

Every surjection to a projective R-module is split

Mario Román

<2018-12-24 Mon 09:07>

The statement comes from an exercise on André Henriques' course on [Homological Algebra](#).

Statement. *Let R be a ring. Prove that an R -module P is projective iff every surjective map $A \rightarrow P$ admits a section.*

Given a projective module P , we can apply the definition to any surjection $f: A \twoheadrightarrow P$ to get the existence of a section.

$$\begin{array}{ccc} & & P \\ & \nearrow \exists s & \downarrow \text{id} \\ A & \xrightarrow{f} & P \end{array}$$

Suppose every surjection admits a section, we can take a surjection from the free module over P , which we call $\pi: F \rightarrow P$ and take a section $s: P \rightarrow F$. Then the sequence $0 \rightarrow \ker(\pi) \rightarrow F \rightarrow P \rightarrow 0$ splits, P is a direct summand of a free module and thus a projective module.