The glass optic

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At the intersection between a **lens** and a **grate** there should be an optic that I have started to call **glass**. If the pseudomonoid action that describes a lens is $c \times (-)$ and the one for glass is $c \to (-)$, the action that we want to get should be the one given by the coproduct pseudomonoid.

We rewrite the action of every word $c_1 d_1 \dots c_k d_k$ as follows, for some $e, f \in \mathbf{C}$ bicartesian closed category.

$$\begin{aligned} a \mapsto\ & d_1 \times (c_1 \to d_2 \times (c_2 \to \dots x)) \\ =\ & d_1 \times (c_1 \to d_2) \times ((c_1 \times c_2) \to \dots x) \\ =\ & e \times (f \to x) \end{aligned}$$

And we derive the concrete representation.

$$\begin{aligned} & \int^{c,d} \mathbf{C}(s, c \times (d \to a)) \times \mathbf{C}((d \to b) \times c, t) \\ \cong\ & \int^{c,d} \mathbf{C}(s, c) \times \mathbf{C}(s \to d \to a) \times \mathbf{C}((d \to b) \times c, t) \\ \cong\ & \int^{d} \mathbf{C}(s \to d \to a) \times \mathbf{C}((d \to b) \times s, t) \\ \cong\ & \int^{d} \mathbf{C}(d \to (s \to a)) \times \mathbf{C}((d \to b) \times s, t) \\ \cong\ & \mathbf{C}(((s \to a) \to b) \times s, t) \\ \cong\ & \mathbf{C}(((s \to a) \to b), s \to t) \end{aligned}$$

The optic in Haskell is `((s -> a) -> b) -> s -> t`. My intuition at the moment is "If from a getter you can create a b, then you can update s to t", which apparently also rhymes.