Electron Heating in Quasi-Relativistic Magnetic Reconnection

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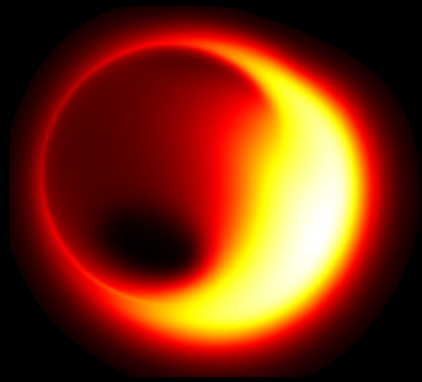
Black holes are highly energetic

- Highly energetic
  - Quasars
  - GRBs
- EHT
  - Image black holes in near future

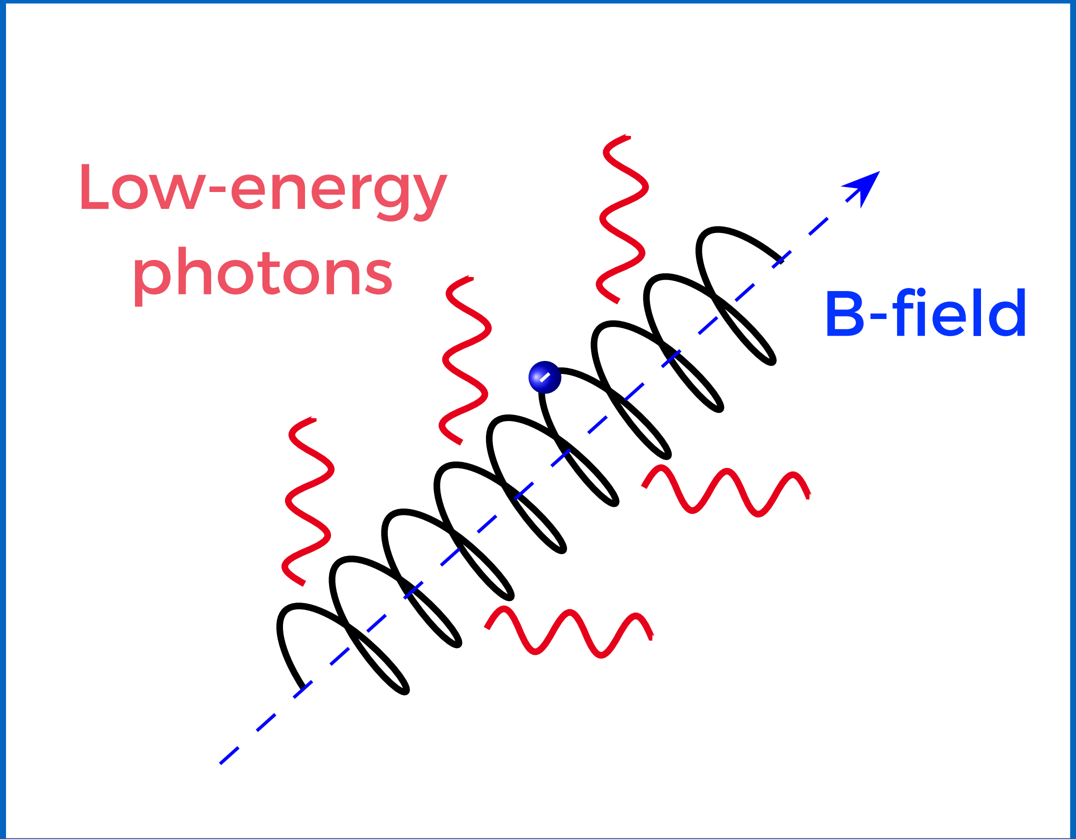


Credit: A. E. Broderick

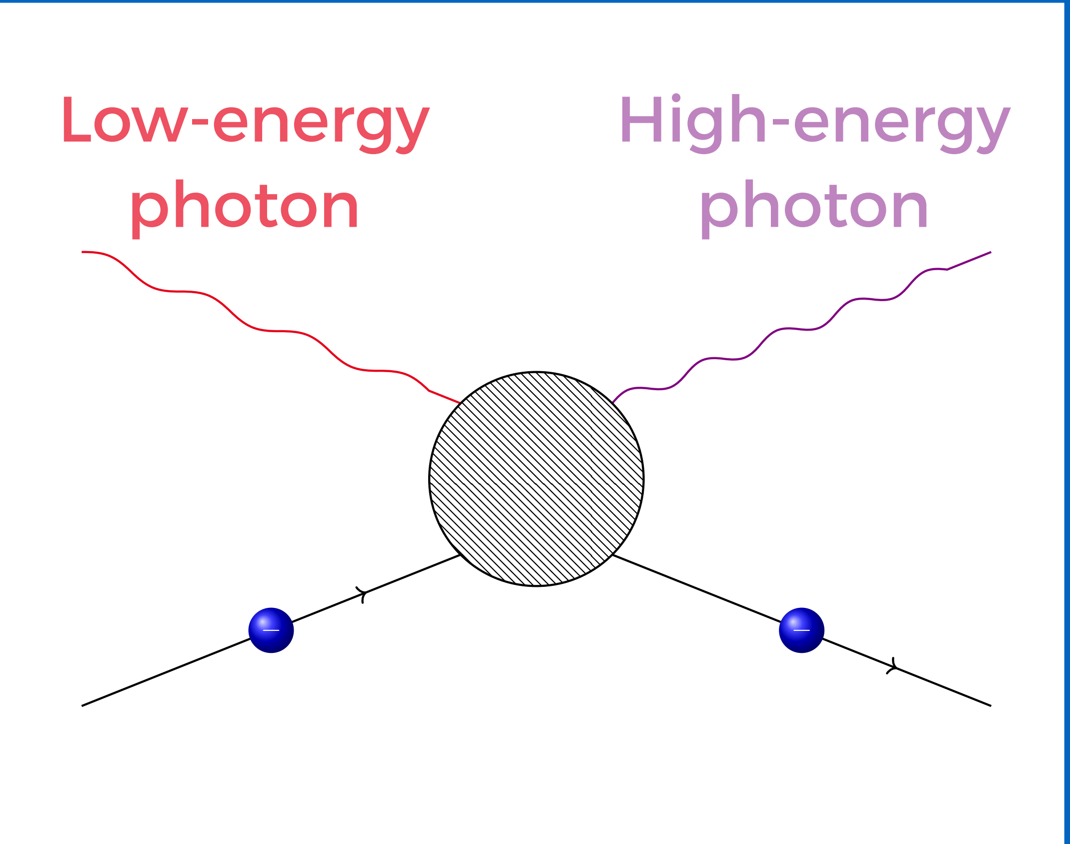
Radiation from BH is produced by electrons...

Energetic electrons can produce radiation through various processes:

Synchrotron



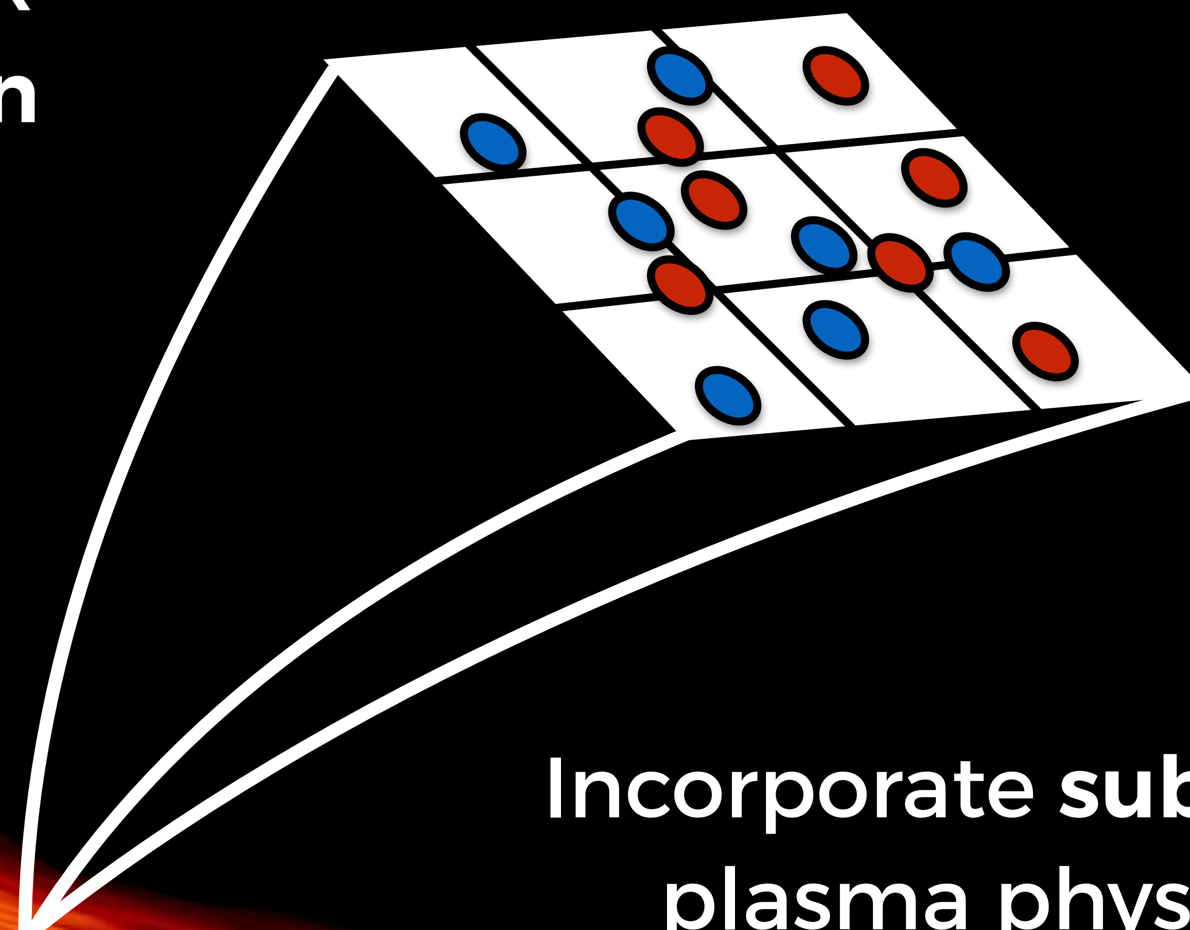
Inverse Compton



Electrons are responsible for observed radiation profiles, yet the production of energetic electrons is **unresolved** in global simulations

…but global simulations don’t resolve microphysics

Global fluid simulations of collisionless accretion disk radiation are **potentially in great error**



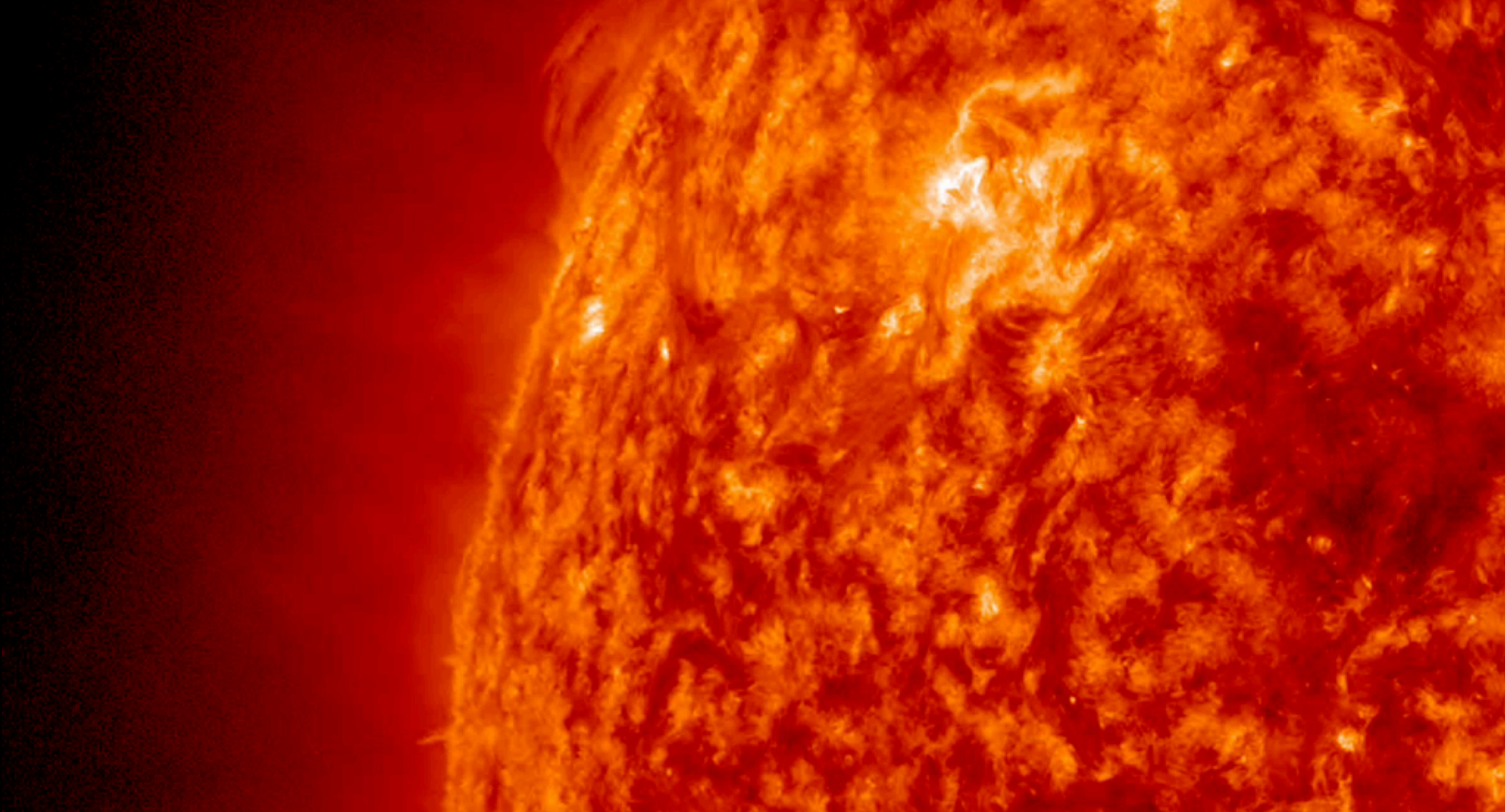
Incorporate **sub-grid** plasma physics

- Reconnection
- Shocks

NASA GSFC/ J. Schnittman

Reconnection can energize particles

- Rearrangement of magnetic field lines
- Magnetic energy → particle kinetic energy



NASA SDO

Reconnection can energize particles

- Rearrangement of magnetic field lines
- Magnetic energy → particle kinetic energy

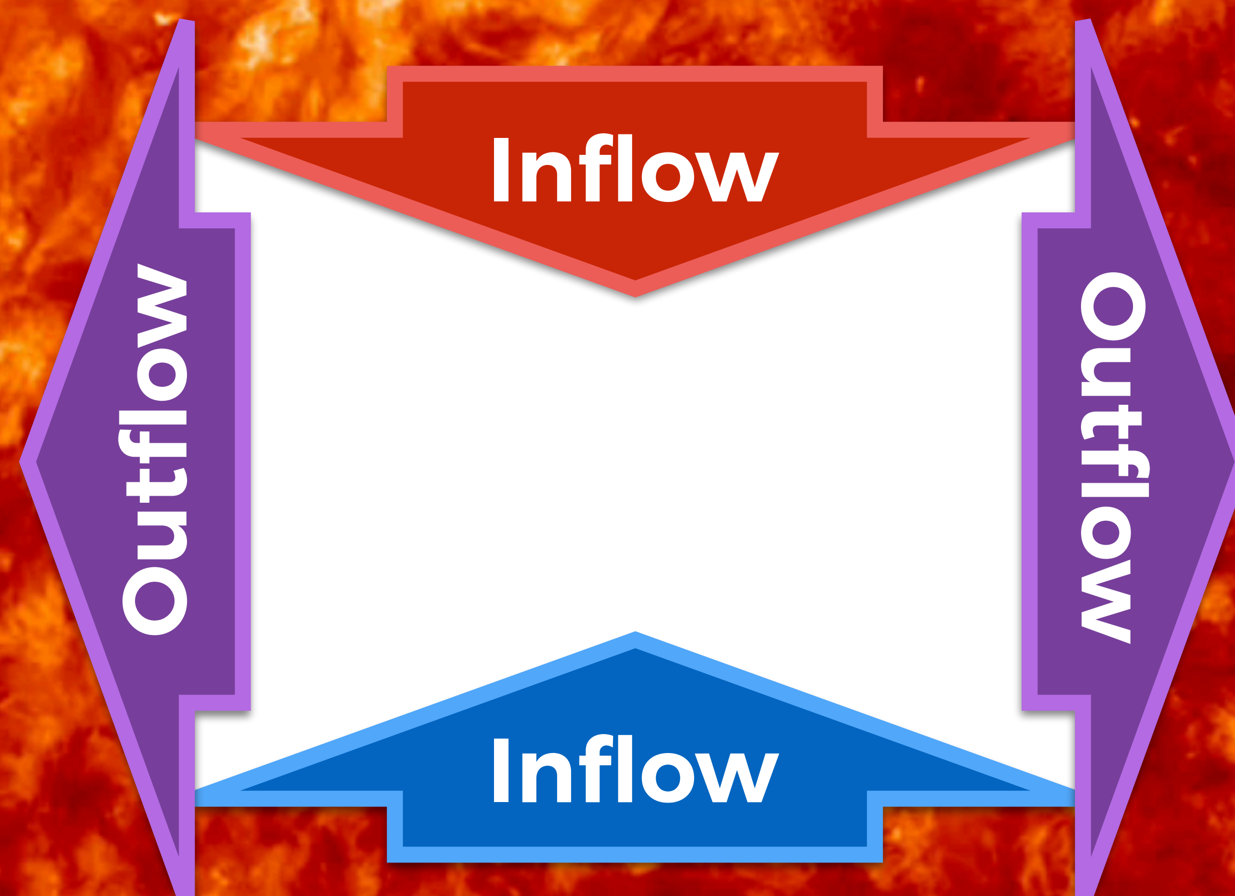
Happens many places:

