On virialization with dark energy

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Abstract. We review the inclusion of dark energy into the formalism of spherical collapse, and the virialization of a two-component system, made of matter and dark energy. We compare two approaches in previous studies. The first assumes that only the matter component virializes, e.g. as in the case of a classic cosmological constant. The second approach allows the full system to virialize as a whole. We show that the two approaches give fundamentally different results for the final state of the system. This might be a signature discriminating between the classic cosmological constant which cannot virialize and a dynamical dark energy mimicking a cosmological constant. This signature is *independent* of the measured value of the equation of state. An additional issue which we address is energy non-conservation of the system, which originates from the homogeneity assumption for the dark energy. We propose a way to take this energy loss into account.

1. Introduction

The top hat spherical collapse formalism dates back to Gunn and Gott [1]. In addition to its beautiful simplicity, it has proven to be a powerful tool for understanding and analysing the growth of inhomogeneities and bound systems in the Universe. It describes how a small spherical patch of homogeneous overdensity decouples from the expansion of the Universe, slows down, and eventually turns around and collapses. One assumes that the collapse is not completed into a singularity, but that the system eventually virializes and stabilizes at a finite size. The definition of the moment of virialization depends on energy considerations. The top hat spherical collapse is incorporated, for example, in the Press-Schecter [2] formalism. It is therefore widely used in present day interpretation of data sets.

For an Einstein-de Sitter Universe (EdS), i.e. a Universe with $\Omega_m = 1$ and $\Omega_\Lambda = 0$ that is composed strictly of non relativistic dust, there is an analytical solution for the spherical collapse. The ratio of the final, virialized radius to the maximal size at turnaround of the bound object is $R_{vir}/R_{ta} = \frac{1}{2}$. The situation becomes more complicated and parameter dependent when one considers a component of dark energy. This was a subject of numerous studies [3, 4, 5, 6, 7, 8, 9]. Lahav *et al* (LLPR) [3] generalized the formalism to a Universe composed of ordinary matter and a cosmological constant. In this case the cosmological constant is 'passive', and only the matter virializes. This leads to $R_{vir}/R_{ta} < \frac{1}{2}$. This scenario also corresponds to the dynamics implemented in N-body simulations for the concordance ΛCDM model, i.e. the cosmological constant only affects the time evolution of the scale factor of the background Universe. Wang and Steinhardt (WS) [4] included quintessence with a constant or slowly varying equation of state. Battye and Weller (BW) [7] included quintessence in a different manner to WS, taking into account its pressure. Mota and Van de Bruck (MB) [9] considered spherical collapse for different potentials of the quintessence field, and checked what happens when one relaxes the common assumption that the quintessence field does not cluster on the relevant scales.

In this work we wish to review the inclusion of a cosmological constant and quintessence into the formalism of spherical collapse. Adding dark energy creates a system with two components - the matter and the additional fluid. Most existing works [3, 4, 7, 8] look at the virialization of the matter component (luminous and dark), which feels an additional potential due to the presence of the dark energy. With this procedure, one implicitly assumes that the dark energy either does not virialize, or does so separately from the matter component. MB on the other hand, included the additional fluid in the virialization equations, the assumption here being that all of the system's components virialize together. However, they did not remark either on the difference in physical understanding of the system between their approach and the one common in the literature, or on the case of the cosmological constant. Our aim here is to critically contrast the two approaches - the assumption that the dark matter component virializes separately (as in LLPR, WS, and BW), and the assumption that the whole system virializes as a whole (as in MB). We wish to consider the meaning of including or not including the additional fluid into the virialization, and point out a few puzzles.

A second issue which we will address here is the use of energy conservation in order to find the condition of virialization. Assuming that the quintessence field does not collapse with the mass perturbation but stays homogeneous as the background means that the system must lose energy as it collapses. Yet, energy conservation between turnaround and virialization is assumed. This inconsistency is for quintessence fields with equation of state $w \neq -1$. Energy is conserved with a cosmological constant ($w = -1$), for reasons that will be discussed later on. We will propose a way to take into account this energy loss for quintessence, and introduce a correction to the equation that defines the final virialized radius of the system.

The inclusion of dark energy in the virialization process changes the results in a fundamental manner. As we will show, the ratio of final to maximal size of the spherical perturbation is *larger* if the dark energy is part of the virialization.

While the results we will show are of the cosmological constant or quintessence with a constant w, the methods we use are applicable to a time dependent equation of state, as well as to models in which quintessence is coupled to matter [10]. We limit the discussion here to $w \geq -1$. While an equation of state which is more negative than -1 is observationally interesting, the physical interpretation of it is unclear, and beyond the scope of this work. We assume throughout that the background is described by a flat, FRW metric, with two energy components - the matter and the dark energy.

