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INSTABILITY IN THE EXPANDING UNIVERSE AND ITS ASTRONOMICAL IMPLICATIONS

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I. *Introduction*

The Universe is formed of nebulae which fill an elliptical space of radius R.

This radius is a solution of FRIEDMANN's equation

$$\left(\frac{dR}{dt}\right)^2 = -1 + \frac{2\mathfrak{M}}{R} + \frac{R^2}{T^2}$$

and depends on the values of the two constants of this equation: $\mathfrak{M} = \frac{4\pi}{3} G\rho R^3$, approximately constant, is the mass of the universe, and T is related to the cosmological constant λ by $1/T^2 = \lambda/3$.

We consider a solution which starts from $R=0$ with a diminishing velocity until the radius reaches, with a minimum velocity, a value R_E, called the equilibrium radius. Then the velocity increases up to the present value. The constant T is not very different from the inverse of HUBBLE's Constant, 4×10^9 years; from a geometrical view point, it is the sub-tangent to the curve (t, R).

The equilibrium $R = R_E$ is unstable. It is some aspect of this instability and its cosmological consequences which we propose to examine in this paper.

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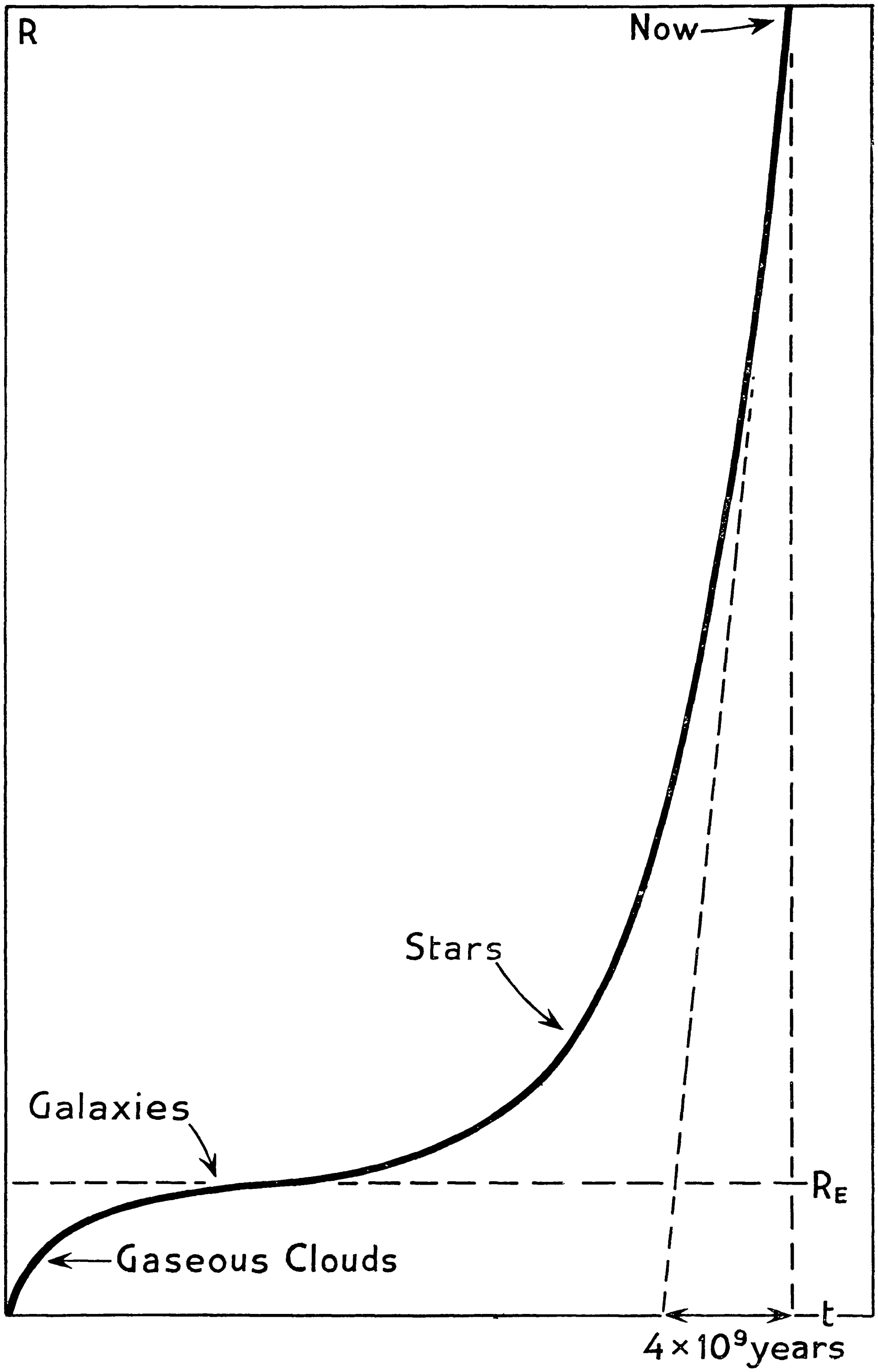

