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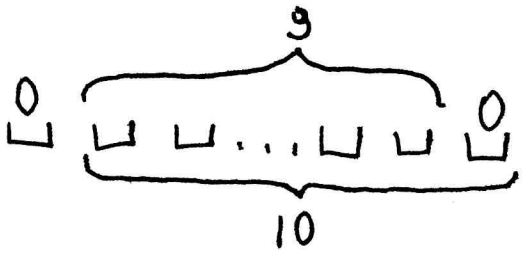
Вариант 9.

Таблица 1. Ответы к задачам.

№	Ответ
1	C_9^6
2	C_{299}^{39}
3	37, 941, 192
4	1495
5	256
6	413100
7	а) $N=22$ или $N=20$ б) $C_{27}^5 - 6 \cdot C_{20}^5$
8	$\frac{454}{455}$

① 

6 единиц, 9 цифр $\Big\} \Rightarrow C_9^6$

Ответ: C_9^6

② $\begin{cases} x_1 + x_2 + x_3 + \ldots + x_{40} = 180 \\ x_i \geq -2 \end{cases}$

$y_i = x_i + 3 \Rightarrow y_i \geq 1$

$\begin{cases} y_1 + y_2 + y_3 + \ldots + y_{40} = 180 + 3 \cdot 40 = 300 \\ y_i \geq 1 \end{cases}$

Ответ: C_{299}^{39}

③ $\underline{8} \cdot \underline{9} \cdot \underline{9} \cdot \underline{9} \cdot \underline{9} \cdot \underline{9} \cdot \underline{9} \cdot \underline{9} - \underline{8} \cdot \underline{8} \cdot \underline{7} \cdot \underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} = 37.941.192$

