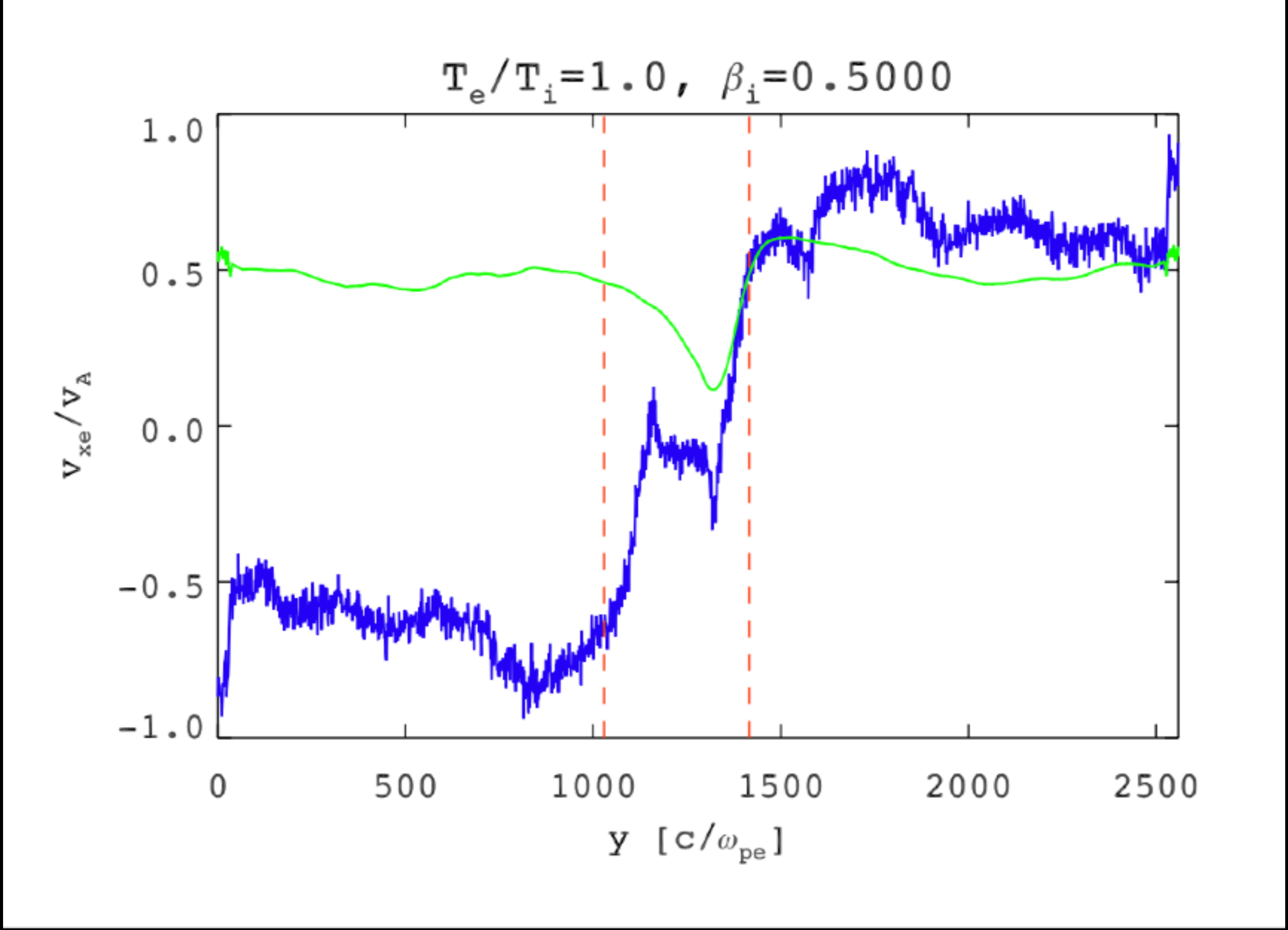
Strange-looking point from plot of L(beta)



E.o.S. derivation

$\boxed{\text{What is our equation of state if the adiabatic index is not constant?}}$

$$dS = \frac{dQ}{\Theta},\ dQ = dH - V\,dp, \Theta = \frac{p}{\rho}$$

$$\Rightarrow dS = \frac{dH}{\Theta} - V\frac{dp}{\Theta} = \frac{dH}{\Theta} - \frac{N}{p}dp$$

$$\boxed{dS = \frac{dh}{\Theta} - d\,\log p}$$

Now integrate, using equation for $h(\Theta)$:

$$h = \frac{5}{2}\Theta + \sqrt{\frac{9}{4}\Theta^2 + 1}$$

$$\Rightarrow \int dS = \int \frac{1}{\Theta}\left(\frac{5}{2} + \left(\frac{9}{4}\Theta^2 + 1\right)^{-1/2}\frac{9}{4}\Theta\right)d\Theta - \log p$$

$$S = \frac{5}{2}\log\Theta + \frac{3}{2}\sinh^{-1}\left(\frac{3}{2}\Theta\right) - \log p$$

$$= \frac{5}{2}\log\Theta + \frac{3}{2}\log\left(\frac{3}{2}\Theta + \sqrt{\left(\frac{3}{2}\Theta\right)^2 + 1}\right) - \log p$$

$$= \frac{3}{2}\left(\log\Theta + \log\left(h - \Theta\right)\right) + \log\left(\frac{\Theta}{p}\right)$$

$$= \frac{3}{2}\log\left(\frac{\Theta^{5/3}}{p^{2/3}}(h - \Theta)\right)$$

$$= \frac{3}{2}\log\left(\frac{p}{\rho^{5/3}}(h - \Theta)\right)$$

$$\boxed{const = \frac{p}{\rho^{5/3}}\left(\frac{3}{2}\Theta + \sqrt{\frac{9}{4}\Theta^2 + 1}\right)}$$

This provides the correct limiting values. For $\Theta \to 0$,

$$const = \frac{p}{\rho^{5/3}}$$

and for $\Theta \gg 1 \Leftrightarrow p \gg \rho$,

$$const \simeq \frac{p}{\rho^{5/3}}3\Theta$$

$$\Rightarrow const' = \frac{p^2}{\rho^{8/3}}$$

$$\Rightarrow const'' = \frac{p}{\rho^{4/3}}$$

The adiabatic index is not the same in the upstream as compared to outflow region. To make a meaningful comparison of the compressive heating, we should compute *const* upstream, then use the variable equation of state (boxed above) to compute the predicted value *const* for given Θ, p, ρ in the outflow. Discrepancy between the predicted and actual values should then be accounted for by 'actual' heating.

The equation we used for specific enthalpy comes from Taub inequality, taking the equals sign:

$$(h - \Theta)(h - 4\Theta) \geq 0$$

$$\Rightarrow h^2 - h5\Theta + 4\Theta^2 - 1 \geq 0$$

$$\Rightarrow h = \frac{5}{2} + \sqrt{\frac{9}{4}\Theta^2 + 1}$$