

Machine Learning and Statistics- Driven Code-Oriented Shear Strength Equation for RC Structural Walls

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Motivation & Challenges

- Equation in ACI is based on limited data and have a large error
- Availability of the UCLA-RC Walls Database
- Significant variance when using the same equation/model to predict values of different databases
- Complexity of ML models and its interpretation

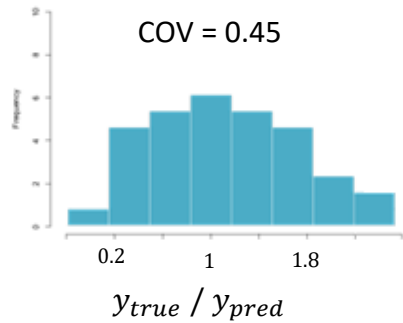


Identified Main Problems - Example

$$y = \frac{a}{\sqrt{x_3}}x_2 + c x_1$$

Mean = 1.08

COV = 0.45



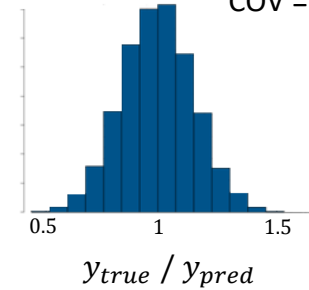
Model Complexity – Acceptable Error

Trade Off

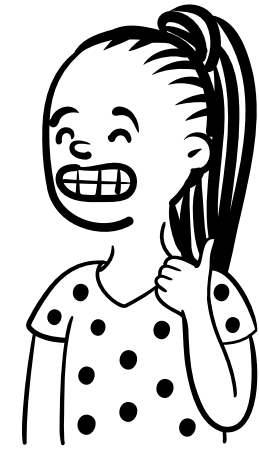


Mean = 1.00

COV = 0.10



Requirements	Model Complexity Level	
	Complex ML Models	Simplified Models
Number of parameters	-	~ 3 - 6
V_{true}/V_{pred} mean ratio	0.99 – 1.01	0.98 – 1.02
COV	≤ 0.12	0.16 – 0.19
Train. vs Test. Error Margin	$\pm 20\%$ of the converging error	$\pm 10\%$ of the converging error





Framework to Set Target Errors for Different Model Complexity Levels

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- 6 generic steps of ML + Sensitivity Analysis with Iterative k-Fold CV
- + Elastic Net Models (ENMs) with engineered features
 - + Establish desired underfitting levels

Framework to Set Target Errors for Different Model Complexity Levels



- (1) Collection and preparation of data
- (2) Feature selection
 - Mechanics + Literature Review
 - Normalized/scaled features + Mechanics → Engineered Features
- (3) Choice of ML algorithm
 - Complex ML model (e.g., ANN, RF regression)
 - ENMs
- (4) Selection hyper-parameters
 - Sensitivity analysis with iterative k-Fold CV
 - Define underfitting levels to be used for the ENMs training
- (5) Model training
- (6) Model performance evaluation

- Algorithm to assess the sensitivity analysis on the hyper parameters

(1) Collection and preparation of data

(2) Feature selection

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- Normalized/scaled features + Mechanics → Engineered Features