The paper is organized as follows. In section 2 we give the general picture of how one calculates the point of virialization of a single component system, and define the relevant fundamental quantities. In section 3 we consider the case of a two component system. Section 4 reviews the case of a clustered quintessence. In section 5 we examine the transition from clustered to homogeneous quintessence and in section 6 we examine the transition toward $w = -1$. We summarize and conclude in section 7.

2. Virialization of a single component system

The spherical collapse provides a mathematical description of how an initial inhomogeneity decouples from the general evolution of the Universe, and expands in a slower fashion, until it reaches the point of turnaround and collapses on itself. The mathematical solution gives a point singularity as the final state. Physically though, we know that objects go through a virialization process, and stabilize towards a finite size.

Since virialization is not 'built in' into the spherical collapse model (see though [11]), the common practice is to *define* the virialization radius as the radius at which the virial theorem holds, and the kinetic energy T is related to the potential energy U by $T_{vir} = \frac{1}{2}(R\, \partial U/\partial R)_{vir}$. Using energy conservation between virialization and turnaround (where $T_{ta} = 0$) gives

$$\left[U + \frac{R}{2}\frac{\partial U}{\partial R}\right]_{vir} = U_{ta} \ . \tag{1}$$

Equation (1) defines R_{vir}. Thus in order to calculate the final size of a bound object, we need to know how to calculate the potential energy of the spherical perturbation, and to use energy conservation between turnaround and the time of virialization. We discuss later the case where energy is not conserved, and how to account for it. For an EdS Universe, $U = -\frac{3}{5}GM/R$ (M is the conserved mass within the spherical perturbation) and $T_{vir} = -\frac{1}{2}U$, so the ratio of final to maximal radii of the system is $x = R_{vir}/R_{ta} = \frac{1}{2}$.

The virial equation, which at equilibrium gives the above results, is usually derived from the Euler Equation for particles. It is worth noting here that one can derive the virial equation for a cosmological fluid. The starting point is the continuity equation of a perfect fluid (derived from $T^{0\nu}{}_{;\nu} = 0$), with equation of state (the ratio of pressure to energy density) $w = p/\rho$:

$$\dot{\rho} + 3\,(1+w)\,\frac{\dot{r}}{r}\rho = 0 \ . \tag{2}$$

Multiplying equation (2) by r^2, taking the time derivative and integrating over a sphere of radius R gives

$$\int \dot{G} dV + \frac{1}{2}\,(1+3w)\left[\int \rho\dot{r}^2 dV + \int r\frac{d}{dt}\,(\rho\dot{r})\,dV\right] = 0 \ , \tag{3}$$

where $G = (d/dt)\left(\frac{1}{2}\rho r^2\right)$. In the classical analogy, $\dot{G} = \ddot{I}$ is the second derivative of the inertia tensor. In a state of equilibrium, $\dot{G} = 0$. The quantity $\int \rho\dot{r}^2 dV$ is twice the kinetic energy, and $\int r(d/dt)\,(\rho\dot{r})\,dV$ is $R\ \partial U/\partial R$. As equation (3) shows, the value of w factorizes out when one is looking for the equilibrium condition. In the case where the fluid does not conserve energy, the right hand side of of equation (2) will be equal to some function Γ. In that case, the virial equation (3) will have an additional surface term. In equilibrium, the surface term and $\dot{G}$ should vanish.

This is the non-relativistic version of the scalar virial theorem. Hence, it is not applicable for a fluid with a relativistic equation of state, $w \to \frac{1}{3}$. It can be shown that when writing the relativistic version, the energy of the radiation field drops out of the virial equation [12].

3. A two - component system

When adding a component to the system, there are three questions to be asked - **(a)** how does the potential induced by the new component affect the system? **(b)** does the new component participate in the virialization? and **(c)** does the new component cluster, or does it stay homogeneous? These questions should be addressed in the framework of a fundamental theory for dark energy. Here we work out the consequences of virialization and clustering of dark energy, if they do take place. We shall try to address each of these questions separately.

(a) In the case where the new component does not cluster or virialize, its sole effect is contributing to the potential energy of component 1. LLPR [3] calculated this contribution to the potential energy using the Tolman-Bondi equation. Their result was generalized for quintessence by WS [4] and, in a different manner, by BW [7]. We follow LLPR and WS in our numerical calculations.

