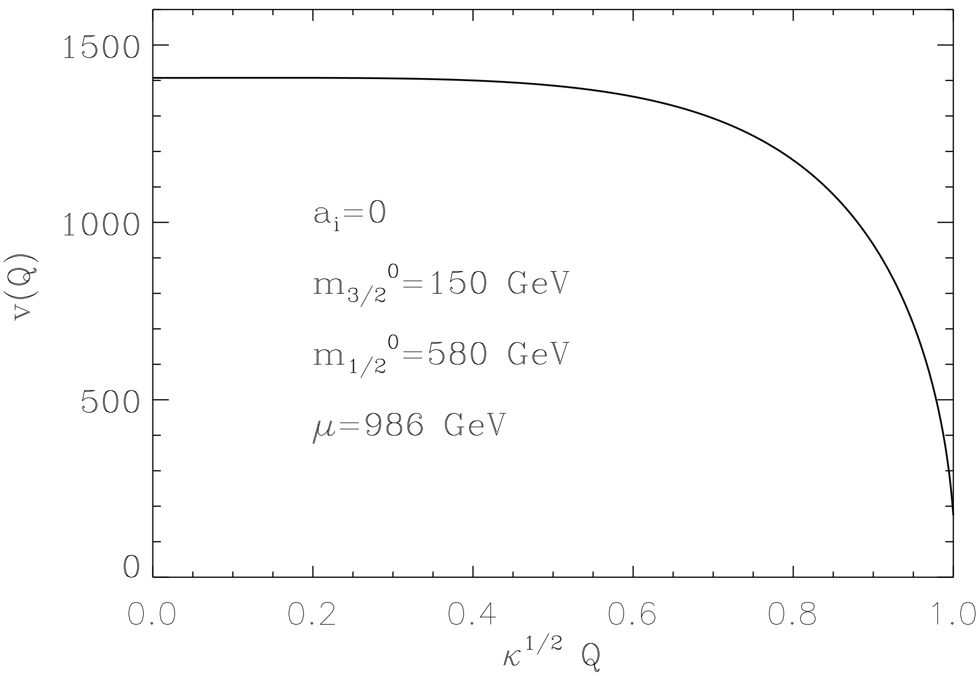

Figure 2: Evolution of v versus the vev of the quintessence field as predicted by Eq. (2.32)

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as the recent Cassini spacecraft experiment, one must require that the Eddington (post-Newtonian) parameter $|\gamma - 1| \leq 5 \times 10^{-5}$, see Ref. [21]. The Eddington parameter γ is related to the parameters $\alpha_{\rm u,d}$ by $\gamma = 1 + \alpha_{\rm u,d}^2$, where $\alpha_{\rm u,d}$ can be expressed as

$$\alpha_{\rm u,d}(Q) \equiv \left|\frac{1}{\kappa^{1/2}}\frac{{\rm d}\ln m_{\rm u,d}^{\rm F}(Q)}{{\rm d}Q}\right| = \left|\frac{1}{\kappa^{1/2}}\frac{{\rm d}\ln\left[{\rm e}^{K_{\rm quint}/2+\sum_i |a_i|^2/2} v_{\rm u,d}(Q)\right]}{{\rm d}Q}\right|, \tag{3.1}$$

Therefore, the coefficients $\alpha_{\rm u,d}$ can be determined explicitly from the formulas giving $m_{\rm u,d}$. The model is free from difficulties if $\alpha_{\rm u,d}^2 \leq 10^{-5}$. Clearly, the coefficients can be calculated explicitly as soon as the functions $a_i(Q)$ and $c_i(Q)$ are known. Here, we perform this calculation for the choice $a_i = 0$ leaving the function $c_i \neq 0$ free to lead to a runaway potential as already discussed at length before. In addition, in order to deal with the simplest model, we consider that the scale $M_{\rm s}$ does not dependent on Q as indicated by

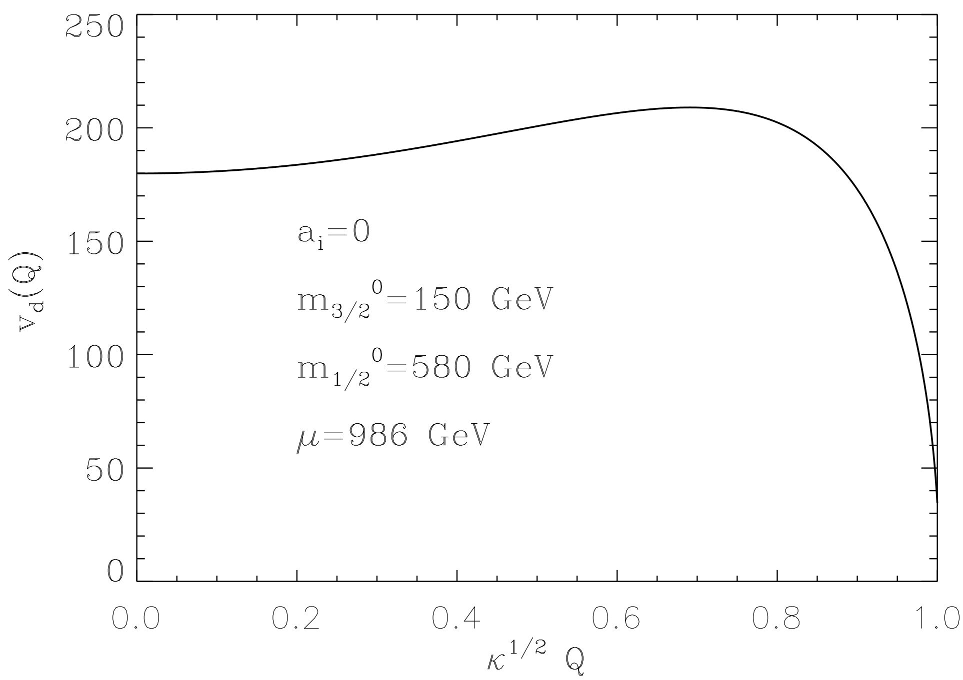

Figure 3: Evolution of the Higgs vev $v_{\rm d}$ versus the vev of the quintessence field as predicted by the second of Eqs. (2.26).

the model of SUSY breaking presented before. One obtains

$$\begin{aligned}
\alpha_{\rm u} &= \frac{\kappa^{1/2}}{2}\partial_Q K_{\rm quint} + \frac{\kappa^{-1/2}}{\tan\beta\,(1+\tan^2\beta)}\frac{{\rm d}\tan\beta}{{\rm d}Q} + \frac{\kappa^{-1/2}}{v}\frac{{\rm d}v}{{\rm d}Q} \\
&= \frac{\kappa^{1/2}}{2}\partial_Q K_{\rm quint} + \frac{\kappa^{-1/2}}{v}\frac{{\rm d}v}{{\rm d}Q} + \mathcal{O}\left(\frac{1}{\tan^2\beta}\right), \qquad (3.2)\\
\alpha_{\rm d} &= \frac{\kappa^{1/2}}{2}\partial_Q K_{\rm quint} - \frac{\kappa^{-1/2}\tan\beta}{1+\tan^2\beta}\frac{{\rm d}\tan\beta}{{\rm d}Q} + \frac{\kappa^{-1/2}}{v}\frac{{\rm d}v}{{\rm d}Q} \\
&= \frac{\kappa^{1/2}}{2}\partial_Q K_{\rm quint} - \frac{\kappa^{-1/2}}{\tan\beta}\frac{{\rm d}\tan\beta}{{\rm d}Q} + \frac{\kappa^{-1/2}}{v}\frac{{\rm d}v}{{\rm d}Q} + \mathcal{O}\left(\frac{1}{\tan^2\beta}\right), \qquad (3.3)
\end{aligned}$$

where the derivative of the function $\tan\beta(Q)$ can be expressed as

$$\frac{\mathrm{d}\tan\beta}{\mathrm{d}Q} = \left(\frac{\mathrm{d}m^2_{\mathrm{H_u}}}{\mathrm{d}Q} + \frac{\mathrm{d}m^2_{\mathrm{H_d}}}{\mathrm{d}Q}\right)\left(2|\mu|^2 + m^2_{\mathrm{H_u}} + m^2_{\mathrm{H_d}}\right)^{-1}\tan\beta - \frac{1}{B(Q)}\frac{\mathrm{d}B(Q)}{\mathrm{d}Q}\tan\beta$$
$$\pm 2\mu\left(2|\mu|^2 + m^2_{\mathrm{H_u}} + m^2_{\mathrm{H_d}}\right)^{-1}\left[1 - 4\mu^2 B^2(Q)\left(2|\mu|^2 + m^2_{\mathrm{H_u}} + m^2_{\mathrm{H_d}}\right)^{-2}\right]^{-1/2}$$
$$\times\left[-\frac{\mathrm{d}B(Q)}{\mathrm{d}Q} + B(Q)\left(\frac{\mathrm{d}m^2_{\mathrm{H_u}}}{\mathrm{d}Q} + \frac{\mathrm{d}m^2_{\mathrm{H_d}}}{\mathrm{d}Q}\right)\left(2|\mu|^2 + m^2_{\mathrm{H_u}} + m^2_{\mathrm{H_d}}\right)^{-1}\right], \tag{3.4}$$
$$\sim \left(\frac{\mathrm{d}m^2_{\mathrm{H_u}}}{\mathrm{d}Q} + \frac{\mathrm{d}m^2_{\mathrm{H_d}}}{\mathrm{d}Q}\right)\left(2|\mu|^2 + m^2_{\mathrm{H_u}} + m^2_{\mathrm{H_d}}\right)^{-1}\tan\beta - \frac{\mathrm{d}\ln B(Q)}{\mathrm{d}Q}\tan\beta\,. \tag{3.5}$$

The last expression is valid at leading order and we use the explicit form of the functions $A(Q)$ and $B(Q)$ for the SUGRA model in order to evaluate the derivatives of the soft terms

$$\frac{\mathrm{d}A(Q)}{\mathrm{d}Q} = \frac{2m^0_{3/2}}{3}\kappa Q\,, \quad \frac{\mathrm{d}B(Q)}{\mathrm{d}Q} = m^0_{3/2}\kappa Q\,, \tag{3.6}$$

since M_{S} is constant. Consequently, the derivatives of the Higgs masses can be expressed as

$$\frac{\mathrm{d}m^2_{\mathrm{H_u}}}{\mathrm{d}Q} \simeq -0.72\times 0.28\frac{\mathrm{d}A(Q)}{\mathrm{d}Q}\left[A(Q) + 2m^0_{1/2}\right]\,, \quad \frac{\mathrm{d}m^2_{\mathrm{H_d}}}{\mathrm{d}Q} \simeq 0\,, \tag{3.7}$$

the symbol "approximate" in the last two equations meaning that we have used the fact that the terms in $1/\tan^2\beta$ have been neglected in the expression of the above formulas. In the previous calculation, we have also used the fact that $m^0_{1/2}$ is constant. This means that we have assumed specific forms for the gauge functions f_G, namely we have considered that they do not depend on Q and z_i but only on the dark sector fields X_α. Then, if we parameterize ξ_α and the derivative of $f = f_G$ as

$$\xi_\alpha = \sqrt{\frac{3}{\kappa}}e_\alpha\,, \quad \kappa^{-1/2}\frac{\partial f}{\partial X_\alpha} = h_\alpha\,, \tag{3.8}$$

where the coefficients e_α and h_α are of order one, one finds that

$$m^0_{1/2} = \sqrt{3}m^0_{3/2}\sum_\alpha e_\alpha h_\alpha\,, \tag{3.9}$$

with no dependence on Q and a model dependent prefactor of $m^0_{3/2}$.

