

E.o.S. derivation

What is our equation of state if the adiabatic index is not constant?

$$dS = \frac{dQ}{\Theta}, dQ = dH - V dp, \Theta = \frac{p}{\rho}$$

$$\Rightarrow dS = \frac{dH}{\Theta} - V \frac{dp}{\Theta} = \frac{dH}{\Theta} - \frac{N}{p} dp$$

$$dS = \frac{dh}{\Theta} - d \log p$$

Now integrate, using equation for $h(\Theta)$:

$$h = \frac{5}{2}\Theta + \sqrt{\frac{9}{4}\Theta^2 + 1}$$

$$\Rightarrow \int dS = \int \frac{1}{\Theta} \left(\frac{5}{2} + \left(\frac{9}{4}\Theta^2 + 1 \right)^{-1/2} \frac{9}{4}\Theta \right) d\Theta - \log p$$

$$S = \frac{5}{2} \log \Theta + \frac{3}{2} \sinh^{-1} \left(\frac{3}{2}\Theta \right) - \log p$$

$$= \frac{5}{2} \log \Theta + \frac{3}{2} \log \left(\frac{3}{2}\Theta + \sqrt{\left(\frac{3}{2}\Theta \right)^2 + 1} \right) - \log p$$

$$= \frac{3}{2} (\log \Theta + \log (h - \Theta)) + \log \left(\frac{\Theta}{p} \right)$$

$$= \frac{3}{2} \log \left(\frac{\Theta^{5/3}}{p^{2/3}} (h - \Theta) \right)$$

$$= \frac{3}{2} \log \left(\frac{p}{\rho^{5/3}} (h - \Theta) \right)$$

$$const = \frac{p}{\rho^{5/3}} \left(\frac{3}{2}\Theta + \sqrt{\frac{9}{4}\Theta^2 + 1} \right)$$

This provides the correct limiting values. For $\Theta \rightarrow 0$,

$$const = \frac{p}{\rho^{5/3}}$$

and for $\Theta \gg 1 \Leftrightarrow p \gg \rho$,

$$const \simeq \frac{p}{\rho^{5/3}} 3\Theta$$

$$\Rightarrow const' = \frac{p^2}{\rho^{8/3}}$$

$$\Rightarrow const'' = \frac{p}{\rho^{4/3}}$$

The adiabatic index is not the same in the upstream as compared to out-flow region. To make a meaningful comparison of the compressive heating, we should compute *const* upstream, then use the variable equation of state (boxed above) to compute the predicted value *const* for given Θ, p, ρ in the outflow. Discrepancy between the predicted and actual values should then be accounted for by 'actual' heating.

The equation we used for specific enthalpy comes from Taub inequality, taking the equals sign:

$$(h - \Theta)(h - 4\Theta) \geq 0$$

$$\Rightarrow h^2 - h5\Theta + 4\Theta^2 - 1 \geq 0$$

$$\Rightarrow h = \frac{5}{2} + \sqrt{\frac{9}{4}\Theta^2 + 1}$$

Sgr A* radiation spectrum

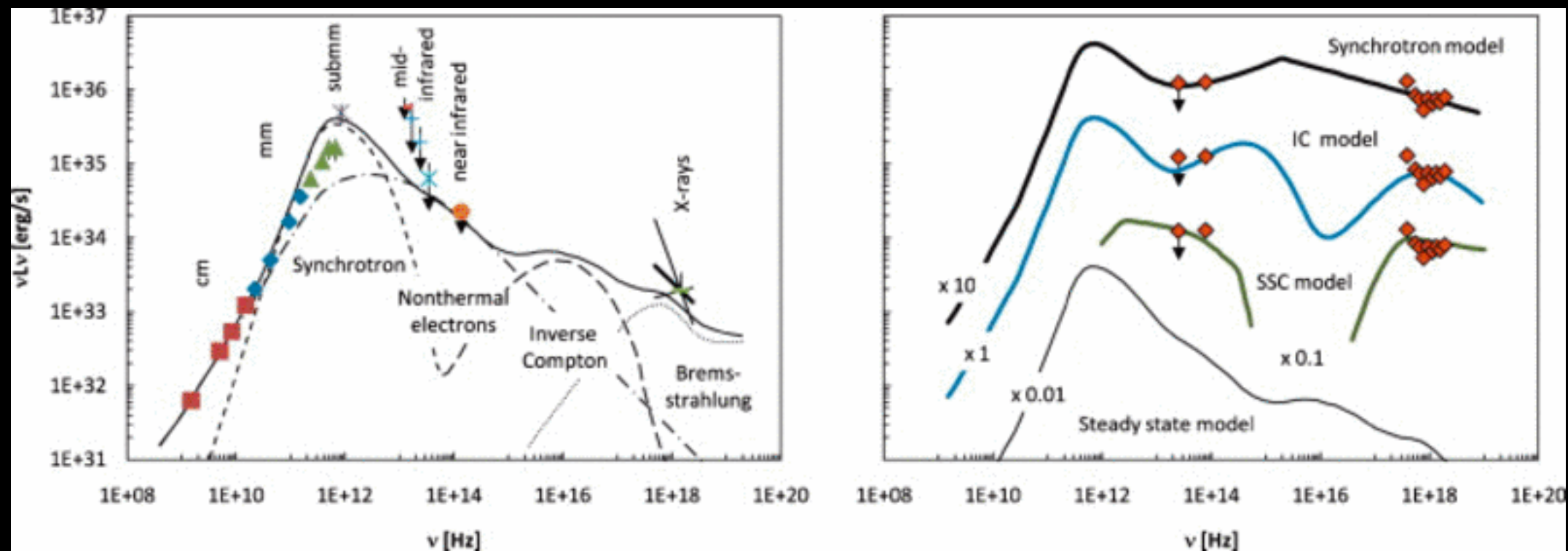


Figure 30(Color) Spectral energy distribution of Sgr A*. All numbers are given for a distance of 8.3kpc of the Galactic Center and are dereddened for interstellar absorption (infrared and x rays) and scattering (x rays). Left: steady state. The Sgr A* radio spectrum follows roughly a power law $\nu L_\nu \sim \nu^{4/3}$. The observed peak flux at submillimeter wavelengths is about $5 \times 10^{35} \text{ erg/s}$. The spectrum then steeply drops toward infrared wavelengths down to less than the detection limit of about $2 \times 10^{34} \text{ erg/s}$ at $2 \mu\text{m}$. The only other unambiguous detection of Sgr A* in its steady state is at x rays with energies from 2–10keV with a flux of about $2 \times 10^{33} \text{ erg/s}$. The figure shows a compilation of data (with increasing frequency) from ■ (560), ♦ (150), ▲ (565), × (487), - (105), + (186), × (475), ● (243), and – (34). Overplotted is a model of the quiescent emission [adapted from 541]: the radio spectrum is well described by synchrotron emission of thermal electrons (short-dashed line). The flattening of the radio spectrum at low frequency is modeled by the additional emission from a nonthermal power-law distribution of electrons, which carry about 1.5% of the total thermal energy (dash-dotted line). The quiescent x-ray emission arises from thermal bremsstrahlung from the outer parts of the accretion flow (dotted line). The secondary maximum (long-dashed line) at frequencies of about 10^{16} Hz is the result of the inverse Compton upscattering of the synchrotron spectrum by the thermal electrons. Right: SED during a simultaneous x ray and infrared flare: while the total