(b) Any energy component with non vanishing kinetic energy is capable of virializing, but the question is: on what time scale? If one imagines that the full system virializes, then the virial theorem should relate the *full* kinetic and potential energies of the system,

$$U = U_{11} + U_{12} + U_{21} + U_{22} = \frac{1}{2}\int (\rho_1 + \rho_2)(\Phi_1 + \Phi_2)\, dV \ , \tag{4}$$

where the potential Φ_x induced by each energy component in a spherical homogeneous configuration is

$$\Phi_x(r) = \ -2\pi G(1 + 3w_x)\rho_x \left(R^2 - \frac{r^2}{3}\right) \ . \tag{5}$$

The kinetic energy at virialization is then

$$T_{tot} = \frac{1}{2} R \frac{\partial}{\partial R} (U_{11} + U_{12} + U_{21} + U_{22}) \ . \tag{6}$$

The expression above is the full potential energy of the system. As we will show, the addition of these new terms to the virial theorem makes a fundamental difference in the final state of the system, so the question of whether dark energy participates in the virialization is crucial.

(c) Every positive energy component other than the cosmological constant is capable of clustering. Even though Caldwell *et al* [13] have shown that quintessence cannot be perfectly smooth, it is assumed that the clustering is negligible on scales less than 100 Mpc. It is therefore common practice to keep the quintessence homogeneous during the evolution of the system. The effects of relaxing this assumption were explored in MB. We will consider both cases here. The continuity equation for a Q component which is kept homogeneous is

$$\dot{\rho}_{Qc} + 3(1 + w)\frac{\dot{a}}{a}\rho_{Qc} = 0 \ , \tag{7}$$

and for clustering Q is

$$\dot{\rho}_{Qc} + 3(1 + w)\frac{\dot{r}}{r}\rho_{Qc} = 0 \tag{8}$$

(a and r are the global and local scale factors, respectively). One can ask what happens if one slowly 'turns on' and enables the possibility of clustering for the Q component.

To enable a slow continuous 'turn on' of the clustering, one can write

$$\dot{\rho}_{Qc} + 3(1+w)\left(\frac{\dot{r}}{r}\right)\rho_{Qc} = \gamma\Gamma \qquad (9)$$

$$\Gamma = 3(1+w)\left(\frac{\dot{r}}{r} - \frac{\dot{a}}{a}\right)\rho_{Qc} \qquad (10)$$

$$0 \le \gamma \le 1 \ , \qquad (11)$$

where ρ_{Qc} is the dark energy's density within the cluster. The notation Γ follows MB. γ is the 'clustering parameter', $\gamma = 0$ gives clustering behaviour and $\gamma = 1$ gives homogeneous behaviour. In the case of $\gamma = 1$, the dark energy inside the spherical region and the background dark energy ρ_Q, behave in similar ways: $\rho_{Qc} = \rho_Q$. A point to bear in mind is that for the case of homogeneous quintessence, the system does not conserve energy as it collapses from turnaround to virialized state.

Putting all this together, the equations governing the dynamics of the spherical perturbation are

$$\left(\frac{\ddot{r}}{r}\right) = -\frac{4\pi G}{3}\left(\rho_{mc} + (1+3w)\,\rho_{Qc}\right) \qquad (12)$$

$$\dot{\rho}_{mc} + 3\left(\frac{\dot{r}}{r}\right)\rho_{mc} = 0 \qquad (13)$$

$$\dot{\rho}_{Qc} + 3\,(1+w)\left(\frac{\dot{r}}{r}\right)\rho_{Qc} = \gamma\Gamma \ . \qquad (14)$$

Our results are going to be presented as a function of q, which is defined as the ratio of the system's dark energy to matter's densities at the time of turnaround, $q(z_{ta}) \equiv \rho_{Qc}(z_{ta})/\rho_{mc}(z_{ta})$. The system's matter density ρ_{mc} at turnaround is $\rho_{mc}(z_{ta}) = \zeta(z_{ta})\rho_m(z=0)(1+z_{ta})^3$, expressed in terms of the background matter density $\rho_m(z_{ta})$ and the density contrast at turnaround, $\zeta(z_{ta})$. In order to estimate which values of q are of interest,we plotted, in figure 1, the dependence of q on the turnaround redshift z_{ta}, for various values of Ω_m and Ω_Λ. As can be seen, q takes typical values no larger than 0.3.

4. A clustered quintessence

In the case of fully clustering quintessence, $\gamma = 0$, the quintessence field responds to the infall in the same way as matter, the only difference being its equation of state which dictates a different energy conservation (in the general relativity sense). However, energy is conserved, and since the quintessence is active in the dynamics of the system, it is quite reasonable to imagine that it takes part in the virialization. We therefore present here the calculation assuming the whole system virializes, matter and dark energy.