↑ все вар-ты

↑ все разные

Ответ: 37.941.192

④ $A = \{a, b, c, d, e\}$

$a = 0$
$b = 1$
$c = 2$
$d = 3$
$e = 4$

$02143_4 \rightarrow X_{10}$

$X_{10} = 4 + 3 \cdot 5 + 4 \cdot 5^2 + 1 \cdot 5^3 + 2 \cdot 5^4 = 1494$

Ответ: 1495

⑤ $\{A\} - 375$

$\{7\} - 206$

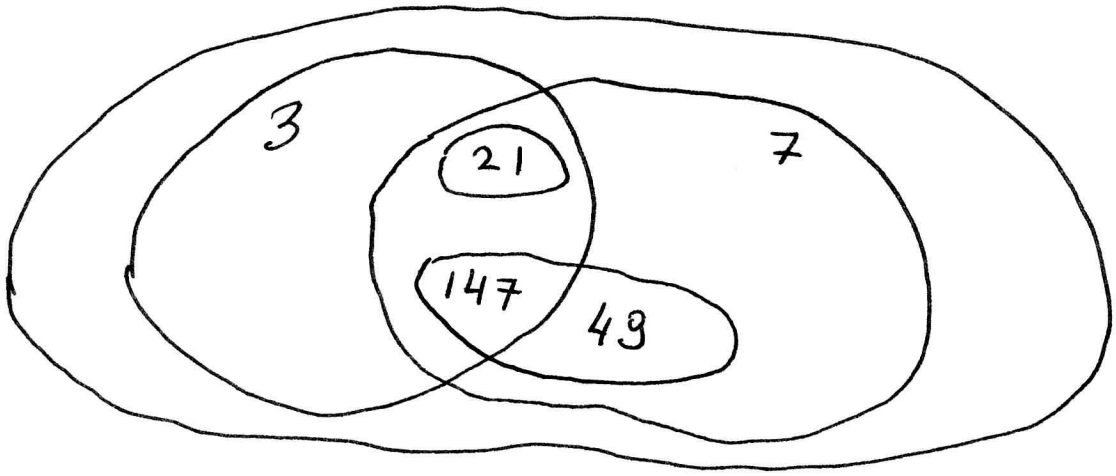
$\{3\} - 115$

$\{49\} - 46$

$\{21\} - 89$

$\{147\} - 42$

$\{7\} \cup \{\overline{3}\} \cup \{\overline{49}\}$



$\{A\} - \{3\} - \{49\} + \{147\} =$

$= 375 - 115 - 46 + 42 = 256$

Ответ: 256

⑥ $3079_{10} \to X_!$

1) $3079 - 1 = 3078$

2) $3078 = 2 \cdot 1539 + 0$

$1539 = 3 \cdot 513 + 0$

$513 = 4 \cdot 128 + 1$

$128 = 5 \cdot 25 + 3$

$25 = 6 \cdot 4 + 1$

$4 = 7 \cdot 0 + 4$

$X = 413100_!$

4	7654321	5
1	764321	2
3	76431	6
1	7431	3
0	741	1
0	74	4
∅	7	7

Ответ: 5263147

(7) $x_1 x_2 x_3 x_4 x_5 x_6 \qquad x_i \in [0; 7]$

а) $x_1 + x_2 + 6 = x_3 + x_4 + x_5 + x_6$

$$\begin{cases} x_i = a_i, & i \le 7 \\ x_i = 7 - a_i, & i > 7 \end{cases}$$

$a_1 + a_2 + 6 = 7 - a_3 + 7 - a_4 + 7 - a_5 + 7 - a_6$

$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 22$

$N = 22$

$$\begin{cases} x_i = 7 - a_i, & i \le 7 \\ x_i = a_i, & i > 7 \end{cases}$$

$7 - a_1 + 7 - a_2 + 6 = a_3 + a_4 + a_5 + a_6$

$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 20$

$N = 20$

$\boxed{N = 20 \text{ или } N = 22}$

б) I способ

$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 22$

1. $a_i \ge 0 \Rightarrow C_{22+6-1}^{6-1} = C_{27}^{5}$

2. $a_i > 7$

$a_1' = a_1 - 7$

$a_1 - 7 + a_2 + a_3 + a_4 + a_5 + a_6 = 22 - 7$

$a_1' + a_2 + a_3 + a_4 + a_5 + a_6 = 15$

$C_{15+6-1}^{6-1} = C_{20}^{5}$

$\boxed{C_{27}^{5} - 6 \cdot C_{20}^{5}}$

II способ

$a_1+a_2+a_3+a_4+a_5+a_6=22$, $a_i \in [0;7]$

$(1+X+X^2+\dots+X^7)^6=\dots+k_{22}X^{22}+\dots$

$X^{22}=X^{y_1}\cdot X^{y_2}\cdot\dots\cdot X^{y_6}=X^{y_1+y_2+\dots+y_6} \Rightarrow 22=y_1+y_2+\dots+y_6$

$f=(1+X+X^2+\dots+X^7)^6=\left(\frac{1-X^8}{1-X}\right)^6=\frac{(1-X^8)^6}{(1-X)^6}$

$\frac{1}{1-X}=1+X+X^2+\dots X^n+\dots$

$f=(1-X^8)^6\cdot\left(\frac{1}{1-X}\right)^6$ $\Rightarrow f=(1-X^8)^6\cdot(1+X_1+X_2+\dots+X^n+\dots)^6$

$(1-X^8)^6=1-6X^8+\dots$

$(1+X+X^2+\dots+X^n+\dots)^6=C_{27}^5X^{22}+C_{20}^5X^{14}$

$(1-6X^8)(\dots+C_{27}^5X^{22}+C_{20}^5X^{14})=\underline{\left(C_{27}^5-6C_{20}^5\right)X^{22}}$

Ответ: $C_{27}^5-6C_{20}^5$, $N=22$ или $N=20$

(8) 3-к
12-3

$1-\frac{3}{15}\cdot\frac{2}{14}\cdot\dots\cdot\frac{1}{13}=1-\frac{1}{455}=\frac{454}{455}$

Ответ: $\frac{454}{455}$