FIG. I — Change in Radius of Universe with Time. The actual radius is assumed to be ten times the equilibrium radius R_E. The time scale is given by HUBBLE's constant (subtangent of the curve). The main evolutionary processes are indicated.

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2. *Physical Aspects*

We have first to make clear the physical aspect of the theory, in order to understand the state of the matter when the radius of space reaches the equilibrium value R_E.

It is an obvious feature of the cosmological problem, that it must be considered under all its aspects and that it becomes meaningless when mutilated.

The physical beginning which fits the solution of FRIEDMANN's equation starting from $R = 0$ is provided by the Primaeval Atom Hypothesis.

Here the word "Atom" should be understood in the primitive Greek sense of the word. It is intended to mean absolute simplicity, excluding any multiplicity. The Atom is so simple that nothing can be said about it and no question raised. It provides a beginning which is entirely inaccessible.

It is only when it has split up into a large number of fragments by filling up a space of small, but not strictly zero, radius, that physical notions begin to acquire some meaning.

The first physical question which has to be considered is whether the resulting assembly of particles has to be described as a gas.

If one gives an affirmative answer to this question, one has to face the difficulty of understanding how such a gas, which presumably filled up an expanding space, has to be able, later on, to divide itself into separate nebulae.

To be more precise, we must make clear what has to be considered as characterizing a gas. It is not enough to have an assembly of a large number of particles. In order to be called a gas, such an assembly must have velocities with a distribution that is strongly concentrated around a mean velocity, the velocity of the gas, and distributed around this velocity according to a law not too different from the Maxwellian distribution which is realised in ordinary gases.

On the other hand, a mere assembly of particles with velocities spreading in every direction with speeds of the same order

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of magnitude could not be considered as a gas. It should be described as an assembly of corpuscular rays, as corpuscular radiation.

It is true that, by collisions, such radiation would finally reach a state of statistical equilibrium and become a gas. But in the extreme condition of expansion, starting (theoretically) with infinite velocity, it is not likely that such a statistical equilibrium would have had time to establish itself.

From that point of view, the problem which cosmology has to face is to understand how gas would finally arise from the primaeval radiation and then organise itself into nebulae and secondly to understand what would arise from the part of this primaeval radiation which would have escaped condensation into gases.

The second point gives an interpretation of the observed cosmic radiation, which may, of course, be only a partial one. In discussing this aspect of the theory, one must take into account for the rays the reduction of intensity due to the expansion. This phenomenon, quite analogous to the red shift of light, reduces the intensity of the rays in proportion to $1/R$. The total intensity of the cosmic rays is about $1/10,000$ of the total energy of matter condensed in the stars. This means that cosmic rays and matter would have been of the same order of magnitude when the radius was only one ten-thousandth of its present value.