Following equation (4), the potential energy of the full system is

$$U = -\frac{3}{5}\frac{GM^2}{R} - (2+3w)\frac{4\pi G}{5}M\rho_{Qc}R^2 - (1+3w)\frac{16\pi^2 G}{15}\rho_{Qc}^2 R^5 \ . \qquad (15)$$

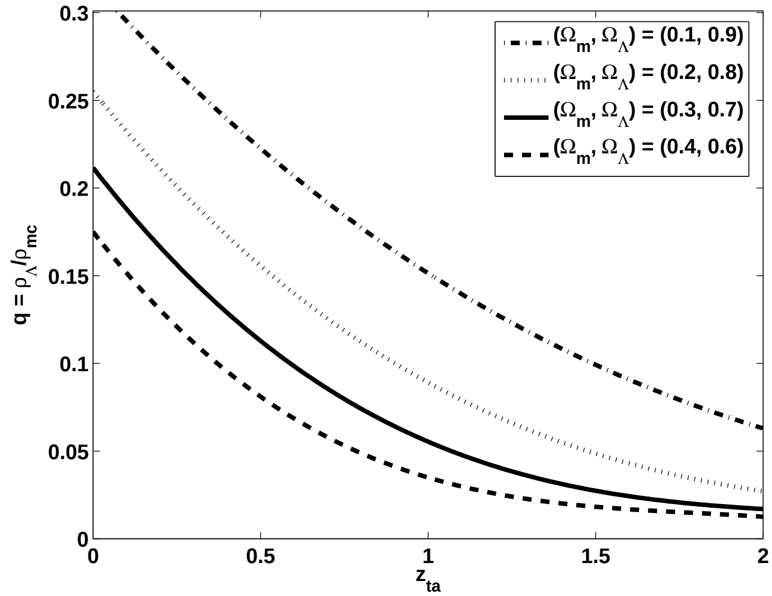

Figure 1. $q = \rho_\Lambda/\rho_{mc}$ as a function of the turnaround redshift z_{ta}, for various values of Ω_m and Ω_Λ.

Once the potential energy has been calculated, virialization is found with the use of equation (1). Expressing it in term of $q = \rho_{Qc}/\rho_{mc}$ at turnaround and $x = R_{vir}/R_{ta}$ gives

$$\left[1 + (2+3w)q + (1+3w)q^2\right] x \\ - \frac{1}{2}(2+3w)(1-3w)qx^{-3w} - \frac{1}{2}(1+3w)(1-6w)q^2x^{-6} = \frac{1}{2}\ . \qquad (16)$$

Equation (16) is valid under the assumption that the whole system virializes.

If, on the other hand, only the matter virializes, then the equation defining x is

$$(1+q)\,x - \frac{q}{2}\,(1-3w)\,x^{-3w} = \frac{1}{2}\ . \qquad (17)$$

In figure 2 we show x as a function of q, for a fluid with $w = -0.8$ that fully clusters, $\gamma = 0$. The dotted line is the ratio in the case where only matter virializes, equation (17). The solid line is the ratio when the whole system, including the dark energy component, have virialized, equation (16). As can be seen, there is a fundamental difference of the solutions: if only the matter virializes then the final ratio is smaller than the EdS $\frac{1}{2}$ value, while when the whole system virializes, the final ratio is larger than $\frac{1}{2}$.

5. Turning off the clustering

It is the usual practice to neglect spatial perturbations of the quintessence field, and to keep it homogeneous [13]. With our generalized notation of the 'clustering parameter'

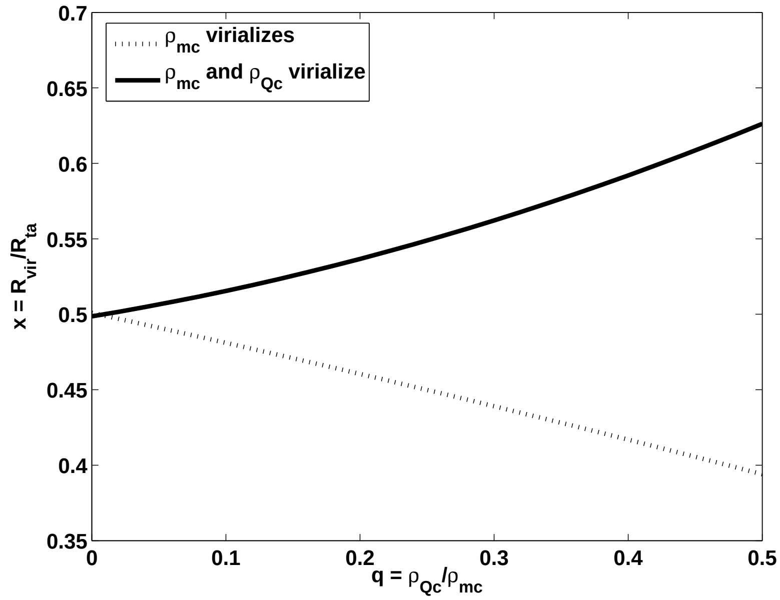

Figure 2. The ratio of final to turnaround radii as a function of $q = \rho_{Qc}/\rho_{mc}$ at turnaround, for quintessence with a constant equation of state, $w = -0.8$, which fully clusters ($\gamma = 0$). The dotted line is the ratio in the case where only matter virializes. The solid line is the ratio when the whole system, including the dark energy component, has virialized.