As an example we have plotted in Fig. 4 the gravitational coupling constants $\alpha_{\mathrm{u,d}}$ for a realistic situation where $a_i = 0$ and the parameters $m^0_{3/2} = 150\,\mathrm{GeV}$, $m^0_{1/2} = 580\,\mathrm{GeV}$ and, hence, $\mu = 986\,\mathrm{GeV}$, that is to say $m^0_{3/2}$ and $m^0_{1/2}$ roughly speaking of the same order of magnitude as indicated by the previous calculation. We see that the limit $\alpha_{\mathrm{d}} \sim 10^{-2.5}$ is reached for relatively large value of the quintessence field vev, or the order of $\kappa^{1/2}\langle Q\rangle \sim 10^{-4} - 10^{-2}$. This implies that the SUGRA model with $a_i = 0$ and $c_i \neq 0$ to obtain a runaway potential with $Q_{\mathrm{now}} \sim m_{\mathrm{Pl}}$ is excluded unless the dependence of the masses on Q in (3.1) involves a Q dependent Yukawa coupling compensating exactly the Q dependence of the Higgs vevs. Although this is not excluded, this is a functional fine–tuning which is hard to explain.

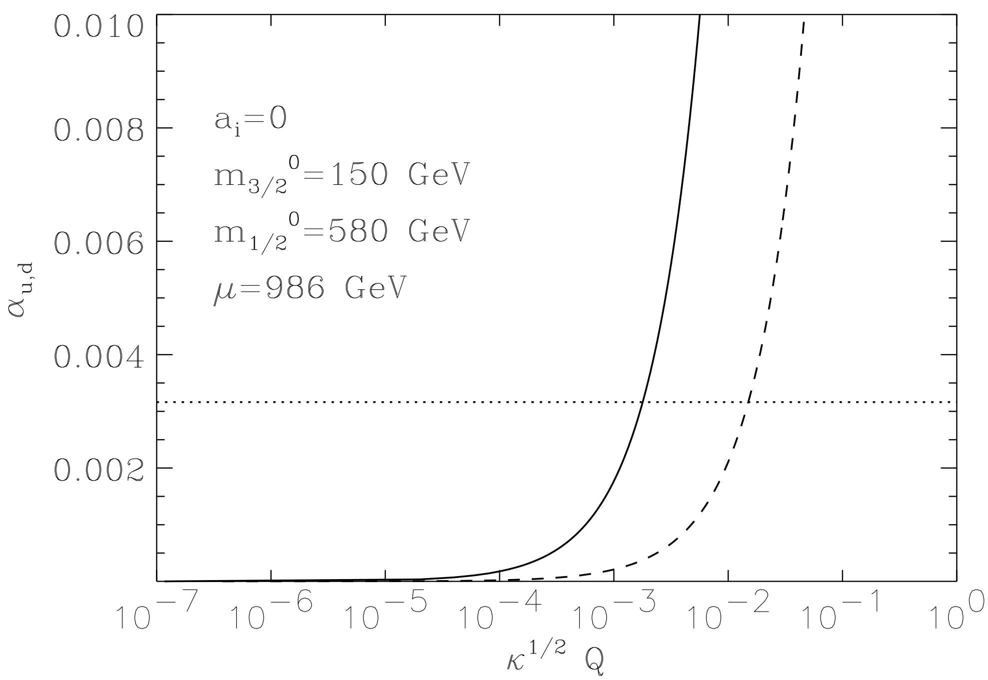

Figure 4: Evolution of the coefficients α_u (solid line) and α_d (dashed line) versus the vev of the quintessence field.

3.2 Violation of the Weak Equivalence Principle

As explained in detail in Ref. [16], the fact that, in the MSSM, the fermions couple differently to the two Higgs doublets H_u and H_d causes, in the presence of dark energy, a violation of the weak equivalence principle. This violation is quantified in terms of the η_{AB} parameter defined by [22–24]

$$\eta_{\mathrm{AB}} \equiv \left(\frac{\Delta a}{a}\right)_{\mathrm{AB}} = 2\frac{a_\mathrm{A} - a_\mathrm{B}}{a_\mathrm{A} + a_\mathrm{B}}\,, \tag{3.10}$$

for two test bodies A and B in the gravitational background of a third one E. Current limits [25] give $\eta_{\mathrm{AB}} = (+0.1 \pm 2.7 \pm 1.7) \times 10^{-13}$. The η_{AB} parameter was computed in Ref. [16] where a general formula was derived. Applying this general result to the case of the SUGRA potential leads to

$$\begin{aligned}\eta_{\mathrm{AB}} = \frac{1}{2}\kappa^{-1/2}\alpha_{\mathrm{E}}\Bigg[&\frac{\partial}{\partial Q}\left(\frac{\sigma'}{\Lambda_{\mathrm{QCD}}}\right)\left(\frac{N_\mathrm{A}+Z_\mathrm{A}}{M_\mathrm{A}} - \frac{N_\mathrm{B}+Z_\mathrm{B}}{M_\mathrm{B}}\right)\\ &+\frac{\partial}{\partial Q}\left(\frac{\delta'}{\Lambda_{\mathrm{QCD}}}\right)\left(\frac{N_\mathrm{A}-Z_\mathrm{A}}{M_\mathrm{A}} - \frac{N_\mathrm{B}-Z_\mathrm{B}}{M_\mathrm{B}}\right)\Bigg]\,,\end{aligned} \tag{3.11}$$

where N_A (respectively N_B) represents the number of neutrons in the atom A, the body A, by definition, being made of this type of atoms (respectively in the atom B) and Z_A (respectively Z_B) represents the number of protons in the atom A (respectively in the atom B). The coefficients $\alpha_{\rm E}$ is defined by $\alpha_{\rm E} \equiv \kappa^{-1/2} {\rm d} \ln m_{\rm E}/{\rm d}Q$ but for two pairs of test bodies, the ratio $\eta_{\rm AB}/\eta_{\rm BC}$ is independent of the background object E. The quantity $\Lambda_{\rm QCD} \sim 180$MeV is the QCD scale. Finally the variation of the coefficients σ' and δ' can be expressed as

$$\frac{\partial}{\partial Q}\left(\frac{\sigma'}{\Lambda_{\rm QCD}}\right) = \frac{1}{2}\frac{\kappa^{1/2}}{\Lambda_{\rm QCD}}\left(b_{\rm u}+b_{\rm d}\right)\left(\alpha_{\rm u} m_{\rm u}+\alpha_{\rm d} m_{\rm d}\right)+\frac{1}{2}\frac{\kappa^{1/2}}{\Lambda_{\rm QCD}}\alpha_{\rm d} m_{\rm e}\,, \tag{3.12}$$

$$\frac{\partial}{\partial Q}\left(\frac{\delta'}{\Lambda_{\rm QCD}}\right) = -\frac{1}{2}\frac{\kappa^{1/2}}{\Lambda_{\rm QCD}}\left(b_{\rm u}-b_{\rm d}\right)\left(\alpha_{\rm u} m_{\rm u}-\alpha_{\rm d} m_{\rm d}\right)-\frac{1}{2}\frac{\kappa^{1/2}}{\Lambda_{\rm QCD}}\alpha_{\rm d} m_{\rm e}\,, \tag{3.13}$$

where $b_{\rm u}+b_{\rm d} \sim 6$, $b_{\rm u}-b_{\rm d} \sim 0.5$, $\sigma'/\Lambda_{\rm QCD} \sim 3.8\times 10^{-2}$, $\delta'/\Lambda_{\rm QCD} \sim 4.2\times 10^{-4}$, $m_{\rm u} \sim 5$MeV, $m_{\rm d} \sim 10$MeV and $m_{\rm e} \sim 0.5$MeV. Let us notice that the coefficient $\alpha_{\rm d}$ appears in front of the mass of the electron $m_{\rm e}$ because, in the MSSM, the electron behaves as a "d" particle. If we compare the above equations with the ones of Ref. [16], one sees that there is no variations of the fine structure constant and/or of the gauge function $f_{\rm QCD}$. This is simply because, in this article, we assume that they are constant.