Chromosphere

Magnetosphere

Black hole coronae

NASA GSFC/ J. Schnittman



NASA SDO

Parameters: physical and computational

beta (of the ions)

$$\beta_i = \frac{n_i k_B T_i}{B^2/(8\pi)} = \frac{\text{thermal pressure}}{\text{magnetic pressure}}$$

sigma (of the ions)

$$\sigma_i = \frac{B^2/(4\pi)}{n_i m_i c^2} = \frac{\text{magnetic pressure}\,(\times 2)}{\text{rest-mass energy density}}$$

temperature ratio

$$\frac{T_e}{T_i} = \frac{\text{electron temperature}}{\text{ion temperature}}$$

Computational

d_{stripe}

dv_{stripe}

n_{stripe}

m_y

n_{times}

m_i/m_e

ppc

c/ω_{pe}

Full relativistic definition of sigma includes enthalpy

sigma, including enthalpy

$$\sigma_w = \frac{\frac{m_i}{m_e}+1}{\underbrace{\frac{m_i}{m_e}\left(1+\frac{\hat{\gamma}_i}{\hat{\gamma}_i-1}\frac{k_B T_i}{m_i c^2}\right)+\left(1+\frac{\hat{\gamma}_e}{\hat{\gamma}_e-1}\frac{k_B T_e}{m_e c^2}\right)}_{\approx 1 \text{ for a cold plasma}}}\underbrace{\frac{B^2}{4\pi(n_i m_i + n_e m_e)c^2}}_{\approx \frac{B^2}{4\pi n_i m_i c^2} \equiv \sigma_i}$$

For a high-beta (thermally 'hot') plasma, the contribution from the thermal pressure is non-negligible

One more important definition: Alfvén velocity, which describes the speed of magnetic waves

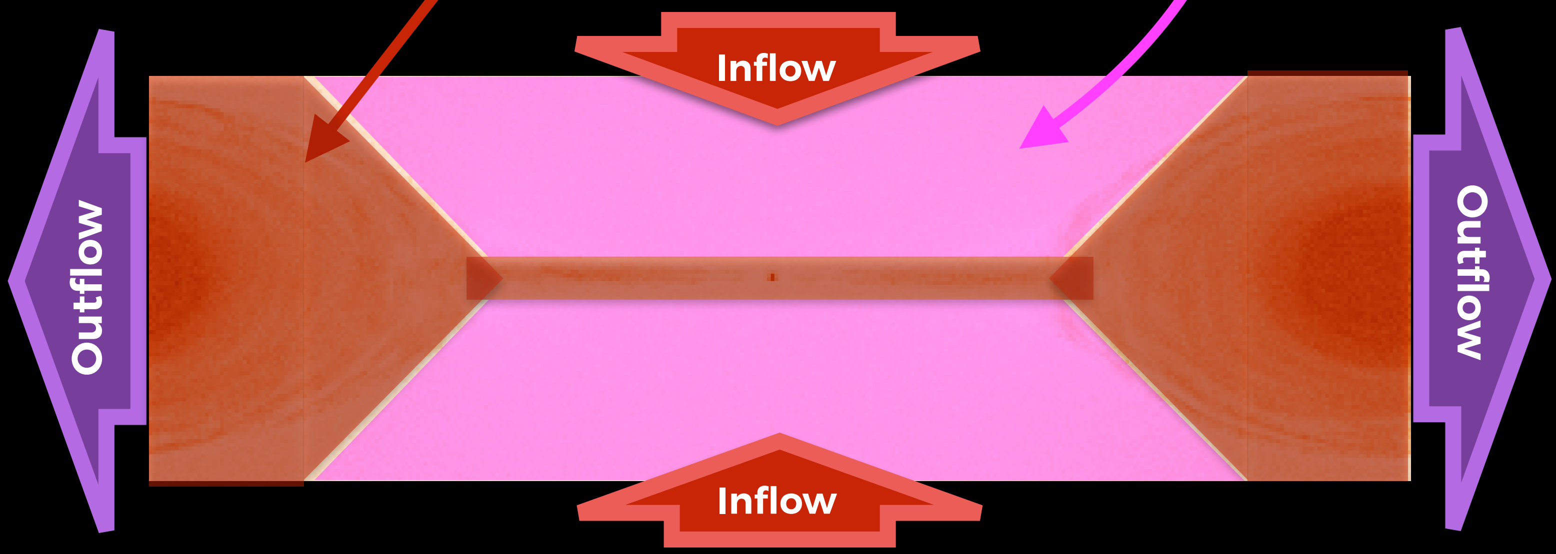
Alfvén velocity

$$\frac{v_A}{c} = \sqrt{\frac{\sigma_w}{1+\sigma_w}}$$

Characterization of heating

A useful number we can extract from each simulation is the following dimensionless ratio:

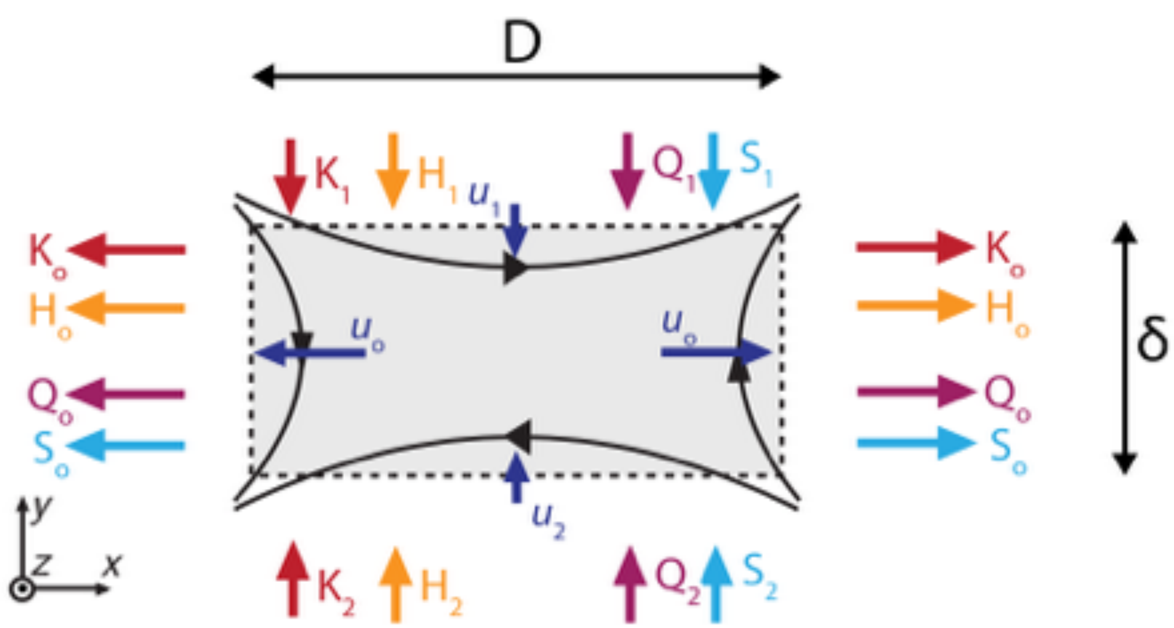
$$M_{Te} = \frac{k_B T_{e,out} - k_B T_{e,in}}{B^2/(4\pi n)}$$



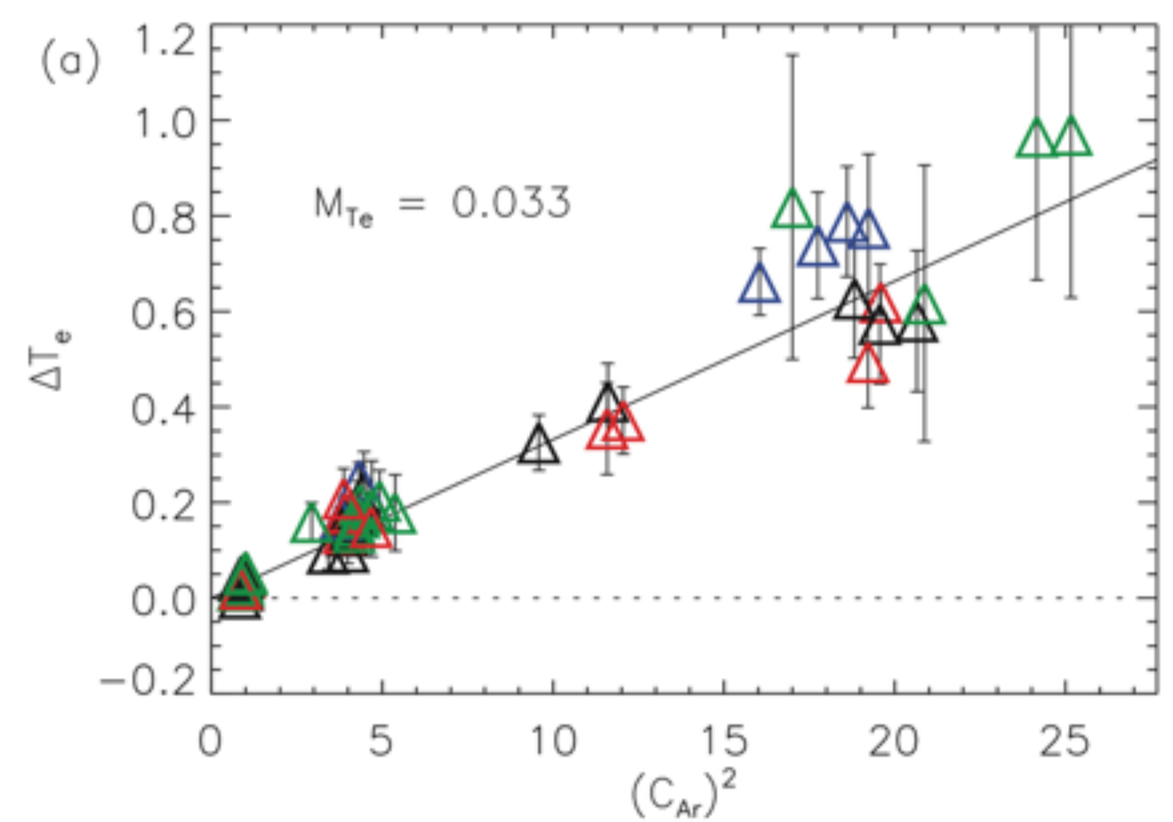
This is the ratio of increase in temperature to magnetic energy available for dissipation. It can be thought of as the 'efficiency' of reconnection.

How much are electrons heated during reconnection?

PIC simulations and observations of magnetic reconnection suggest that a constant fraction of inflowing magnetic energy is given to electrons



$$M_{Te} = \frac{k_B T_{e,out} - k_B T_{e,in}}{m_i v_A^2}$$

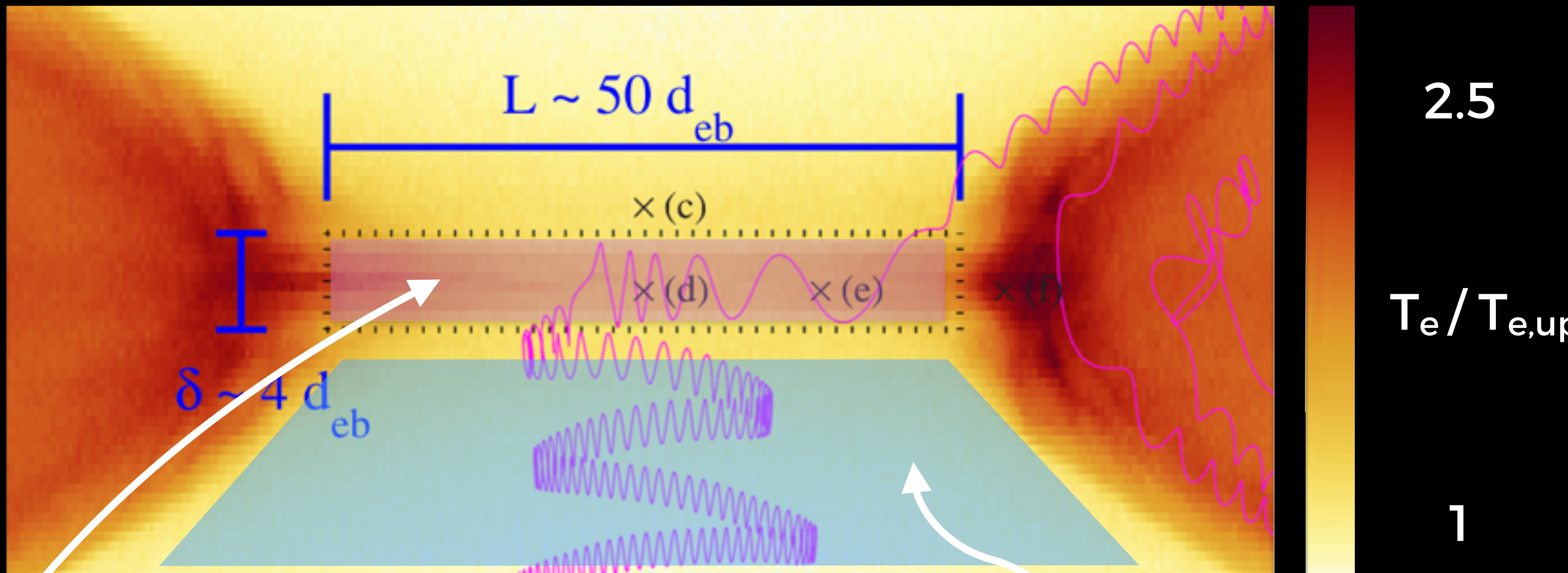


(Drake et al., 2014)

This fraction M_{Te} is remarkably independent of plasma parameters in the inflowing region

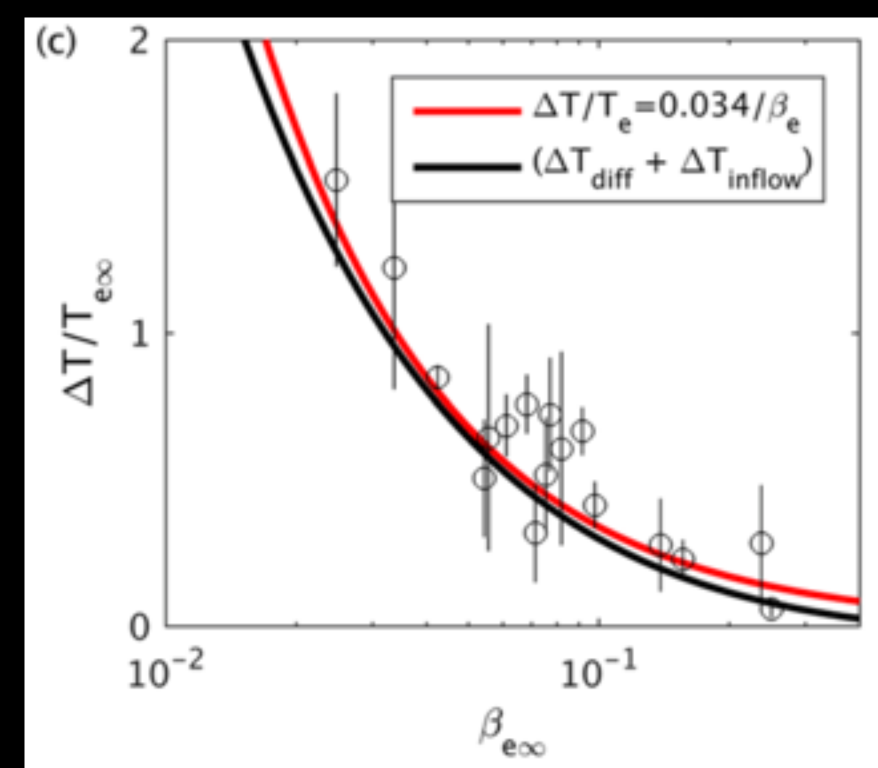
A model for the heating mechanism exists

(Le et al., 2016)



$$\boxed{\frac{\Delta T_{e,tot}}{T_{e,up}} = \frac{\Delta T_{e,in}}{T_{e,up}} + \frac{\Delta T_{e,diff}}{T_{e,up}} \simeq \frac{0.034}{\beta_{e,up}}}$$

The model (middle terms) agrees with the empircal scaling (last term)



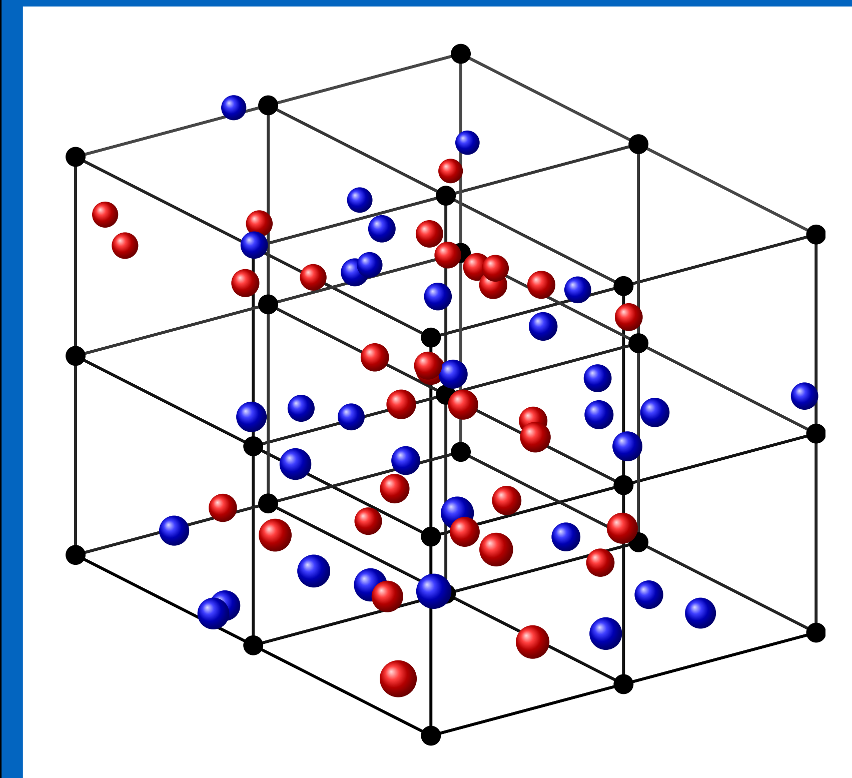
(Le et al., 2016)

