# SMG spherical collapse notes

MG

August 17, 2018

**Abstract**

$\gamma$ depends on the value of the scalar field which is at the minimum of the effective potential. Let's find the value for this!

## 1 Introduction

The thin shell factor is given by

$$\frac{\Delta R}{R} = \frac{|\phi_\infty - \phi_c|}{6\beta M_{\rm Pl}\Phi}\,, \tag{1}$$

where $\phi_\infty$ and $\phi_c$ are the values of the scalar field far away and at the center of the object, respectively. The Newtonian potential of the sphere $\Phi$ in code units is simply given by

$$\Phi = \frac{R^2\rho}{6M_{\rm Pl}^2} \tag{2}$$

so all we need is $\phi_\infty$ and $\phi_c$ which is residing at the minimum of the effective potential.

## 2 Hu-Sawicky $f(R)$

As we have discussed[1], $f(R)$-models can be treated as scalar-tensor theories, if we redefine the metric the following way:

$$\tilde{g}_{\mu\nu} = \mathrm{e}^{2\beta\phi/M_{\rm Pl}} g_{\mu\nu} \quad \text{with } \beta = \frac{1}{\sqrt{6}}\,. \tag{3}$$

Under this transformation to the Einstein frame metric $g_{\mu\nu}$, a scalar field appears with

$$f_R = \mathrm{e}^{-2\beta\phi/M_{\rm Pl}} \tag{4}$$

and a potential

$$V(\phi) = M_{\rm Pl}^2 \frac{f_R R - f}{2f_R^2}\,. \tag{5}$$

One of these is the previously introduced model by Hu+2007, in which

$$f(R) = R - m^2 \frac{c_1 (R/m^2)^n}{1 + c_2 (R/m^2)^n} \tag{6}$$

where $c_1$, $c_2$, $m$ and $n$ are positive constants. Furthermore, $m$ is given by

$$m^2 = M_{\rm Pl}^2 \bar{\rho}_{m0}/3 = \frac{H_0^2}{\Omega_{m0}} \tag{7}$$

with $\bar{\rho}_{m0}$ being the average matter density today.

Taking the high curvature limit, that is expanding Eq. (6) around $m^2/R \to 0$ results in

$$f(R) \approx R - \frac{c_1}{c_2} m^2 + \frac{c_1}{c_2^2} m^2 \left(\frac{m^2}{R}\right)^n\,. \tag{8}$$

[1] We can go through some of the derivations of this if you want to. One can be, e.g., found at https://en.wikipedia.org/wiki/F(R)_gravity#Equivalent_formalism.

Notice how the constant second term plays the role of a cosmological constant and in the case $c_1/c_2^2 \to 0$ and $c_1/c_2 m^2 \to 2\Lambda$ we recover the standard $\Lambda CDM$ case.

As we can see the model has three free parameters: $c_1$, $c_2$ and $n$. In order to obtain background ΛCDM evolution we set

$$\frac{c_1}{c_2} m^2 = 2\Lambda = 16\pi G \rho_\Lambda \ . \tag{9}$$

Furthermore, it is convenient to introduce the background curvature today $f_{R0} \equiv \mathrm{d}f/\mathrm{d}R\big|_{a=a_0}$ and write Eq. (8) as

$$f(R) = R - 16\pi G \rho_\Lambda - \frac{f_{R0} - 1}{n} \frac{R_0^{n+1}}{R^n} \ . \tag{10}$$

The two remaining parameters $n$ and $f_{R0}$ specify the evolution of the Compton wavelength of the scalar field and the deviation from ΛCDM respectively. For $f_{R0} \to 1$ ΛCDM is recovered. *Note that some (most?) people define $f_{R0}$ to be $f_{R0} - 1$, i.e., this is close to 0 then!*

Using now Eqns. (4) and (10) we obtain in leading order

$$\frac{2\beta\phi}{M_{\rm Pl}} = (f_{R0} - 1) \frac{R_0^{n+1}}{R^{n+1}} \ . \tag{11}$$

Therefore, the potential Eq. (5) can be written as

$$V(\phi) = \left(1 + \frac{4\beta\phi}{M_{\rm Pl}}\right) \rho_\Lambda + \frac{n+1}{2n} M_{\rm Pl}^2 (f_{R0} - 1) R_0 \left(\frac{2\beta\phi}{M_{\rm Pl}(1 - f_{R0})}\right)^{n/(n+1)} \ . \tag{12}$$

The curvature today can be found using the Einstein equations with a FLRW metric and is given by

$$\begin{aligned} R_0 &= M_{\rm Pl}^{-2} \sum_i \rho_i (1 - 3w_i) \\ &= 3H_0^2 \left(\Omega_{m,0} + 4\Omega_{\Lambda,0}\right) \end{aligned} \tag{13}$$

where in the first line the sum is over all matter species and in the second line radiation is ignored as it has a negligible energy content today.