γ, one can also allow a small but non-zero amount of clustering for the quintessence. For any $\gamma \neq 0$, the quintessence field within the system does not conserve energy. As equation (1) assumes energy conservation, the problem with not allowing the quintessence to fully cluster is how to find the radius of virialization. We will now propose a correction to equation (1), that will take into account the loss of energy.

We denote the potential energy at turnaround as U_{ta}, and at virialization as U_{vir}. We also define a function $\tilde{U}$ as the system's potential energy *had* it conserved energy. Thus by construction the energy that the system lost is

$$\Delta U \equiv \tilde{U} - U \ . \tag{18}$$

Equation (1) which describes energy conservation between turnaround and virialization now needs to be corrected. Accounting for the lost energy gives

$$\left[U + \frac{R}{2}\frac{\partial U}{\partial R}\right]_{vir} + \Delta U_{vir} = \left[\tilde{U} + \frac{R}{2}\frac{\partial U}{\partial R}\right]_{vir} = U_{ta} \ . \tag{19}$$

We are now set to calculate ΔU.

Looking at equation (15), one can treat U as $U(\rho_x, R)$ (ρ_x being the various energy density components). In order to calculate $\tilde{U}(\rho_x, R)$, one needs to replace ρ_x with $\tilde{\rho}_x$ in the expression for U, which has $\gamma = 0$ and conserves energy. The continuity equation for $\tilde{\rho}_x$ is then

$$\dot{\tilde{\rho}}_x + 3\left(\frac{\dot{R}}{R}\right)(1+w_x)\,\tilde{\rho}_x = 0 \ , \tag{20}$$

and we impose boundary conditions such that $\tilde{\rho}_x(a_{ta}) = \rho_x(a_{ta})$.
For a constant equation of state, this gives

$$\tilde{\rho}_x = \tilde{\rho}_x(a_{ta})\left(\frac{R_{ta}}{R_{vir}}\right)^{3(1+w_x)} = \rho_x(a_{ta})\left(\frac{R_{ta}}{R_{vir}}\right)^{3(1+w_x)} \ . \tag{21}$$

For a time dependent w one needs to use the integral expression for $\tilde{\rho}$.
We therefore have

$$\tilde{U}(\rho_x, R) = U(\tilde{\rho}_x, R) \ . \tag{22}$$

Equation (19) is now a function of R_{vir} and values determined at turnaround time (such as U_{ta} and $\rho_x(a_{ta})$), and defines R_{vir} in the same manner as equation (1) did. With the definitions of q, x, $y = (a_{vir}/a_{ta})^{1+w}$ and $p = x/y$, the final form of equation (1) for a quintessence with a general value of γ is then

$$\begin{aligned}
&\frac{q^2}{2}(1+3w)\Big[1+6w-6\gamma(1+w)\Big]\Big[(1-\gamma)\,x^{-3w}+\gamma p^3\Big]^2 \\
&+\frac{q}{2}(2+3w)\Big[1+3w-3\gamma(1+w)\Big]\Big[(1-\gamma)\,x^{-3w}+\gamma p^3\Big] \\
&+\Big[1+(2+3w)\,q+(1+3w)\,q^2\Big]x-(2+3w)\,qx^{-3w}-(1+3w)\,q^2x^{-6w} \\
&=\frac{1}{2} \ .
\end{aligned} \tag{23}$$

For the case (common in literature) of a completely homogeneous quintessence, $\gamma = 1$, the virialization condition (23) is reduced to

$$\begin{aligned}
&\Big[1+(2+3w)q+(1+3w)q^2\Big]x- \\
&(2+3w)q\left(p^3+x^{-3w}\right)-(1+3w)q^2\left(\frac{5}{2}p^6+x^{-6w}\right)=\frac{1}{2} \ .
\end{aligned} \tag{24}$$

The equation for x when only the matter virializes does not need to be corrected for energy conservation, as it counts only the energy associated with the matter. Its general form is

$$(1+q)\,x-\frac{q}{2}\Big[1-3w+3\gamma(1+w)\Big]\Big[(1-\gamma)\,x^{-3w}+\gamma p^3\Big]=\frac{1}{2} \ , \tag{25}$$

and for $\gamma = 1$ it reproduces the solution of WS,

$$(1+q)\,x-2qp^3=\frac{1}{2} \ , \tag{26}$$

which is a generalization of LLPR's results (to be discussed later on, see equation (28)).