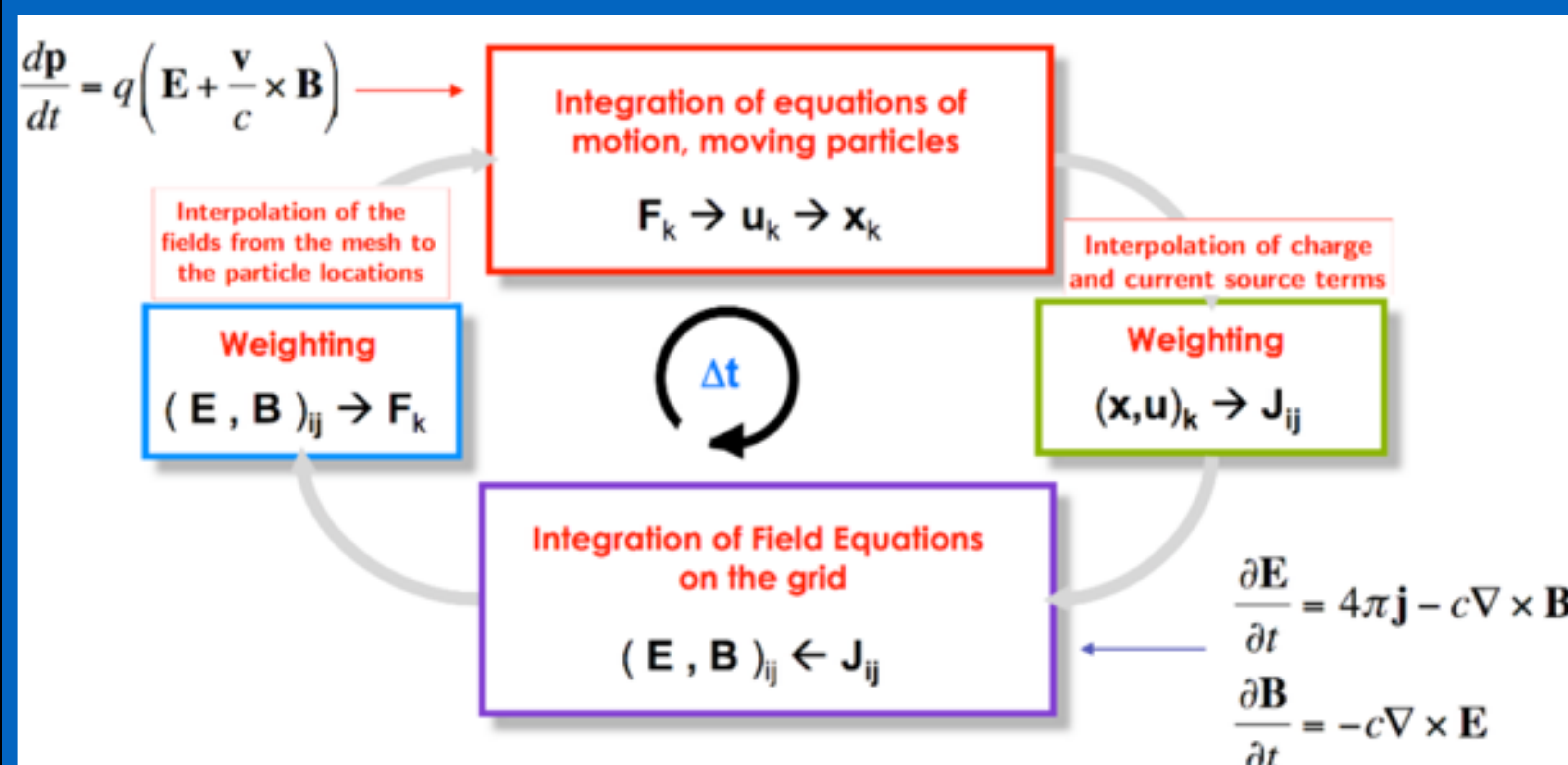
The quasi-relativistic regime is relatively unexplored

Parameters

σ_w	β_i	T_e/T_i	$\Delta\gamma_i$
0.1	0.0078125	0.1	0.000406687
0.1	0.0078125	0.3	0.000406767
0.1	0.0078125	1	0.000407051
0.1	0.03125	0.1	0.00163203
0.1	0.03125	0.3	0.00163334
0.1	0.03125	1	0.00163818
0.1	0.125	0.1	0.00661497
0.1	0.125	0.3	0.00663803
0.1	0.125	1	0.00673223
0.1	0.5	0.1	0.0280133
0.1	0.5	0.3	0.0285164
0.1	0.5	1	0.0308345
0.1	2.	0.1	0.155222
0.1	2.	0.3	0.178254
0.1	2.	1	0.394336
0.3	0.0078125	0.1	0.0012227
0.3	0.0078125	0.3	0.00122343
0.3	0.0078125	1	0.0012261
0.3	0.03125	0.1	0.00493921
0.3	0.03125	0.3	0.00495179
0.3	0.03125	1	0.00500182
0.3	0.125	0.1	0.0205981
0.3	0.125	0.3	0.0208554
0.3	0.125	1	0.022019
0.3	0.5	0.1	0.102084
0.3	0.5	0.3	0.110952
0.3	0.5	1	0.163062

Use PiC simulation. Choose parameters so that inflow/outflow electrons are **moderately** relativistic





The Vlasov equation

$$\frac{Df}{Dt} \equiv \frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \dot{\mathbf{v}} \cdot \nabla_{\mathbf{v}} f = 0$$

- This is a statement of Liouville's theorem
- Phase-space distribution function f is constant along particle trajectories in phase space
- This is abstract, so let's inject some physics, i.e. Lorentz force law

$$\frac{d\mathbf{p}}{dt} = q\left(\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B}\right)$$

The Vlasov-Maxwell equations

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{q}{m}\left(\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B}\right) \cdot \nabla_{\mathbf{v}} f = 0$$

$$\frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{B} - \frac{4\pi}{c}\mathbf{j}$$

$$\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\nabla \cdot \mathbf{E} = 4\pi\rho$$

$$\nabla \cdot \mathbf{B} = 0$$

Source terms are calculated from moments of the distribution function:

$$\rho = \sum_{species} q \int f(\mathbf{x}, \mathbf{v}, t) d^3 v$$

$$\mathbf{j} = \sum_{species} q \int \mathbf{v} f(\mathbf{x}, \mathbf{v}, t) d^3 v$$

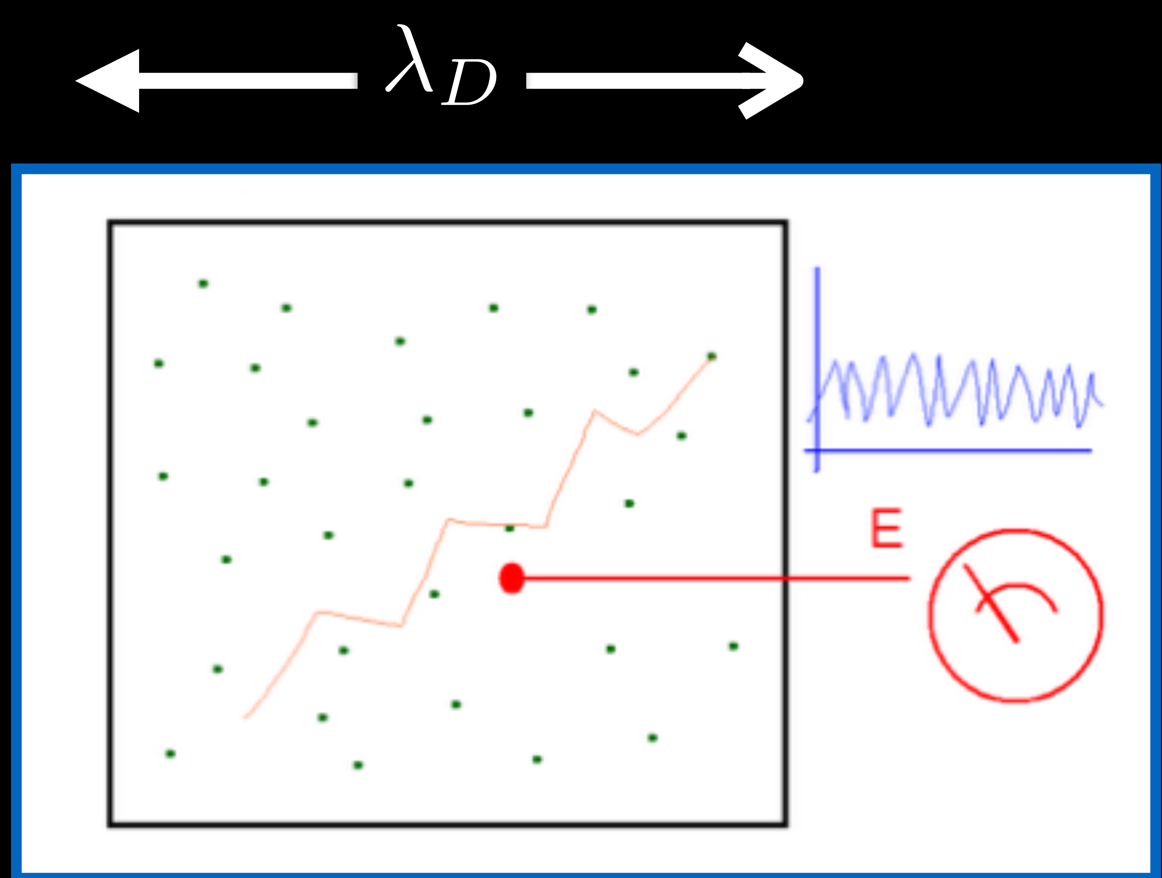
Rather than brute-force solve the 6+1dimensional equation, PIC simulation uses 'super-particles' as Lagrangian tracers of phase space

