

# 1. Solve for the fields

- In electromagnetic PIC codes, only two equations need to be solved.

- The other two are satisfied as initial conditions, and they continue to be satisfied for appropriate choices of the numerical scheme.

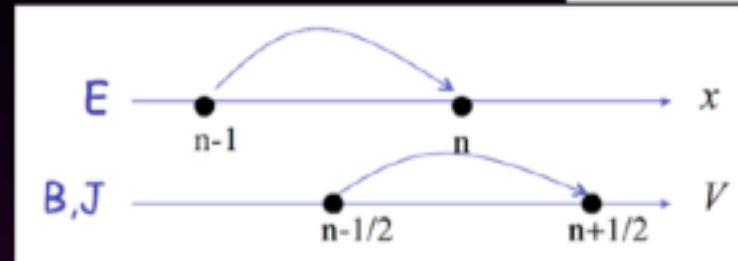
$$\begin{aligned}\partial_t B &= -\nabla \times E, \\ \partial_t E &= \nabla \times B - J.\end{aligned}$$

$$\begin{aligned}\nabla \cdot E &= \rho, \\ \nabla \cdot B &= 0.\end{aligned}$$

STAGGERING in time (leapfrog):

- second-order accurate in time

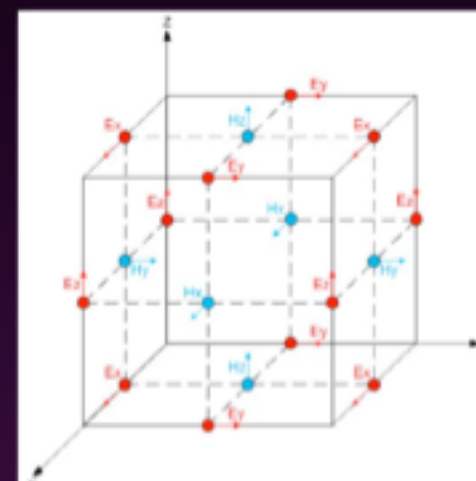
$$\begin{aligned}E^{n+1/2} &= E^{n-1/2} + \Delta t [c(\nabla \times B^n) - 4\pi J^n] \\ B^{n+1} &= B^n - c\Delta t \nabla \times E^{n+1/2}.\end{aligned}$$



STAGGERING in space (Yee's mesh):

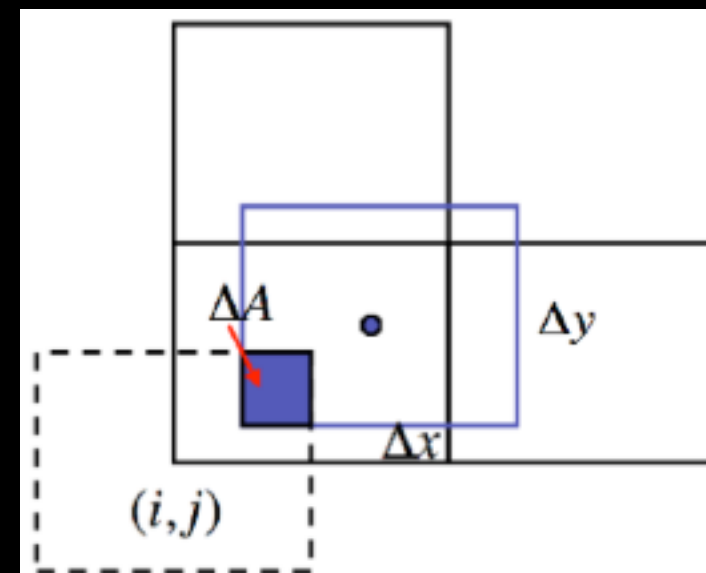
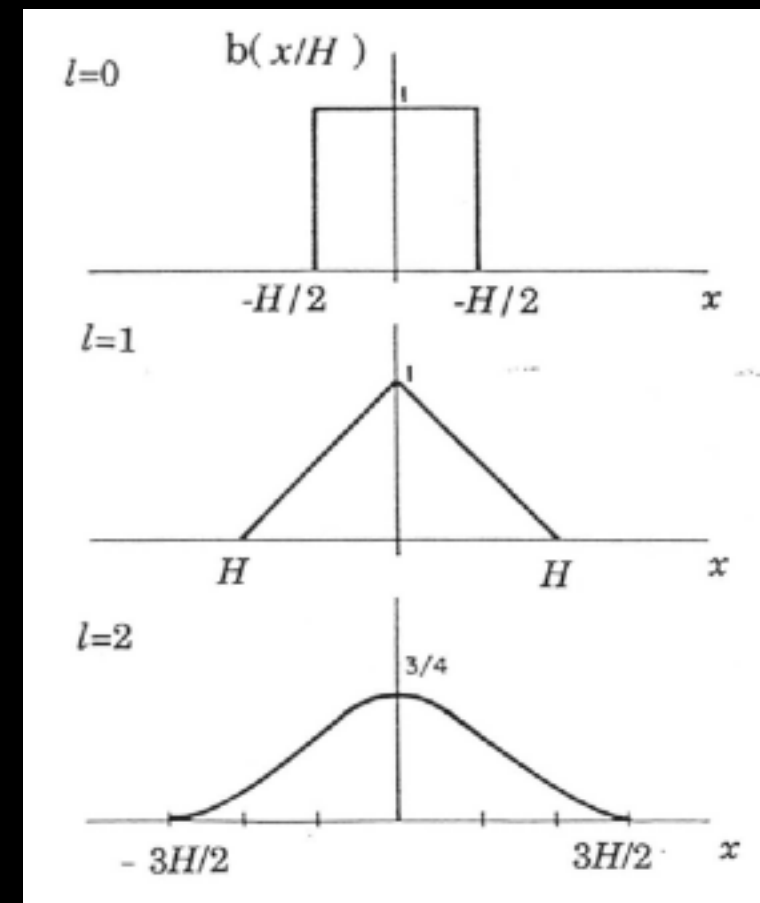
- electric fields on cell edges, magnetic fields on cell faces
- second-order accurate in space
- maintains divergence-free B

$$\begin{aligned}\partial_t B &= -\nabla \times E, \\ \partial_t E &= \nabla \times B - J\end{aligned}$$



## 2. Interpolate fields to particle positions

- The fields obtained from Maxwell's equations are determined only at the grid points, they need to be interpolated to the particle positions.
- The interpolation is done by assuming a particle shape function.
- The shape function needs to be:
  1. isotropic
  2. zero outside some range
  3. higher order B-splines are computationally more expensive, but more accurate and less "collisional"



$$E(\vec{x}_k) = \sum_{i,j} E_{ij} S(\vec{x}_k - \vec{X}_{ij})$$

$$S(\vec{x}_k - \vec{X}_{ij}) = \frac{\Delta A}{\Delta x \Delta y}$$

$$\sum_{i,j} S(\vec{x}_k - \vec{X}_{ij}) = 1$$