Вариант № 24

№ 1

$742x - 777y = 35$

Заменим $(-y)$ на y':

$742x + 777y' = 35$

Найдём значения x_0 и y'_0 с помощью алгоритма Евклида:

$777 = 742 \cdot 1 + 35$

$742 = 35 \cdot 21 + 7$

$35 = 7 \cdot 5$

$7 = 742 - 35 \cdot 21 = 742 -$

$- (777 - 742) \cdot 21 = 742 \cdot$

$\cdot 22 - 777 \cdot 21$

$x_0 = 22$; $y'_0 = (-21)$

$x = 22 \cdot 5 = 110$

$y' = (-21) \cdot 5 = (-105)$

$y = -y' = 105$

Проверка:

$772 \cdot 110 - 777 \cdot 105 = 35$

$81620 - 81585 = 35$

$35 = 35$

Ответ: $\begin{cases} x = 110 + 111k, & k \in \mathbb{Z} \\ y = 105 - 106k, & k \in \mathbb{Z} \end{cases}$

№ 2

$$\sqrt{395} = 19 + \sqrt{395} - 19 =$$

$$= 19 + \cfrac{1}{\cfrac{1}{\sqrt{395} - 19}} = 19 + \cfrac{1}{\cfrac{\sqrt{395} + 19}{34}} =$$

$$= 19 + \cfrac{1}{1 + \cfrac{\sqrt{395} - 15}{34}} = 19 + \cfrac{1}{1 + \cfrac{1}{\cfrac{34}{\sqrt{395} - 15}}} =$$

$$= 19 + \cfrac{1}{1 + \cfrac{1}{\cfrac{34(\sqrt{395} + 15)}{170}}} =$$

$$= 19 + \cfrac{1}{1 + \cfrac{1}{\cfrac{\sqrt{395} + 15}{5}}} = 19 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{\sqrt{395} - 15}{5}}} =$$

$$= 19 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{\cfrac{5}{\sqrt{395} - 15}}}} =$$

$$= 19 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{\cfrac{5\left(\sqrt{395} + 15\right)}{170}}}} =$$

$$= 19 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{\cfrac{\sqrt{395} + 15}{34}}}} =$$

$$= 19 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{\sqrt{395} - 19}{34}}}} =$$

$$= 19 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{\cfrac{34\left(\sqrt{395} + 19\right)}{34}}}}} =$$

$$= 19 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{38 + \sqrt{395} - 19}}}}$$

$$\left[19;\ \overline{1,\ 2,\ 1,\ 38}\right]$$

Проверка:

$$19 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{38}}}} =$$

$$= 19 + \cfrac{1}{1 + \cfrac{1}{2 + \frac{38}{39}}} = 19 + \cfrac{1}{1 + \frac{39}{116}} =$$

$$= 19 + \frac{116}{155} = \frac{2945}{155} = 19 = \sqrt{395}$$

Ответ: $\left[19;\ \overline{1,\ 2,\ 1,\ 38}\right]$

$= 6859 \mod 66$

№ 4

$19^{3^{45}} \mod 66$

$\varphi(66) = \varphi(2) \cdot \varphi(3) \cdot \varphi(11) =$
$= 1 \cdot 2 \cdot 10 = 20$

Так как $D(19, 66) = 1$, то
$19^{20} \equiv 1 \mod 66$

$3^{45} = 20n + b$

$19^{3^{45}} = 19^{20n + b} = 19^{20n} \cdot 19^{b} =$
$= (19^{20})^{n} \cdot 19^{b} = 1^{n} \cdot 19^{b} = 19^{b}$

$\varphi(20) = \varphi(2^2) \cdot \varphi(5) = 2 \cdot 4 =$
$= 8$

Так как $D(3, 20) = 1$, то

$3^{8} = 1 \mod 20$
$b = 3^{45} = 3^{40} \cdot 3^{5} = (3^{8})^{5} \cdot 3^{5} =$
$= 243 \mod 20 = 3$

