Initialized feedback for nonempty lists

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The category of cofree coalgebras of the nonempty list comonad can be used for modelling causal streams, as done by Vene and Uustalu [UV08]. We can show it is a category with initialized feedback, a variation of the categories with feedback of Katis, Sabadini and Walters [KSW02]. As a consequence, we can use intuitive graphical syntax for it.

Let us call $\mathbf{N}$ to the category of cofree coalgebras of the nonempty list comonad. A morphism $\mathbf{N}(A, B)$ is a family of functions $A^n \to B$ indexed by $n \in \mathbb{N}^+$. The feedback operator will take one such family $f \in \mathbf{N}(A \times X, B \times X)$ to a family of functions $g \in \mathbf{N}(A, B)$, given some $x \in X$.

In order to construct $g_n(a_1, \ldots, a_n)$, we start by inductively constructing a list $x_1, \ldots, x_n$, where $x_0 \coloneqq x$ and $x_{n+1} \coloneqq \pi_2 f_n(a_1, x_1, \ldots, a_n, x_n)$; we then define $g_n(a_1, \ldots, a_n) \coloneqq \pi_1 f_n(a_1, x_1, \ldots, a_n, x_n)$.

References

[KSW02] Piergiulio Katis, Nicoletta Sabadini, and Robert F. C. Walters. Feedback, trace and fixed-point semantics. *ITA*, 36(2):181–194, 2002.

[UV08] Tarmo Uustalu and Varmo Vene. Comonadic notions of computation. In Jirí Adámek and Clemens Kupke, editors, *Proceedings of the Ninth Workshop on Coalgebraic Methods in Computer Science, CMCS 2008, Budapest, Hungary, April 4-6, 2008*, volume 203 of *Electronic Notes in Theoretical Computer Science*, pages 263–284. Elsevier, 2008.