Coinduction

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<2018-08-15 Wed 23:08>

The following article by Jacobs and Rutten is a really nice introduction to the notion of coinduction. It assumes almost no prior knowledge of categories and details algebras, initiality, coalgebras, finality, induction and bisimulation.

- A tutorial on (Co)algebras and (Co)induction - Bart Jacobs, Jan Rutten

Conatural numbers can be implemented in Agda using coinductive records as in the following example. If you are interested in understanding coinduction, it might be a good idea to experiment in Agda; I learnt a lot writing basic coinductive definitions.

```
data Maybe (A : Set) : Set where
  Nothing : Maybe A
  Just : A -> Maybe A

record coNat : Set where
  coinductive
  field
    pred : Maybe coNat
open coNat public

coZero : coNat
pred coZero = Nothing

coInf : coNat
pred coInf = Just coInf

succ : coNat -> coNat
pred (succ n) = Just n

infixl 20 _+_
_+_ : coNat -> coNat -> coNat
pred (a + b) with pred a
```

```
pred (a + b) | Nothing = pred b
pred (a + b) | Just a' = Just (a' + b)
```