Let us recall that when the mass of the quintessence field is of the order of the gravitino mass, all the gravity tests, among which is the equivalence principle violation are trivially satisfied. However, it is interesting to compute the $\eta_{\rm AB}$ in the SUGRA model with the non trivial hidden sector. One obtains

$$\eta_{\rm AB} \sim \frac{1}{2}\alpha_{\rm E}\left(0.084\alpha_{\rm u}+0.168\alpha_{\rm d}\right)\left(\frac{N_{\rm A}+Z_{\rm A}}{M_{\rm A}}-\frac{N_{\rm B}+Z_{\rm B}}{M_{\rm B}}\right)+\frac{1}{2}\alpha_{\rm E}\left(-0.0069\alpha_{\rm u}+0.013\alpha_{\rm d}\right) \times\left(\frac{N_{\rm A}-Z_{\rm A}}{M_{\rm A}}-\frac{N_{\rm B}-Z_{\rm B}}{M_{\rm B}}\right). \tag{3.14}$$

when $a_i = 0$ and $c_i \neq 0$ to lead to a runaway potential. It is interesting to notice that the calculations of the equivalence principle violation is directly related to the supergravity Lagrangian and, therefore, originates from first principle. A good order of magnitude estimate of the parameter $\eta_{\rm AB}$ is simply, see Fig. 4

$$\eta_{\rm AB} \sim \alpha_{\rm u}^2\,, \tag{3.15}$$

where we have assumed $\alpha_{\rm E} \sim \alpha_{\rm u} \gg \alpha_{\rm d}$. In order to comply with the currently available experimental limits this implies

$$\kappa^{1/2} Q_{\rm now} \ll 10^{-6}\,. \tag{3.16}$$

The above number is obtained for $m_{3/2}^0 = 150\,{\rm GeV}$ and $m_{1/2}^0 = 580\,{\rm GeV}$, which implies $\mu = 985\,{\rm GeV}$, *i.e.* the values used in Fig. 4. This means that constraints on the weak equivalence principle are able to rule out this version of the SUGRA model. In other words, if the hidden sector is chosen such that it leads to an interesting cosmological model, then local tests of gravity are able to kill the scenario (at least for these values of the parameters).

3.3 The Proton-Electron Mass Ratio

Another consequence of coupling the MSSM to dark energy is that this will cause the proton to electron mass ratio, $r \equiv m_{\rm p}/m_{\rm e}$, to vary with time, that is to say with the redshift. This is an important consequence since, experimentally, there is an indication for a possible variation, $\Delta r/r \sim (2.0 \pm 0.6) \times 10^{-5}$ [27]. The proton mass can be written as

$$m_{\rm p} = C_{\rm QCD}\Lambda_{\rm QCD} + b_{\rm u}m_{\rm u} + b_{\rm d}m_{\rm d} + C_{\rm p}\alpha_{\rm QED}\,, \tag{3.17}$$

where $m_{\rm u}$ and $m_{\rm d}$ are the mass of the u and d quarks, $C_{\rm QCD} \sim 5.2$ and $C_{\rm p}\alpha_{\rm QED} \sim 0.63$MeV. Recalling that the electron behaves as a "d" particle, this leads to

$$r = b_{\rm u}\frac{\lambda_{\rm u}}{\lambda_{\rm e}}\tan\beta + b_{\rm d}\frac{\lambda_{\rm d}}{\lambda_{\rm e}} + \sqrt{1+\tan^2\beta}\,\frac{C_{\rm QCD}\Lambda_{\rm QCD} + C_{\rm p}\alpha_{\rm QED}}{\lambda_{\rm e}v(Q)}{\rm e}^{-\kappa K_{\rm quint}/2}\,, \tag{3.18}$$

where $\lambda_{\rm u}$, $\lambda_{\rm d}$ and $\lambda_{\rm e}$ are respectively the Yukawa couplings of the quarks u, d and of the electron. We also assume, for simplicity, that the gauge functions are constant, see above. Using Eqs. (2.25) and (2.26), one can express the Yukawa couplings in terms of the fermion masses at present time, generically denoted "m^0". This leads to

$$\begin{aligned} r \, = \, & b_{\rm u}\frac{m_{\rm u}^0}{m_{\rm e}^0}\frac{\tan\beta(Q)}{\tan\beta(Q_{\rm now})} + b_{\rm d}\frac{m_{\rm d}^0}{m_{\rm e}^0} + \frac{C_{\rm QCD}\Lambda_{\rm QCD} + C_{\rm p}\alpha_{\rm QED}}{m_{\rm e}^0}\sqrt{\frac{|\mu|^2 + m_{H_{\rm u}}^2(Q_{\rm now})}{|\mu|^2 + m_{H_{\rm u}}^2(Q)}} \\ & \times \frac{\tan\beta(Q)}{\tan\beta(Q_{\rm now})}\,. \end{aligned} \tag{3.19}$$

As before, in order to be consistent, we have to expand the above equation in terms of $1/\tan\beta$. Then, one obtains

$$r \sim r_{\rm now} + \left(b_{\rm u}\frac{m_{\rm u}^0}{m_{\rm e}^0} + \frac{C_{\rm QCD}\Lambda_{\rm QCD} + C_{\rm p}\alpha_{\rm QED}}{m_{\rm e}^0}\right)\left[\frac{\tan\beta(Q)}{\tan\beta(Q_{\rm now})} - 1\right]\,. \tag{3.20}$$

One can also Taylor expand the tangent function and use the expression of the derivative of the tangent at leading order. This gives

$$\frac{\Delta r}{r} \sim \left[\frac{1}{\kappa^{1/2}}\left(\frac{{\rm d}m_{\rm H_u}^2}{{\rm d}Q} + \frac{{\rm d}m_{\rm H_d}^2}{{\rm d}Q}\right)\left(2|\mu|^2 + m_{\rm H_u}^2 + m_{\rm H_d}^2\right)^{-1} - \frac{1}{\kappa^{1/2}}\frac{{\rm d}\ln B}{{\rm d}Q}\right]\kappa^{1/2}\Delta Q\,. \tag{3.21}$$

For instance, if one uses our fiducial model with $m_{3/2}^0 = 150$GeV and $m_{1/2}^0 = 580$GeV, one has $\mu = 985.7\,{\rm GeV}$, $m_{\rm H_u}^2 \sim -971650.6\,{\rm GeV}^2$ and $m_{\rm H_d}^2 \sim 190431.3\,{\rm GeV}^2$ where we recall, for order of magnitude estimate, we have taken $\kappa^{1/2}Q_{\rm now} = 1$ which is consistent with a runaway behavior. This leads to

$$\left|\frac{\Delta r}{r}\right| \sim 0.69\kappa^{1/2}\Delta Q\,, \tag{3.22}$$

This means that, in order to comply with the recent bound, one should have

$$\kappa^{1/2}\Delta Q < 10^{-5}\,, \tag{3.23}$$

in the range $z \in [0,3]$. One sees that this constraint is less strong than the one obtained from the equivalence principle, at least for this choice of parameters, but is also sufficient to rule out the model.

3.4 Cold Dark and Baryonic Energy Densities

The coupling between the observable sector (radiation and matter) and the quintessence sector also affects the way the energy densities scale with the Friedmann-Lemaitre-Robertson-Walker (FLRW) scale factor a. In a minimal setting, it turns out the quintessence field couples to matter and not to radiation since the gauge functions f_G are chosen to be constant. This implies that the radiation energy density still behaves like

$$\rho_{\rm rad} \propto \frac{1}{a^4}\,. \tag{3.24}$$

In particular, in this case, there is no variation of the fine–structure constant. On the contrary since the masses of the particles depend on κQ^2, the matter density behaves as

$$\rho_{\rm mat} = \sum_a n_a m^{\rm F}_{\rm u,d}\left(\kappa Q^2\right)\,, \tag{3.25}$$

where $m^{\rm F}_{\rm u,d}(\kappa Q^2)$ is the mass of non relativistic species the expression of which has been obtained before. The quantity n_a is the number of non-relativistic particles which is conserved $\dot{n}_a + 3Hn_a = 0$, where $H = \dot{a}/a$ is the Hubble parameter. This leads to

$$\rho_{\rm mat} \propto \frac{1}{a^3}\sum_a n_a^0 m^{\rm F}_{\rm u,d}\left(\kappa Q^2\right)\,, \tag{3.26}$$

where n_a^0 is just a constant. Again, in the most general case, the scaling of the cold dark and baryonic energy densities depends on the functions $a_i(Q)$ and $c_i(Q)$ and is completely explicit in the case $a_i = 0$. Let us also notice that cosmological consequences of Eq. (3.26) have been studied in Ref. [20]. As one can see, the coupling between matter and quintessence induces a modification of the quintessence potential

$$V_{\rm eff}(Q) = V_{\rm DE}(Q) + A_{\rm CDM}(Q)\frac{\rho^0_{\rm CDM}}{a^3}\,, \tag{3.27}$$

where $A_{\rm CDM}(Q) \equiv m_{\rm CDM}(Q)/m_{\rm CDM}(0)$, $m_{\rm CDM}$ being the mass of the dark matter particle, typically the lightest supersymmetric particle. When the effective potential admits a time-dependent minimum, the model is known as a chameleon model [28] and the cosmological evolution is changed. However, in the present context, this correction is negligible in comparison with the correction coming from the susy breaking, *i.e.* $m^2_{3/2}Q^2$, see Eq. (2.12). In fact, one must compare the derivatives of the two corrections since this is the derivative of the potential which appears in the Klein-Gordon equation. For the new term coming from the dark matter energy density, one has

$$\frac{\rho^0_{\rm mat}}{a^3}\frac{1}{A_{\rm CDM}}\frac{\partial A_{\rm CDM}}{\partial Q}A_{\rm CDM} \sim \frac{\rho^0_{\rm mat}}{a^3}\kappa^{1/2}\alpha_{\rm u,d} \sim \frac{\rho^0_{\rm mat}}{a^3}\kappa Q\,, \tag{3.28}$$

and for the susy breaking term, this is simply $m^2_{3/2}Q$. Since $m^2_{3/2} \gg \rho_{\rm mat}/m^2_{\rm Pl}$, one can ignore the correction coming from the cold dark matter energy density as announced previously. This is essentially due to the susy breaking term as the field happens to settle down at a minimum where its vev is very small. The situation would drastically change if the potential kept its runaway shape as could be the case, for instance, if the hidden sector were such that $a_i(Q)$ and $c_i(Q)$ were non trivial. In the following, we do not consider this situation and, therefore, we neglect the correction coming from the non trivial dependence of the dark matter energy density.

