The Vlasov equation

if they are at about the same position and share about the same velocity. Hence, we define $f(\boldsymbol{x}, \boldsymbol{v}, t)$ as the particle distribution function, which represents the number density of particles found near the point $(\boldsymbol{x}, \boldsymbol{v})$ in phase space. Specifically, the number of particles located within intervals d^3x about $\boldsymbol{x}$ and d^3v about $\boldsymbol{v}$ is given by

$$dN = f(\boldsymbol{x}, \boldsymbol{v}, t) d^3x d^3v \ . \tag{2}$$

$\boldsymbol{X} \equiv (\boldsymbol{x}, \boldsymbol{v})$ is phase space coordinates and $\dot{\boldsymbol{X}} = (\boldsymbol{v}, \boldsymbol{a})$

$$\dot{\boldsymbol{x}}_i = \boldsymbol{v}_i \ , \quad \dot{\boldsymbol{v}}_i = \boldsymbol{a}_i = \frac{q}{m}\left(\boldsymbol{E} + \frac{\boldsymbol{v}_i \times \boldsymbol{B}}{c}\right)$$

$$\frac{\partial f}{\partial t} + \nabla \cdot (f\boldsymbol{v}) + \nabla_{\boldsymbol{v}} \cdot (f\boldsymbol{a}) = 0$$

$$\frac{Df}{Dt} \equiv \left(\frac{\partial}{\partial t} + \dot{\boldsymbol{X}}\frac{\partial}{\partial \boldsymbol{X}}\right) f = 0$$

$Df/Dt = 0$ is known as *Liouville's theorem*. It states that the distribution function f is constant along particle trajectories in phase space (when $\nabla_{\boldsymbol{v}} \cdot \boldsymbol{a} = 0$).

The Vlasov-Maxwell equations

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \nabla f + \frac{q}{m}\left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c} \times \boldsymbol{B}\right) \cdot \nabla_{\boldsymbol{v}} f = \left(\frac{\partial f}{\partial t}\right)_{\mathrm{coll}}$$

$$\frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = \nabla \times \boldsymbol{B} - \frac{4\pi}{c}\boldsymbol{j} \ ,$$

$$\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E} \ ,$$

$$\nabla \cdot \boldsymbol{E} = 4\pi\rho \ ,$$

$$\nabla \cdot \boldsymbol{B} = 0 \ .$$

$$\rho = \sum_{\mathrm{species}} q \int f(\boldsymbol{x}, \boldsymbol{v}, t) d^3 v \ ,$$

$$\boldsymbol{j} = \sum_{\mathrm{species}} q \int \boldsymbol{v} f(\boldsymbol{x}, \boldsymbol{v}, t) d^3 v \ .$$