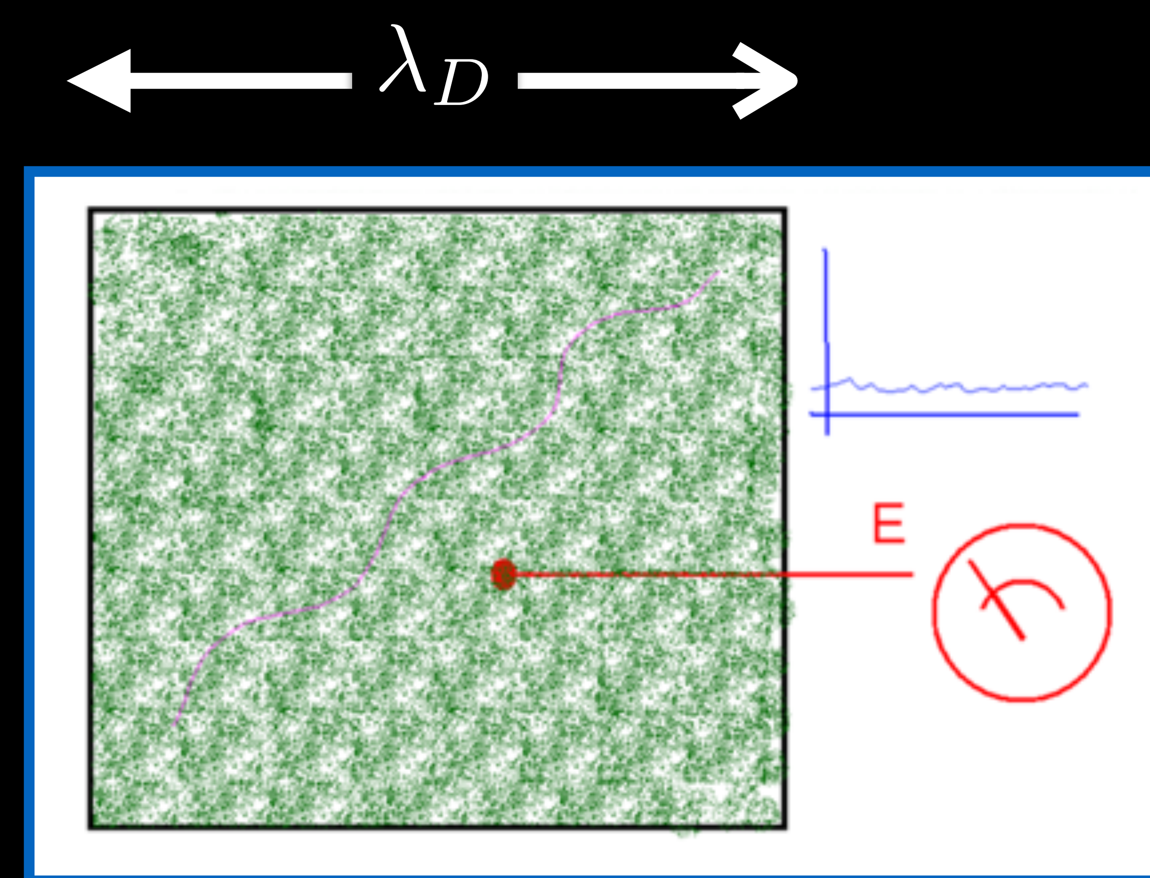
Collisionless plasma

Collisionless plasma is characterized by **collective behavior**

Plasma Parameter

$$\Lambda = \frac{E_{th}}{E_{pot}} = \frac{a k_B T}{q^2}$$

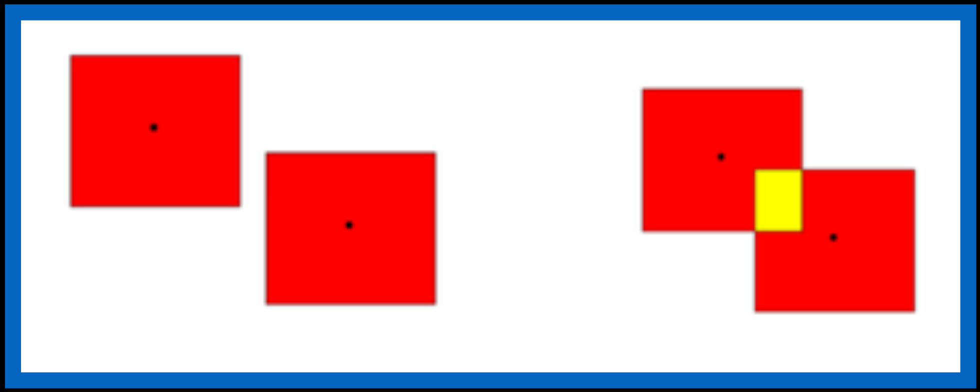


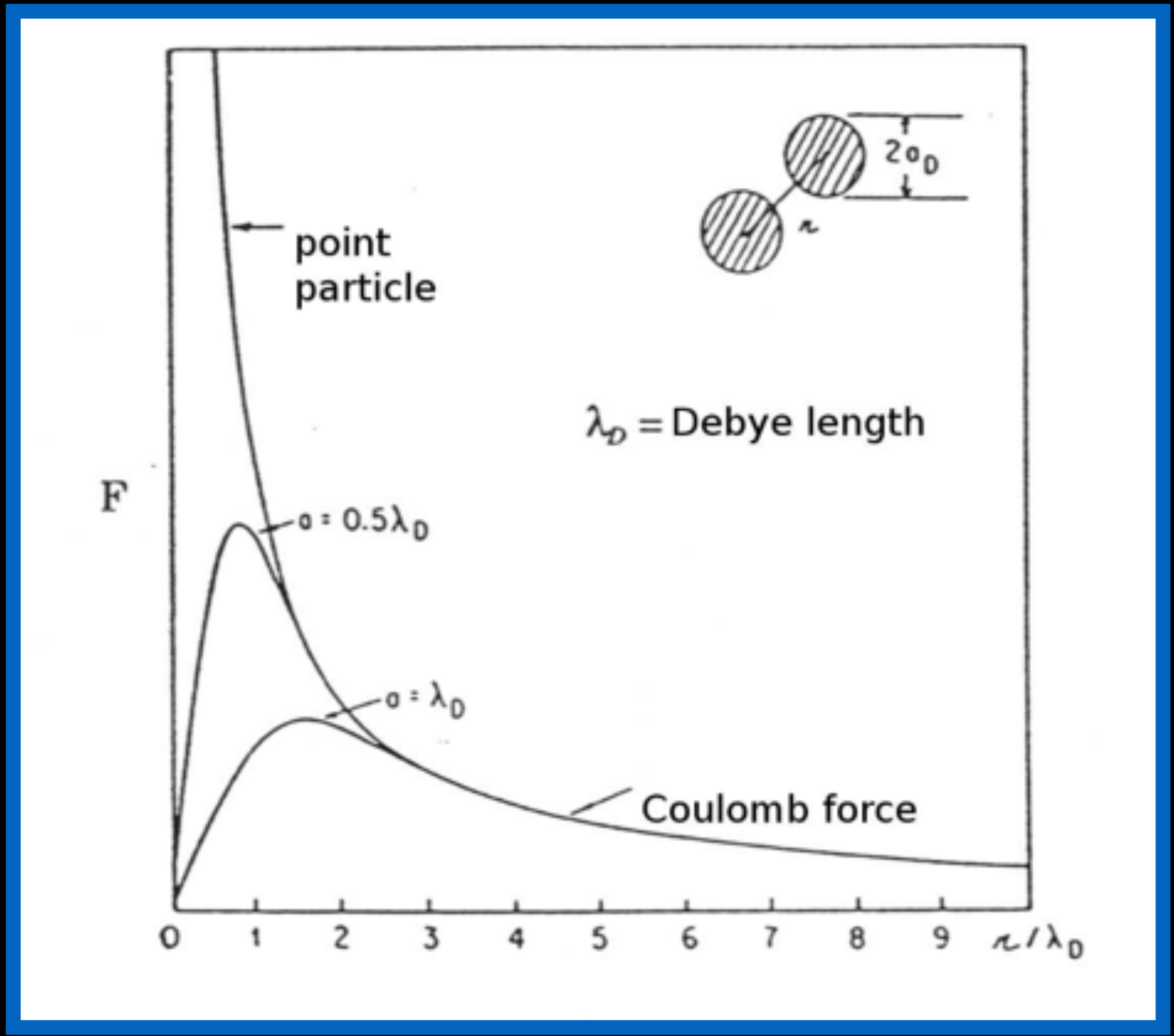


Credit: G. Lapenta

Macro-particles vs. real particles

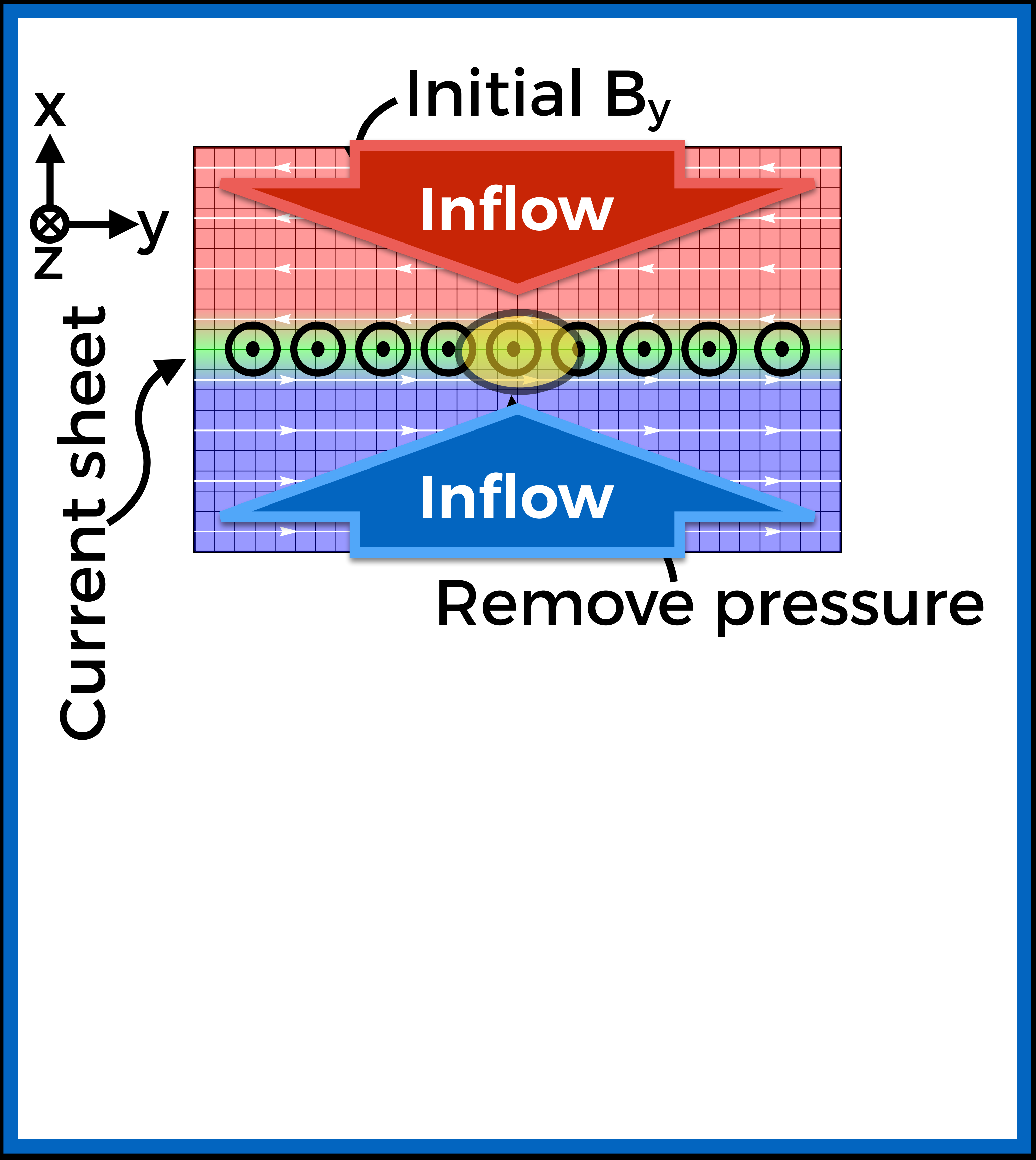
- In PIC simulations, the particles have finite size
- This suppresses the effect of Coulomb collisions
- Collisionless plasma limit can be achieved with fewer macro-particles than real particles



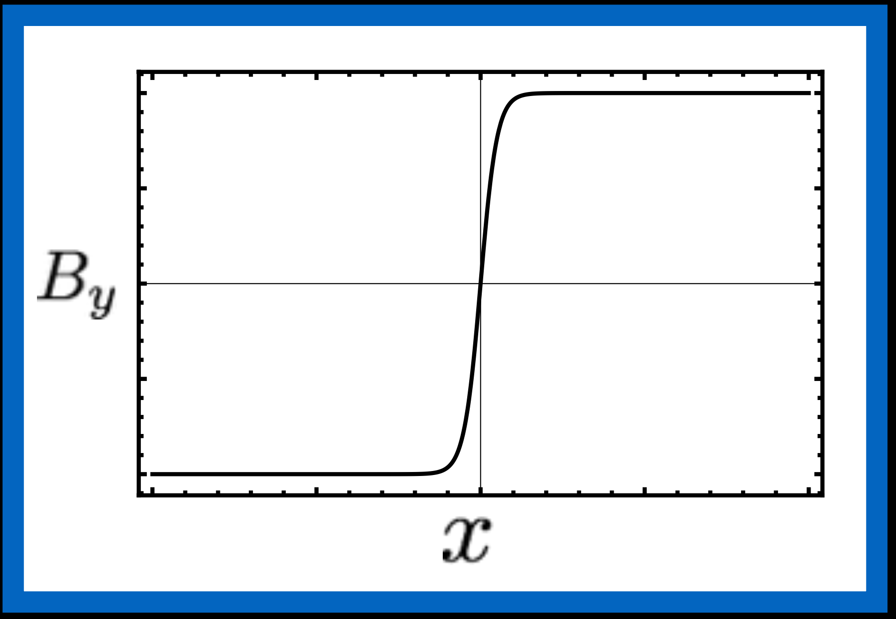


Credit: G. Lapenta

Start with alternating $\vec{B}$-field and trigger reconnection



- B-field initialized in Harris equilibrium



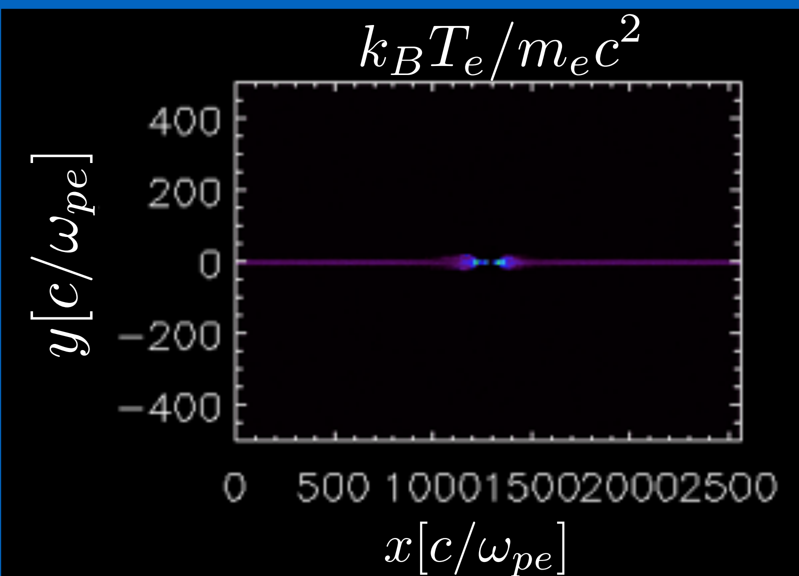
$$\mathbf{B} = B_0 \tanh{(x/L)}\mathbf{e}_y$$

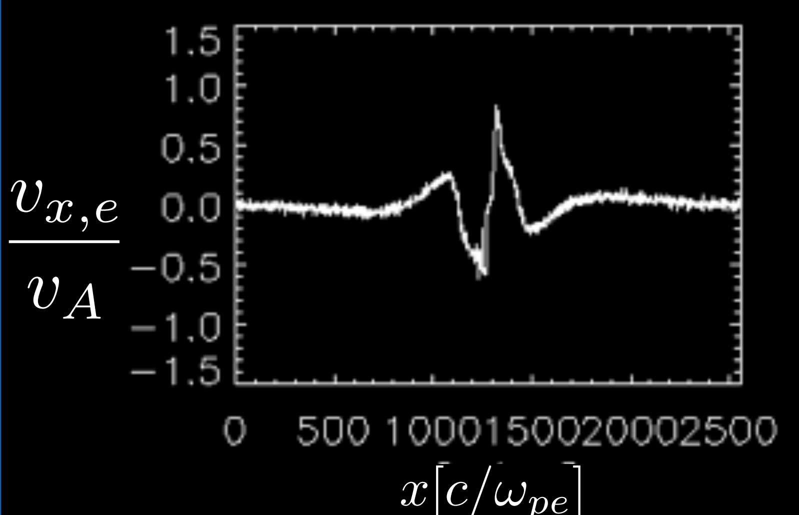
- Hot, overdense strip of particles at beginning (green)
- Remove the particle pressure in center to drive reconnection

Boundary conditions

Outflow

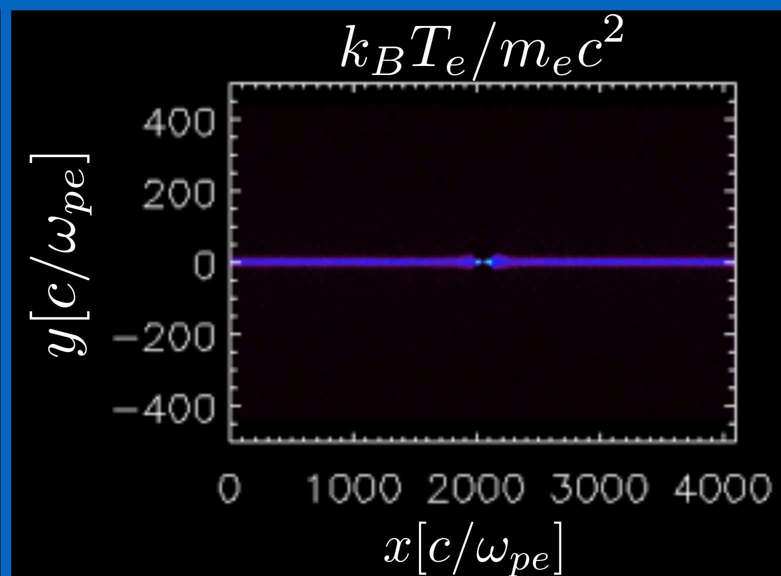
- Particles escape along x-dir.
- Allows for study of long-term evolution of system

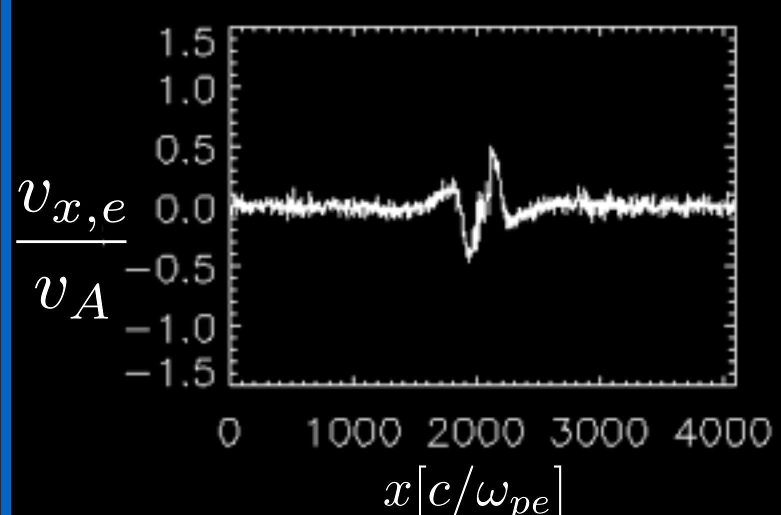




Periodic

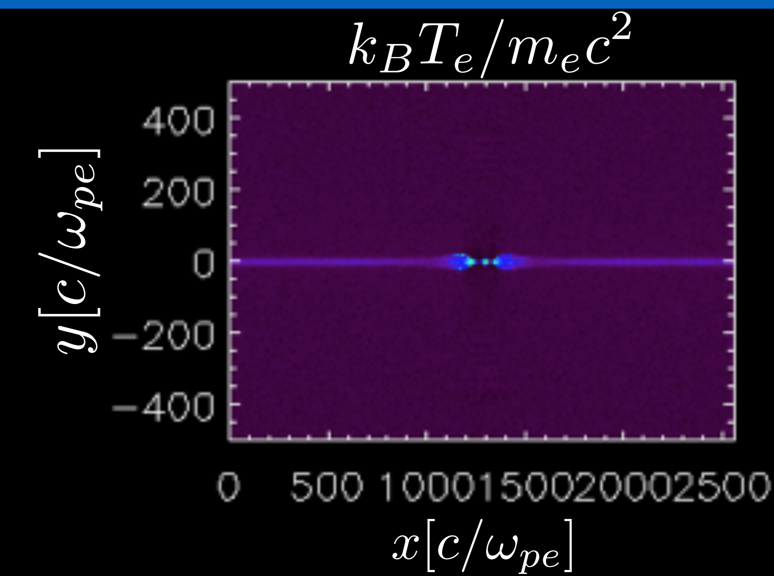
- No particles are lost
- However, sensitive to boundaries after $^1/_2$ Alfvén crossing-time

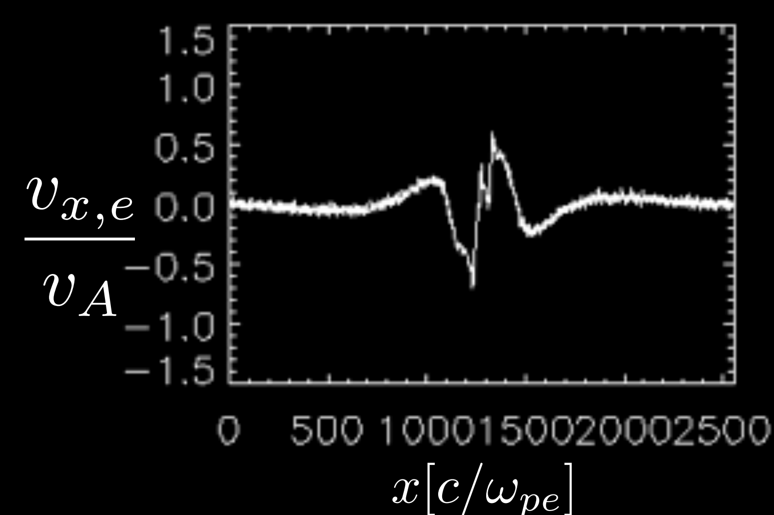




Adaptive

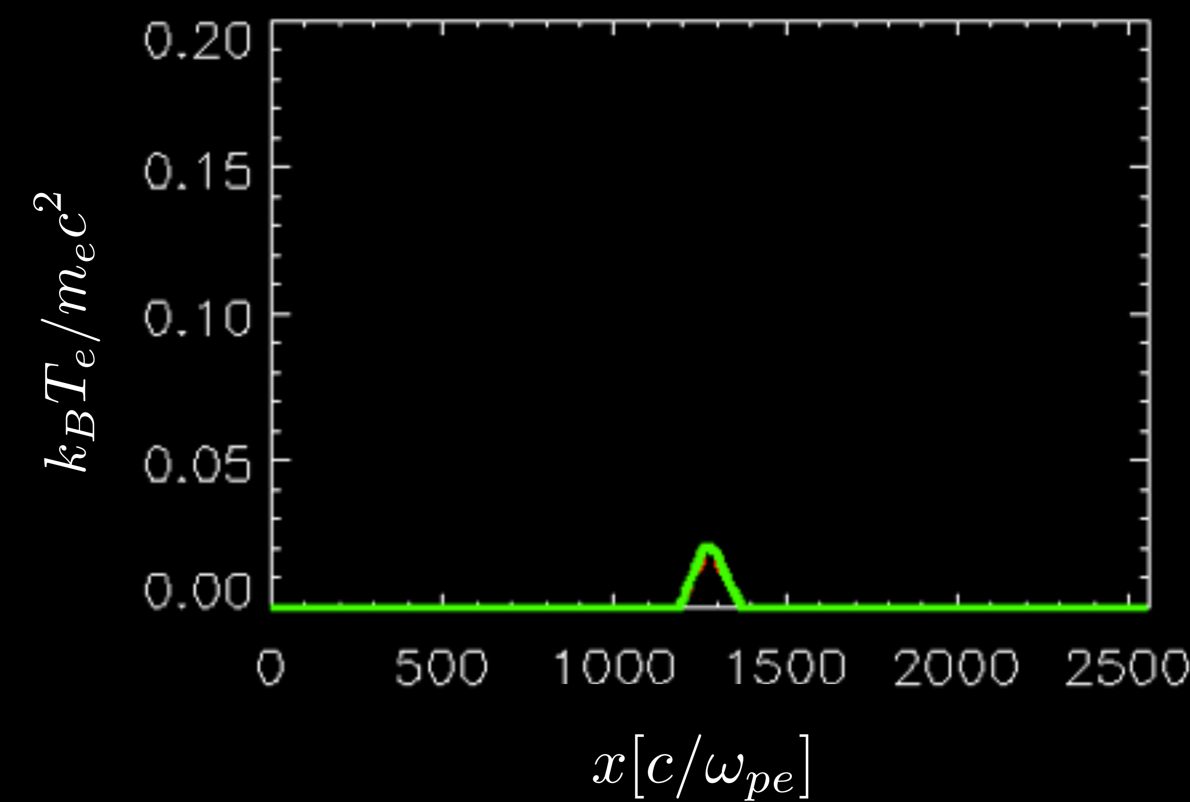
- Modified version of outflow boundary condition
- Includes additional controls necessary for high-beta case





