Tightening QC Relaxations of AC Optimal Power Flow Problems via Coordinate Transformations

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Abstract

Optimal Power Flow (OPF) is a challenging non-convex optimization problem [1]. Various convex relaxation methods have been proposed to address the nonconvexities in OPF problems. This poster describes advances to the QC relaxation of the OPF problem by exploiting coordinate transformations of the power flow equations.

Optimal Power Flow (OPF) Formulation

$$\min \sum_{i\in\mathcal{G}} c_{2,i}\left(P_i^g\right)^2 + c_{1,i}\,P_i^g + c_{0,i}$$

subject to $\quad (\forall i \in \mathcal{N},\ \forall (l,m)$

$$P_{lm} = g_{lm}V_l^2 - g_{lm}V_lV_m\cos\left(\theta_{lm}\right) - b_{lm}V_lV_m\sin\left(\theta_{lm}\right)$$

$$Q_{lm} = -\left(b_{lm} + b_{sh,lm}/2\right)V_l^2 + b_{lm}V_lV_m\cos\left(\theta_{lm}\right) - g_{lm}V_lV_m\sin\left(\theta_{lm}\right)$$

$$P_{ml} = g_{lm}V_m^2 - g_{lm}V_lV_m\cos\left(\theta_{lm}\right) + b_{lm}V_lV_m\sin\left(\theta_{lm}\right)$$

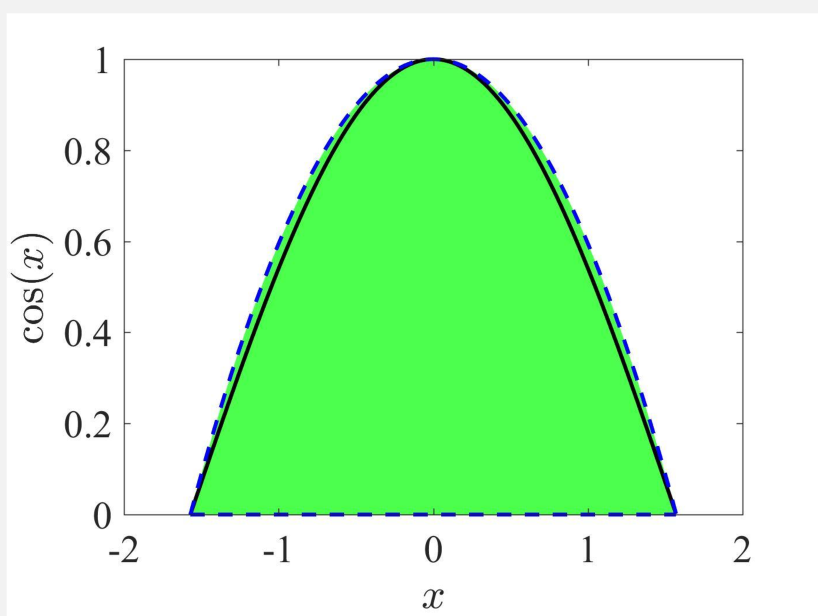
$$Q_{ml} = -\left(b_{lm} + b_{sh,lm}/2\right)V_m^2 + b_{lm}V_lV_m\cos\left(\theta_{lm}\right) + g_{lm}V_lV_m\sin\left(\theta_{lm}\right)$$