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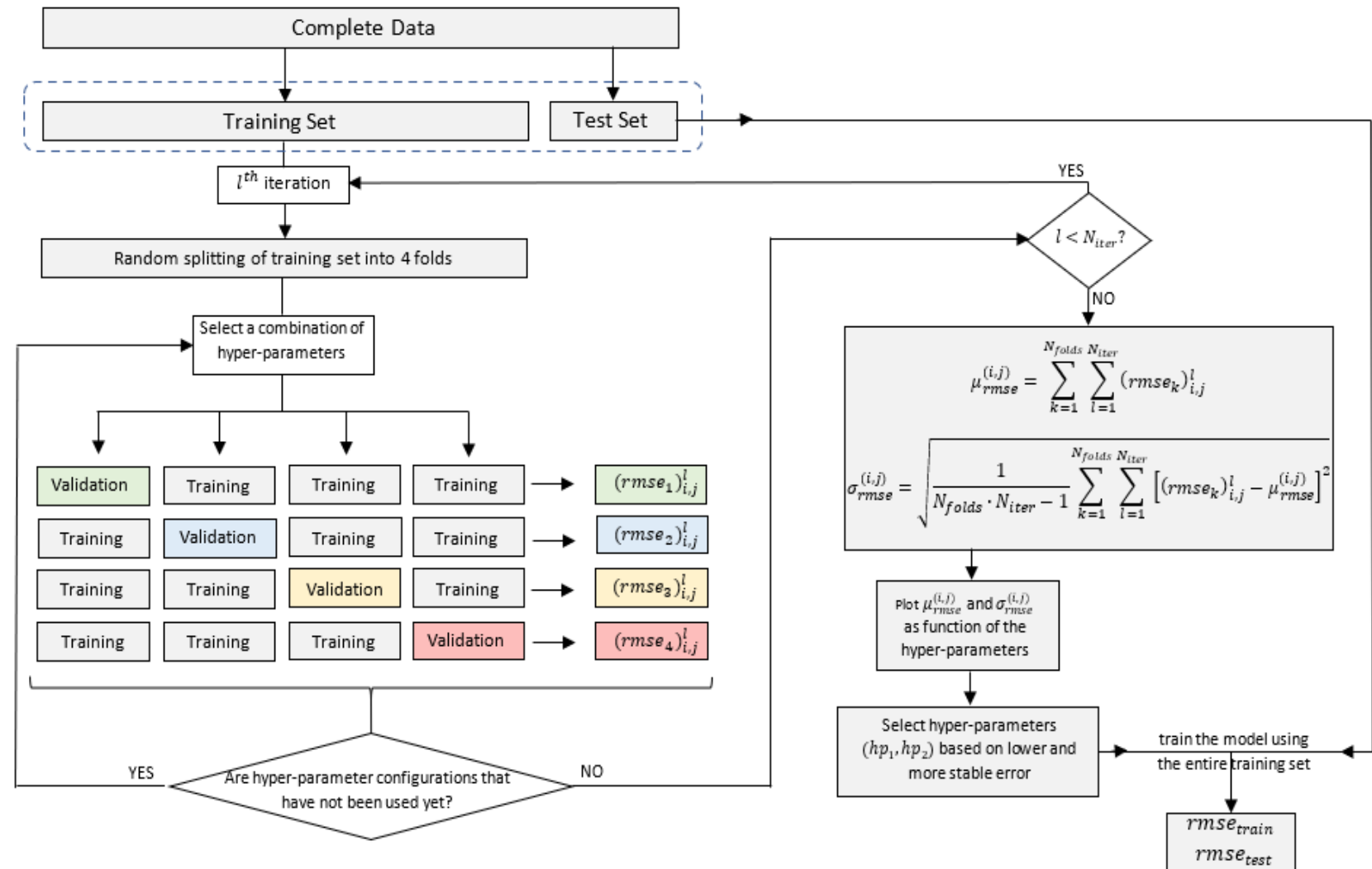
(4) Selection hyper-parameters

- Sensitivity analysis with iterative k-Fold CV
- Define underfitting levels to be used for the ENMs training

