Exercise 1 Halvor Nafstad

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$$* \quad \frac{k_b T}{m_{DM} r^2} \frac{d}{dr} r^2 \frac{d}{dr} \ln \rho(r) = 4\pi G \rho(r)$$

$$(1) \quad \rho(r) = \frac{A}{r^2}, \quad A = \frac{k_b T}{2\pi G m_{DM}}$$

Set (1) into $*$:

$$-\frac{\cancel{k_b T}}{m_{DM} \cancel{r^2}} \frac{d}{dr} r^2 \frac{d}{dr} (\ln A - 2 \ln r) = \cancel{4\pi} \cancel{G} \frac{\cancel{k_b T}}{\cancel{2\pi} \cancel{G} m_{DM}} \frac{1}{\cancel{r^2}}$$

$$\Rightarrow \quad \cancel{2} \frac{d}{dr} r^2 \frac{d}{dr} \ln r = \cancel{2}$$

$$\Rightarrow \quad \frac{d}{dr} r = 1$$

thus (1) is a solution of $*$

For an isothermal gas we have $P = n k_b T = \frac{\rho}{m_p} k_b T_0$

$$\Rightarrow \quad P = \frac{k_b^2 T_0^2}{2\pi G m_{DM} m_p} \frac{1}{r^2}$$

$$\frac{dP}{dr} = -\frac{4\pi G \left(\int_0^r x^2 dx \rho(x)\right)}{r^2} \rho(r)$$

$$r^2 \frac{dp}{dr} = -4\pi G \rho(r) \int_0^r x^2 dx \rho(x)$$

spherical symmetry

$$r^2 \frac{dp}{dr} = -4\pi G \rho(r) \int_0^r r'^2 dr' \rho(r')$$

$$\frac{1}{r^2 dr \rho(r)} r^2 \frac{dp}{dr} = -4\pi G \rho(r)$$

$$\frac{dp}{dr} = \frac{k_b T_0}{m_p} \frac{d\rho(r)}{dr}$$

$$\frac{1}{\rho(r)} \frac{d\rho(r)}{dr} = \frac{d \ln \rho(r)}{dr}$$

$$\frac{k_b T_0}{m_p r^2} \frac{d}{dr} r^2 \frac{d}{dr} \ln \rho(r) = -4\pi G \rho(r)$$

$$\underset{m_p = m_{DM}}{\Longleftrightarrow} \quad \frac{k_b T_0}{m_{DM} r^2} \frac{d}{dr} r^2 \frac{d}{dr} \ln \rho(r) = -4\pi \rho(r) \quad \square$$