Voltage, line flow, generation constraints

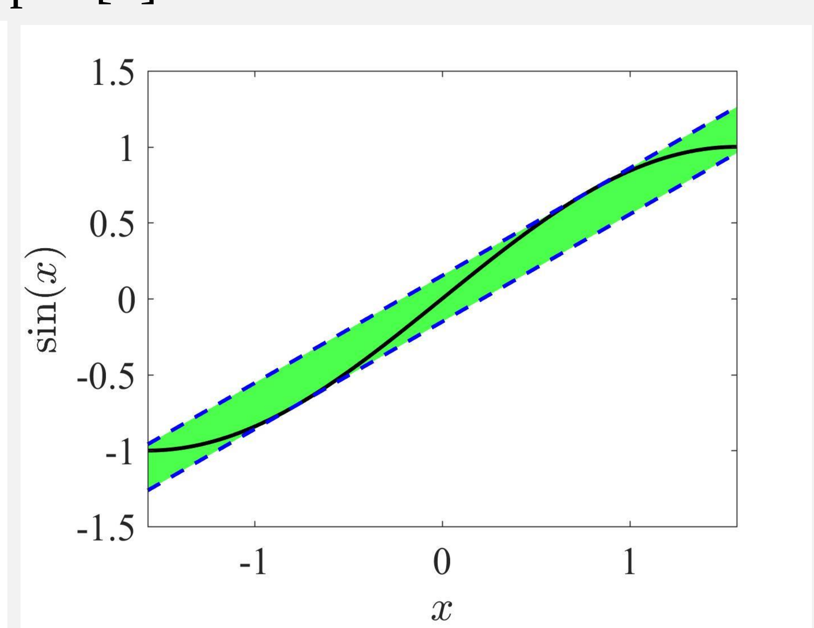
Power balance constraints

QC Relaxation of the OPF Problem

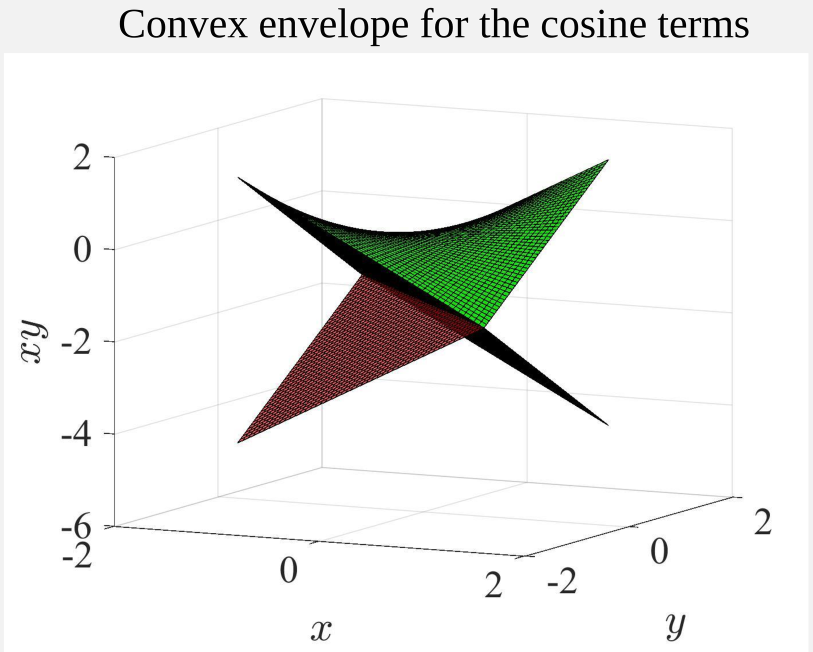
- The QC relaxation convexifies the OPF problem by enclosing the nonconvex terms in convex envelopes [2].



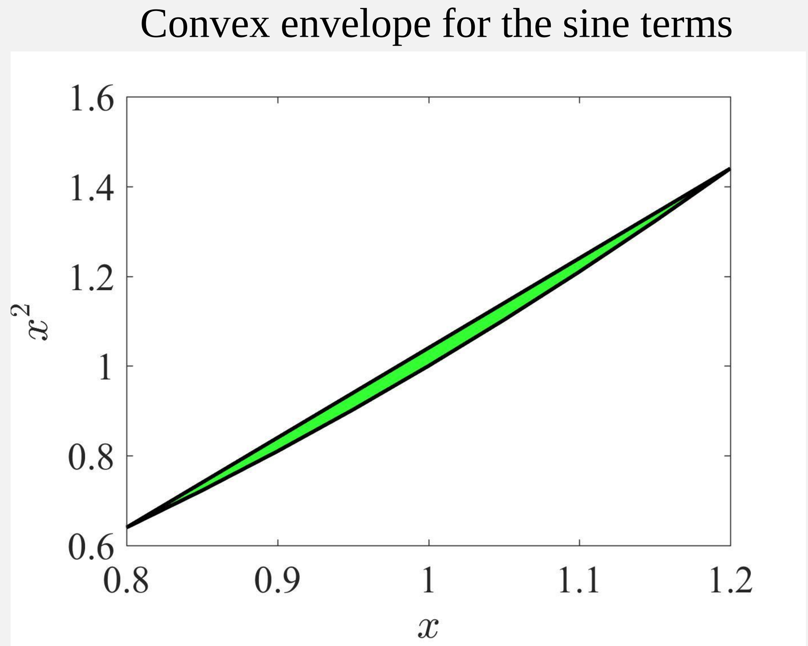
Convex envelope for the cosine terms



Convex envelope for the sine terms



Convex lower bounds for bilinear terms



Convex envelope for squared terms

- QC relaxation's accuracy strongly depends on the tightness of envelopes.
- Convex hull for trilinear terms improves QC relaxation's accuracy [3-5].