(5) Model training

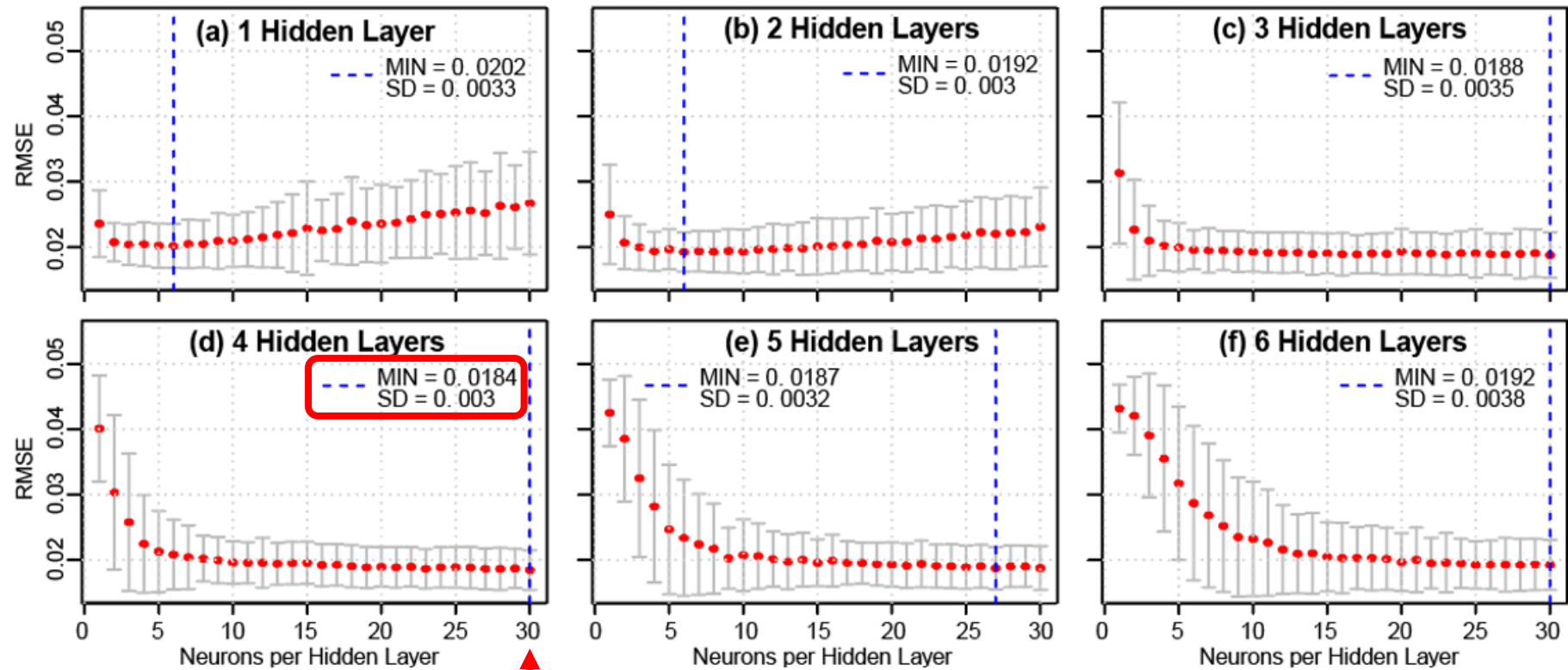
(6) Model performance evaluation

(7) Set target errors for different model complexity levels



- Artificial Neural Network (ANN)

(Sensitivity Analysis → 180 ANN were trained)



- RF Regression (same)

(Sensitivity Analysis → 120 RF Regressions were trained)

- ENMs (same)

(Sensitivity Analysis → (10 models) x (4 complexity levels) = 40 ENMs were trained)

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- Sensitivity analysis with iterative k-Fold CV
- Define underfitting levels to be used for the ENMs training

(5) Model training

(6) Model performance evaluation

(7) Set target errors for different model complexity levels

• Selected ENMs

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(2) Feature selection

- Mechanics + Literature Review
- Normalized/scaled features + Mechanics → Engineered Features

(3) Choice of ML algorithm

- Complex ML model (e.g., ANN, RF regression)
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(7) Set target errors for different model complexity levels

Optimum LASSO models

Model	RMSE _{train}	RMSE _{test}	RMSE _{conv}	Acceptable?
6	0.0117	0.0137	0.0127	YES
5	0.0114	0.0167	0.0141	YES
7	0.0112	0.0177	0.0145	NO
3	0.0131	0.0181	0.0156	YES
8	0.0120	0.0206	0.0163	NO
4	0.0148	0.0186	0.0167	YES
2	0.0134	0.0202	0.0168	YES
1	0.0197	0.0216	0.0207	YES
9	0.0122	0.0631	0.0377	NO
10	0.0122	0.3469	0.1796	NO