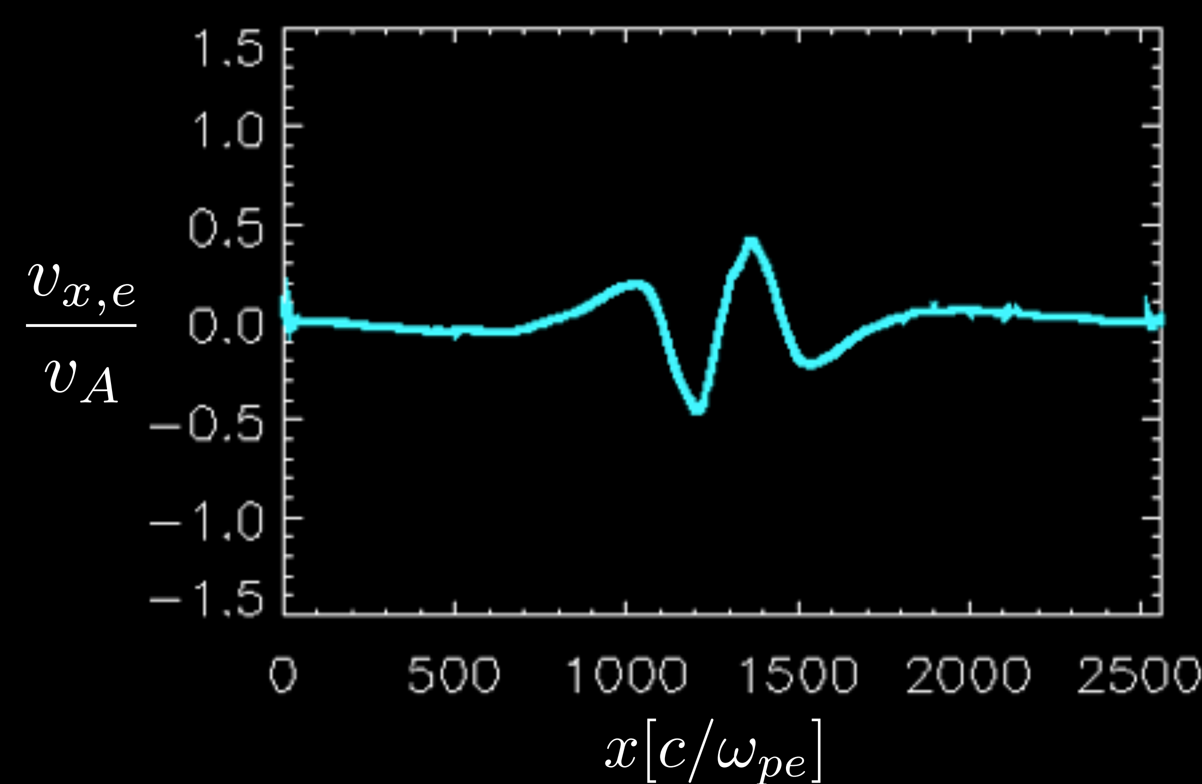
The plasma reaches a quasi-steady state

- To extract a meaningful outflow temperature, temperature profile should be flat
- Alfvén velocity should be saturated in current sheet



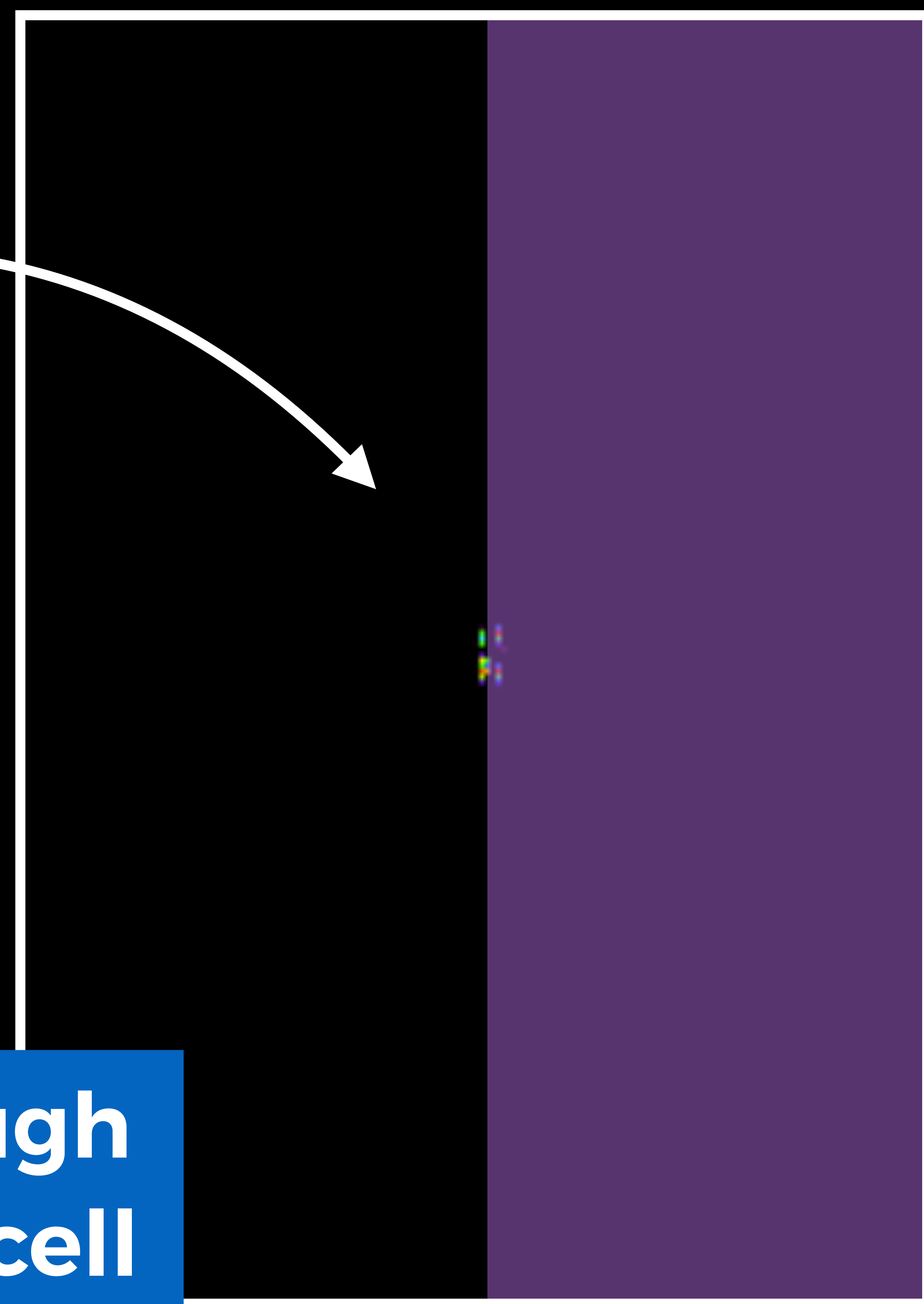
Alfvén velocity

$$\frac{v_A}{c} = \sqrt{\frac{\sigma_w}{1+\sigma_w}}$$



How to identify where reconnection has happened?

Track tagged particles; measure ratio of tagged to total density in each cell



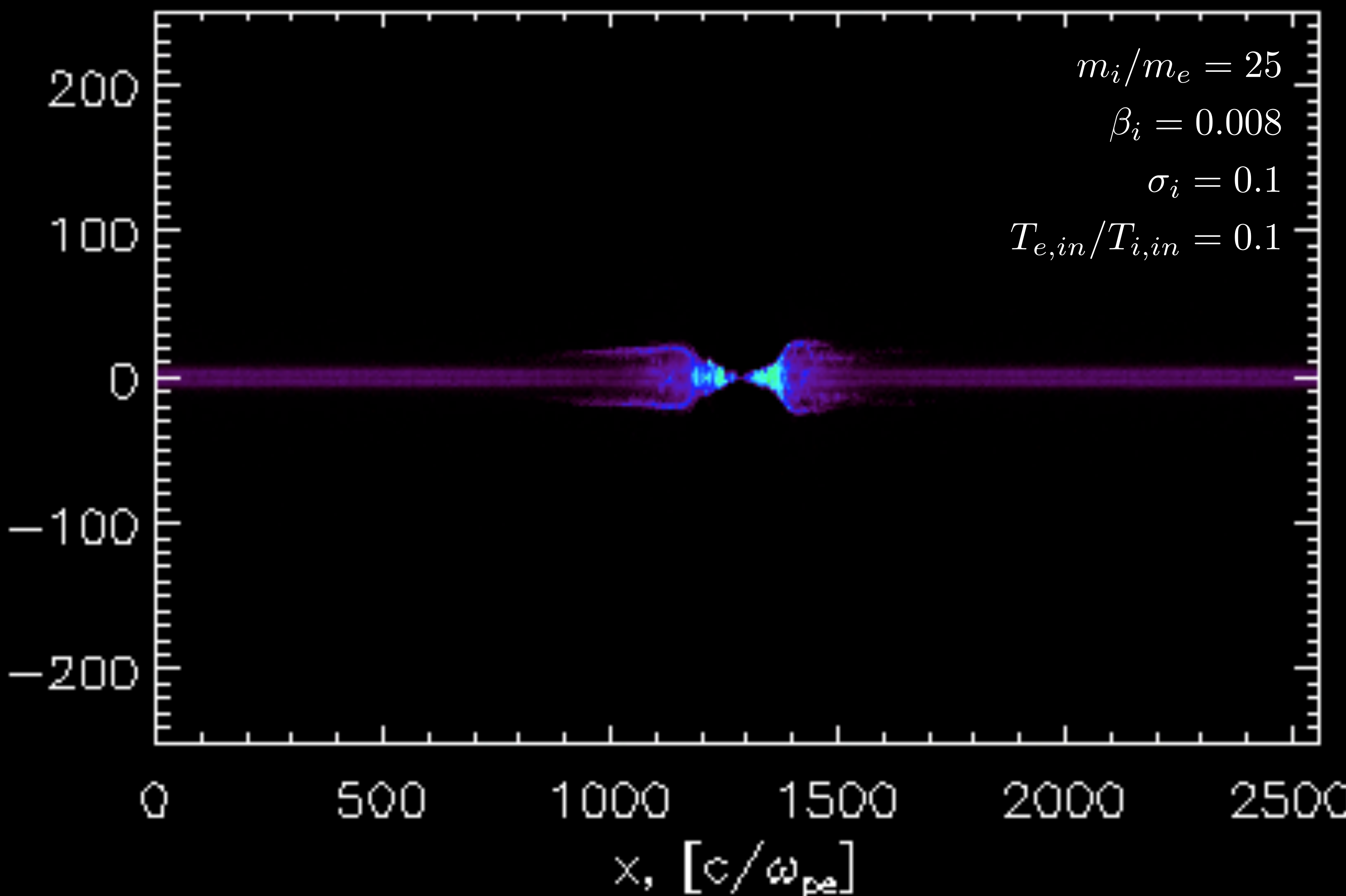
Tag particles on right

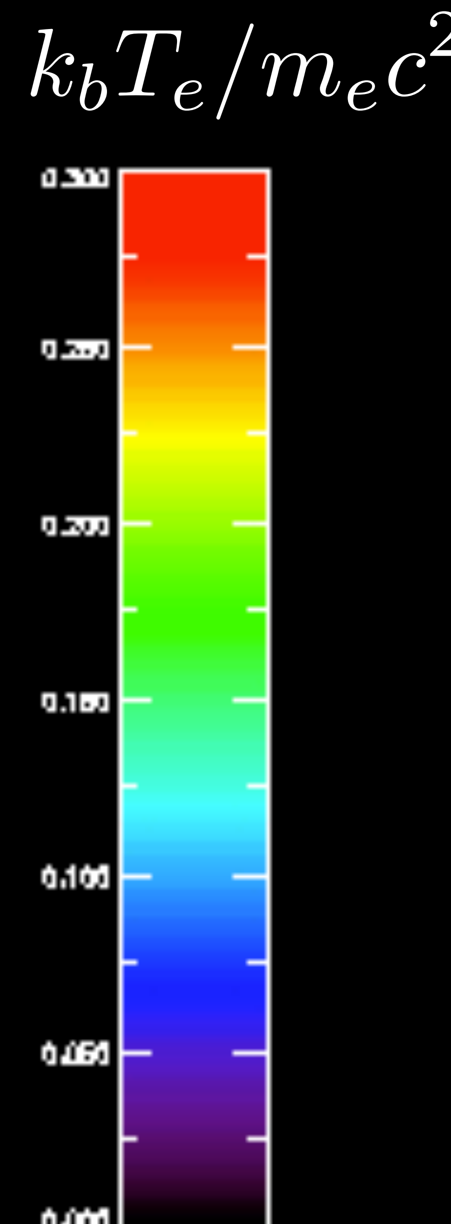
Region selected is insensitive to the particular threshold value

If there is enough mixing, count cell as reconnected

This is a low-beta plasma...

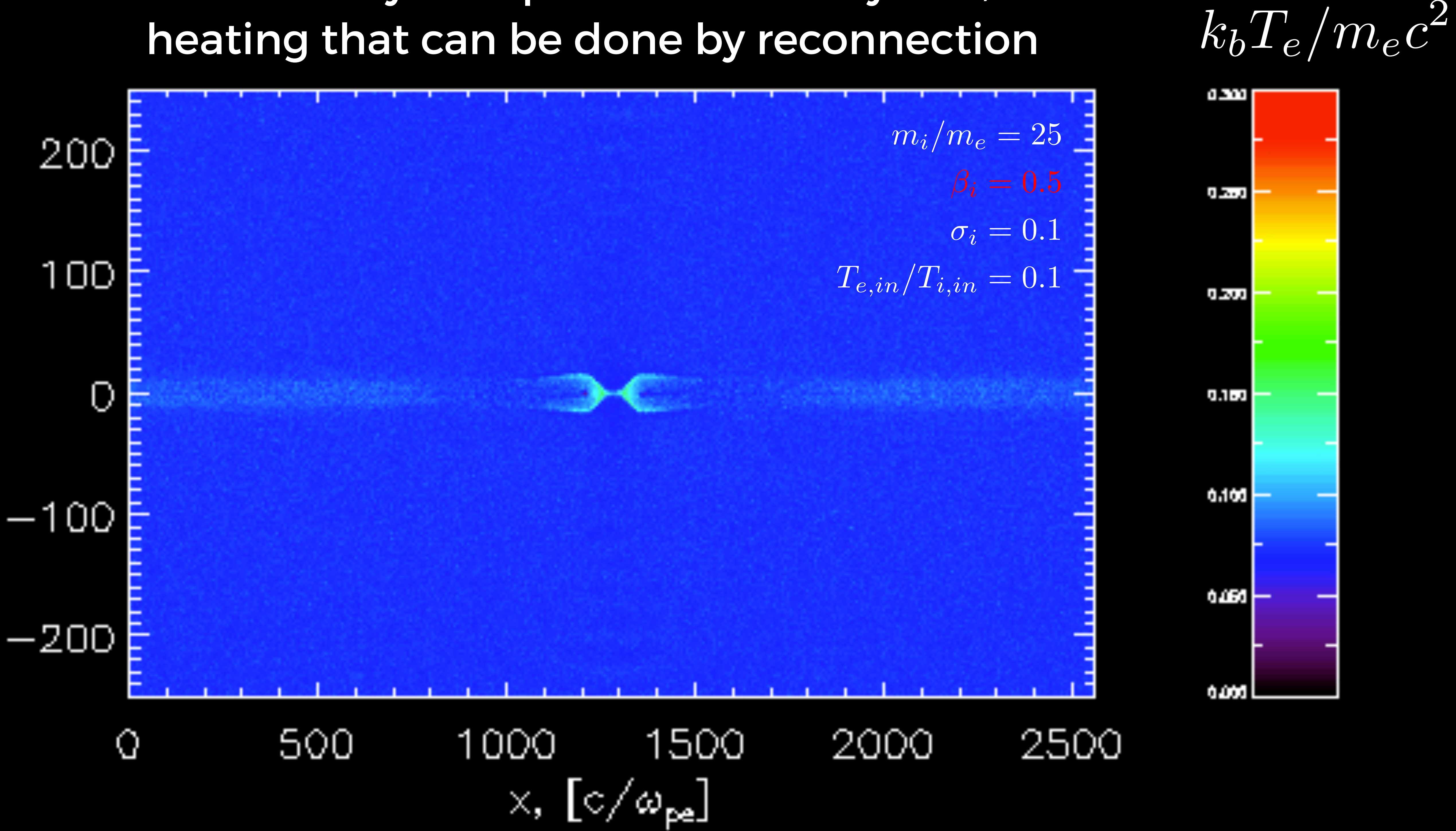
- The circular substructures are ‘magnetic islands’
- They form due to the tearing instability
- Islands heat particles by ‘bouncing’





…and this is a high(er)-beta plasma

- Islands are absent; thermal pressure suppresses tearing mode
- Heuristically: the plasma is already 'hot,' so there is less heating that can be done by reconnection



We carefully extract the temperature increase

$T^{\mu\nu}_{lab}$ → **For each cell, compute lab-frame stress tensor** ← Recorded in code as: $\frac{1}{N_p}\sum_{particles}\frac{p^\mu p^\nu}{E}$

We carefully extract the temperature increase

$T^{\mu\nu}_{lab}$

For each cell, compute lab-frame stress tensor

Recorded in code as:

$$\frac{1}{N_p}\sum_{particles}\frac{p^\mu p^\nu}{E}$$

$$T^{\alpha\beta}_{CMRF} = T^{\mu\nu}_{lab}\Lambda^{\alpha}_{\mu}\Lambda^{\beta}_{\nu}$$

Boost to fluid frame

We carefully extract the temperature increase

$T^{\mu\nu}_{lab}$ → **For each cell, compute lab-frame stress tensor** ← Recorded in code as: $\frac{1}{N_p}\sum_{particles}\frac{p^\mu p^\nu}{E}$

↓

$T^{\alpha\beta}_{CMRF} = T^{\mu\nu}_{lab}\Lambda^\alpha_\mu\Lambda^\beta_\nu$ → **Boost to fluid frame**

↓

$e_0 = \rho c^2 + u_{int}$

$$T^{\alpha\beta}_{CMRF} \overset{?}{=} \begin{pmatrix} e_0 & 0 & 0 & 0 \\ 0 & p_0 & 0 & 0 \\ 0 & 0 & p_0 & 0 \\ 0 & 0 & 0 & p_0 \end{pmatrix}$$

→ **Assume perfect fluid** ← E.o.S: $(\hat{\gamma}(T) - 1)u_{int} = nT$

↑

Diagonal?
Isotropic?

We carefully extract the temperature increase

$T^{\mu\nu}_{lab}$

For each cell, compute lab-frame stress tensor

Recorded in code as:

$$\frac{1}{N_p}\sum_{particles}\frac{p^\mu p^\nu}{E}$$

$$T^{\alpha\beta}_{CMRF} = T^{\mu\nu}_{lab}\Lambda^\alpha_\mu\Lambda^\beta_\nu$$

Boost to fluid frame

$$e_0 = \rho c^2 + u_{int}$$

$$T^{\alpha\beta}_{CMRF} \overset{?}{=} \begin{pmatrix} e_0 & 0 & 0 & 0 \\ 0 & p_0 & 0 & 0 \\ 0 & 0 & p_0 & 0 \\ 0 & 0 & 0 & p_0 \end{pmatrix}$$

Diagonal?
Isotropic?