$19^{3^{45}} = 19^3 \mod 66 = 6859 \mod 66 =$

$= 61$

Ответ: 61

№ 5

$p(1) = 10 \qquad p(-5) = 40$

$p(-3) = -34 \qquad p(-2) = -11 \qquad p(-1) = 4$

$$p(x) = \frac{(x+5)(x+3)(x+2)(x+1)}{6 \cdot 4 \cdot 3 \cdot 2} \cdot 10 +$$

$$+ \frac{(x-1)(x+3)(x+2)(x+1)}{(-6)\cdot(-2)\cdot(-3)\cdot(-4)} \cdot 40 +$$

$$+ \frac{(x-1)(x+5)(x+2)(x+1)}{(-4)\cdot 2 \cdot(-1)\cdot(-2)} \cdot (-34) +$$

$$+ \frac{(x-1)(x+5)(x+3)(x+1)}{(-3)\cdot 3 \cdot 1 \cdot (-1)} \cdot (-11) +$$

$$+ \frac{(x-1)(x+5)(x+3)(x+2)}{(-2)\cdot 4 \cdot 2 \cdot 1} \cdot 4 =$$

$$= \frac{5(x^2+8x+15)(x^2+3x+2)}{72} +$$

$$+\frac{5(x^2+2x-3)(x^2+3x+2)}{18}+$$

$$+\frac{17(x^2+4x-5)(x^2+3x+2)}{8}-$$

$$-\frac{11(x^2+4x-5)(x^2+4x+3)}{9}-$$

$$-\frac{(x^2+4x-5)(x^2+5x+6)}{4}=$$

$$=\frac{5(x^4+11x^3+41x^2+61x+30)}{72}+$$

$$+\frac{5(x^4+5x^3+5x^2-5x-6)}{18}+$$

$$+\frac{17(x^4+7x^3+9x^2-7x-10)}{8}-$$

$$-\frac{11(x^4+8x^3+14x^2-8x-15)}{9}-$$

$$-\frac{(x^4+9x^3+21x^2-x-30)}{4}=$$

$$= \frac{5x^4 + 55x^3 + 205x^2 + 305x + 150 +}{72}$$

$$\underline{+ 20x^4 + 100x^3 + 100x^2 - 100x -}$$

$$\underline{- 120 + 153x^4 + 1071x^3 + 1377x^2 -}$$

$$\underline{- 1071x - 1530 - 88x^4 - 704x^3 -}$$

$$\underline{- 1232x^2 + 704x + 1320 - 18x^4 -}$$

$$\underline{- 162x^3 - 378x^2 + 18x + 540} =$$

$$\cancel{= \frac{72x^4 + 360x^3 + 72x^2 - 144x -}{72}} =$$

$$\cancel{= x^4 + 5x^3 + x^2 - \frac{101}{24}x - \frac{45}{8}}$$

$$= \frac{72x^4 + 360x^3 + 72x^2 - 144x + 360}{72} =$$

$$= x^4 + 5x^3 + x^2 - 2x + 5$$

Проверка:

$p(1)$ $\quad 1 + 5 + 1 - 2 + 5 =$

$= 7 + 3 = 10$

$p(-5)$ $\quad 625 - 625 + 25 + 10 + 5 =$

$= 40$

$p(-3)$ $\quad 81 - 135 + 9 + 6 + 5 =$

$= -54 + 20 = -34$

$p(-2)$ $\quad 16 - 40 + 4 + 4 + 5 =$

$= -24 + 13 = -11$

$p(-1)$: $1 - 5 + 1 + 2 + 5 =$

$= -4 + 8 = 4$

Ответ: $x^4 + 5x^3 + x^2 - 2x + 5$

$\cancel{x \quad 8}$

$\cancel{\frac{17}{59} \text{ в } \mathbb{Z}_{67}}$

$\cancel{17 \cdot \frac{1}{59} \text{ в } \mathbb{Z}_{67}}$

$\cancel{x = \frac{1}{59}}$

$\cancel{59x = 1} \vdots$

№ 8

$\frac{17}{59}$ в $\mathbb{Z}_{67}$

$$59x - 67y = 17$$

$$59x_0 + 67y_0' = 1$$

$67 = 59 \cdot 1 + 8$

$59 = 8 \cdot 7 + 3$

$8 = 3 \cdot 2 + 2$

$3 = 2 \cdot 1 + 1$

$1 = 3 - 2 = 3 - (8 - 3 \cdot 2) =$

$= 3 \cdot 3 - 8 = (59 - 8 \cdot 7) \cdot 3 - 8 =$

$= 59 \cdot 3 - 8 \cdot 22 = 59 \cdot 3 -$

$- (67 - 59) \cdot 22 = 59 \cdot 25 -$

$- 67 \cdot 22$

$x_0 = 25$ $\quad$ $y_0' = -22$

$y_0 = -y_0' = 22$

В $\mathbb{Z}_{67}$: $\frac{17}{59} = 25 \cdot 17 = 425$

$425 \bmod 67 = 23$

Ответ: 23

№ 9

I способ:

$$\frac{955}{727} = 1 + \frac{228}{727} = 1 + \frac{1}{\frac{727}{228}} =$$

$$= 1 + \frac{1}{3 + \frac{43}{228}} = 1 + \frac{1}{3 + \frac{1}{\frac{228}{43}}} =$$

$$= 1 + \frac{1}{3 + \frac{1}{5 + \frac{13}{43}}} = 1 + \frac{1}{3 + \frac{1}{5 + \frac{1}{\frac{43}{13}}}} =$$

$$= 1 + \frac{1}{3 + \frac{1}{5 + \frac{1}{3 + \frac{4}{13}}}} =$$

$$= 1 + \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{3 + \cfrac{1}{\frac{13}{4}}}}} = 1 + \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{3 + \cfrac{1}{3 + \frac{1}{4}}}}}$$

$$[1;\ 3,\ 5,\ 3,\ 3,\ 4]$$

II способ:

$955 = 727 \cdot 1 + 228$

$727 = 228 \cdot 3 + 43$

$228 = 43 \cdot 5 + 13$

$43 = 13 \cdot 3 + 4$

$13 = 4 \cdot 3 + 1$

$4 = 1 \cdot 4$

НОД $(955,\ 727) = 1$

Ответ: $[1;\ 3,\ 5,\ 3,\ 3,\ 4]$