4. Implications for Cosmology

4.1 Fixing the Free Parameters of the Potential

In this section, we study the cosmological implications for the case $a_i = c_i = 0$ as we have just seen that this case cannot be ruled out by the gravity constraints. The previous quintessence potential with $a_i = c_i = 0$ is characterized by three free parameters, M, $m_{3/2}^0$ and Υ. As a consequence, the potential can be rewritten in terms of dimensionless quantities as

$$V_{\rm quint}(Q) = m^4 {\rm e}^{\bar{Q}^2/2}\left(\bar{Q}^{-\alpha} + A\bar{Q}^2 - B\right) , \tag{4.1}$$

where $\bar{Q} \equiv \kappa^{1/2}Q$ is dimensionless and (obviously, the A and B below have nothing to do with the soft terms calculated in the previous subsection)

$$\left(\frac{M}{m_{\rm Pl}}\right)^{4+\alpha} = (8\pi)^{-\alpha/2}\left(\frac{m}{m_{\rm Pl}}\right)^4 , \quad A = \frac{1}{2}(8\pi)^{1-\alpha/2}\left(\frac{m_{3/2}^0}{m_{\rm Pl}}\right)^2\left(\frac{M}{m_{\rm Pl}}\right)^{-(4+\alpha)} , \tag{4.2}$$

$$B = (8\pi)^{-\alpha/2}\left(\frac{\Upsilon}{m_{\rm Pl}}\right)^4\left(\frac{M}{m_{\rm Pl}}\right)^{-(4+\alpha)} . \tag{4.3}$$

The above potential has a minimum, $\bar{Q}_{\rm min}$. Let us assume that we are in a situation where $\bar{Q}_{\rm min} \ll 1$. In this case, an explicit expression of $\bar{Q}_{\rm min}$ can be found. Since the value of the field at the minimum is small, the exponential factor will be close to one. If so, it is easy to show that

$$\bar{Q}_{\rm min} \simeq \left(\frac{p}{A}\right)^{1/(\alpha+2)} . \tag{4.4}$$

We see that the location of the minimum is now controlled by the gravitino mass through the constant A. It is interesting to compare this value with the minimum of the usual SUGRA potential which reads $\bar{Q}_{\rm min} = \sqrt{\alpha}$ and is controlled by the Planck mass only. As we show in the following, the value of the minimum in the case where supersymmetry breaking is taken into account is much smaller than the minimum of the SUGRA potential. If we define the reduced field q by $q \equiv \bar{Q}/\bar{Q}_{\rm min}$, then the potential becomes

$$V_{\rm quint}(q) = \tilde{m}^4 {\rm e}^{\bar{Q}_{\rm min}^2 q^2/2}\left(q^{-\alpha} + pq^2 - \tilde{B}\right) . \tag{4.5}$$

The new scale $\tilde{m}$ is of course directly related to the scale m. The relation reads $\tilde{m}^4 = \bar{Q}_{\rm min}^{-\alpha} m^4$. The potential is still characterized by three parameters which are now $\tilde{m}$, $\tilde{B}$ and $\bar{Q}_{\rm min}$. The constant $\tilde{m}$ is chosen such that the quintessence energy density be approximately 70% of the critical energy density today. If we assume that the field is, today, at its minimum, *i.e.* $q = 1$, (this will be shown in the following) this leads to

$$\tilde{m}^4 \sim \frac{\Omega_Q}{1 + \alpha/2 - \tilde{B}}\rho_{\rm cri} \simeq \mathcal{O}(1)\rho_{\rm cri} . \tag{4.6}$$

Then, one can link the remaining parameters with the particle physics parameters. For the gravitino mass, one obtains

$$\left(\frac{m_{3/2}^0}{m_{\rm Pl}}\right)^2 \approx \frac{\alpha}{8\pi}\bar{Q}_{\rm min}^{-2}\frac{\rho_{\rm cri}}{m_{\rm Pl}^4}\,. \tag{4.7}$$

As an example, one can take $m_{3/2}^0 \simeq 100{\rm GeV}$ which gives $\bar{Q}_{\rm min} \sim 2.6\times 10^{-45}$, where we have used $m_{\rm Pl} \sim 1.22\times 10^{19}{\rm GeV}$ and $\rho_{\rm cri} \sim 8.1h^2 10^{-47}{\rm GeV}^4$ with $h \sim 0.72$ (h is the reduced present time Hubble parameter). If $m_{3/2}^0 = 1{\rm eV}$, one obtains $\bar{Q}_{\rm min} \simeq 2.6\times 10^{-34}$. Therefore, the minimum is located at tiny values of the quintessence vev (compared to the Planck mass). This makes a very important difference compared to the standard SUGRA case where, as already mentioned before, the minimum is close to $m_{\rm Pl}$. Notice that, despite the fact that the vev of the quintessence field is now very small in comparison to the Planck scale, this does not mean that supergravity is no longer a necessary ingredient in the present context. This is because, when susy breaking is taking into account, the global susy limit is more subtle than simply taking the limit $m_{\rm Pl} \rightarrow +\infty$.

Then, the scale M is given by

$$\left(\frac{M}{m_{\rm Pl}}\right)^{4+\alpha} = (8\pi)^{-\alpha/2}\bar{Q}_{\rm min}^{\alpha}\left(\frac{\tilde{m}}{m_{\rm Pl}}\right)^4 \simeq (8\pi)^{-\alpha/2}\bar{Q}_{\rm min}^{\alpha}\frac{\rho_{\rm cri}}{m_{\rm Pl}^4}\,. \tag{4.8}$$

If one uses the expression of $\bar{Q}_{\rm min}$ deduced from Eq. (4.7) and the expression of the mass M, $M^{4+\alpha} \sim \lambda^2\xi^4 m_{\rm c}^{\alpha}$, one obtains (assuming, as usual, $\lambda \sim 1$) the expression of the scale ξ, namely

$$\frac{\xi}{m_{\rm Pl}} \sim \left(\frac{\rho_{\rm cri}}{m_{\rm Pl}}\right)^{1/4}\left(\frac{m_{\rm Pl}}{m_{\rm c}}\right)^{\alpha/4}\left[\left(\frac{m_{3/2}^0}{m_{\rm Pl}}\right)^{-2}\frac{\rho_{\rm cri}}{m_{\rm Pl}^4}\right]^{\alpha/8}\,. \tag{4.9}$$

This expression should be compared with Eq. (1.3). We see that the difference lies in the presence of the term in the squared bracket which contains the gravitino mass and the critical energy density. This has disastrous consequences. If, as before, one chooses $m_{3/2}^0 = 100{\rm GeV}$, then for, say, $\alpha = 4$ one gets $\xi/m_{\rm Pl} \sim 10^{-70}$. Even more serious, it is clear that there is no value of α which, for a reasonable value of the gravitino mass, would allow us to obtain a scale ξ greater than the TeV as it was the case for Eq. (1.3). Recall that the energy scale associated to the cosmological constant is $\Lambda/m_{\rm Pl} \sim 10^{-30}$. This means that, in the case of quintessence, we have to build a complicated model and that, in addition, we have to fine tune the basic scale of the model even more than in the case of the cosmological constant where nothing else is needed. The origin of the problem can really be traced back to the fact that modeling quintessence in a more realistic fashion (breaking supersymmetry properly, taking into account the interaction between the various sectors etc ...) has modified the shape of the potential such that the original success of finding reasonable ξ from the "see-saw" formula (1.3) has vanished.

Finally, the scale Υ can be expressed as

$$\left(\frac{\Upsilon}{m_{\rm Pl}}\right)^4 \approx \tilde{B}\frac{\rho_{\rm cri}}{m_{\rm Pl}^4}\,, \tag{4.10}$$

and this implies that $\sum_{\alpha=1}^{n} \xi_\alpha^2 \sim 3/\kappa$. This equation was used in Ref. [16].

The previous estimates rest on the assumption that, very quickly, the quintessence field is stabilized at its minimum. Considering to the drastic consequences evoked before, we now carefully check that this is indeed the case.

4.2 Dynamics of the Quintessence Field

Let us assume that the initial value of the field is such that $Q_{\rm ini} \ll Q_{\rm min}$. In this case, the potential studied above approximatively reduces to the Ratra-Peebles potential. In fact, it seems necessary to start from the branch $Q^{-\alpha}$ rather from the branch Q^2 in order to have insensibility to the initial conditions. Indeed, if the potential is made of a series of monomial with positive powers, it is known that a fine-tuning of the initial conditions becomes necessary. In the case of the Ratra-Peebles potential, there is a particular attractor solution given by

$$Q_{\rm attra} = Q_{\rm p} \left(\frac{a}{a_{\rm p}}\right)^{3(1+\omega_{\rm B})/(\alpha+2)} , \tag{4.11}$$

where we have assumed that the field is a test field and that the background behaves as

$$a(\eta) = a_{\rm p} \left(\frac{\eta}{\eta_{\rm p}}\right)^{2/(1+3\omega_{\rm B})} , \tag{4.12}$$

η being the conformal time and $\omega_{\rm B}$ the equation of state parameter of the background fluid (radiation or matter, that is to say $\omega_{\rm B} = 1/3$ or $\omega_{\rm B} = 0$). The time $\eta_{\rm p}$ is arbitrary and can be chosen at convenience. In practice, we will often consider that this is the time of reheating. The constant $Q_{\rm p}$ can be expressed as

$$Q_{\rm p}^{-\alpha-2} = \frac{18}{\alpha^2 a_{\rm p}^2 \eta_{\rm p}^2 M^{4+\alpha}} \frac{1-\omega_{\rm Q}^2}{(1+3\omega_{\rm B})^2} = \frac{9\left(1-\omega_Q^2\right)}{2\alpha^2} \frac{H_{\rm p}^2}{M^{4+\alpha}} . \tag{4.13}$$

where $H_{\rm p}$ denotes the Hubble parameter at time $\eta = \eta_{\rm p}$. ω_Q is the equation of state parameter of the attractor and is given by $\omega_Q = (-2 + \alpha\omega_{\rm B})/(\alpha+2)$.

At this point, it is worth recalling the following point, already discussed at the end of the previous section. For $\omega_{\rm B} = 0$, the solution (4.12) $a(\eta) \propto \eta^2$ is obtained from $\rho_{\rm mat} \propto 1/a^3$. However, in the present context, one has to be more precise as in Eq. (3.25) a dependence in the quintessence field is present. However, the Q dependence, at very small values of Q, is very weak. In other words $m_a\left(\kappa Q^2\right) \sim$ cte if $Q \ll m_{_{\rm Pl}}$. Therefore, in the regime under consideration, one can safely assume that $\rho_{\rm mat} \propto 1/a^3$ and the corresponding behavior of the scale factor follows. In other words, as already discussed, one can safely neglect the fact that the model is a chameleon in the present context.

