

Let \mathcal{C} be traced. We know $\mathbf{Int}(\mathcal{C})$ is the free compact closed category over \mathcal{C} , and there is a fully faithful $\mathcal{C} \rightarrow \mathbf{Int}(\mathcal{C})$.

Lemma 1. *Let $F : \mathcal{C} \rightarrow \mathcal{D}$ be strong monoidal and fully faithful, then $\mathbf{Optic}(F) : \mathbf{Optic}(\mathcal{C}) \rightarrow \mathbf{Optic}(\mathcal{D})$ is faithful.*

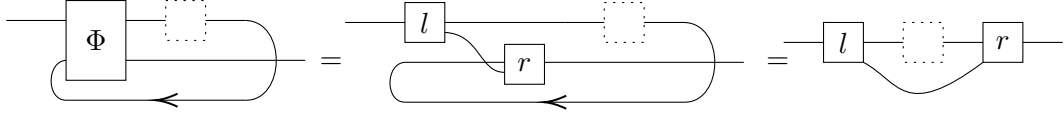
Proof. Assume $\langle Fl \mid Fr \rangle \sim \langle Fl' \mid Fr' \rangle$, this means

$$\begin{aligned} & \exists m. (Fl = (m \otimes \text{id}) \circ Fl') \wedge (Fr \circ (m \otimes \text{id}) = Fr') \\ & \quad (\text{by fullness}) \\ & \exists n. (Fl = (Fn \otimes \text{id}) \circ Fl') \wedge (Fr \circ (Fn \otimes \text{id}) = Fr') \\ & \quad (\text{by strong monoidal}) \\ & \exists n. (Fl = F((n \otimes \text{id}) \circ l')) \wedge (F(r \circ (n \otimes \text{id})) = Fr') \\ & \quad (\text{by faithfulness}) \\ & \exists n. (l = (n \otimes \text{id}) \circ l') \wedge (r \circ (n \otimes \text{id}) = r') \end{aligned}$$

then, $\langle l \mid r \rangle \sim \langle l' \mid r' \rangle$. □

Lemma 2. *There is a function $\mathcal{C}(S \otimes B, A \otimes T) \rightarrow \mathbf{Optic}(\mathbf{Int}(\mathcal{C}))((S, T), (A, B))$ such that $\mathbf{Optic}(\mathcal{C})((S, T), (A, B)) \rightarrow \mathcal{C}(S \otimes B, A \otimes T) \rightarrow \mathbf{Optic}(\mathbf{Int}(\mathcal{C}))((S, T), (A, B))$ is the induced by $\mathcal{C} \rightarrow \mathbf{Int}(\mathcal{C})$.*

Proof. We put $\phi \in \mathcal{C}(S \otimes B, A \otimes T)$ on the left side of the optic, and we connect the two B using the compact structure of $\mathbf{Int}(\mathcal{C})$. A diagram explains this better.



These equalities are not between optics over \mathcal{C} , but optics in the free compact closed category over it. □

Finally, putting together the two lemmas, if two optics are the same after embedding in \mathbf{Int} , they should be the same in $\mathbf{Optic}(\mathbf{Int}(\mathcal{C}))$ with the function on Lemma 2, and thus (Lemma 1) they should be the same.