In the second place, one might have to take into account some absorption of the rays by the intergalactic gases, if any, and surely by the gas when they go through the galaxy. If one take as an estimate of the density of the intergalactic gases the density 10^{-30} of matter observed as stars, one finds an absorption which is quite negligible.

Nevertheless, it might provide qualitative changes in the composition of the rays. It may be that a part of the hydrogen and helium which is observed in the cosmic radiation is due

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to secondary phenomena owing to interaction with intergalactic gas.

In any case, it is significant that, even if such correction is not made, the observed cosmic radiation contains less hydrogen and helium in comparison with heavy elements than is the case for stellar matter.

One may think that hydrogen was not present, at least in such abundance, in the primaeval radiation of which cosmic radiation provides a sample, and to inquire what is the origin of this hydrogen [1] [2].

Coming to the question of the formation of gas from the primaeval radiation, one might expect that the radiation was formed of charged particles (as in the cosmic rays). Then these rays would act as electric currents and accordingly produce magnetic fields.

It might occur that, occasionally, such magnetic fields might have some stability and form magnetic regions moving with some definite velocity without too large change of the field.

Incoming particles which reach such regions with moderate relative velocities would not be able to escape. They would be kept together long enough for elastic collisions to be able to realize some approach to a Maxwellian distribution of velocities.

In this way, it seems possible to understand the formation of local gaseous clouds from the primaeval radiation.

When such a region of some extent has been formed it would easily increase by capture of the incoming radiation, even if this radiation reaches it, as the cosmic rays at the top of the atmosphere, with large velocities.

It is conceivable that, in this process of capture, the large kinetic energy of the incoming rays would be materalised, not only into electrons and mesons, but even into hydrogen and helium nuclei [1] [2].

One could understand, in this way, that the matter of the gaseous clouds, that will eventually form the stars, should be

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richer in hydrogen than the primaeval radiation, and therefore than the observed cosmic rays.

When comparing the intensity of the rays to the density of matter, one would have to take into account the metallic part of the stars, which is of the order of only one percent of the whole.

The relative velocity of the gaseous clouds must therefore be very great even if one takes into account that it is reduced as a consequence of the expansion. It is a well-known fact, so cleverly utilised by MILNE, that objects moving with abnormal velocities in the expanding space have a tendency to sort themselves and reach regions where their velocities are not so abnormal. This kinematic effect reduces the peculiar velocities in the proportion of $1/R$.

But, even so, one must expect that, when the radius of space reaches the equilibrium value R, the gaseous clouds will have large relative velocities.

3. *The Formation of Nebulae*

We must now consider the effect of the instability of the equilibrium on the assembly of rapid gaseous clouds.

In order to do that, we may restrict ourselves to the Newtonian approximation which is known to be valid for regions not too large in which Euclidean geometry and Newtonian mechanics are good approximations.

The only essential modification which has to be introduced in classical mechanics is that POISSON's equation must be modified by the introduction of a constant ρ_0 related to the cosmical constant λ or T.

One must write

$$\Delta V = 4 \pi G (\rho - \rho_0)$$

and the equilibrium consists in the fact that the real density ρ is nearly equal to the supplementary constant ρ_0, the so called cosmical density.

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Detailed computations are not necessary to understand what will be the effect of such dynamical circumstances on our assembly of gaseous clouds.

There will occur ocasionally permanent attractive regions moving with some velocity. Every gaseous cloud reaching one of theses regions with small relative velocity will not be able to escape and would contribute to increase the incipient condensation.

The condensation will sort the clouds according to their velocities and the instability of the general field will result in the formation of an assembly of clouds which can be considered as the origin of the present nebulae.

4. *Initial Distribution*

It is easy to compute a model [3] for such an assembly of gaseous clouds and this may provide a natural starting point for a discussion of the further evolution of the system.

One assumes spherical symmetry and a static distribution which fills up uniformly the phase-space, that is the product of ordinary space by the velocity space. This uniform distribution is abruptly limited by the representative points of particles which are just able to escape from the condensation.

Some simple condition may be set up which is sufficient (but not necessary) to prevent such an escape.

