Distributive Laws of Data Accessors

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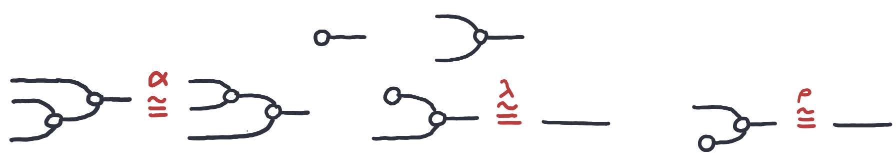
August 7, 2020

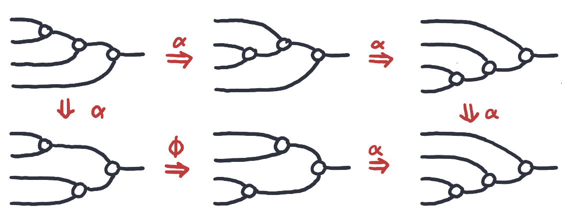
1 Pseudomonoid actions

1.1 Pseudomonoids

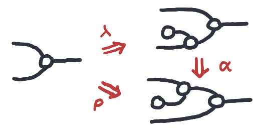
Definition 1.1. Let **K** be a monoidal bicategory. The data for a pseudomonoid is given by a 0-cell A together with two 1-cells representing *unit* and *multiplication*, and 2-cells representing *associativity*, *left unitality* and *right unitality*,



such that the two following diagrams commute. The first diagram corresponds to the pentagon equation of monoidal categories.



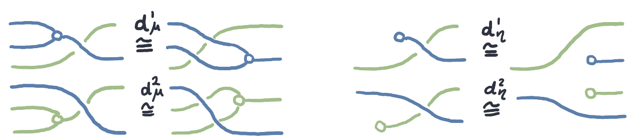
The second diagram corresponds to the triangle equation of monoidal categories. [DS97]



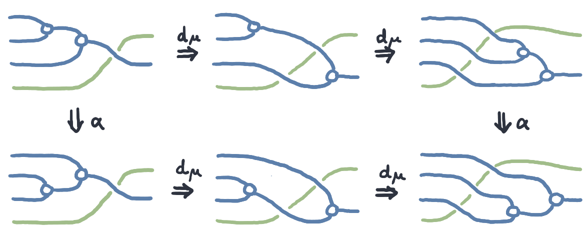
1.2 Distributive laws

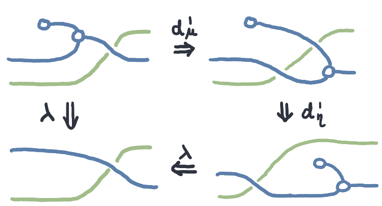
Definition 1.2. A distributive law between two pseudomonoids A and B is given by a 1-cell $d\colon A \otimes B \to B \otimes A$

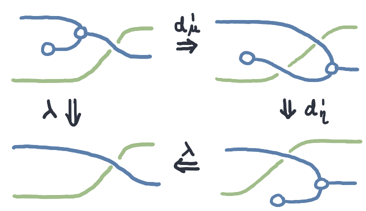
together with transformations



such that the following two sets of diagrams commute. For the pseudomonoid structure of A, we consider a diagram for associativity, left unitality and right unitality.





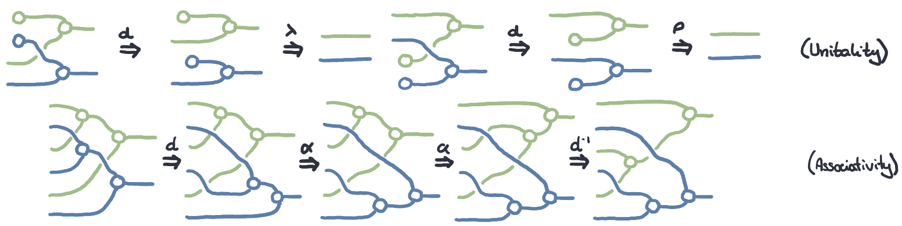


The second set of diagrams concerns the monoidal structure on B instead of A.

1.3 Composition via distributive laws

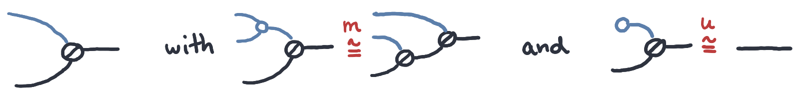
Assume A and B are pseudomonoids with a distributive law. We will show this induces a pseudomonoid structure on $A \otimes B$. Multiplication and unit are given by the following diagrams.