One should consider which of the solutions is more plausible. Ultimately the choice between the two solutions should be dictated by the theory with which the dark energy

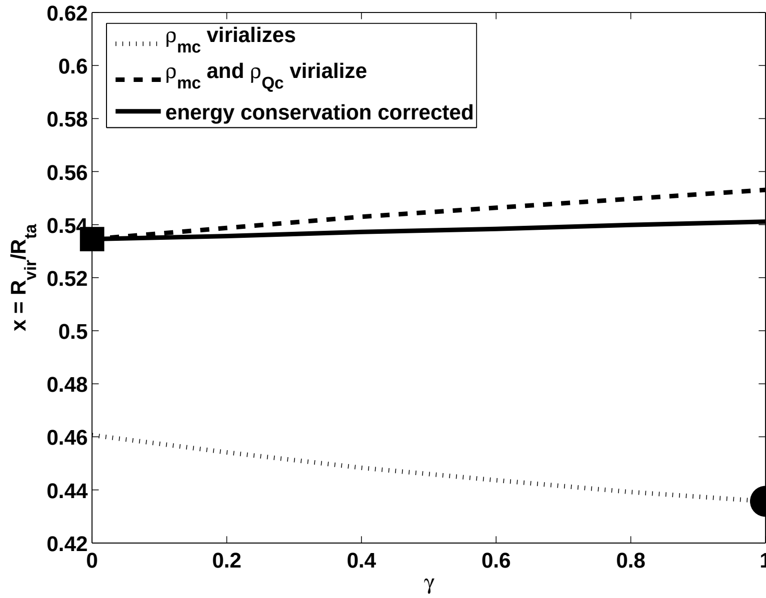

Figure 3. R_{vir}/R_{ta} as a function of γ, for $w = -0.8$ and $q = 0.2$. $\gamma = 0$ describes the case of a fully clustering Q field, and $\gamma = 1$ is the case of a homogeneous Q, allowing only the matter component to cluster. For $\gamma = 0$, taking the dark energy into the virialization is highly plausible, (see square on left). If one assumes that only the matter component virializes for $\gamma = 1$ (see circle on right), it is unclear how to extrapolate in a smooth way between the two cases. This will produce a discontinuity in the transition from the 'clustering' to the 'non-clustering' behaviour.

is modelled. As we are looking at an effective description of the dark energy as a perfect fluid and not with a fundamental theory, this information is lost.

For the case of $\gamma = 1$, when one keeps the evolution of the dark energy in the system identical to that of the background, it is reasonable to assume that it does not participate in the local processes that lead to virialization. This gives credibility to the solution of equation (26), allowing only the matter to virialize. However, this presents a question of continuity, presented in figure 3. The figure shows the solutions of x as a function of γ, with fixed $w = -0.8$ and $q = 0.2$. The circle on the right is the WS's result when the quintessence is kept completely homogeneous. The square on the left is the result when both the matter and the quintessence virialize, for the fully clustering case. The 'clustering parameter' allows us to think of a continuous transition between the two cases. One would expect the transition in the behavior of the system along γ to be smooth. Allowing the dark energy to virialize for the clustering case, $\gamma = 0$, and keeping it out of the virialization process when $\gamma = 1$, raises the question of how one should extrapolate smoothly between the two cases. As figure 3 suggests, there will be a discontinuity.

In figure 4 we compare the solution of equation (26) and (24) for $w = -0.8$ and $\gamma = 1$, and show the effect of the energy correction. As can be seen, taking into account

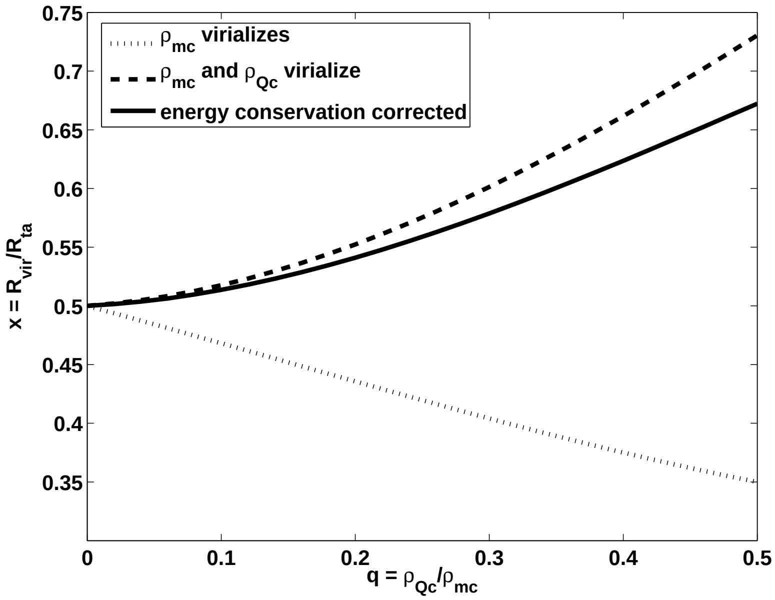

Figure 4. The ratio of final to turnaround radii as a function of $q = \rho_{Qc}/\rho_{mc}$ at turnaround, for quintessence with a constant equation of state, $w = -0.8$, which stays homogeneous ($\gamma = 1$). The dotted line follows WS's calculation, assuming only the matter component virializes. The dashed line is the ratio when the whole system, including the dark energy component, has virialized. The solid line takes into account the loss of energy between turnaround and virialization.