Power Flow Equations with Admittance in Polar Form

- Model line flows using a polar representation of the line's admittance.

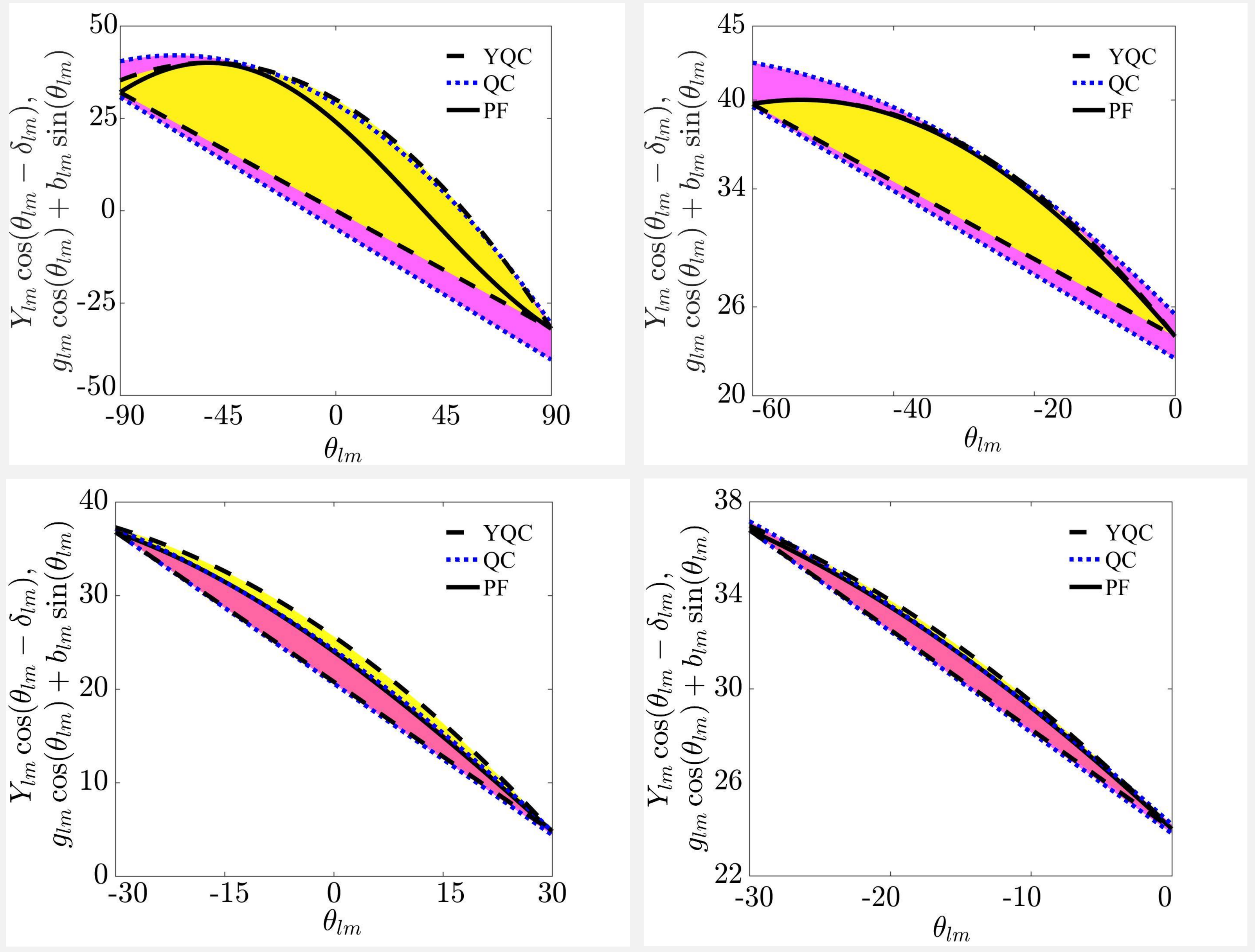
$$P_{lm} = Y_{lm}\cos(\delta_{lm})V_l^2 - Y_{lm}V_lV_m\cos(\theta_{lm} - \delta_{lm})$$

$$Q_{lm} = -\left(Y_{lm}\sin(\delta_{lm}) + \frac{b_c}{2}\right)V_l^2 - Y_{lm}V_lV_m\sin(\theta_{lm} - \delta_{lm})$$

$$P_{ml} = Y_{lm}\cos(\delta_{lm})V_m^2 - Y_{lm}V_lV_m\cos(\theta_{lm} + \delta_{lm})$$

$$Q_{ml} = -\left(Y_{lm}\sin(\delta_{lm}) + \frac{b_c}{2}\right)V_m^2 + Y_{lm}V_lV_m\sin(\theta_{lm} + \delta_{lm})$$

- Different envelopes for trigonometric terms in the QC relaxation.