6-feature LASSO models

Model	RMSE _{train}	RMSE _{test}	RMSE _{conv}	Acceptable?
3	0.0153	0.0194	0.0173	NO
9	0.0152	0.0201	0.0177	NO
4	0.0164	0.0190	0.0177	YES
8	0.0147	0.0209	0.0178	NO
10	0.0164	0.0198	0.0181	YES
6	0.0203	0.0212	0.0207	YES
1	0.0203	0.0219	0.0211	YES
5	0.0200	0.0222	0.0211	YES
7	0.0220	0.0233	0.0226	YES
2	0.0213	0.0240	0.0227	YES

1-SD away LASSO models

Model	RMSE _{train}	RMSE _{test}	RMSE _{conv}	Acceptable?
9	0.0137	0.0147	0.0142	YES
6	0.0140	0.0175	0.0157	YES
10	0.0139	0.0185	0.0162	NO
3	0.0137	0.0189	0.0163	NO
5	0.0146	0.0181	0.0163	YES
4	0.0144	0.0183	0.0164	NO
2	0.0138	0.0199	0.0168	NO
7	0.0140	0.0213	0.0176	NO
8	0.0145	0.0244	0.0194	NO
1	0.0203	0.0219	0.0211	YES

3-feature LASSO models

Model	RMSE _{train}	RMSE _{test}	RMSE _{conv}	Acceptable?
10	0.0192	0.0218	0.0205	YES
8	0.0184	0.0256	0.0220	NO
4	0.0218	0.0237	0.0227	YES
9	0.0208	0.0291	0.0250	NO
2	0.0245	0.0292	0.0268	YES
3	0.0249	0.0288	0.0269	YES
1	0.0273	0.0289	0.0281	YES
5	0.0264	0.0310	0.0287	YES
6	0.0279	0.0320	0.0299	YES
7	0.0276	0.0360	0.0318	NO

Target Errors for Different Model Complexity Levels

(1) Collection and preparation of data

(2) Feature selection

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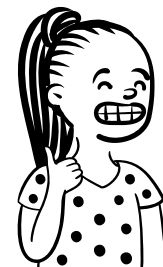
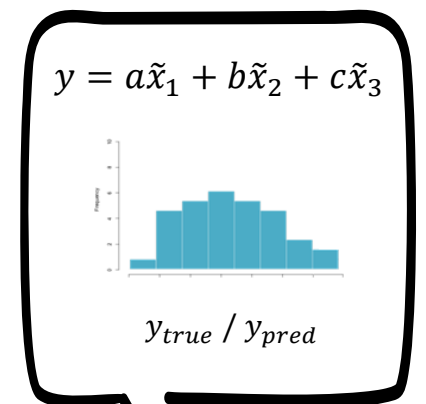
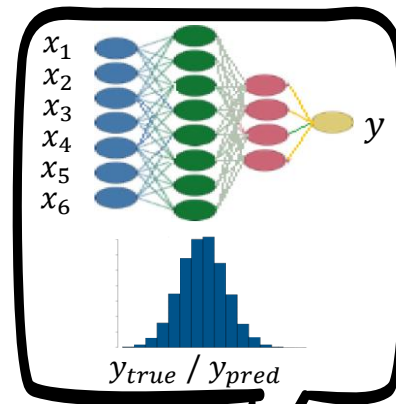
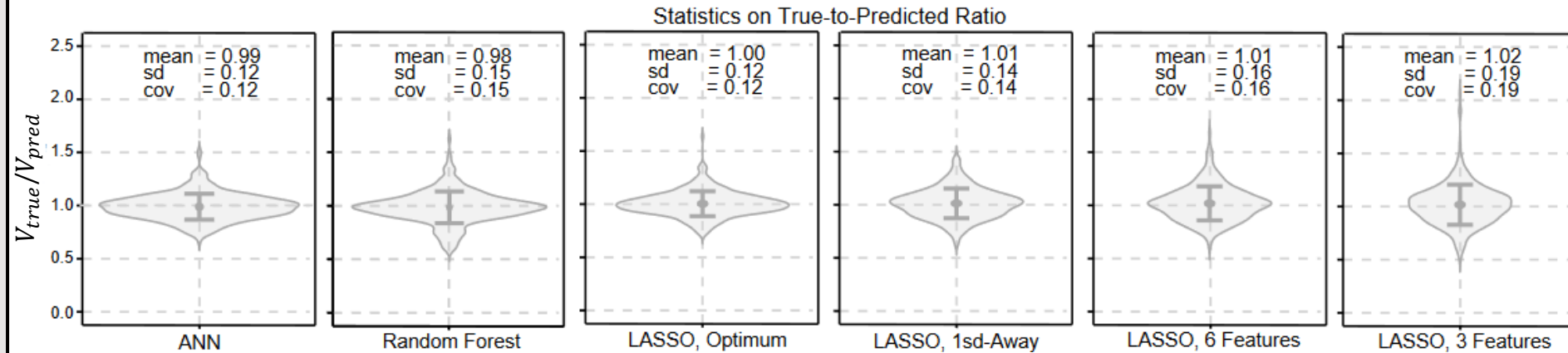
(4) Selection hyper-parameters