the loss of energy produces a small quantitative correction, but keeps the general feature of enlarging the final size of the system if the dark energy is allowed to virialize.

6. The limit of a cosmological constant

Equations (23) and (25) are valid for any constant w. As w approaches -1, we get that $p \to x$, and the dependency on γ vanishes. The reason that γ plays no role in the limit of $w \to -1$ is that the question whether such a fluid is allowed to cluster ($\gamma = 0$) or not ($\gamma = 1$) is rather abstract. It stays homogeneous in any case, because of its equation of state, $w_\Lambda = -1$ (which leads to $\Gamma = 0$). Accordingly, energy is automatically conserved.

In that limit, equation (23) which assumes that the whole system to virialize, is simplified into

$$7q^2x^6 + 2qx^3 + \left(1 - q - 2q^2\right)x = \frac{1}{2} \ . \tag{27}$$

Taking the same limit for equation (25) yields the familiar result of LLPR:

$$(1+q)\,x - 2qx^3 = \frac{1}{2} \qquad (LLPR) \ , \tag{28}$$

which is valid under the assumption that the matter component alone virializes ‡ (notice

‡ One can look at a test particle feeling an inverse square force and an additional repulsive Λ force.

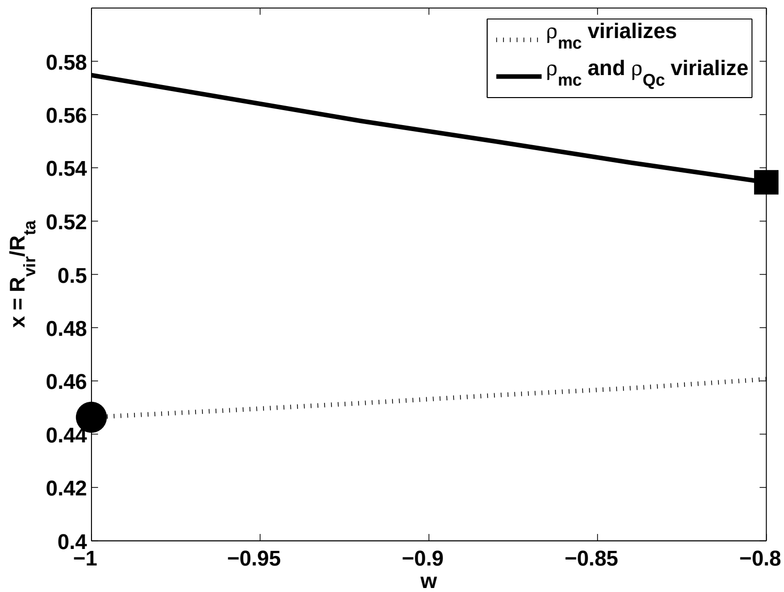

Figure 5. R_{vir}/R_{ta} as a function of w, for $q = 0.2$ and $\gamma = 0$. The dotted line is the ratio when the matter alone virializes, and the solid is for the case where the whole system virializes. The circle on the left is LLPR's solution for the cosmological constant. The square on the right is an example of a clustered quintessence, where we expect to take into account the whole system in the virialization. The figure suggests we should expect a discontinuity in the behaviour of quintessence fields and a true cosmological constant.

that our definition of q differs by a factor of 2 from the definition of η of LLPR, $q = \frac{1}{2}\eta$).

Again, we wish to consider the plausibility of the two solutions. If one considers the cosmological constant as a true constant of Nature, $\rho_\Lambda = \Lambda/(8\pi G)$, it is hard to imagine it participating in the dynamics that lead to virialization, as it is a true constant. In this case, one could categorically say that the right procedure is to look at the virialization of the matter fluid only, and follow LLPR's solution, equation (28). The sole effect of the cosmological constant, then, is to modify the potential that the matter fluid feels.

If, on the other hand, one considers the origin of a perfect fluid with $w \approx -1$ as a special case of quintessence, which is indistinguishable from a cosmological constant, it is reasonable to expect continuity in the behaviour of the system as one slowly changes the value of w toward -1. In other words, if the physical interpretation of the fluid with $w \approx -1$ is of a dynamical field that *mimics* a constant, the idea of including it in the dynamics of the system has a physical meaning.

