

Double Rel and Peircean Negation

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Let us define a category **DRel** where objects are sets, morphisms $(P, N): A \rightarrow B$ are pairs of relations which we call the “positive” and the “negative”. Two morphisms $(P, N): A \rightarrow B$ and $(P', N'): B \rightarrow C$ compose into $(P \diamond P', N \blacklozenge N')$ as follows

$$\begin{aligned}(P \diamond P')(a, c) &= \exists b \in B. P(a, b) \wedge P'(b, c), \\ (N \blacklozenge N')(a, c) &= \forall b \in B. N(a, b) \vee N'(b, c).\end{aligned}$$

This is a locally posetal category where $(P, N) \leq (P', N')$ when $P \leq P'$ and $N' \leq N$.

There is a functor $\mathbf{Rel} \rightarrow \mathbf{DRel}$ that sends a relation $R: A \rightarrow B$ to the pair of relations $(R, \neg R)$. There is a negation operator on 2-cells that exchanges the positive and negative sides. There are two projections both on the positive and the negative.

This construction looks similar to the Chu construction [Shu18], it can be applied to the problem described by [HS20].

References

- [HS20] Nathan Haydon and Paweł Sobociński. Compositional diagrammatic first-order logic. *In Peer Review*, 2020.
- [Shu18] Michael Shulman. Linear logic for constructive mathematics, 2018.