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Wiring Diagrams and Feedback.

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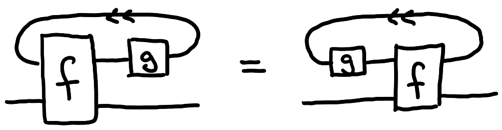
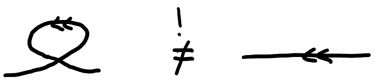
Categories with feedback were defined by Katis, Sabadini and Walters [KSW] as a weakening of the axioms of a traced symmetric monoidal category.



The sliding axiom still holds.



The yanking axiom does not hold.

The free such category over a symmetric monoidal category $\mathbb{C}$ is called $\mathrm{Fbk}\,\mathbb{C}$, and its hom-sets can be explicitly constructed as

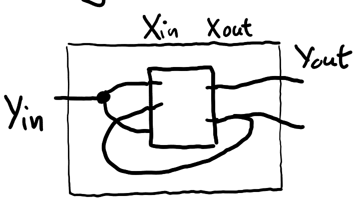
$$\mathrm{Fbk}(A,B) = \int^{M\in\mathbb{C}} \hom(M\otimes A, M\otimes B).^{[1]}$$

[LGRSS]

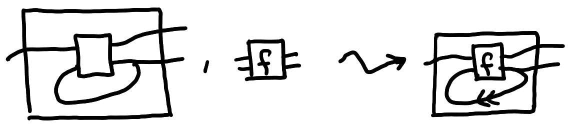
For instance, $\mathrm{Fbk}(\mathrm{Set})$ is a category of Moore transition systems $M\times A \to M\times B$.

Wiring Diagrams

Wiring Diagrams [SSV], say, in the free cartesian category with one object, $(\mathrm{FinSet}^{op}, +)$, can be thought of as "diagrams in FinSet, with feedback and a hole".



Even if the usual way of "filling" these diagrams is different, this suggests that we should be able to "plug" some morphism $X_{in} \to X_{out}$ inside the wiring diagram and get back a morphism $Y_{in} \to Y_{out}$.



After filling that hole, the resulting diagram still uses feedback, so the best we can say is the following proposition.

Proposition. A wiring diagram
$$w\colon (X_{in}, X_{out}) \to (Y_{in}, Y_{out})$$
with $X_{in}, X_{out}, Y_{in}, Y_{out} \in \mathbb{C}$, together with a morphism $f \in \mathbb{C}(X_{in}, X_{out})$, determines a morphism $w \,\square\, f \in \mathrm{Fbk}_{\mathbb{C}}(Y_{in}, Y_{out})$.

Proof. We can prove something more general, there is a function

$$\int^{M} \mathbb{C}(M\otimes Y_{in}, X_{in}) \times \mathbb{C}(X_{out}, M\otimes Y_{out}) \times \mathbb{C}(X_{in}, X_{out}) \longrightarrow \mathbb{C}(Y_{in}, Y_{out}),$$

given by composition along X_{in} and X_{out}.

The formula

$$\int^{M} \mathbb{C}(M\otimes Y_{in}, X_{in}) \times \mathbb{C}(X_{out}, M\otimes Y_{out})$$

is an optic [Ril] and particularizes to wiring diagrams in the cartesian case.

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[1]: The original definition is actually with M ranging over $\mathrm{Core}\,\mathbb{C}$, the isomorphisms of $\mathbb{C}$, but that is not relevant to this discussion.

References.

[SSV]: Spivak, Schultz, Vasilakopoulou.

[KSW]: Katis, Sabadini, Walters.

[LGRSS]: Di Lavore, Gianola, Román, Sabadini, Sobociński.

[Ril]: Riley.