The attractor solution is completely specified once the fact that quintessence represents 70% of the critical energy density today has been imposed. Let us evaluate its value just after inflation. At reheating, $z_{\rm reh} = 10^{28}$, the value of the field is

$$\bar{Q}_{\rm attra}\left(z_{\rm reh}\right) \simeq 10^{-117/(\alpha+2)} \times \bar{Q}_{\rm min}^{\alpha/(\alpha+2)} , \tag{4.14}$$

where, for simplicity, we have not considered the factor $9(1-\omega_Q^2)/(2\alpha^2)$ which is, roughly speaking, of order one (in the following we will always neglect this kind of factors since we are interested in order of magnitude estimates only). We have also taken the radiation contribution today to be $\Omega_{\rm rad}^0 \sim 10^{-5}$. For $m_{3/2}^0 = 100\,{\rm GeV}$ and $\alpha = 6$, $\bar{Q}_{\rm attra}(z_{\rm reh}) \sim 8.6 \times 10^{-49} m_{\rm Pl}$. For $m_{3/2}^0 = 1{\rm eV}$ and the same parameters, $\bar{Q}_{\rm attra}(z_{\rm reh}) \sim 1.5 \times 10^{-40} m_{\rm Pl}$. If we now compare the initial quintessence energy density with the energy density of the background (*i.e.* the energy density of radiation), one obtains

$$\Omega_{Q_{\rm attra}}(z_{\rm reh}) \sim 10^{-107/(p+1)} \times \bar{Q}_{\rm min}^{2p/(p+1)} \,. \tag{4.15}$$

For $m_{3/2}^0 = 100\,{\rm GeV}$ and $\alpha = 6$, one obtains $\Omega_{Q_{\rm attra}}(z_{\rm reh}) \sim 2.4 \times 10^{-94}$. For $m_{3/2}^0 = 1{\rm eV}$ and $\alpha = 6$, one obtains $\Omega_{Q_{\rm attra}}(z_{\rm reh}) \sim 7.4 \times 10^{-78}$. For comparison, we recall that, at reheating, one has $\rho_{\rm B} \sim 10^{107} \rho_{\rm cri}$. The previous estimates show that the energy density of the attractor is always initially very small in comparison with that of the background. This is not the case in the "standard" Ratra-Peebles case. This difference is due to the fact that the scale M now depends on $\bar{Q}_{\rm min}$ which turns out to be a very small quantity.

The solution given in Eq. (4.11) is valid as long as $Q_{\rm attra}$ is small in comparison with $Q_{\rm min}$ and breaks down when $Q_{\rm attra} \sim Q_{\rm min}$. This happens when

$$\frac{a_{\rm min}}{a_{\rm p}} \sim \left(\frac{H_{\rm p}}{m_{3/2}^0}\right)^{2/(3+3\omega_{\rm B})} \,, \tag{4.16}$$

that is to say at a redshift given approximately by (strictly speaking, this expression is valid if $z_{\rm min} > 10^4$ since we used that $\omega_{\rm B} = 1/3$; otherwise, one has to take into account that the background becomes matter dominated)

$$1 + z_{\rm min} \sim 10^{32} \times \left(\frac{m_{3/2}^0}{m_{\rm Pl}}\right)^{1/2} \sim 10 \times \bar{Q}_{\rm min}^{-1/2} \,. \tag{4.17}$$

For our typical examples with $m_{3/2}^0 \simeq 100\,{\rm GeV}$, one obtains $z_{\rm min} \simeq 2.4 \times 10^{23}$ to be compared with the reheating redshift, $z_{\rm reh} \sim 10^{28}$. For $m_{3/2}^0 = 1{\rm eV}$, one gets $z_{\rm min} \simeq 7.7 \times 10^{17}$. In both cases, the field reaches the minimum well before Big Bang Nucleosynthesis.

Now, the field is of course not necessarily on the attractor initially. It is therefore important to estimate at which redshift the attractor is joined and to compare this redshift to $z_{\rm min}$. If $Q_{\rm ini} > Q_{\rm attra}$ (undershoot) then the field remains frozen. Therefore, the redshift at which the attractor is joined is given by the condition $Q_{\rm ini} = Q_{\rm attra}$ which results in

$$\frac{a_{\rm under}}{a_{\rm p}} \sim \left(Q_{\rm ini}^{2p+2} \frac{H_{\rm p}^2}{M^{4+2p}}\right)^{1/(3+3\omega_{\rm B})} \,. \tag{4.18}$$

Using this relation, one finds

$$1 + z_{\rm under} \sim 10 \times \left(\frac{\bar{Q}_{\rm min}}{\bar{Q}_{\rm ini}}\right)^{p/2} \bar{Q}_{\rm ini}^{-1/2} \,. \tag{4.19}$$

One can check that, if the initial value of the field is the value on the attractor given by Eq. (4.14), then $z_{\rm under} \sim z_{\rm reh}$ as expected. More generally, we conclude that each time the field starts in a undershoot situation, that is to say initially $Q_{\rm attra} < Q_{\rm ini} < Q_{\rm min}$, it will join the attractor before reaching the minimum.

Let us now consider the case of an overshoot, *i.e.* $Q_{\rm ini} < Q_{\rm attra}$. Then, the field is first kinetic dominated until the potential energy becomes equal to the kinetic energy. When this happens, the field becomes frozen until the attractor is joined. During the phase dominated by the kinetic energy, the field behaves as

$$Q = Q_{\rm ini} + m_{_{\rm Pl}}\sqrt{\frac{3\Omega_{\rm Q_{ini}}}{4\pi}}\left(1-\frac{a_{\rm ini}}{a}\right) . \tag{4.20}$$

This allows us to estimate the redshift at which the field becomes frozen, $z_{\rm froz}$, and to compare it with $z_{\rm min}$. The redshift at which the field becomes frozen can be estimated to be

$$1 + z_{\rm froz} \sim 10^{10} \times \bar{Q}_{\rm min}^{p/3} \times \Omega_{Q_{\rm ini}}^{-(p+1)/6} . \tag{4.21}$$

On the other hand, if the quintessence field behaves as in Eq. (4.20), it reaches the minimum at a redshift given by

$$1 + z_{\rm kin\to min} \sim 10^{10} \times \Omega_{Q_{\rm ini}}^{-1/6} . \tag{4.22}$$

Therefore, the field becomes frozen before reaching the minimum if $1+z_{\rm froz} > 1+z_{\rm kin\to min}$, a condition which amounts to $\bar{Q}_{\rm min} > \Omega_{\rm Q_{ini}}^{1/2}$. As an illustration, let us consider again the model for which $m_{3/2}^0 = 100\,{\rm GeV}$ and $\alpha = 6$. From the previous considerations, one knows that $\bar{Q}_{\rm min} \simeq 2.6\times 10^{-45}$ and that, in order to have overshoot, $10^{-94} \lesssim \Omega_{Q_{\rm ini}} \lesssim 10^{-4}$, this last bound corresponding to equipartition initially. Therefore, for the above condition to be satisfied, one needs $\Omega_{Q_{\rm ini}} \lesssim 10^{-90}$.

We conclude that, on more general grounds, the above condition cannot be satisfied unless the field is, at the beginning, very close to the attractor. Therefore, generically, when we have overshoot, the field reaches the minimum in the kinetic dominated phase and has no time to freeze out.

4.3 Approaching the Minimum

We now describe the behavior of the quintessence field when it starts feeling that the potential has developed a minimum. When the field is close to the minimum, the potential can be approximated by

$$V(Q) \simeq V_{\rm min} + \frac{1}{2}\left(m_{3/2}^0\right)^2 (Q - Q_{\rm min})^2 . \tag{4.23}$$

The mass of the field is the gravitino mass as established before and in Ref. [16]. Let us first consider the situation where the quintessence field is a test field. The Klein-Gordon equation, written with the number of e-folds N as the time variable, reads

$$\frac{{\rm d}^2}{{\rm d}N^2}\left(\frac{Q}{m_{_{\rm Pl}}}\right) + \left(3 + \frac{1}{H}\frac{{\rm d}H}{{\rm d}N}\right)\frac{{\rm d}}{{\rm d}N}\left(\frac{Q}{m_{_{\rm Pl}}}\right) + \frac{m_{_{\rm Pl}}^2}{H^2}\frac{\partial f}{\partial (Q/m_{_{\rm Pl}})} = 0\,, \tag{4.24}$$

where $V(Q) \equiv m_{_{\rm Pl}}^4 f(Q)$. The Hubble parameter is given by $H = H_{\rm p}\exp[-3(1+\omega_{_{\rm B}})N_{\rm p}/2]$, where $N_{\rm p}$ is the number of e-folds counted from the time $\eta = \eta_{\rm p}$. The above equation can be easily integrated and the solution reads

$$\frac{Q}{m_{_{\rm Pl}}} = \frac{Q_{\rm min}}{m_{_{\rm Pl}}} + {\rm e}^{-3(1-\omega_{_{\rm B}})N_{\rm p}/4}\left\{A_1 J_{(1-\omega_{_{\rm B}})/(2+2\omega_{_{\rm B}})}\left[\frac{2}{3(1+\omega_{_{\rm B}})}\frac{m_{3/2}^0}{H_{\rm p}}{\rm e}^{3(1+\omega_{_{\rm B}})N_{\rm p}/2}\right]\right.$$
$$\left. + A_2 J_{-(1-\omega_{_{\rm B}})/(2+2\omega_{_{\rm B}})}\left[\frac{2}{3(1+\omega_{_{\rm B}})}\frac{m_{3/2}^0}{H_{\rm p}}{\rm e}^{3(1+\omega_{_{\rm B}})N_{\rm p}/2}\right]\right\}, \tag{4.25}$$

where $J_\nu(z)$ is a Bessel function of order ν. As expected the field start oscillating around its minimum when its mass equals the Hubble parameter. Using the expression of the gravitino mass given before, one easily checks that this happens at a redshift of

$$1 + z_{\rm osci} \sim 10 \times \left(\bar{Q}_{\rm min}\right)^{-1/2}. \tag{4.26}$$

When the mass is smaller than the Hubble parameter, the field is essentially frozen. Using the asymptotic expression of the Bessel functions for small arguments, one obtains

$$\frac{Q}{m_{_{\rm Pl}}} \simeq \frac{Q_{\rm min}}{m_{_{\rm Pl}}} + \bar{A}_1 + \bar{A}_2 {\rm e}^{-3(1-\omega_{_{\rm B}})N_{\rm p}/2}, \tag{4.27}$$

where $\bar{A}_1$ and $\bar{A}_2$ are two new constants, different from A_1 and A_2. Very rapidly, the branch proportional to $\bar{A}_2$ becomes negligible. In the opposite situation, *i.e.* when the mass is much larger than the Hubble parameter, one can use the asymptotic expansion of the Bessel functions for large values of the arguments and one obtains

$$\frac{Q}{m_{_{\rm Pl}}} \simeq \frac{Q_{\rm min}}{m_{_{\rm Pl}}} + {\rm e}^{-3(1-\omega_{_{\rm B}})N_{\rm p}/2}\left\{\tilde{A}_1 \cos\left[\frac{2}{3(1+\omega_{_{\rm B}})}\frac{m_{3/2}^0}{H_{\rm p}}{\rm e}^{3(1+\omega_{_{\rm B}})N_{\rm p}/2} - \frac{\pi(1-\omega_{_{\rm B}})}{4(1+\omega_{_{\rm B}})} - \frac{\pi}{4}\right]\right.$$
$$\left. + \tilde{A}_2 \cos\left[\frac{2}{3(1+\omega_{_{\rm B}})}\frac{m_{3/2}^0}{H_{\rm p}}{\rm e}^{3(1+\omega_{_{\rm B}})N_{\rm p}/2} + \frac{\pi(1-\omega_{_{\rm B}})}{4(1+\omega_{_{\rm B}})} - \frac{\pi}{4}\right]\right\}, \tag{4.28}$$

where $\tilde{A}_1$ and $\tilde{A}_2$ are new constants. The oscillations are damped by a factor $a^{-3(1-\omega_{_{\rm B}})/2}$.