- Sensitivity analysis with iterative k-Fold CV
- Define underfitting levels to be used for the ENMs training

(5) Model training

(6) Model performance evaluation

(7) Set target errors for different model complexity levels



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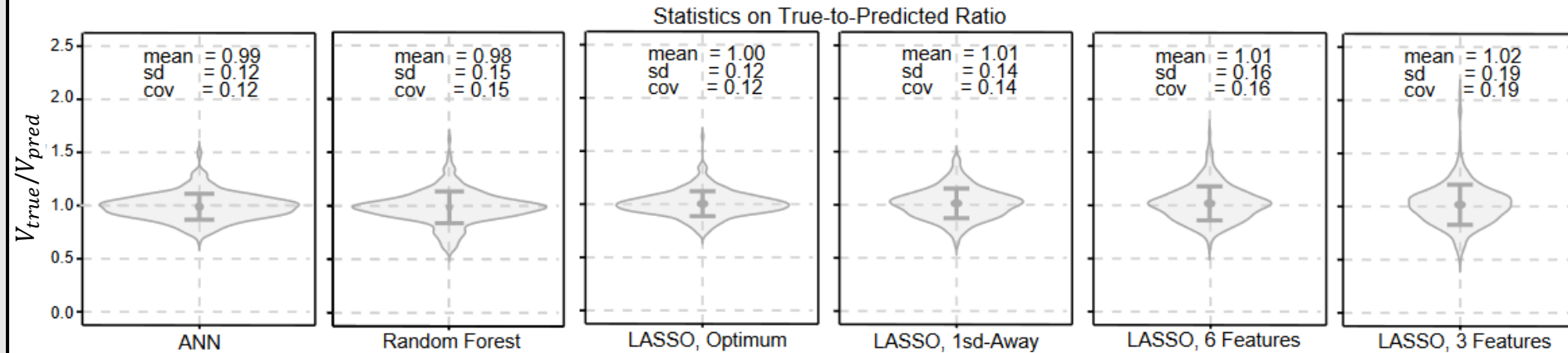
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Methodology to Get an Equation with a Code-Oriented Format

(Preliminary Results)

Methodology to Get an Equation with a Code-Oriented Format

(1) Identification of relevant parameters

(2) Differentiate parameters between:

- Materiality-related parameters (V_i)
- Other parameters ($\gamma_{j,i}$)

(3) Re-arrange parameters (" $V_c + V_s$ " format):

$$V_n = \beta_0 V_c + \sum_i^{N_i} \beta_i \left(\prod_j^{N_j} \gamma_{j,i} \right) V_i$$

(4) Re-write equation in its normalized version

(5) Train model

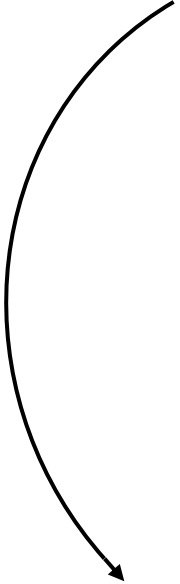
(6) Drop less significant variable until reaching target error