The result, then, is that we possibly have a signature differentiating between a

Consider two possible orbits of the particle: one circular, and one in which its kinetic energy can vanish. The ratio of the circular radius to the radius of zero kinetic energy ('turnaround') is described exactly by equation (28). This assumes that the test particle does not contribute to the forces of the system. We thanks Martin Rees for pointing out this similarity.

cosmological constant which is a true constant, and something else which *mimics* a constant. This point is shown in figure 5. The figure shows x as a function of w, with $q = 0.2$ and $\gamma = 0$. The dotted line follows the solution of equation (25), with the matter alone virializing. The circle on the left is LLPR's solution for the cosmological constant. The solid line follows the solution of equation (23). The square on the right is an example of a clustered quintessence, where we expect to take into account the whole system in the virialization. As with figure 3, there is a suggested discontinuity, but here one can associate the discontinuity with a clear physical meaning: a true cosmological constant is not on the continuum of perfect fluids with general w, as its physical behaviour is different.

An observational detection of virialized objects with $R_{vir} > \frac{1}{2}R_{ta}$ would be a strong evidence against a cosmological constant which is a true constant, regardless of the measured value of the equation of state.

7. Conclusions

In this work, we have reconsidered the inclusion of a dark energy component into the formalism of spherical collapse. We compared existing results (such as those of LLPR and WS) which implicitly assume that only the dark matter virializes, to the case where the whole system's energy is taken into account for virialization, implying that the dark energy component also participates in the process (MB). While previous studies allow the dark energy component either to fully cluster or keep completely homogeneous, we generalized and allowed a smooth transition between the two cases. Additionally, we addressed the issue of energy non-conservation when the dark energy is kept homogeneous.

Our main conclusions are:

- In the case of a true cosmological constant only the matter component virlializes and the LLPR solution is valid.
- If both components of the system virialize, two additional terms to the potential energy appear. These are the self - energy of the additional energy source, and its reaction to the presence of the matter.
- The inclusion of these terms results in a fundamentally different behaviour of the system. If only dark matter virializes, the final size of the system is *smaller* than half of its maximal size. When the whole system virializes, its final size is *bigger* than half of its maximal size.
- It is hard to understand the physical meaning of a cosmological constant 'virializing', if it is a true constant. Accordingly, observational evidence for $R_{vir} > \frac{1}{2}R_{ta}$ would be strong evidence in favour of a dynamical field for the dark energy, regardless of the measured value of the equation of state. On the other

	ρ_{mc} **virializes**	ρ_{mc} **and** ρ_{Qc} **virialize**
general case	(25)	(23)
$\gamma = 0$	(17)	(16)
$\gamma = 1$	(26)	(24)
$w \to -1$	(28)	(27)
Cosmological Constant	(28)	–

Table 1. A summary of the relevant equations defining x for the various cases that we considered.

hand, $R_{vir} < \frac{1}{2}R_{ta}$ is compatible with both a true constant and a field mimicking the cosmological constant.

- Keeping the dark energy component homogeneous implies that the overdense region does not conserve energy. The exception here is the case of the cosmological constant, for which the non-clustering behaviour is exact and not an approximation. The equation defining virialization needs to be corrected, in order to account for the energy lost by the Q field between turnaround and virialization. It should read

$$\left[\tilde{U} + \frac{R}{2}\frac{\partial U}{\partial R}\right]_{vir} = U_{ta} \ .$$

This introduces a small quantitative correction.

Table 1 gives a summary of the relevant solution for the different cases that we considered in this work.

Our work has consequences for the linear theory as well. As we have not altered any of the equations governing the evolution of the system, the linear equation of growth will not be altered either. Nonetheless, reconsidering the energy budget changed the time in which we perceive virialization to happen, and as a consequence the linear contrast at virialization (1.686 for the EdS case) will change. In practice though, the numerical change is rather small. We find that for the cosmological constant, the maximal deviation from the EdS value is a rise of about 3%.

A future work would be to incorporate the possible virialization of dark energy into numerical simulations and into analyses of observations. It would be particularly interesting to see how it affects cluster abundances, and which approach provides a better fit to the observations. Several works are pursuing such directions [14].

For various models of coupled quintessence [10], it is very likely that the dark energy component clusters and virializes. For models in which the dark energy doesn't cluster, one could ask how plausible the scenario of the dark energy participating in the virialization is. Of course, should one argue that the dark energy virializes and not just the dark matter component, a mechanism of how it physically happens would be needed. An additional direction to pursue is the actual mechanism of virialization, which at the moment is still rather obscure. Understanding what the physical process by which the system virializes is will hopefully give us a clue as to whether we should include the dark energy or not.

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