Let us summarize the two possibilities. In case of an undershoot, the field joins the attractor and then reaches the minimum without any oscillatory phase as $z_{\rm osci} \sim z_{\rm min}$. If there is an overshoot, the field has no time to freeze out and goes directly from the kinetic dominated phase to the oscillatory phase.

At some point, the quintessence field starts dominating the matter content of the Universe. In this case, the above treatment breaks down since the quintessence field is no longer a test field. Assuming, for simplicity, that $\rho \sim V_{\rm min}$, the Klein-Gordon equation can still be solved explicitly. The solution reads

$$\frac{Q}{m_{_{\rm Pl}}} = \frac{Q_{\rm min}}{m_{_{\rm Pl}}} + {\rm e}^{-3N/2}\left\{B_1 \cos\left[\sqrt{\frac{3}{8\pi}}\frac{m_{3/2}^0/m_{_{\rm Pl}}}{(V_{\rm min}/m_{_{\rm Pl}}^4)^{1/2}}N\right]\right.$$
$$\left. + B_2 \sin\left[\sqrt{\frac{3}{8\pi}}\frac{m_{3/2}^0/m_{_{\rm Pl}}}{(V_{\rm min}/m_{_{\rm Pl}}^4)^{1/2}}N\right]\right\} \tag{4.29}$$

and we still have very rapid oscillations, damped by a factor $a^{-3/2}$. For $m_{3/2}^{0} = 100\,\mathrm{GeV}$, the dimensionless frequency (N being the time variable) is $\sim 10^{43}$. Again the oscillations stop rapidly as their amplitude decays exponentially with the number of e–folds.

4.4 Numerical Integration

Of course, rather than the approximate considerations developed above, a full numerical integration would allow us to obtain the exact solution. Unfortunately, the realistic values of $\bar{Q}_{\rm min}$ are so small that a simple Fortran code cannot handle the corresponding solution. However, one can check the previous analytical estimates for values of $\bar{Q}_{\rm min}$ which are numerically reasonable (but not physically realistic). Then, having checked and validated the previous estimates, we will use them in a physically relevant situation.

Let us consider a situation where the free parameters of the potential are given by $\bar{Q}_{\rm min} = 10^{-4}$, $\alpha = 6$ and $\tilde{B} = 1$. This implies that the mass scale $M \sim 8 \times 10^{-16} m_{\rm Pl}$, the gravitino mass $m_{3/2}^{0} \sim 2 \times 10^{-58} m_{\rm Pl}$ and $\Upsilon \sim 2 \times 10^{-31} m_{\rm Pl}$. According to the previous estimates, the initial value of the field on the attractor is $\bar{Q}_{\rm attra} \sim 2.3 \times 10^{-18} m_{\rm Pl}$ corresponding to $\Omega_{Q_{\rm attra}} \sim 1.8 \times 10^{-33}$. On the attractor, the minimum of the potential is felt at a redshift of $1 + z_{\rm min} \sim 1000$.

Let us now consider the initial conditions corresponding to equipartition, *i.e.* $\Omega_{Q_{\rm ini}} = 10^{-4}$. This implies that $\bar{Q}_{\rm ini} \sim 6.8 \times 10^{-22} m_{\rm Pl}$ and we have overshoot since $\bar{Q}_{\rm ini} < \bar{Q}_{\rm attra}$. As a consequence, as explained before, the initial evolution is dominated by the kinetic energy and we have

$$Q = Q_{\rm ini} + m_{\rm Pl} \sqrt{\frac{3\Omega_{\rm Q_{ini}}}{4\pi}} \left(1 - {\rm e}^{-N}\right) , \tag{4.30}$$

where N is the total number of e-folds counted from reheating. Very quickly, we have $Q = \sqrt{3\Omega_{\rm Q_{ini}}/(4\pi)} \sim 0.00489 m_{\rm Pl}$ (or $\bar{Q} \sim 0.0245 m_{\rm Pl}$). Let us notice that this value is greater than the value of the minimum. Although the field seems to be frozen, its time variation is still sufficient for the corresponding kinetic energy to be greater than the critical energy (a similar situation arises in the standard Ratra-Peebles scenario, see Ref. [14]). In fact, the field rolls down the potential so quickly that it goes through the minimum while the kinetic regime goes on (when the kinetic energy dominates, the fact that the potential has a minimum is irrelevant). The kinetic energy reaches the critical energy, by definition at $z_{\rm kin \to min}$, when the field is on the other side of the potential. As a consequence, the field will approach the minimum "from the right".

In the Ratra-Peebles case, the kinetic regime comes to an end at the redshift $1 + z_{\rm froz} \sim 4.64 \times 10^{8}$ while, in the present case where the potential possesses a minimum, this one is felt by the field at $z_{\rm kin \to min} \sim 4.64 \times 10^{10}$. Therefore, in this case, the field does not enter the potential dominated regime at all and directly goes from the kinetic dominated regime to a regime where the minimum is felt and where the solution (4.25) is relevant. However, the solutions before $z_{\rm kin \to min} \sim 4.64 \times 10^{10}$ given by Eq. (4.30) and after, given by Eq. (4.27) are the same. As a consequence, the coefficients $\bar{A}_1$ and $\bar{A}_2$ are such that the solution after $z_{\rm kin \to min}$ is still given by Eq. (4.30). So, even after $z_{\rm kin \to min}$, the field remains "frozen" at $Q \sim 0.00489 m_{\rm Pl}$. Nevertheless, the evolution of the energy density changes and, instead of $\rho \propto a^{-6}$, we have $\rho \sim \mathrm{cte}$.

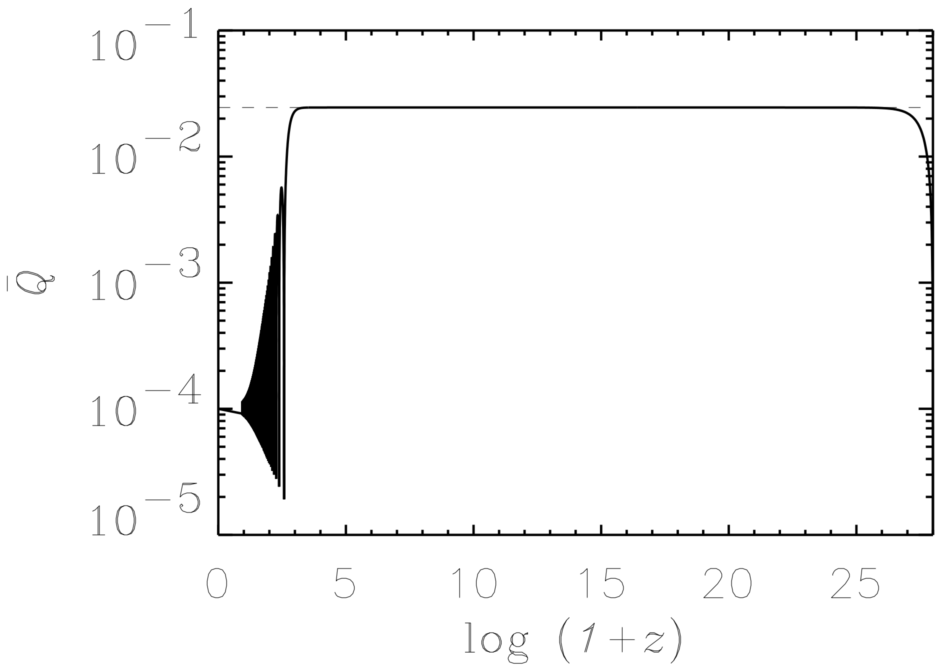

Figure 5: Evolution of the scalar field for the parameters, $\bar{Q}_{\rm min} = 10^{-4}$, $\tilde{B} = 1$ and $\alpha = 6$ with the initial conditions such that $\Omega_{Q_{\rm ini}} = 10^{-4}$ (equipartition and overshoot). The dashed horizontal represents the value $Q = \sqrt{3\Omega_{Q_{\rm ini}}/(4\pi)}$. It is clear from this plot that the numerical estimates presented in the text are fully compatible with the numerical evolution. In particular, one can check that the oscillations start at $z \sim 10^3$ and that, eventually, after rapid oscillations, the field stabilizes at its minimum.

At $1+z_{\rm osci} \sim 10^3$, the Hubble parameter is equal to the gravitino mass and the damped oscillations of Eq. (4.28) start. Then, the field stabilizes at its minimum. The evolution of the field is represented in Fig. 5 and the corresponding quintessence energy density is plotted in Fig. 6.