In first order the effective potential reads

$$V_{eff} = V(\phi) + \frac{\beta\phi}{M_{\rm Pl}} \rho$$

and the minimum is given by $V'_{eff}\big|_{\phi=\phi_{min}} = 0$ and can be written as

$$\begin{aligned} \phi_{min} &= \frac{M_{\rm Pl}|1 - f_{R0}|}{2\beta} \left[\frac{M_{\rm Pl}^2 R_0}{\rho + 4\rho_\Lambda}\right]^{n+1} \\ &= \frac{M_{\rm Pl}|1 - f_{R0}|}{2\beta} \left[\frac{\Omega_{m,0} + 4\Omega_{\Lambda,0}}{\Delta_\rho + 4\Omega_{\Lambda,0}}\right]^{n+1} \end{aligned} \tag{14}$$

with $\Delta_\rho = \rho/\rho_{c,0}$. It is to note that $\phi_{min}$ approaches zero with a growing $\rho$ which causes the screening of the fifth force as seen before.

The thin shell factor depends on the value of the scalar field inside and far away from the sphere. Therefore, Eq. (14) is simply applied for $\Delta_\rho = \Omega_{m,0} a^{-3}$ in order to obtain $\phi_\infty$ and $\Delta_\rho = \rho/\rho_{c,0} = 3M_c/4\rho_{c,0}\pi R_c^3$ for the center of the halo. This leaves the thin shell factor as

$$\begin{aligned} \frac{\Delta R}{R} &= \frac{|\phi_\infty - \phi_c|}{6\beta M_{\rm Pl}\Phi} \\ &= \frac{|1 - f_{R0}|}{12\beta^2\Phi} \left[\frac{\Omega_{m,0} + 4\Omega_{\Lambda,0}}{\rho/\rho_{c,0} - \Omega_{m,0} a^{-3}}\right]^{n+1} \ . \end{aligned} \tag{15}$$

Again, $\Phi = R^2\rho/6M_{\rm Pl}^2$ is the Newtonian potential of the sphere.

# 3 Symmetron

For the Symmetron, we can do a similar thing. This can be found in Hinterbichler & Khoury (2010) or Hinterbichler et al. (2011), and this section follows their lead.

As in the chameleon theory presented in the previous section, the action in the Einstein frame is given by $S = S_{EH} + S_\phi + S_{\text{matter}}$:

$$S = \int \mathrm{d}^4x \left\{ \sqrt{-g} \left( \frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi - V(\phi) \right) + \sqrt{-\tilde{g}} \mathcal{L}_m(\psi^{(i)}, \tilde{g}_{\mu\nu} \right\} \tag{16}$$

where the matter fields $\psi^{(i)}$ couple to the Jordan frame metric given by[2]

$$\tilde{g}_{\mu\nu} \equiv A^2(\phi) g_{\mu\nu} \ . \tag{17}$$

This leads to a equation of motion for the scalar field given by

$$\Box\phi - V_{,\phi} + A^3(\phi) A_{,\phi}(\phi) \tilde{T} = 0 \tag{18}$$

with the trace of the energy-momentum tensor is

$$\tilde{T} = \tilde{g}^{\mu\nu} \tilde{T}_{\mu\nu} = -\frac{2\tilde{g}^{\mu\nu}}{\sqrt{-\tilde{g}}} \frac{\delta \mathcal{L}_m}{\delta \tilde{g}^{\mu\nu}} \ . \tag{19}$$

The coupling to matter $A(\phi)$ as well as the potential $V(\phi)$ have to be of a form so that they are symmetric under $\phi \to -\phi$. A commonly considered coupling is

$$A(\phi) = 1 + \frac{\phi^2}{2M^2} + \mathcal{O}\left(\frac{\phi^4}{M^4}\right) \tag{20}$$

and the simplest potential can be stated as

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4 \ . \tag{21}$$

Here, the variables $\mu$ and $M$ have the unit of mass and $\lambda$ is a positive dimensionless coupling constant.

This leaves the effective potential from Eq. (18) using non relativistic matter ($\tilde{T} \approx -\tilde{\rho}$) as

$$V_{\text{eff}} = V(\phi) + A^3(\phi) A_{,\phi}(\phi) \tilde{\rho} \tag{22}$$

$$= \frac{1}{2}\left(\frac{\rho}{M^2} - \mu^2\right)\phi^2 + \frac{1}{4}\phi^4 \ , \tag{23}$$

where the density $\rho = A^3(\phi)\tilde{\rho}$, which is conserved in the Einstein frame, has been used. In this equation non-$\phi$ dependent terms have been neglected as they do not change the behavior of the scalar field.

One can now in a similar manner as above find $\phi_c$ and $\phi_\infty$ and use it in the spherical collapse code. Yeah!

[2]This is a conformal transformation as mentioned above.