Assume perfect fluid

E.o.S:

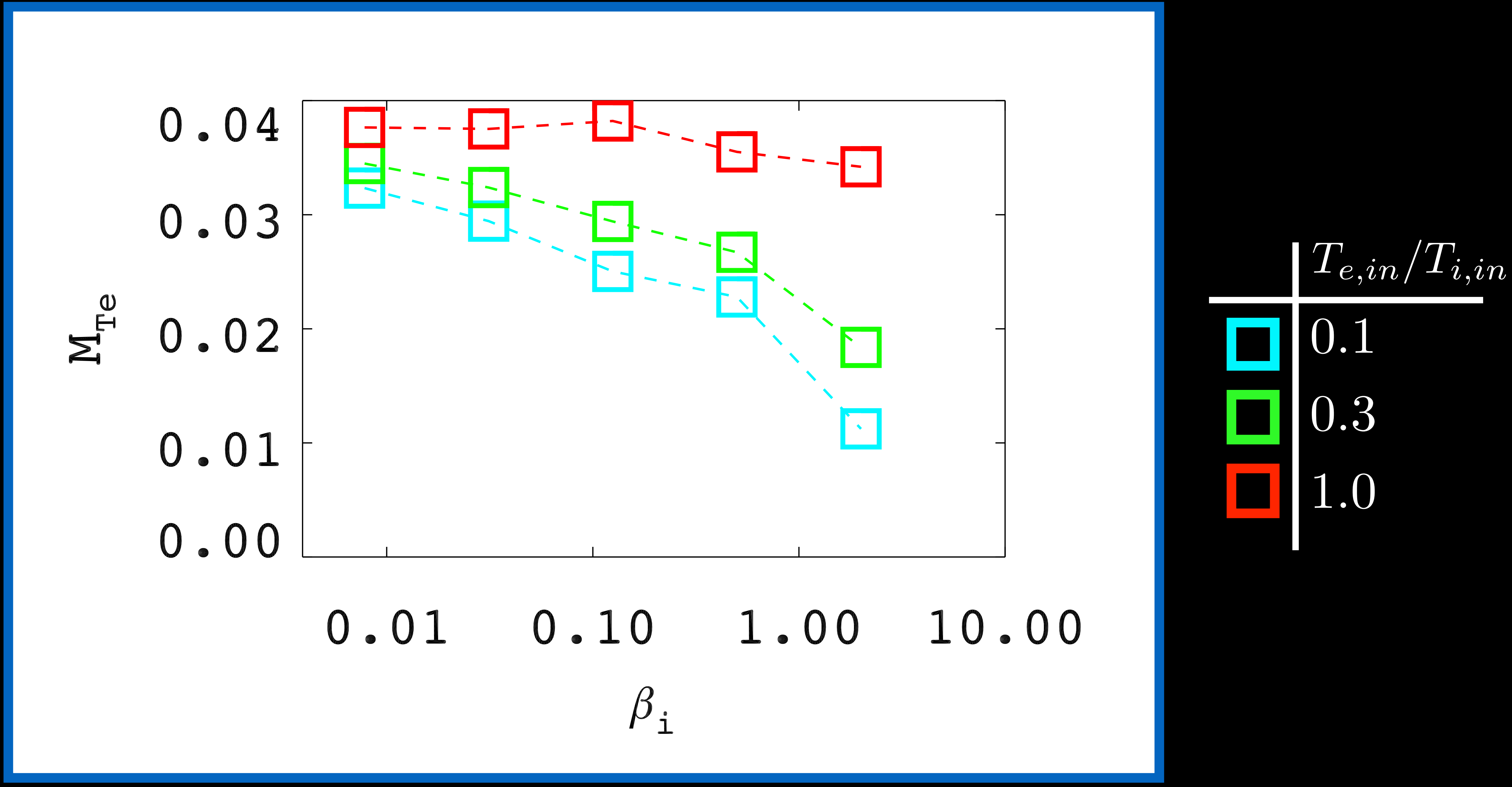
$$(\hat{\gamma}(T) - 1)u_{int} = nT$$

Iterate E.o.S. and adiabatic index eq.

Temperature

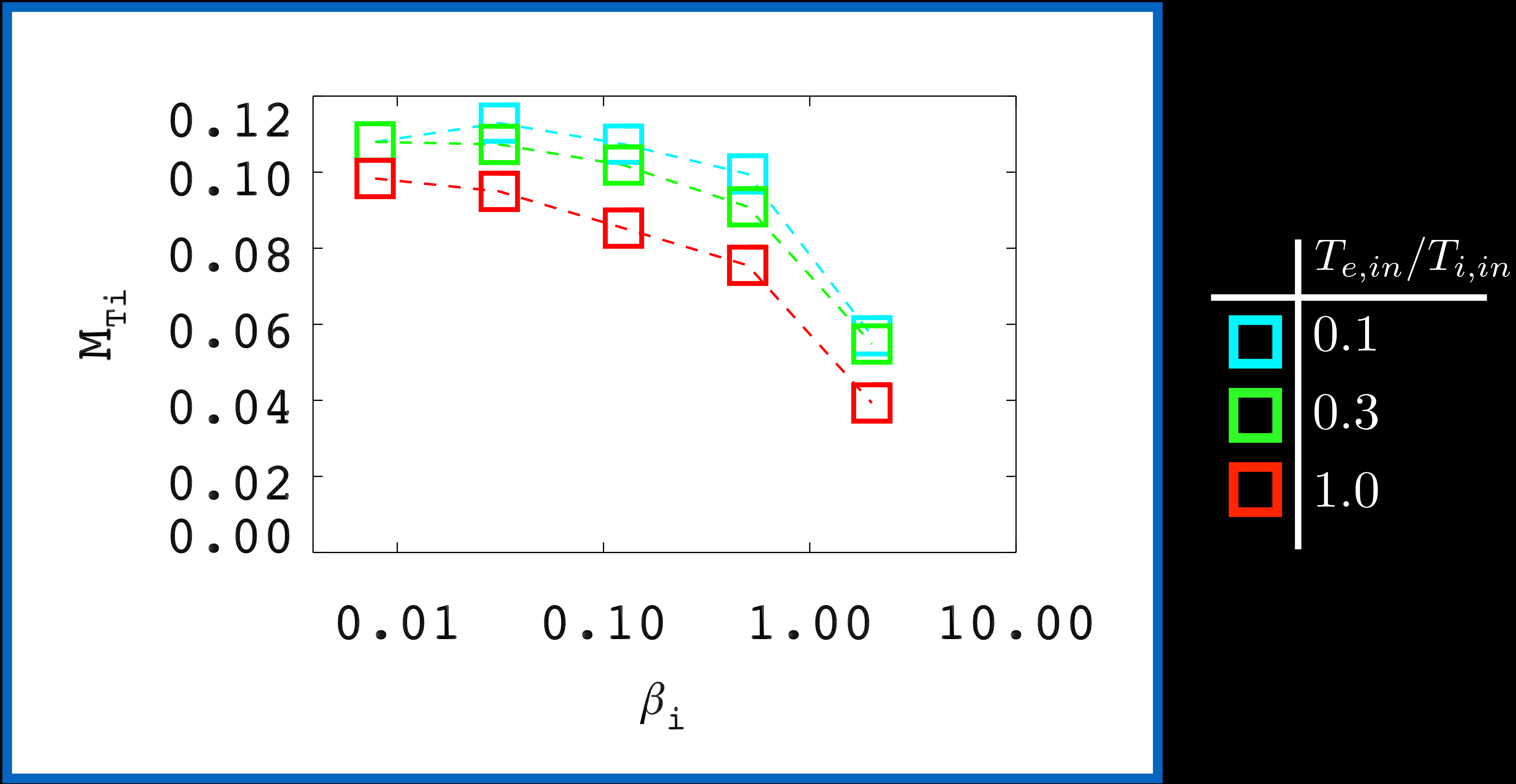
Electron gross heating

- For low-beta, the fraction of magnetic energy that ends up as electron heating is around 3%
- Dependence on initial temperature ratio



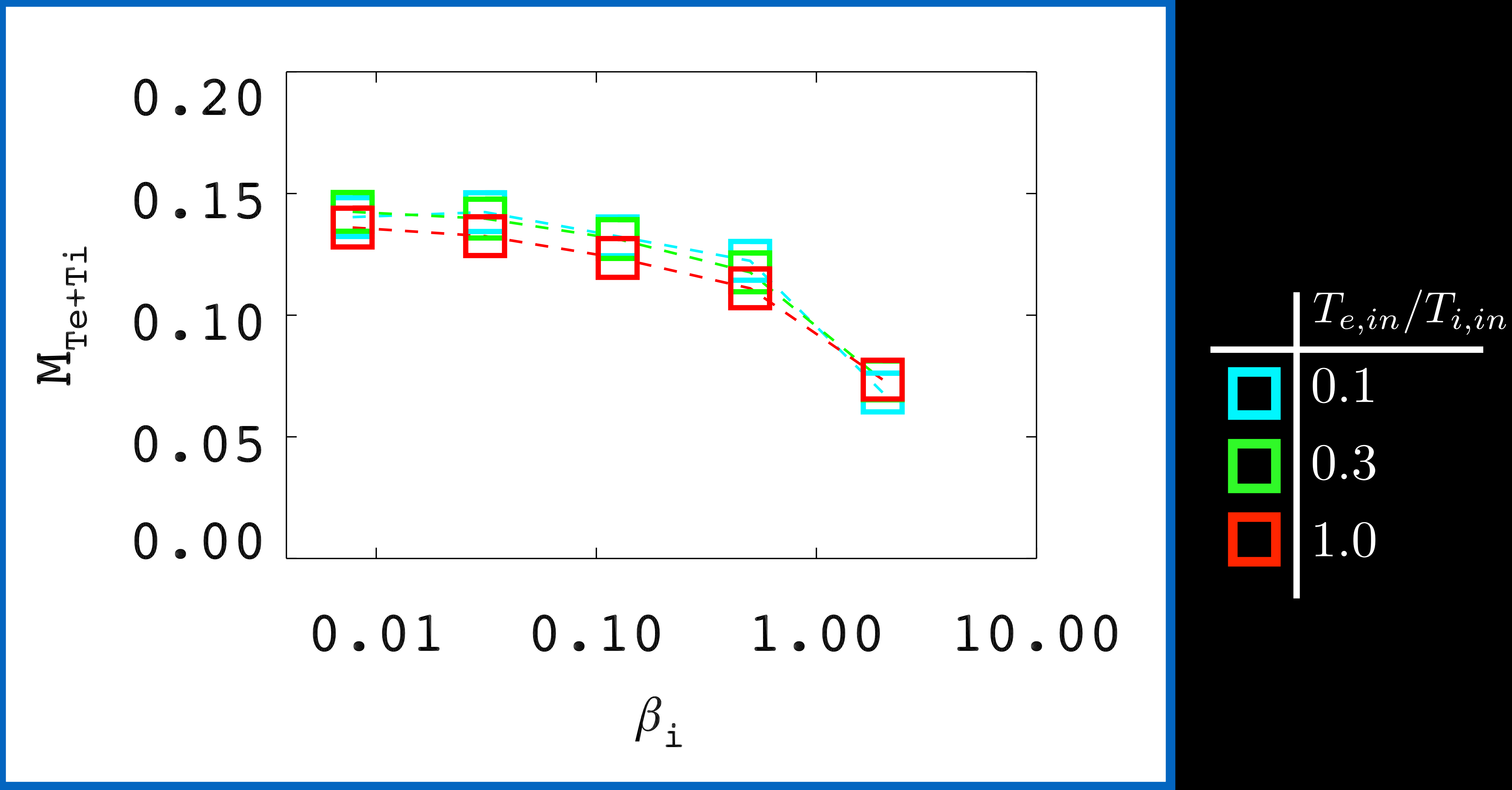
Ion gross heating

- Free magnetic energy that ends up as ion heating: ~10-12%
- This gives us a rough value for electron:ion heating as 1/3 in the low-beta cases

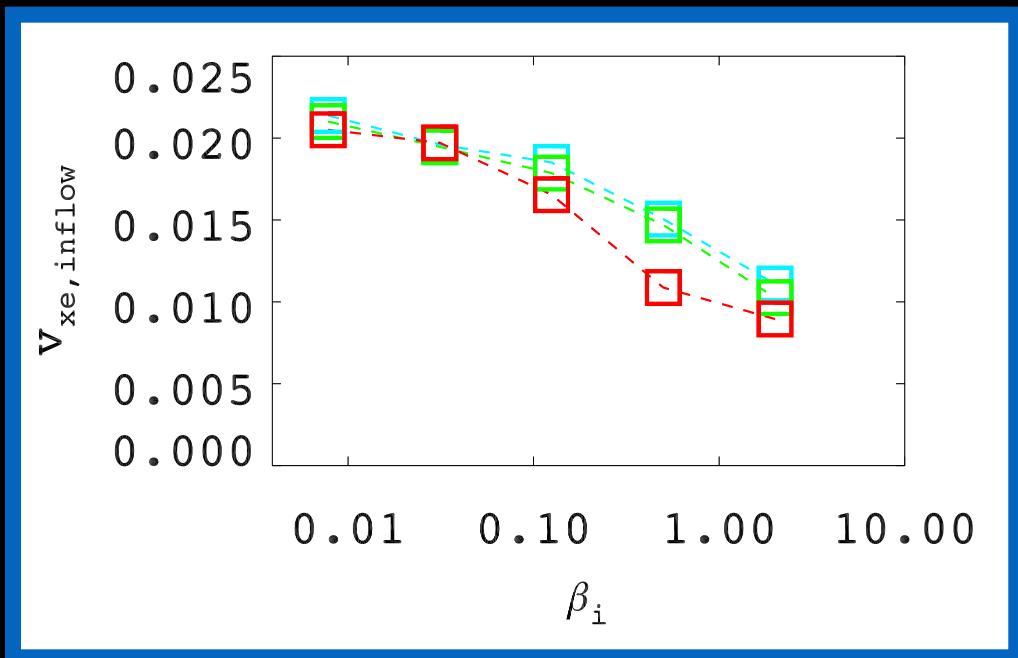


Particle gross heating

- Now look at the sume of M_{Te}+M_{Ti}
- Less free magnetic energy is converted to particle heating in a hotter plasma

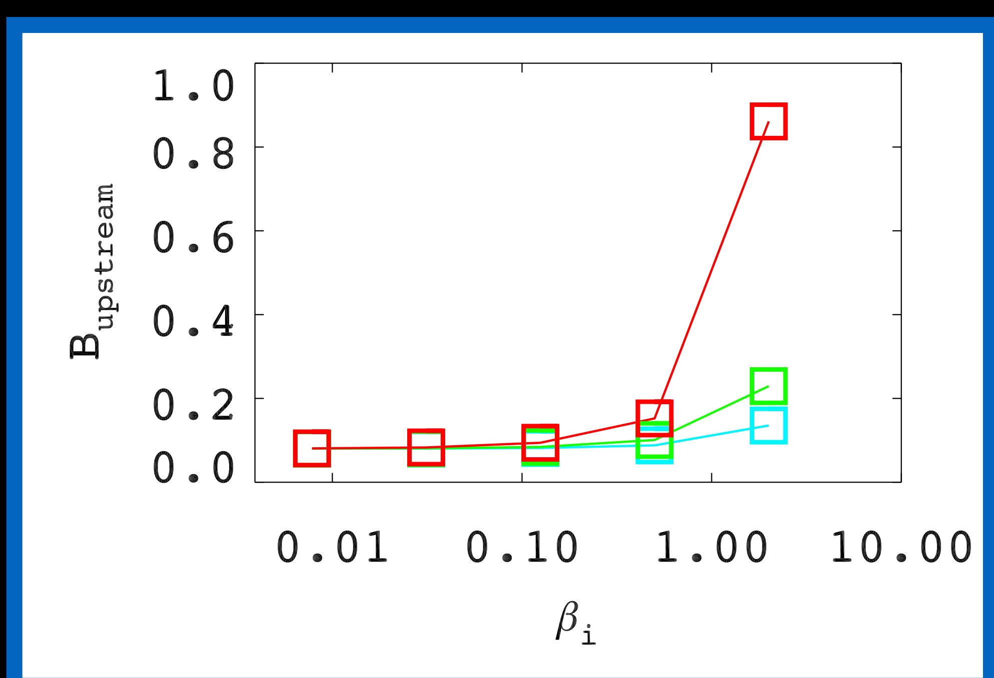


A simple model for the electron heating



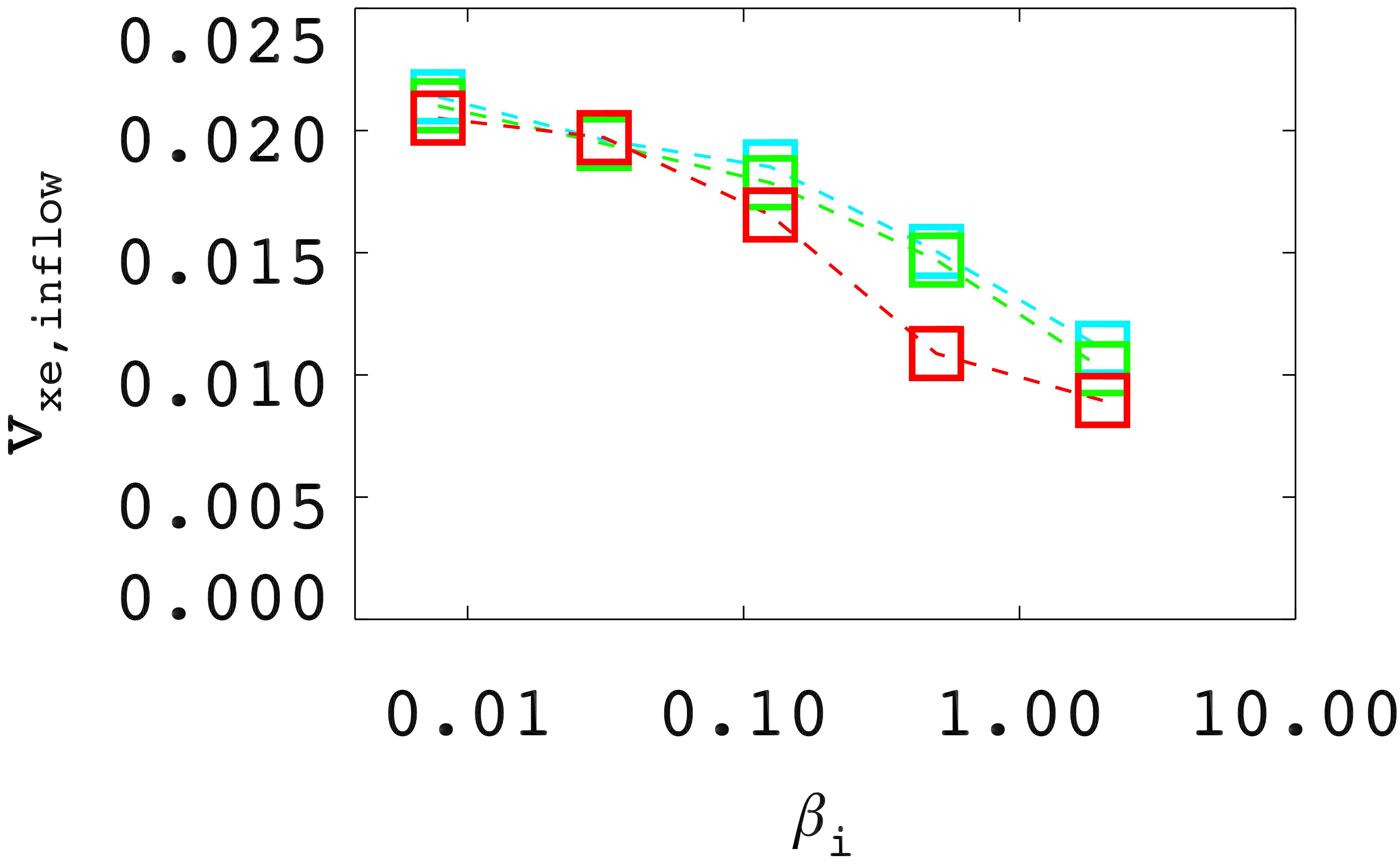
?