Let us now consider the case of an undershoot, *i.e.* the case where the initial value of the field is greater than the initial value on the attractor. We take $\Omega_{Q_{\rm ini}} = 10^{-45}$ which implies that $Q_{\rm ini} \sim 4.6 \times 10^{-15} m_{_{\rm Pl}}$. The corresponding numerical evolution is represented in Figs. 7 and 8. Since we have undershoot, the field will be initially frozen until it reaches the attractor. This happens at $z_{\rm under} \sim 4.6 \times 10^{23}$. On the attractor, the effective equation of state is $\omega_Q = (-2 + \alpha\omega_{_{\rm B}})/(\alpha + 2) = 0$ for $\alpha = 6$ and $\omega_{_{\rm B}} = 1/3$. As a consequence, the energy density of quintessence scales as matter, as can be checked on Fig. 8, until the minimum is reached. The minimum is felt at $z_{\rm min} \sim 1000$ which is also $z_{\rm osci}$, the redshift at which the Hubble parameter is equal to the gravitino mass and the oscillations start. Therefore, we expect no phase where the field is frozen, as predicted by Eq. (4.27) but expect the oscillations to start immediately after the field has left the attractor. However, in the case of an undershoot, when the minimum is felt we necessarily have $Q \sim Q_{\rm min}$.

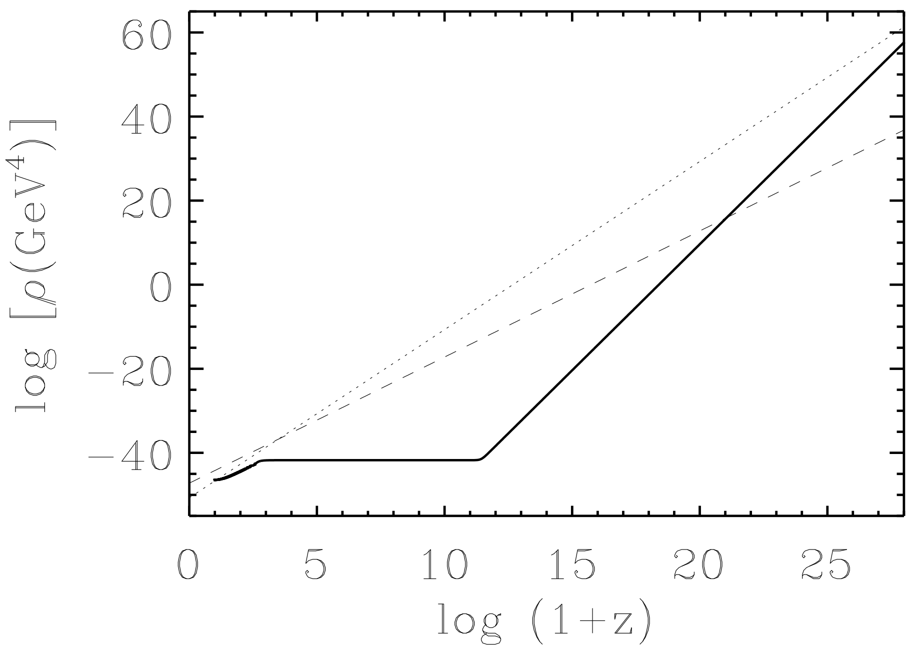

Figure 6: Evolution of the quintessence energy density (solid line) for $\bar{Q}_{\min} = 10^{-4}$, $\tilde{B} = 1$ and $\alpha = 6$ with the initial conditions such that $\Omega_{Q_{\rm ini}} = 10^{-4}$. The dotted line represents the evolution of the radiation energy density while the dashed line is the matter energy density. The energy density freezes at $z_{\rm kin\to min}$ and not at $z_{\rm froz}$ as predicted in the text since the minimum of the potential is felt before the kinetic energy becomes equal to the potential energy. The analytical estimate $z_{\rm kin\to min} \sim 5\times 10^{10}$ is in relatively good agreement with the actual value observed in the figure. The difference (about one order of magnitude) is probably due to the fact that the minimum is felt before the energy density is equal to $\rho_{\rm cri}$ (which was the criterion used in order to derive the expression of $z_{\rm kin\to min}$) which has the effect to increase $z_{\rm kin\to min}$.

In the previous case of an overshoot this was not the case because the kinetic energy of the field was dominating the potential energy at the moment where the presence of the minimum is seen by the field. Therefore, the amplitude of the oscillations is very small and, in practice, we see no oscillations at all. The field directly stabilizes at this minimum. This behavior is indeed observed in Fig. 7.

As a conclusion of the subsection, we have checked that the numerical estimates derived before are confirmed by a numerical calculation in the (physically unrealistic) case where this one is possible.

Let us summarize our findings in the realistic cases where the gravitino mass is of the eV or 100 GeV scales. In both cases the minimum of the potential is at extremely small values compared to the Planck scale. This implies that we can safely neglect the Q dependence of the particle masses and trust the supergravity expansion scheme in $1/m_{\rm c}$. On the other hand, the existence of a minimum with a mass of the order of the gravitino mass leads to a drastic modification of the evolution of the quintessence field. In both

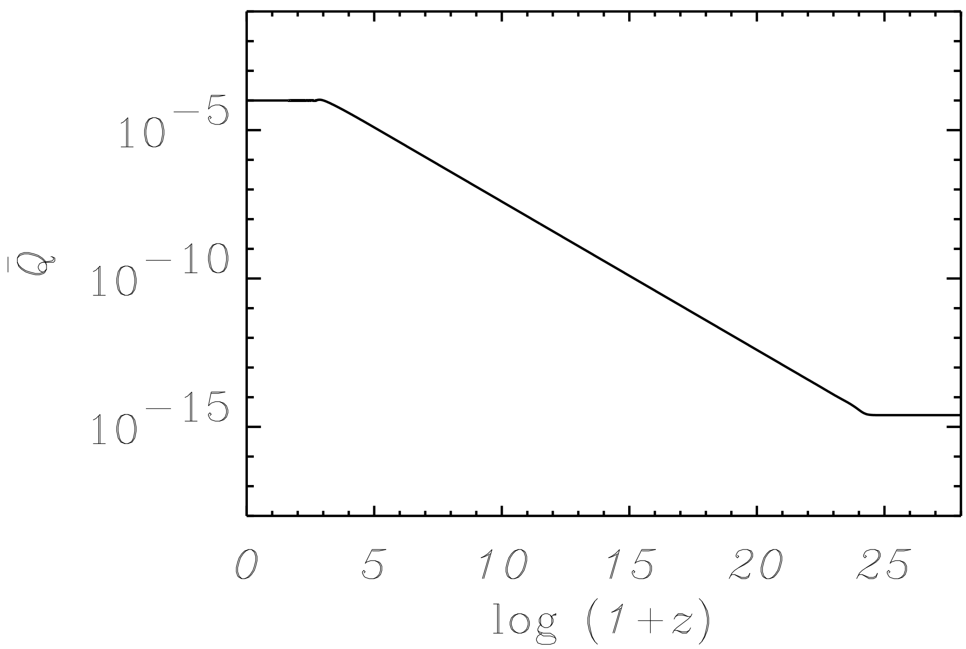

Figure 7: Evolution of the scalar field for the parameters, $\bar{Q}_{\rm min} = 10^{-4}$, $\tilde{B} = 1$ and $\alpha = 6$ with the initial conditions such that $\Omega_{Q_{\rm ini}} = 10^{-45}$ (undershoot). The numerical estimates presented in the text are in good agreement with the numerical evolution.

cases, either overshoot or undershoot, we find that the quintessence field settles down at the bottom of the potential before the beginning of BBN. From the point of view of late time physics, the quintessence sector plays the role of an effective cosmological constant with equation of state $\omega_Q = -1$. Observationally this is not a problem since a cosmological constant is perfectly compatible with the currently available data. However, conceptually, we consider it as a second fundamental difficulty (the first one was he magnitude of ξ) of the realistic model where supersymmetry breaking is taken into account. Indeed, it seems clear that the justification for building a complicated model which is just equivalent to a cosmological constant is very weak. One possible way out is to study whether the behavior of the perturbed quantities allows us to distinguish this model from a pure cosmological constant. In particular, although the equation of state is $\omega_Q = -1$, the mass of the quintessence field is now $m_{3/2}^0$ and, therefore, the corresponding Jeans length is very small compared to the usual case where it is the Hubble scale. This is why, in the next section, we analyze whether the perturbations of the quintessence field can give information on the dynamics of quintessence prior to BBN.

4.5 Cosmological Perturbations

In this section, we study how the perturbations of the quintessence field behave. In con-

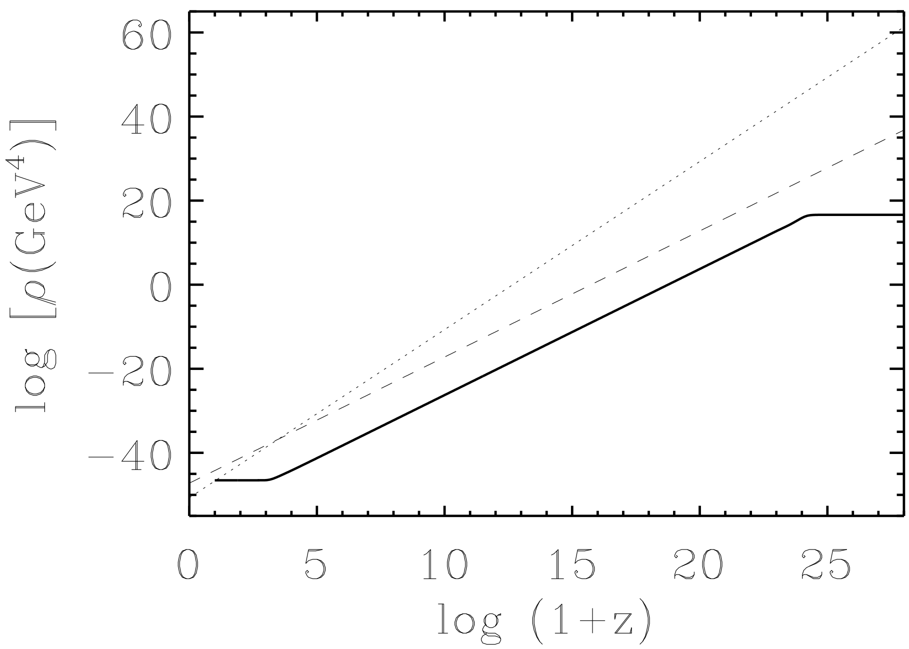

Figure 8: Evolution of the quintessence energy density (solid line) for $\bar{Q}_{\min} = 10^{-4}$, $\tilde{B} = 1$ and $\alpha = 6$ with the initial conditions such that $\Omega_{Q_{\rm ini}} = 10^{-45}$. The dotted line represents the evolution of the radiation energy density while the dashed line is the matter energy density. Initially, the energy density is frozen until the attractor is joined at $z_{\rm under} \sim 4.6 \times 10^{23}$. At $z_{\min} = z_{\rm osci}$, the presence of the minimum is felt by the field which stabilizes at its minimum.