Using this condition, one can compute a first approximation of the model. This approximation can be refined by perturbation if we wish to take account of the clouds which do not satisfy the condition introduced above and are otherwise prevented from escaping.

One finds a model in which particles with great angular momentum are in great number. It is well known that this is the condition which would lead to the occurrence of a disc in the nebula. These particles with great angular momentum are not able to take part in some primary process of collapse towards

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the center of the system and it is left to them to coalesce by some kind of z collisions.

5. *Radial Models*

It is also possible to work out simplified models [4] which may illustrate the main process of central collapse.

The simplest conceivable model is formed of gaseous clouds which move along the radii, and such that each cloud oscillates with the same amplitude to and fro along a radius. Phases have to be adjusted in order to make the system stationary.

There is a singularity at the center which provides a kind of dynamical nucleus. There is also a rather unsatisfactory singularity at the edge where the clouds have instantaneous rest and infinite congestion. This last singularity does not occur in another model where the velocity distribution at each point extends uniformly between two limits. In that case the velocity distribution is a line extending from one to the other of two opposite radial velocities, the one towards, the other away from the center.

In these radial models, the cosmological density has been neglected.

Both models provide distributions very similar to the distribution found by Hubble for the elliptical nebulae.

This result has been independently confirmed by the investigation of Belzer, Gamow and Keller [5]. Their treatment of the question is in some way more realistic than ours. They start from the observed distribution. Our model is over-simplified, but it is perfectly clear from the mathematical standpoint.

6. *Clusters of Nebulae*

Besides the effect of selection due to the instability that we have described already and which sorts the clouds with the required velocity in order to form an assembly of gaseous clouds from which the nebulae might have evolved, there are

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also other effects occuring on a larger scale which help us to understand the clustering tendency and generally the departures from homogenity in the distribution of the nebulae.

In order to study these effects, it is convenient to consider, in place of the slowly expanding universe passing through equilibrium, a universe staying in equilibrium like EINSTEIN's original universe [6].

Nevertheless, we have to take into account the fact that matter is endowed with large internal velocities.

We may suppose that the distribution of the velocities is such that the phase-space distribution would be uniform. According to LIOUVILLE's theorem, this situation will remain permanently without any interaction between the moving material (identified with the gaseous clouds, and later on with the nebulae which have evolved from them).

The problem will be to compute the motion of maximum velocity in every direction under the action of the gravitational field. This gravitational field is due to a density measured by the volume obtained by plotting in each direction the corresponding maximum velocity. The Newtonian approximation can still be applied with due allowance to the cosmical density ρ_0.

When spherical symmetry is accepted as a simplification, the angular momentum k will remain constant for each moving particle and the distribution shall be defined by the radial component of the velocity in both directions as a function of the distance r from the center, the time t, and the angular momentum k.

Partial differential equations, involving an integral evaluating the velocity-volume, can then be written up.

In the statical case, precise solutions can be obtained. They depend on two functions, the first of r the other of k; both are connected by a rather simple integral equation.

The most interesting case is when the function of k vanishes. In that case the distribution of velocities at each point is isotropic.

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It is found that statical concentrations do occur, with central velocity up to about 2.3 times the velocity for uniformity.

They are kinds of stationary waves where the congestion remains permanent while the individual components pass through.

The size of the condensation is related to the velocity considered. In fact it is convenient to measure the radius r using as unit the length travelled during the cosmic time $T/\sqrt{3}$ with the maximum velocity postulated for the case of uniform distribution.

When one takes velocities of the order of the standard deviation in large clusters, say 600 km/sec, one finds radii of 4 to 5 million light years, corresponding to the theoretical values of the radius from 1.8 to 1.5.

These are comparable (though rather too large) to the size of the clusters of nebulae.

This raises the question whether the clusters of nebulae are not evolved from such standing waves formed during equilibrium.