$$M_{Te,ideal} \sim \frac{eE_{rec}Ln}{B^2/8\pi} \sim \boxed{\frac{e\left(\frac{v_{in}}{c}\right)Ln}{B}}$$

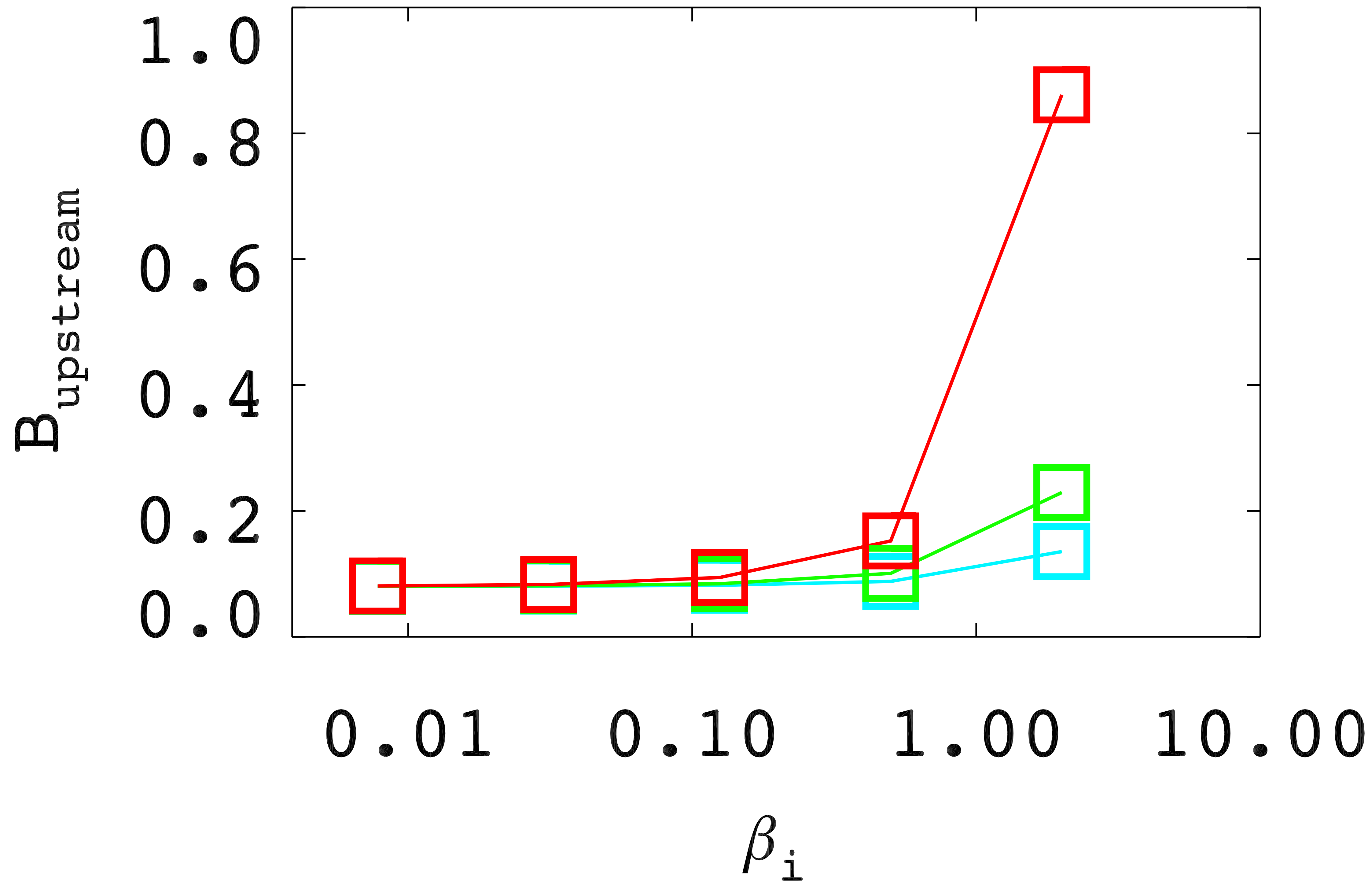


- The expression is roughly the work done by reconnection E field compare to inflow magnetic energy
- Treat B, L, and v_{in} as functions of beta, T_e/T_i - don’t just assume constant

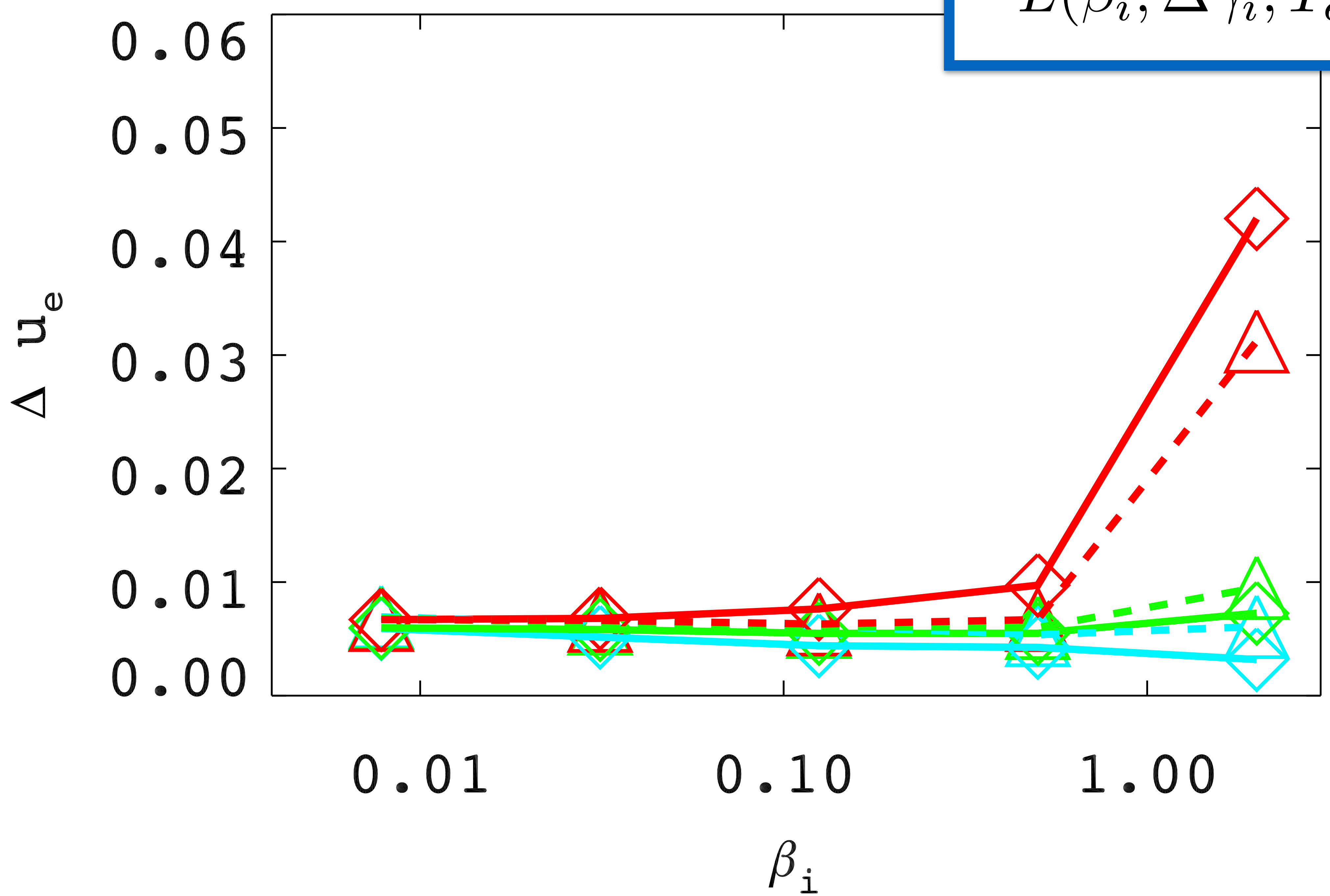
A simple model for the electron heating



A simple model for the electron heating



Model (~$v_{in}B_{in}$) vs. Simulations



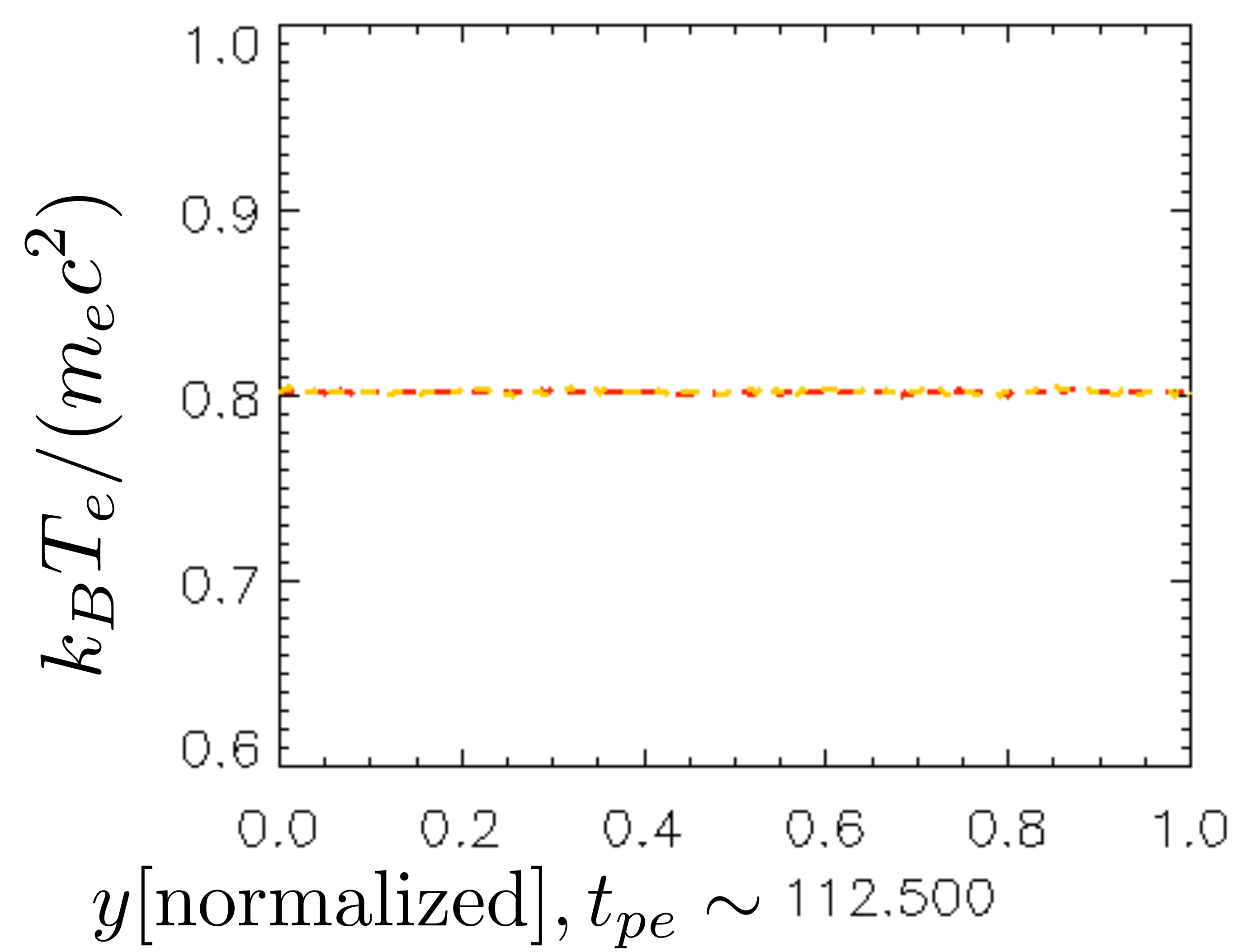
Need to incorporate

$L(\beta_i, \Delta\gamma_i, T_e/T_i)$

Numerics: effect of particles-per-cell

- Check for convergence by varying computational parameters
- To trust the numerical results, need to make sure numerical heating is relatively small
- Particles per cell, domain size, boundary conditions, etc.

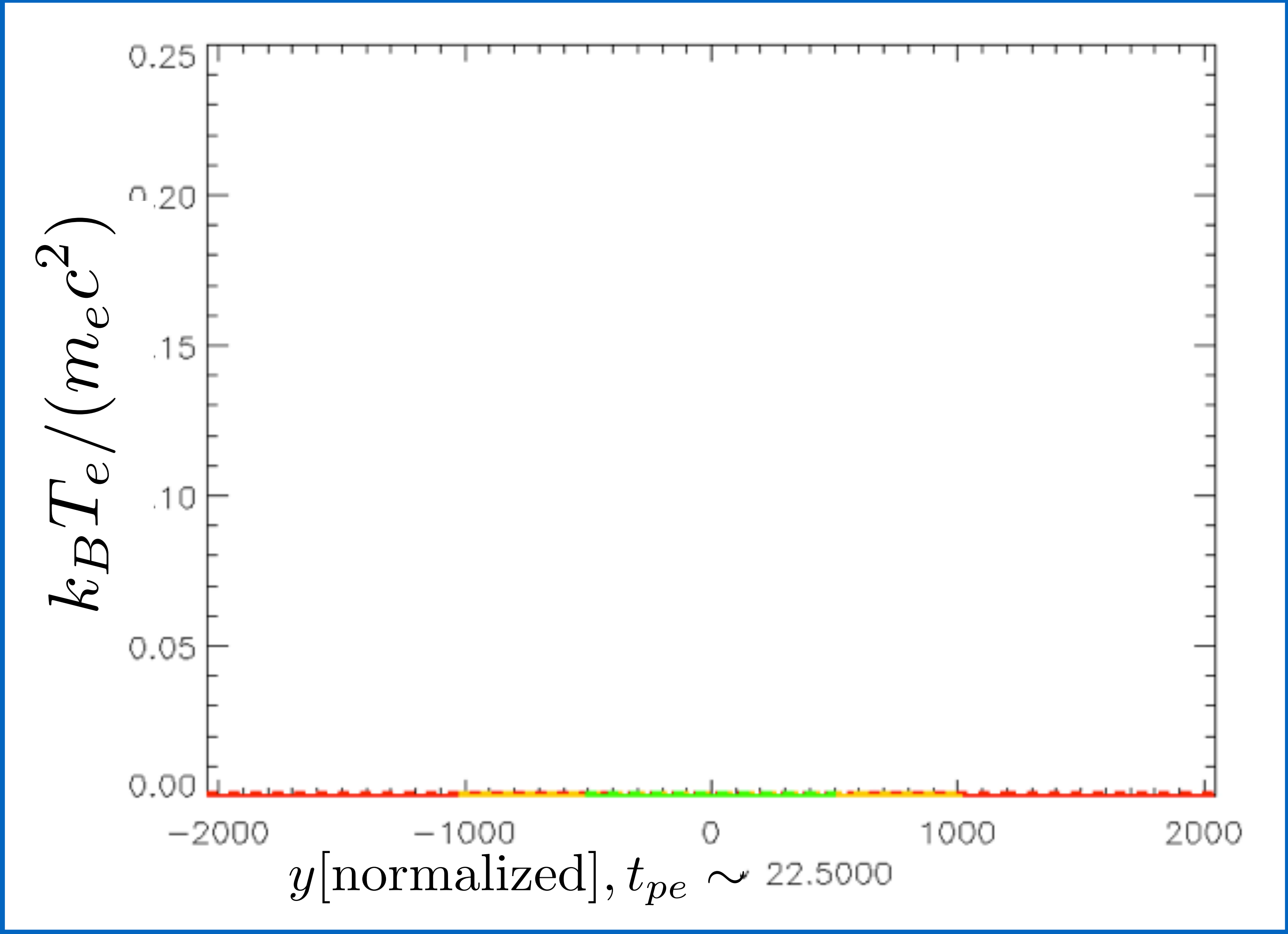
ppc = 64 (yellow), ppc = 256 (red)



Numerics: effect of domain size

- Check for convergence by varying computational parameters
- To trust the numerical results, need to make sure numerical heating is relatively sm
- Particles per cell, domain size, boundary conditions, etc.

m_y = 4096 (green), 8192 (yellow), 16384 (red)