- Differing admittance representations enable coordinate transformation.

Rotated Base QC Relaxation

- Complex-valued choices for the base power [4].

$$\tilde{S}_{lm} = \frac{S_{lm}}{e^{j\psi}}, \qquad \tilde{S}_{ml} = \frac{S_{ml}}{e^{j\psi}}$$

- A degree of freedom (i.e.,ψ) the arguments of trigonometric terms.

$$\tilde{P}_{lm} = \left(Y_{lm}\cos(\delta_{lm}+\psi) - \frac{b_c}{2}\sin(\psi)\right)V_l^2 - Y_{lm}V_lV_m\cos(\theta_{lm} - \delta_{lm} - \psi)$$

$$\tilde{Q}_{lm} = -\left(Y_{lm}\sin(\delta_{lm}+\psi) + \frac{b_c}{2}\cos(\psi)\right)V_l^2 - Y_{lm}V_lV_m\sin(\theta_{lm} - \delta_{lm} - \psi)$$

$$\tilde{P}_{ml} = \left(Y_{lm}\cos(\delta_{lm}+\psi) - \frac{b_c}{2}\sin(\psi)\right)V_m^2 - Y_{lm}V_mV_l\cos(\theta_{lm} + \delta_{lm} + \psi)$$

$$\tilde{Q}_{ml} = -\left(Y_{lm}\sin(\delta_{lm}+\psi) + \frac{b_c}{2}\cos(\psi)\right)V_m^2 + Y_{lm}V_mV_l\sin(\theta_{lm} + \delta_{lm} + \psi)$$

- Relationships between power generation in different coordinates.

$$\begin{bmatrix}\tilde{P}_i^g\\ \tilde{Q}_i^g\end{bmatrix} = \begin{bmatrix}\cos(\psi) & \sin(\psi)\\ -\sin(\psi) & \cos(\psi)\end{bmatrix}\begin{bmatrix}P_i^g\\ Q_i^g\end{bmatrix}$$

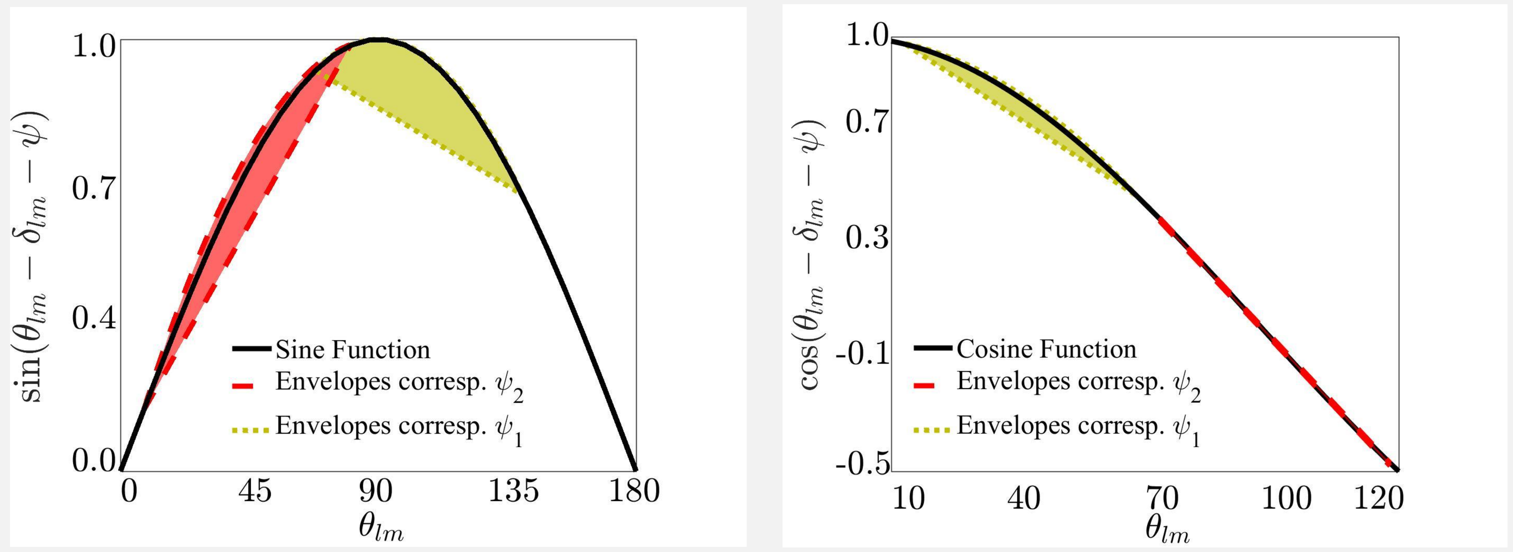
- The analogous relationship applies for the load demands.