It would be very important to be able to compute the evolution of such condensations during the general expansion. Unfortunately the equations to be solved are somewhat complicated. It would be necessary to introduce convenient boundary conditions [8]. Some progress has been obtained for distributions infinitely near Einstein equilibrium by finding solutions separating the time in an exponential factor $e^{\theta t}$ which can be described by Taylor developments in r and k [7].

It has been found that the distribution compatible with the boundary conditions strongly departs from isotropy and that some simplified assumptions which had been introduced under the name of quasi-isotropy are not admissible.

7. *General Clustering*

Nevertheless some results found under this condition seem to have at least a qualitative value in the general case.

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It has been shown that the ϑ factor in $e^{\vartheta t}$ increases with the size of the condensation. It is of course equal to 0 for the static case and goes up by definition to 1 for the uniform expansion of the universe.

This gives some understanding of the fact that instability in a universe in equilibrium (approximate or exact), which could break down just as well towards contraction as towards expansion, does not contract in some part of space and expand in another.

Some clue to this arises in the present investigation, which is of course restricted to spherical symmetry. The distribution can be analysed in elementary solutions with time exponential factors. The greater this factor, the greater is the size of the fluctuation; thus, it is the larger one which must finally prevail even if locally the conditions had been in the reverse direction.

Thus a universe which will begin to collapse in the central region, while at a larger scale it expands, will eventually expand as a whole.

Nevertheless it must be expected that some trace of the initial collapsing region will remain in the final distribution and that would be observed as a cluster of nebulae.

Of course these conclusions depend essentially on the postulated large velocities of the constituent of the universe.

It implies that the clusters of nebulae would be of the nature of more or less stationary waves and that constituent nebulae would escape from the clusters and be replaced by field nebulae. The density would have to be proportional to the cube of the peculiar velocity.

Another consequence is that one would have to expect large scale fluctuations in the distribution of the densities and of the velocities of the nebulae.

In fact the larger a fluctuation due to the instability of the equilibrium through which the universe is supposed to have passed, the more permanent it is.

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This may be important in the interpretation of astronomical observations which are generally discussed under the hypothesis of a strict uniformity in the distribution of densities and velocities. Such a perfect homogeneity does not seem very probable.

It may be emphasized that the present discussion depends entirely on the sign and value assumed for the cosmical constant; this itself depends on the essential correctness of the estimates of the mass of the nebulae.

This may be questioned, on the theoretical side as well as on the astronomical side.

Conversely, the great possibilities afforded by these considerations which are based on the instability and essentially depend on the cosmological term of EINSTEIN's equations, may induce cosmologists not to reject it too lightly, even if its theoretical significance remains somewhat of an enigma.

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DISCUSSION

CHAIRMAN: O. HECKMANN

HOYLE

Is it at all possible that the clusters of galaxies should be gravitationally bound units?

LEMAÎTRE

It depends on the fact that their density is of the order of the equilibrium density 10^{-27}. The cosmological term balances the gravitational term.

SPITZER

What is the mean density of the Coma cluster?

OORT

If the mass to light ratio is about 100 the density must be nearly equal to that corresponding to the virial theorem.

SPITZER

Can LEMAÎTRE account then for the observed presence of neutral hydrogen in the Coma cluster with a mass about equal to the mass of the galaxies?

LEMAÎTRE

There is no theoretical objection to this.

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SALPETER

Have you considered in your early stages the creation of anti-matter?

LEMAÎTRE

If there is any sense in a beginning of multiplicity, the sign must be decided in the first event. The theory is not reconcilable with a large region of anti-mass.

HOYLE

Is there any significance in the fact that LEMAÎTRE indicates on his diagram stars older than the Hubble constant?

LEMAÎTRE

The extent of the graph is quite indeterminate. The answer to questions of this kind must come from observations.

HECKMANN

Those of us who believe in the usefulness of the idea of an expanding universe must be interested in every attempt of this kind to draw a model in which expansion on the one side and the formation of stars and galaxies on the other side are essentially connected.

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VIII.

SUMMARY OF THE DISCUSSIONS

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