ИДЗ.

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Вариант 5.

Номер задания	Ответы
1	$x = 114 + 124t;\ y = -126 - 137t$
2	$[13; \overline{2, 26}]$
3	77209
4.	37
5	$x^4 + 4x^3 - x^2 + 2x + 1$
6	$-\frac{1}{3}$
7	43_7
8	15
9	[4, 1, 2, 1, 2, 2]
10	$4x^2 + 2x + 1$

1. $2603x + 2356y = -114$

$2603 = 2356 \cdot 1 + 247$

$2356 = 247 \cdot 9 + 133$

$247 = 133 \cdot 1 + 114$

$133 = 114 \cdot 1 + 19$

$114 = 19 \cdot 6 + 0$

$137x + 126y = -6$

$137x_0 + 126y_0 = 1$

r	137	124	13	7	6	1
q		1	9	1	1	
x	1	0	1	-9	10	-19
y	0	1	-1	10	-11	21

$x_0 = -19 \qquad y_0 = 21$

$137(-6x_0) + 124(-6y_0) = -6$

$137(114 + 124t) + 124(-126 - 137t) = -6$

Ответ: $x = 114 + 124t$

$y = -126 - 137t$

Проверка: $137 \cdot 114 + 124(-126) =$

$= 15618 - 15624 = -6$ верно

2. $\sqrt{180} = 13 + (\sqrt{180} - 13) = 13 + \frac{1}{\left(\frac{1}{\sqrt{180}-13}\right)} =$

$= 13 + \frac{1}{\left(\frac{\sqrt{180}+13}{180-169}\right)} = 13 + \frac{1}{\left(\frac{\sqrt{180}+13}{11}\right)} =$

$= 13 + \frac{1}{2 + \frac{\sqrt{180}+13-26}{11}} = 13 + \frac{1}{2 + \frac{\sqrt{180}-13}{11}} =$

$= 13 + \frac{1}{2 + \frac{1}{\left(\frac{11}{\sqrt{180}-13}\right)}} = 13 + \frac{1}{2 + \frac{1}{\left(\frac{\cancel{11}(\sqrt{180}+13)}{\cancel{11}}\right)}} =$

$= 13 + \frac{1}{2 + \frac{1}{26}} = [13; \overline{2, 26}]$

Проверка:

$[a; \overline{\ldots 2a}]$

$26 = 13 \cdot 2$.

Ответ: $[13; \overline{2, 26}]$

3. $x \equiv 11 \bmod 29$ $\quad M = 29 \cdot 17 \cdot 15 \cdot 13 = 96135$

$x \equiv 12 \bmod 17$ $\quad M_1 = 17 \cdot 15 \cdot 13 = 3315$

$x \equiv 4 \bmod 15$ $\quad M_2 = 29 \cdot 15 \cdot 13 = 5655$

$x \equiv 2 \bmod 13$ $\quad M_3 = 29 \cdot 17 \cdot 13 = 6409$

$M_4 = 29 \cdot 17 \cdot 15 = 7395$

1) $3315 x_1 = 1 \bmod 29$

$3315 x_1 - 29 y_1 = 1$

r	3315	29	9	2	1
q		114	3	4	2
x	1	0	1	-3	13

$x_1 = 13 \bmod 29 = 13$

2) $5655 x_2 = 1 \bmod 17$

$5655 x_2 - 17 y_2 = 1$

r	5655	17	11	6	5	1
q		332	1	1	1	1
x	1	0	1	-1	2	-3

$x_2 = -3 \bmod 17 = 14 \bmod 17 = 14$

3) $6409 x_3 = 1 \bmod 15$

$6409 x_3 - 15 y_3 = 1$

r	6409	15	4	3	1
q		427	3	1	3
x	1	0	1	-3	4

$x_3 = 4 \bmod 15 = 4$

4) $7395 x_4 = 1 \bmod 13$

$7395 x_4 - 13 y_4 = 1$

r	7395	13	11	2	1
q		568	1	5	2
x	1	0	1	-1	6

$x_4 = 6 \bmod 13 = 6$

i	x	M	m	c
1	13	3315	29	11
2	14	5655	17	12
3	4	6409	15	4
4	6	7395	13	2

$x = (13 \cdot 3315 \cdot 11 + 14 \cdot 5655 \cdot 12 + 4 \cdot 6409 \cdot 4 + 6 \cdot 7395 \cdot 2) \bmod 96135$

$x = 1615369 \bmod 96135$

$x = 77209 \bmod 96135$

Проверка:

$77209 \bmod 29 = 11$

$77209 \bmod 17 = 12$

$77209 \bmod 15 = 4$

$77209 \bmod 13 = 2$

$77209 : 29 = 2662$; 58; 192; 174; 180; 174; 69; 58; 11

$77209 : 17 = 4541$; 68; 92; 85; 70; 68; 29; 17; 12

$77209 : 15 = 5147$; 75; 22; 15; 70; 60; 109; 105; 4

$77209 : 13 = 5939$; 65; 122; 117; 50; 39; 119; 117; 2

Ответ: 77209.

4. $5^{3^{37}} \bmod 44$

$3^{37} = k \Rightarrow 5^k \bmod 44$

$\varphi(44) = \varphi(4)\cdot\varphi(11) = \varphi(2^2)\cdot\varphi(11) = 2^2\left(1-\frac{1}{2}\right)\cdot 10 =$

$= 20$

$k = 20n + b$

$5^k \bmod 44 = 5^{20n+b} \bmod 44 = 5^b \bmod 44$

$k = 3^{37} = 20n + b$

$b = 3^{37} \bmod 20$

$\varphi(20) = \varphi(2^2)\cdot\varphi(5) = 2\cdot 4 = 8$

$b \equiv 3^{2z+a} \bmod 20$

$b \equiv 3^a \bmod 20$

$37 = 2z + a$

$a = 37 \bmod 2 = 1$

$b \equiv 3^1 \bmod 20 \equiv 3 \bmod 20 \equiv 3$

$5^3 \bmod 44 = 125 \bmod 44 = 37 \bmod 44 = 37$

Ответ: 37.