Further Tightening the Rotated Based QC Relaxation

- Enforce the deanship between trigonometric terms associated with sending and receiving terminals of each line.

$$\begin{bmatrix}\sin(\theta_{lm}+\hat{\delta}_{lm})\\ \cos(\theta_{lm}+\hat{\delta}_{lm})\end{bmatrix} = \begin{bmatrix}\alpha_{lm} & \beta_{lm}\\ -\beta_{lm} & \alpha_{lm}\end{bmatrix}\begin{bmatrix}\sin(\theta_{lm}-\hat{\delta}_{lm})\\ \cos(\theta_{lm}-\hat{\delta}_{lm})\end{bmatrix}$$

$$\hat{\delta}_{lm} = \delta_{lm} + \psi \qquad \alpha_{lm} = (\cos(\hat{\delta}_{lm}))^2 - (\sin(\hat{\delta}_{lm}))^2 \qquad \beta_{lm} = 2\cos(\hat{\delta}_{lm})\sin(\hat{\delta}_{lm})$$



- Additionally enforcing the original trigonometric envelopes further tightens the relaxation. Empirical results show limited impacts on solution times.

Results and Conclusion

- Applied to the IEEE PES PGLib AC-OPF v18.08 benchmark library.
- Optimality gap is used to measure the relaxations' tightness.

$$\textit{Optimality gap} = \left(\frac{\textit{Local solution} - \textit{QC bound}}{\textit{QC bound}}\right)$$

Test Cases	QC gap (%)	YQC gap (%)	TRQC gap (%) (Best ψ)
pglib_opf_case3_lmbd	0.97	0.97	0.69
pglib_opf_case14_ieee__sad	19.16	21.45	17.91
pglib_opf_case24_ieee_rts__api	11.02	8.75	8.30
pglib_opf_case30_ieee	18.67	16.42	16.18
pglib_opf_case30_ieee__sad	5.66	5.89	5.25
pglib_opf_case39_epri__api	1.71	1.63	1.51
pglib_opf_case73_ieee_rts__api	9.54	8.44	8.01
pglib_opf_case118_ieee	0.77	0.73	0.69
pglib_opf_case179_goc__api	5.86	6.01	4.31
pglib_opf_case300_ieee__api	0.83	0.81	0.75
pglib_opf_case6468_rte__api	0.59	0.58	0.57
pglib_opf_case6495_rte__sad	14.98	14.97	13.78

YQC: QC relaxation with polar representation of admittance.
TRQC: Tightened RQC by original trigonometric function.

- Coordinate transformation provides a degree of freedom for the arguments of the trigonometric terms to strengthen the RQC relaxation of the OPF problem [5].

References

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[3] M. R. Narimani, D. K. Molzahn, and M. L. Crow, "Improving QC Relaxations of OPF Problems via Voltage Magnitude Difference Constraints and Envelopes for Trilinear Monomials," Power Systems Computation Conference (PSCC), June 2018.

[4] M. R. Narimani, D. K. Molzahn, H. Nagarajan, and M. L. Crow, "Comparison of Various Trilinear Monomial Envelopes for Convex Relaxations of Optimal Power Flow Problems," IEEE Global Conference on Signal and Information Processing (GlobalSIP), November 2018.

[5] M. R. Narimani, D. K. Molzahn, and M. L. Crow, "Tightening QC Relaxations of AC Optimal Power Flow Problems via Coordinate Transformations," arXiv preprint arXiv:1912.05061.