(7) Verify performance



- (1) Identification of relevant parameters
- (2) Differentiate parameters between:
 - Materiality-related parameters (V_i)
 - Other parameters ($\gamma_{j,i}$)
- (3) Re-arrange parameters into this equation format:

$$V_n = \beta_0 V_c + \sum_i^{N_i} \beta_i \left(\prod_j^{N_j} \gamma_{j,i} \right) V_i$$
- (4) Re-write the equation in its normalized version
- (5) Use iterative k-fold analysis to find regression coefficients and p-values
- (6) Drop less significant parameter and go back to step (5) until reaching the target error
- (7) Verify performance of the proposed equation

$$\begin{aligned}
 y = & \beta_0 + \beta_1 \left(\frac{M_u}{V_u l_w} \right)^{a_c} \left(\frac{h_w}{l_w} \right)^{b_c} \left(1 + \frac{P_u}{A_g f'_c} \right)^{c_c} \\
 & + \beta_2 \left(\frac{M_u}{V_u l_w} \right)^{a_{be}} \left(\frac{h_w}{l_w} \right)^{b_{be}} \left(1 + \frac{P_u}{A_g f'_c} \right)^{c_{be}} \frac{\rho_{be} f_{ybe} A_{be}}{A_g f'_c} \\
 & + \beta_3 \left(\frac{M_u}{V_u l_w} \right)^{a_{wh}} \left(\frac{h_w}{l_w} \right)^{b_{wh}} \left(1 + \frac{P_u}{A_g f'_c} \right)^{c_{wh}} \frac{\rho_{wh} f_{ywh} A_{cv}}{A_g f'_c} \\
 & + \beta_4 \left(\frac{M_u}{V_u l_w} \right)^{a_{wv}} \left(\frac{h_w}{l_w} \right)^{b_{wv}} \left(1 + \frac{P_u}{A_g f'_c} \right)^{c_{wv}} \frac{\rho_{wv} f_{ywv} A_{cv}}{A_g f'_c}
 \end{aligned}$$


$$V_n = \alpha_c A_g f'_c + \alpha_s (A_{sb} f_y + \rho_{wh} f_{ywh} A_{cv})$$

- (1) Identification of relevant parameters
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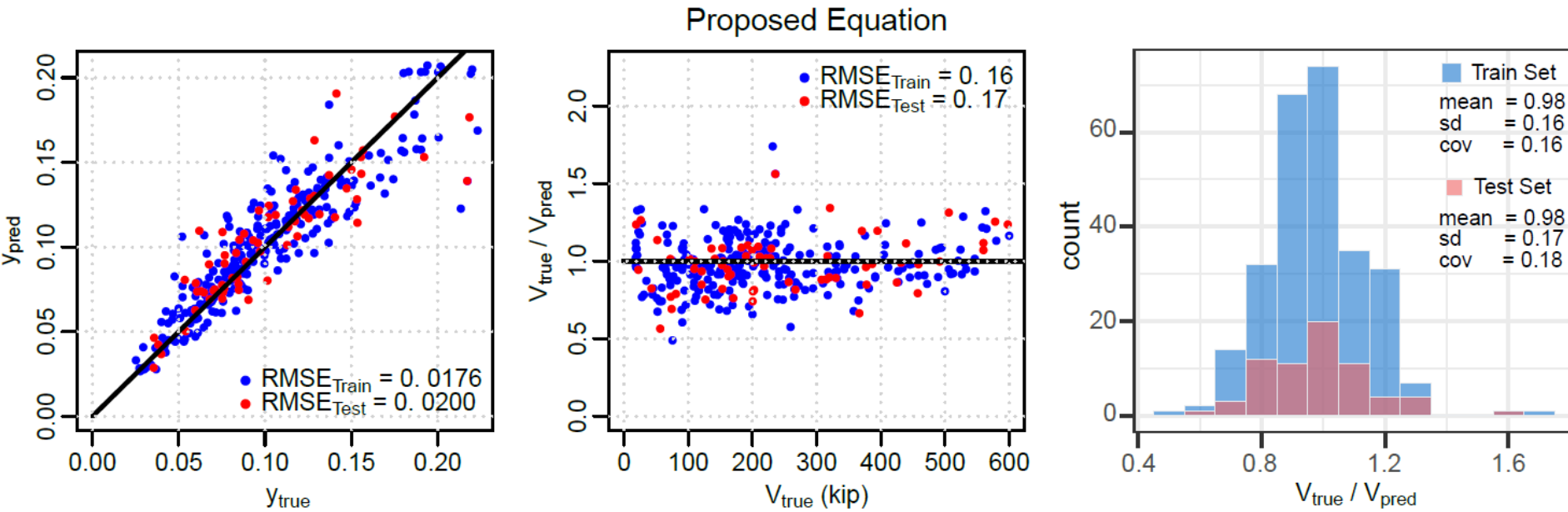
$$V_n = \alpha_c A_g f'_c + \alpha_s (A_{sb} f_y + \rho_{wh} f_{ywh} A_{cv})$$