Associativity and unitality are given by the following transformations.

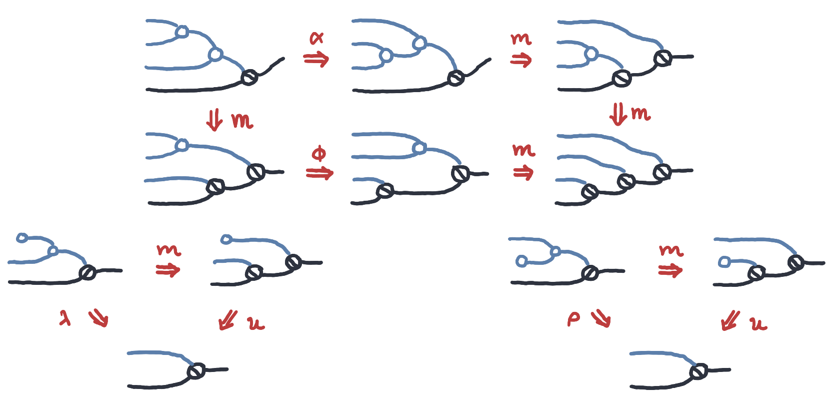


1.4 Pseudomonoid actions

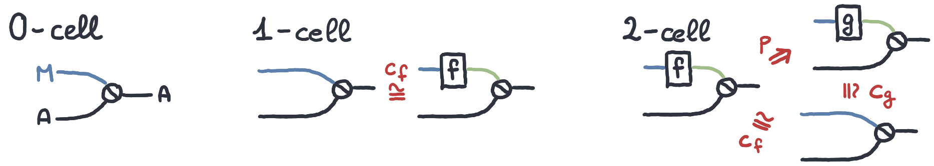
Definition 1.3. A pseudomonoid action of M in A is given by a 1-cell $(\oslash) \colon M \otimes A \to A$ together with 2-cells representing *unitality* and *multiplicativity*.



These cells must be such that the following coherence conditions hold. The first one concerns multiplicativity, the second and third ones concern unitality.



Pseudomonoid actions can be regarded as pseudomonoids on a monoidal bicategory of actions.



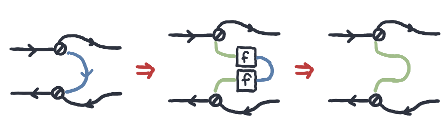
This means we can also consider distributive laws of pseudomonoid actions. We ask the cells determining the distributive law to be in the monoidal bicategory of actions.

The conditions on a distributive law of pseudomonoid actions now follow from the general definition of distributive law of pseudomonoids. A distributive law of monoidal actions endows the composite of the monoidal actions with the same structure.

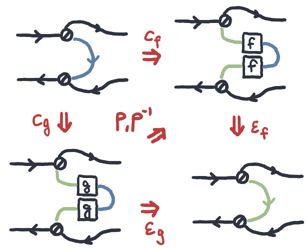
2 Data accessors

We consider a map pseudomonoid, where the pseudomonoid structure is left adjoint to a pseudocomonoid structure. Let $(\oslash) \colon M \otimes A \to A$ be a map action. We define the *accessors* 1-cell to be given by the following diagram.

Every 1-cell between actions translates into a 2-cell in this category.

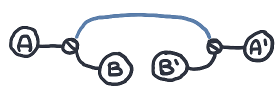


Every 2-cell between actions corresponds to an equality of 2-cells.



And moreover this correspondence is monoidal, taking the monoidal product of actions to the composition of 1-cells. There exists a monoidal pseudofunctor from actions to accessors, considered as a monoidal bicategory with trivial 2-cells.

When the monoidal bicategory is one of profunctors, we call the accessors arising from monoidal actions *optics,* as they particularlize in the usual notion of optics [Mil17, PGW17, BG18, Ril18, CEG+20]. Examples of optics include *lenses* or *prisms.*



A problem that arises in practice from this description of data accessors is how to obtain practical descriptions of composite data accessors that are still optics. We have shown that a distributive law between the monoidal actions describing the optics induces a distributive law between the promonads of accessors themselves, endowing with promonad structure to the composite accessors.

3 Acknowledgements

The author wants to thank Bartosz Milewski for the suggestion of considering generalized accessors without a monoidal action.

References

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