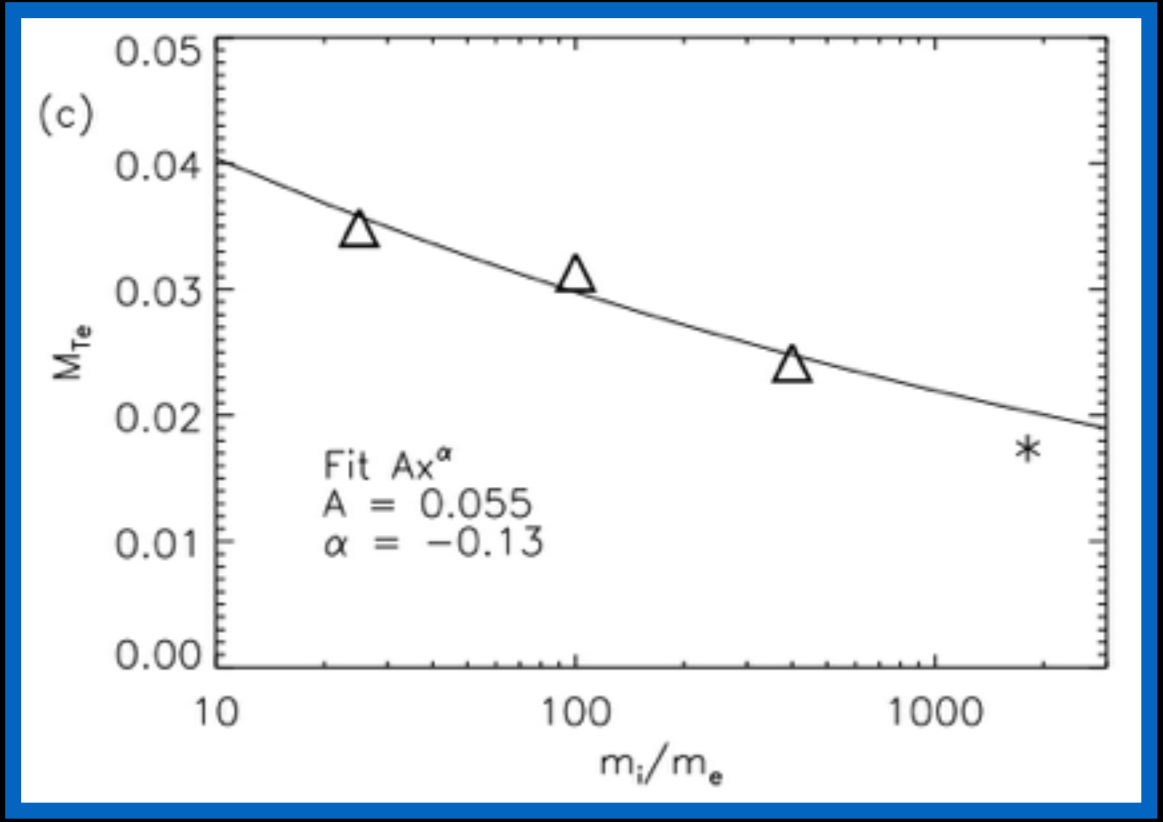


Electron heating will decrease with higher m_i/m_e

- In our simulations, we use an artificial mass ratio of $m_i/m_e = 25$
  - Why? This makes the problem computationally tractable
- We can expect our measured heating will decrease with higher mass ratio;

$$M_{Te} \sim (m_i/m_e)^{-0.13}$$

(Drake et al., 2014)



- Note: this scaling is consistent with the analytical model of Egedal et al.

Connection to black-hole physics

- Two main aspects to our investigation
  - Plasma physics
    - Explore a relatively unstudied region of plasma parameter space
  - Astrophysics
    - Provide (eventually) a lookup table for global simulations of black-hole accretion flows
    - Even if it turns out that the dependence on input values is weak, at least this will be known from a first-principles investigation
    - Might the EHT see ‘black-hole flares’ ?

Summary and future directions

Summary:

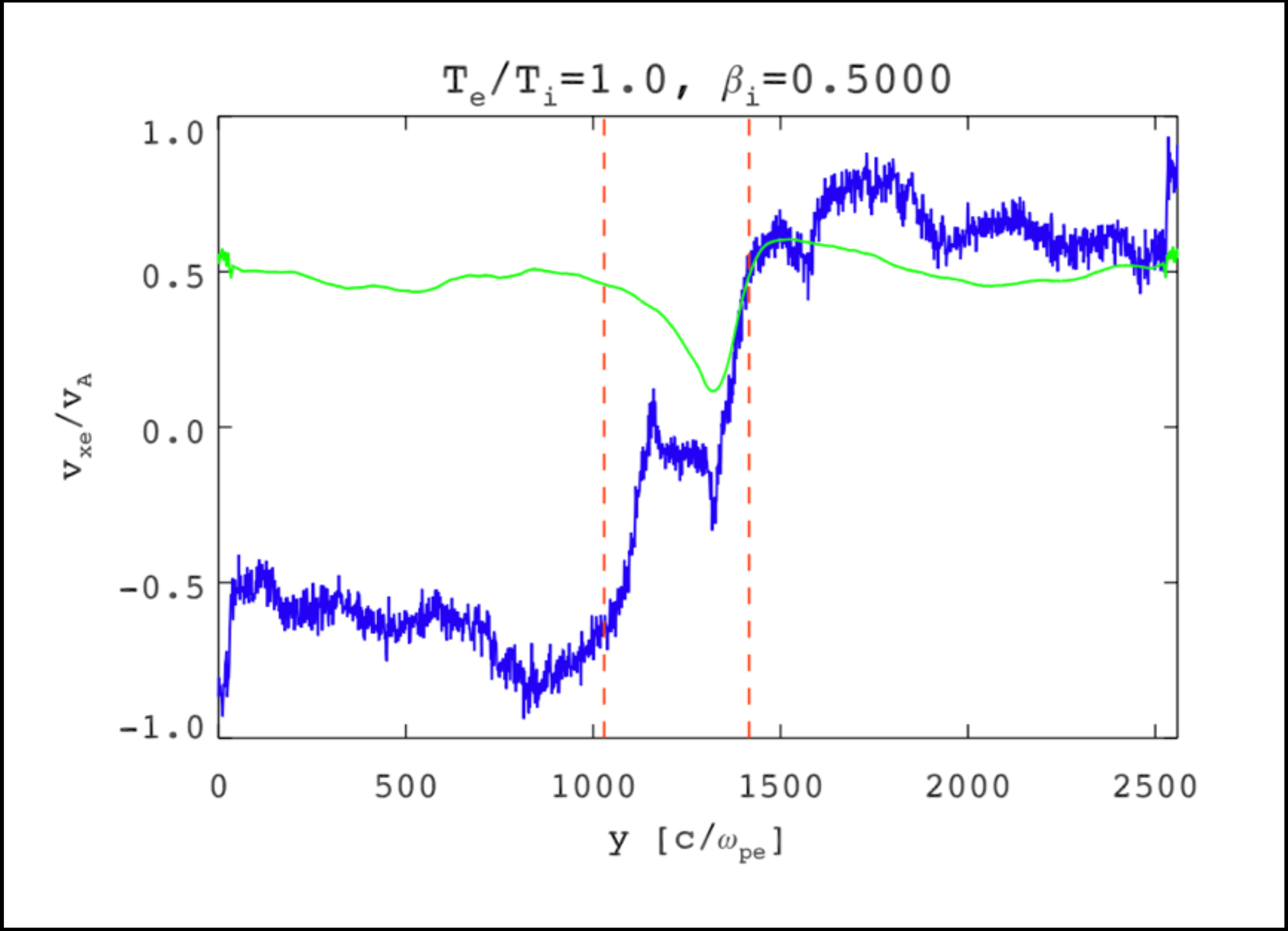
- Reconnection provides less net heating for high-beta compared to low-beta; $T_{e,\,out} / T_{e,in}$ approaches 1 for high beta
- Low-beta: ~3% of the magnetic energy ends up as electron heating, and ~10-12% ands up as ion heating

For the future:

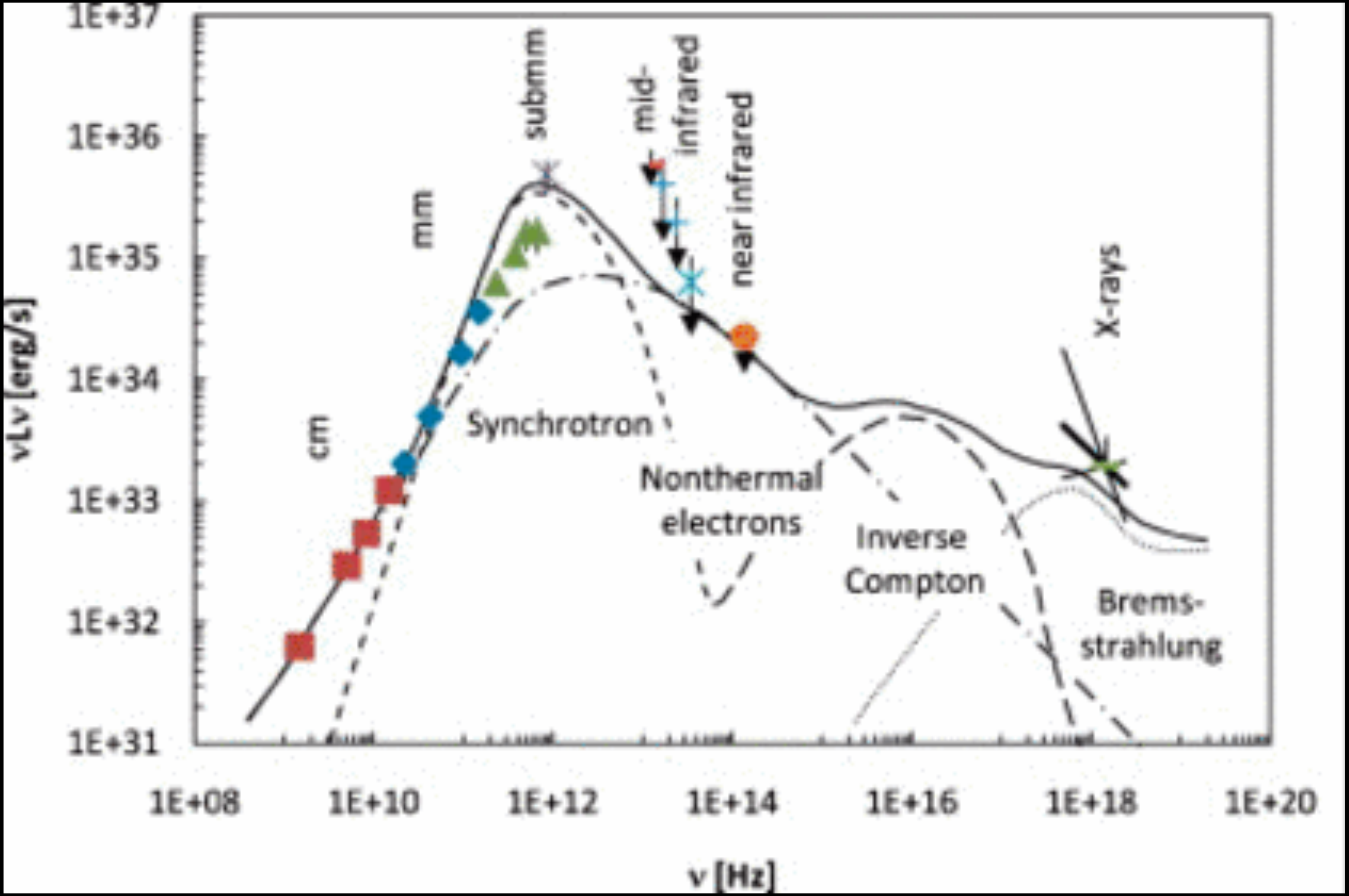
- Explore guide field reconnection
- Push to higher beta
- Vary the mass ratio
- Run with wider range of sigma
- Use particle orbits to study heating mechanism
  - Is this the same as in the non-relativistic case?
- 3D simulations

Thank you for your attention

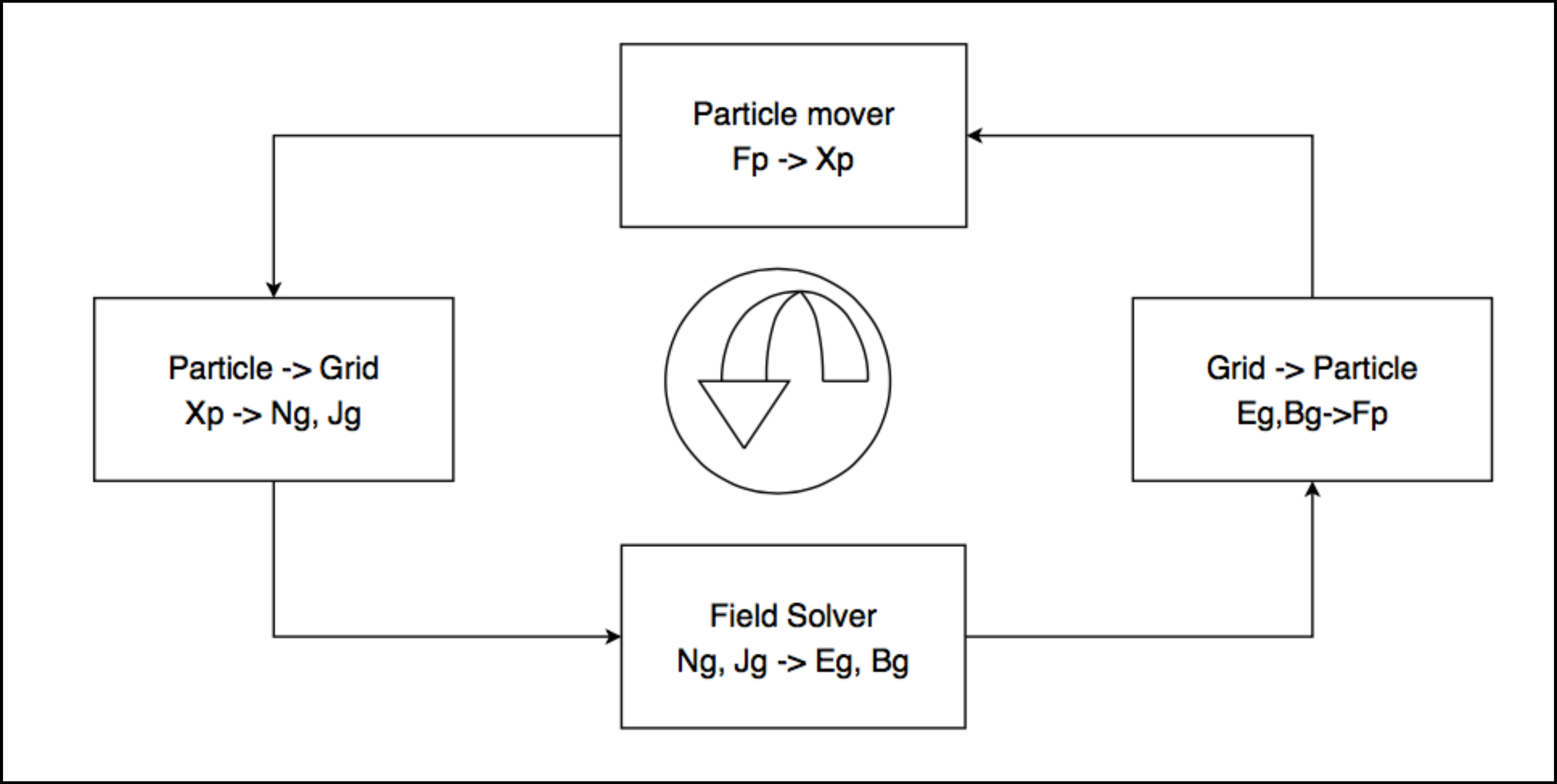
Strange-looking point from plot of v_{in} vs beta



Sgr A* radiation spectrum



Particle-in-cell loop



E.o.S. derivation

$$\boxed{\text{What is our equation of state if the adiabatic index is not constant?}}$$

$$dS = \frac{dQ}{\Theta},\ dQ = dH - V dp, \Theta = \frac{p}{\rho}$$

$$\Rightarrow dS = \frac{dH}{\Theta} - V\frac{dp}{\Theta} = \frac{dH}{\Theta} - \frac{N}{p}dp$$

$$\boxed{dS = \frac{dh}{\Theta} - d\log p}$$

Now integrate, using equation for $h(\Theta)$:

$$h = \frac{5}{2}\Theta + \sqrt{\frac{9}{4}\Theta^2 + 1}$$

$$\Rightarrow \int dS = \int \frac{1}{\Theta}\left(\frac{5}{2} + \left(\frac{9}{4}\Theta^2 + 1\right)^{-1/2}\frac{9}{4}\Theta\right) d\Theta - \log p$$

$$S = \frac{5}{2}\log\Theta + \frac{3}{2}\sinh^{-1}\left(\frac{3}{2}\Theta\right) - \log p$$

$$= \frac{5}{2}\log\Theta + \frac{3}{2}\log\left(\frac{3}{2}\Theta + \sqrt{\left(\frac{3}{2}\Theta\right)^2 + 1}\right) - \log p$$

$$= \frac{3}{2}\left(\log\Theta + \log\left(h - \Theta\right)\right) + \log\left(\frac{\Theta}{p}\right)$$

$$= \frac{3}{2}\log\left(\frac{\Theta^{5/3}}{p^{2/3}}(h - \Theta)\right)$$

$$= \frac{3}{2}\log\left(\frac{p}{\rho^{5/3}}(h - \Theta)\right)$$

$$\boxed{const = \frac{p}{\rho^{5/3}}\left(\frac{3}{2}\Theta + \sqrt{\frac{9}{4}\Theta^2 + 1}\right)}$$

This provides the correct limiting values. For $\Theta \to 0$,

$$const = \frac{p}{\rho^{5/3}}$$

and for $\Theta \gg 1 \Leftrightarrow p \gg \rho$,

$$const \simeq \frac{p}{\rho^{5/3}}3\Theta$$

$$\Rightarrow const' = \frac{p^2}{\rho^{8/3}}$$

$$\Rightarrow const'' = \frac{p}{\rho^{4/3}}$$

The adiabatic index is not the same in the upstream as compared to outflow region. To make a meaningful comparison of the compressive heating, we should compute *const* upstream, then use the variable equation of state (boxed above) to compute the predicted value *const* for given Θ, p, ρ in the outflow. Discrepancy between the predicted and actual values should then be accounted for by 'actual' heating.

The equation we used for specific enthalpy comes from Taub inequality, taking the equals sign:

$$(h - \Theta)(h - 4\Theta) \geq 0$$

$$\Rightarrow h^2 - h5\Theta + 4\Theta^2 - 1 \geq 0$$

$$\Rightarrow h = \frac{5}{2} + \sqrt{\frac{9}{4}\Theta^2 + 1}$$