$$\alpha_c = \frac{1}{100} \left(14 \frac{\left(1 + \frac{P_u}{A_g f'_c} \right)^2}{\left(\frac{M_u}{V_u l_w} \right)^2} \right) \quad \alpha_s = \frac{1}{3 \left(\frac{M_u}{V_u l_w} \right)^{1/4}}$$

$$V_n = \beta_0 V_c + \sum_i^{N_i} \beta_i \left(\prod_j^{N_j} \gamma_{j,i} \right) V_i$$

- (4) Re-write the equation in its normalized version
- (5) Use iterative k-fold to find regression coefficients and p-values
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(7) Verify performance of the proposed equation



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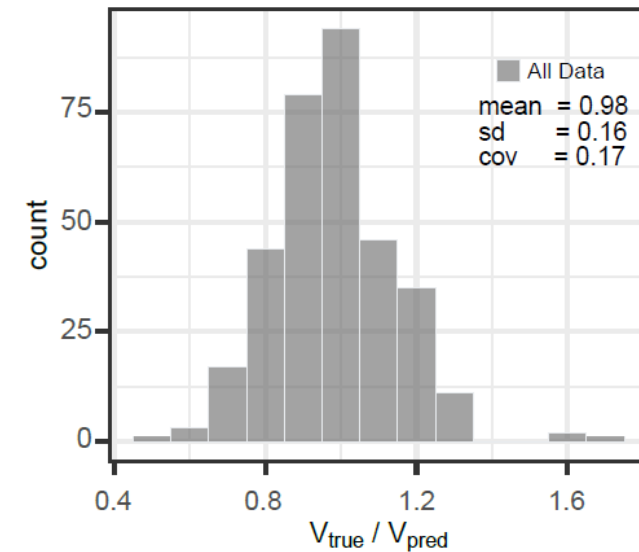
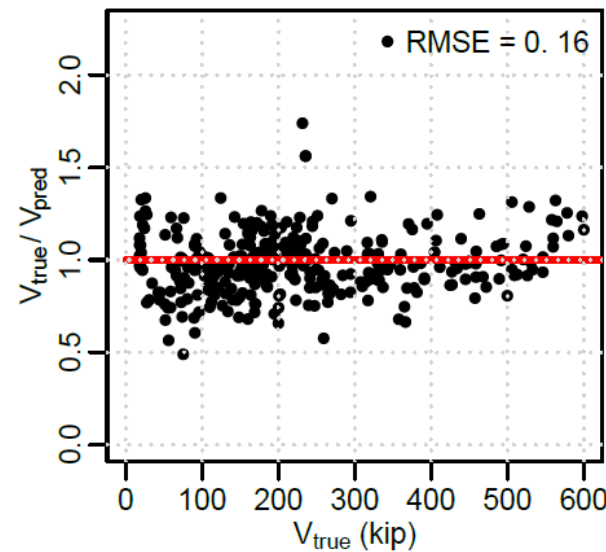
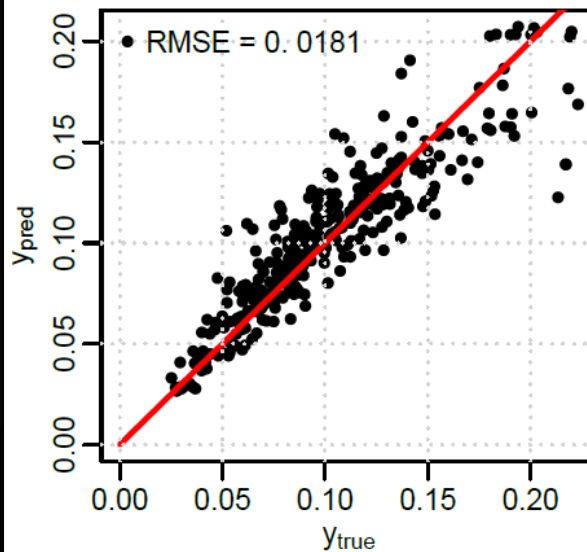
$$\alpha_c = \frac{1}{100} \left(14 \frac{\left(1 + \frac{P_u}{A_g f'_c} \right)^2}{\left(\frac{M_u}{V_u l_w} \right)^2} \right) \quad \alpha_s = \frac{1}{3 \left(\frac{M_u}{V_u l_w} \right)^{1/4}}$$