formal time, the perturbed Klein-Gordon equation Fourier space reads

$$\delta Q''_{\mathbf{k}} + 2\mathcal{H}\delta Q'_{\mathbf{k}} + \left(k^2 + a^2\frac{\mathrm{d}^2 V}{\mathrm{d}Q^2}\right)\delta Q_{\mathbf{k}} = 4Q'\Phi'_{\mathbf{k}} - 2a^2\frac{\mathrm{d}V}{\mathrm{d}Q}\Phi_{\mathbf{k}}\,, \tag{4.31}$$

where $\mathcal{H} \equiv a'/a$ and Φ is the Bardeen potential which describes the metric perturbations. We want to study the situation where the background field stands at its non-vanishing minimum, this field being a test field, *i.e.* the scale factor being given by the expression (4.12). In this case, the right hand side of the above expression vanishes and the perturbed Klein-Gordon equation reduces to

$$\delta Q''_{\mathbf{k}} + \frac{4}{(1+3\omega_{\rm B})\,\eta}\delta Q'_{\mathbf{k}} + \left[k^2 + \left(m^0_{3/2}\right)^2 a_{\rm p}^2\left(\frac{\eta}{\eta_{\rm p}}\right)^{\frac{4}{1+3\omega_{\rm B}}}\right]\delta Q_{\mathbf{k}} = 0\,. \tag{4.32}$$

It is clear that the Jeans mass of the field is now $(m^0_{3/2})^{-1}$. Since this scale is much smaller than the Hubble length, *i.e.* the Jeans length in the standard Ratra-Peebles case, this raises the question as whether quintessence could collapse and develop structures at very small scales. Unfortunately, the above equation cannot be solved explicitly. However, it can be analyzed in the case where the wavelength of the fluctuation is either larger or smaller than the Jeans length.

Let us start by assuming that $k_{\rm ph} \gg m_{3/2}^0$, *i.e.* the wavelength of the Fourier mode is smaller than the Jeans length. Then the term proportional to $(m_{3/2}^0)^2 a^2$ in the perturbed Klein-Gordon equation can be neglected. In this case, the solution reads

$$\delta Q_{\mathbf{k}} = A_1(k)\eta^{\nu} \mathrm{J}_{\nu}(k\eta) + A_2(k)\eta^{\nu}\mathrm{J}_{-\nu}(k\eta)\,, \tag{4.33}$$

where $A_1(k)$ and $A_2(k)$ are two integration constants fixed by the initial conditions which are not important for us in the present context and where J_{ν} is a Bessel function of order ν which is a function of the background equation of state only

$$\nu = \frac{3(\omega_{\rm B}-1)}{2(1+3\omega_{\rm B})}\,. \tag{4.34}$$

Moreover, $k_{\rm ph} \gg m_{3/2}^0$ implies $k_{\rm ph} \gg H$ and then using the asymptotic behavior of Bessel functions for large values of their argument, one obtains the approximate expression

$$\delta Q_{\mathbf{k}} \sim \sqrt{\frac{2}{\pi}}\eta^{\nu-1/2}\left[A_1(k)\cos\left(k\eta - \frac{\pi\nu}{2} - \frac{\pi}{4}\right) + A_2(k)\cos\left(k\eta + \frac{\pi\nu}{2} - \frac{\pi}{4}\right)\right]. \tag{4.35}$$

We have oscillations since we consider modes of wavelength of smaller than the Jeans length. The overall amplitude behaves as $\eta^{\nu-1/2}$. Since one has

$$\nu - \frac{1}{2} = -\frac{2}{1+3\omega_{\rm B}} < 0\,, \tag{4.36}$$

there is no growing mode in this situation. This result makes sense as the pressure forces counter balance the gravitational force which tends to make the system collapse.

Let us now consider the situation where $k_{\rm ph} \ll m_{3/2}^0$, *i.e.* where the wavelength of the Fourier mode under consideration is larger than the Jeans length $(m_{3/2}^0)^{-1}$. In this case, ignoring the term k^2 in Eq. (4.32), the equation for $\delta Q_{\mathbf{k}}$ can also be integrated exactly. It is interesting to compare this equation with the standard equation (*i.e.* in the case where the potential is the Ratra-Peebles one, $V = M^{4+\alpha}Q^{-\alpha}$) for $\delta Q_{\mathbf{k}}$ on large scales when the field is on the attractor. This case was studied in Ref. [14]. On the attractor, the second derivative of the potential is given by $\mathrm{d}^2V/\mathrm{d}Q^2 = 9H^2(\alpha+1)(1-\omega_Q^2)/(2\alpha)$, w_Q being a constant, and, as a result, the terms $a^2\mathrm{d}^2V/\mathrm{d}Q^2$ scales as η^{-2}. This is why the homogeneous equation admits simple power-law solutions. Here, the term $\mathrm{d}^2V/\mathrm{d}Q^2$ is a constant equal to $m_{3/2}^0$ and, therefore the time dependence of the term $a^2\mathrm{d}^2V/\mathrm{d}Q^2$ is now given by a^2, *i.e.* a power-law of the conformal time, namely η^2 for the radiation dominated era and η^4 for the matter dominated epoch. Therefore, we expect the solutions for $\delta Q_{\mathbf{k}}$ on large scales to be different from the Ratra-Peebles case. Indeed, the result reads

$$\begin{aligned}\delta Q_{\mathbf{k}} &= B_1(k)\eta^{\frac{3(\omega_{\rm B}-1)}{1+3\omega_{\rm B}}}\mathrm{J}_{\mu}\left[\frac{1+3\omega_{\rm B}}{3(1+\omega_{\rm B})}m_{3/2}^0\eta_{\rm p}^{\frac{-2}{1+3\omega_{\rm B}}}\eta^{\frac{3(1+\omega_{\rm B})}{1+3\omega_{\rm B}}}\right] \\ &+B_2(k)\eta^{\frac{3(\omega_{\rm B}-1)}{1+3\omega_{\rm B}}}\mathrm{J}_{-\mu}\left[\frac{1+3\omega_{\rm B}}{3(1+\omega_{\rm B})}m_{3/2}^0\eta_{\rm p}^{\frac{-2}{1+3\omega_{\rm B}}}\eta^{\frac{3(1+\omega_{\rm B})}{1+3\omega_{\rm B}}}\right]\,,\end{aligned} \tag{4.37}$$

where $B_1(k)$ and $B_2(k)$ are two arbitrary constants and where the order μ of the Bessel functions can be written as

$$\mu = -\frac{\omega_{\rm B} - 1}{\omega_{\rm B} + 1}\,. \tag{4.38}$$

The same expression can be also rewritten in a form for which the physical interpretation is easier

$$\delta Q_{\mathbf{k}} = B_1(k)\eta^{\frac{3(\omega_{\rm B}-1)}{1+3\omega_{\rm B}}} \mathrm{J}_\mu \left[\frac{2}{3(1+\omega_{\rm B})}\frac{m_{3/2}^0}{H}\right] + B_2(k)\eta^{\frac{3(\omega_{\rm B}-1)}{1+3\omega_{\rm B}}} \mathrm{J}_{-\mu} \left[\frac{2}{3(1+\omega_{\rm B})}\frac{m_{3/2}^0}{H}\right]\,. \tag{4.39}$$

where $m_{3/2}^0 \gg H$. Therefore, in the above equation, one can use the asymptotic expression of the Bessel functions for large values of their argument. This means that the overall amplitude behave as $\eta^{3(\omega_{\rm B}-1)/(1+3\omega_{\rm B})-1/2}$. This gives $\eta^{-7/2}$ and $\eta^{-3/2}$ for $\omega_{\rm B} = 0, 1/3$. Again this function is a decreasing function of η. Hence no growing mode is generated by perturbation of the quintessence scalar field.

Together with the fact that for most of the history of the Universe, the quintessence field sits at the minimum of its potential, this implies that the dynamics of quintessence when coupling to supersymmetry breaking in a hidden sector leads to an absence of deviations from a pure cosmological constant in the late Universe, even at the perturbed level and even though the Jeans length of the field is now considerably smaller than in the standard Ratra-Peebles case.

5. Conclusions

We have presented a cosmological analysis of quintessence models in supergravity coupled to matter. For the quintessence sector, we have used the SUGRA model whose main feature is to reduce to the Ratra–Peebles potential at small values of the quintessence field. This is effectively the only property which is required here. Hence our results generalize to quintessence models with the same type of potentials. Supersymmetry is broken in a hidden sector which couples gravitationally both to the observable sector, *i.e.* the MSSM, and the quintessence sector. Requiring that the gravitino mass is large enough to lead to acceptable masses for the sparticles implies that the quintessence potential is drastically modified by the presence of the hidden sector. In particular, as soon as the hidden sector fields are stabilized and supersymmetry is broken, we find that the quintessence potential develops a minimum acting as an attractor for the quintessence field in the very early universe. Indeed the quintessence field settles down at the minimum of the potential before BBN and both at the background and perturbation level, the model becomes equivalent to a pure cosmological constant scenario. This is the first quintessential difficulty as no observational consequences seems to spring from such a scenario. Moreover, the energy scales of the quintessence sector required to obtain the correct vacuum energy now are minute as a consequence of the tiny value of the quintessence field now. As a result, one must introduce a highly fine-tuned scale in the quintessence sector which is as difficult to explain as the smallness of the vacuum energy. On the other hand, one may hope to preserve the SUGRA potential runaway shape by tuning the hidden sector dynamics. In this case, we find that the local test of gravity (fifth force, weak equivalence principle and proton to electron mass ratio variation) are incompatible with the vev of the quintessence field implied by the presence of an attractor which, again, is required if we want insensitivity to the initial conditions. Hence it seems difficult to build models of quintessence in supergravity where the cosmological, gravitational and particle physics aspects are compatible. One of the plausible possibilities consists in getting rid of the regularity of the Kähler potential for small values of the quintessence field. This is what happens in no–scale models for instance. The analysis of no-scale models leading to both supersymmetry breaking and quintessence is left for future work [17].

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