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`http://www.uio.no/studier/emner/matnat/fys/FYS4160/v17/`

Lecture spring 2017:

General Relativity

Problem sheet 12

⇝ These problems are scheduled for discussion on **Thursday, 4 May 2017**.

Problem 39

The *helicity* of a particle is defined as its spin along the direction of motion. To measure this spin, one can rotate the polarization vector by an angle θ around the axis defined by the 3-momentum $\mathbf{k}$. A polarization vector with helicity λ is then an eigenstate of the rotation matrix with eigenvalue $\exp[i\lambda\theta]$.

Consider now a gravitational wave propagating in x_3 direction which, as we have seen, can be described by two polarizations (h_+ and $h_\times$). Introduce circular polarizations $h_{R,L} \equiv \frac{1}{\sqrt{2}}(h_+ \pm h_\times)$. Now transform to a new coordinate system that is related to the original one via a rotation by an angle θ in the $x_1 - x_2$ plane. How do the polarization vectors $h'_{R,L}$ in the new system look like, and what does this imply for the helicity of gravitational waves?

Problem 40

As we have discussed, linearized gravity takes the form of a symmetric rank-2 tensor field $h_{\mu\nu}$ propagating in Minkowski space, where the Lagrangian is given in Eq. (7.9) in the book.

a) Use the transverse traceless gauge. Show that for a free gravitational wave, propagating in x_3 direction, this Lagrangian simplifies to an expression that is proportional to $(\partial^\mu h_{11})(\partial_\mu h_{11}) + (\partial^\mu h_{12})(\partial_\mu h_{12})$.

b) Calculate the canonical energy momentum tensor from the Lagrangian obtained in a), i.e. the one that is obtained from Noether's theorem:

$$t_\mu^{\ \nu} = h_{\rho\sigma,\mu} \frac{\partial \mathcal{L}}{\partial h_{\rho\sigma,\nu}} - \delta_\mu^\nu \mathcal{L}\,.$$

What is the energy flux in direction x^i?

c) Now apply this result to a gravitational wave source with quadrupole tensor $Q_{ij} \equiv \int d^3x \left(x_i x_j - \frac{1}{3}\delta_{ij} r^2\right) \rho$. What is the energy flux as measured far away from the source?

Problem 41

Consider linearized gravity in the transverse traceless gauge. Show that the time dependence of the metric is such that any (2D) *area* perpendicular to the direction of motion of the gravitational wave stays constant (while, as we have seen in the lecture, *lengths* change as gravitational waves pass through)!

Problem 42

Imagine that the Chinese space station will be ready at some point, at a size similar to that of the ISS. Unfortunately there is a bug in the code keeping it in orbit, which leads to a head-on collision of the two space stations. Estimate the amplitude of the gravitational waves produced, and discuss whether they could be observed on the surface of the earth!

[The ISS weighs approximately 400 tons, and orbits Earth with a speed of roughly 30 000 km/h at a (varying) altitude of around 400 km. Assume that the same goes for the Chinese station, and that the two stations decelerate at a constant rate during the collision, coming to rest in about one millisecond. You may treat the space stations as point masses, and neglect the effect of Earth's gravitational field.]