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Prpposed Equation Applied on Complete Dataset



- (1) Identification of relevant parameters
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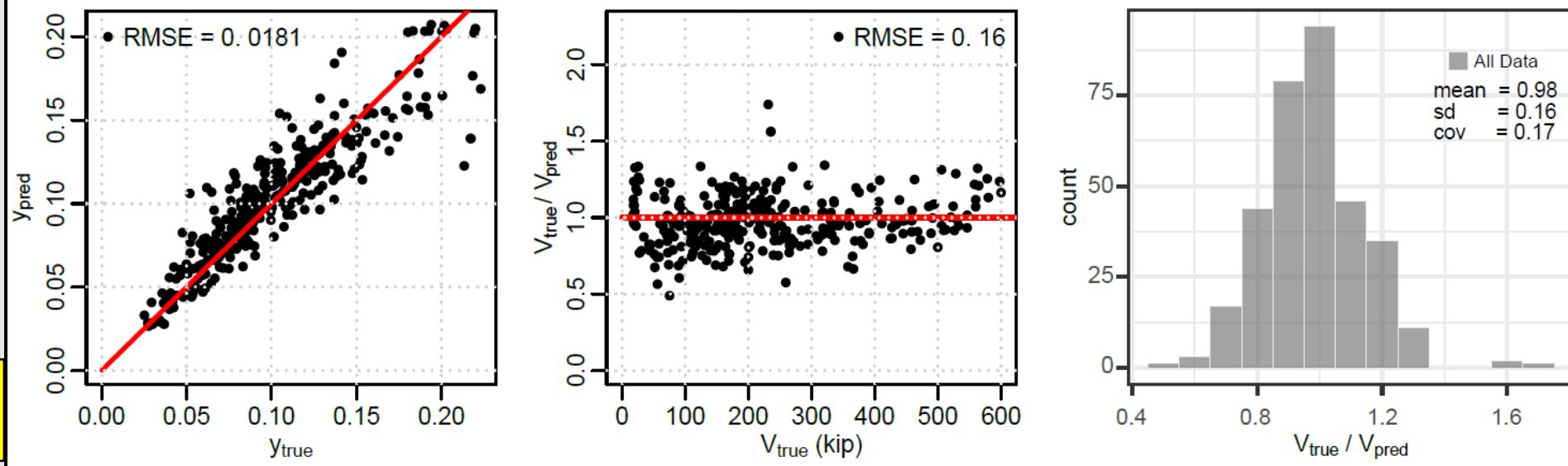
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(7) Verify performance of the proposed equation

Requirements	Simple Models
Number of parameters	~ 3 - 6
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Prpposed Equation Applied on Complete Dataset



Relevant points to investigate:

- (1) Does the equation make sense?
(Contribution from each term, parameter ranges, etc.)
- (2) How to use the equation on unsymmetrical cross-section shaped walls?
- (3) Modification of wall shear strength upper limit?
- (4) Is the model complexity level acceptable?
- (5) How to use the equation in the design?
- (6) Consequences of the equation in the design





Upcoming articles with more details on the results for both:

- (1) Target errors for different model complexity levels
- (2) Proposed RC wall shear strength equation

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