5. $p(-4) = -23$ $p(-1) = -5$

$p(-3) = -41$ $p(2) = 49$

$p(-2) = -23$

$$P(x)=\frac{(x+3)(x+2)(x+1)(x-2)}{-1\cdot(-2)(-3)(-6)}\cdot(-23)+$$

$$+\frac{(x+4)(x+2)(x+1)(x-2)}{1\cdot(-1)(-2)(-5)}\cdot(-41)+\frac{(x+4)(x+3)(x+1)(x-2)}{2\cdot 1\cdot(-1)(-4)}\cdot(-23)+$$

$$+\frac{(x+4)(x+3)(x+2)(x-2)}{3\cdot 2\cdot 1\cdot(-3)}(-5)+\frac{(x+4)(x+3)(x+2)(x+1)}{6\cdot 5\cdot 4\cdot 3}\cdot 49=$$

$$=\frac{(x^2+5x+6)(x^2-x-2)}{36}(-23)+\frac{(x^2+6x+8)(x^2-x-2)}{10}\cdot 41+$$

$$+\frac{(x^2+7x+12)(x^2-x-2)}{8}\cdot(-23)+\frac{(x^2+7x+12)(x^2-x-2)}{18}\cdot 5+$$

$$+\frac{(x^2+7x+12)(x^2+3x+2)}{360}\cdot 49=$$

$$=-\frac{230}{360}(x^4-x^3-2x^2+5x^3-5x^2-10x+6x^2-6x-12)+$$

$$+\frac{1476}{360}(x^4-x^3-2x^2+6x^3-6x^2-12x+8x^2-8x-16)-$$

$$-\frac{1035}{360}(x^4-x^3-2x^2+7x^3-14x+12x^2-12x-24)+$$

$$+\frac{100}{360}(x^4-4x^2+7x^3-28x+12x^2-48)+$$

$$+\frac{49}{360}(x^4+3x^3+2x^2+7x^3+21x^2+14x+12x^2+36x+24)=$$

$$=-\frac{230}{360}(x^4+4x^3-x^2-16x-12)+\frac{1476}{360}(x^4+5x^3-20x-16)-$$

$$-\frac{1035}{360}(x^4+6x^3+3x^2-26x-24)+\frac{100}{360}(x^4+7x^3+8x^2-28x-48)+$$

$$+\frac{49}{360}\left(x^4+10x^3+35x^2+50x+24\right)=$$

$$=\frac{x^4(-230+1476-1035+149)+x^3(-920+7380-6210+900+490)}{360}+$$

$$+\frac{x^2(230-3105+800+1715)+x(3680-29520+26910+2800+2450)}{360}+$$

$$+\frac{2760-23616+24840-4800+1170}{360}=$$

$$=\frac{360x^4+1440x^3-360x^2+720x+360}{360}=$$

$$=x^4+4x^3-x^2+2x+1$$

Проверка:

$$p(-1)=1+4(-1)-\cancel{1}-2+\cancel{1}=-5$$

$$p(-4)=256-256-16-8+1=-23$$

$$p(2)=16+32-\cancel{4}+\cancel{4}+1=49$$

$$p(-2)=16-32-4-4+1=-23$$

$$p(-3)=81-108-9-6+1=-41$$

Ответ: $x^4+4x^3-x^2+2x+1$

6. $6x^4+14x^3-2x^2+7x+3=0$

$$\frac{p}{q}=\frac{\pm 3;\ \pm 1}{\pm 6;\ \pm 2;\ \pm 3;\ \pm 1}$$

x	6	14	-2	7	3	
1	6	20	18	25	28	
-1	6	8	-10	17	-14	
$\frac{1}{2}$	6	17	$\frac{13}{2}$	$\frac{28+13}{4}$	...	
$-\frac{1}{2}$	6	11	$-\frac{15}{2}$	$\frac{29}{4}$	...	
3	6	32	94	...		
-3	6	-4	10	-23	...	
$\frac{3}{2}$	6	23	$\frac{65}{2}$	...		
$-\frac{3}{2}$	6	5	$-\frac{19}{2}$	$\frac{85}{4}$...		
$-\frac{1}{3}$	6	12	-6	9	0	✓

$$\left(x+\frac{1}{3}\right)\left(6x^3+12x^2-6x+9\right)=0$$

$$\left(x+\frac{1}{3}\right)\left(2x^3+4x^2-2x+3\right)=0$$

Проверка:

$$f\left(-\frac{1}{3}\right)=\frac{6}{81}-\frac{14}{27}^{3}-\frac{2}{9}^{9}-\frac{7}{3}^{27}+3^{81}=$$

$$=\frac{6-42-18-189+243}{81}=0$$

Ответ: $-\frac{1}{3}$.

7. $6X + 142_7 = 526_7$

1) $\overset{2\,1\,0}{142_7} = 49 + 4 \cdot 7 + 2 = 79_{10}$

$6_7 = 6_{10}$

$\overset{2\,1\,0}{526_7} = 49 \cdot 5 + 14 + 6 = 265_{10}$

$6X + 79 = 265$

$6X = 186$

$X = 31_{10}$

$X = 43_7$

$$\begin{array}{r|l}
31 & 7 \\
\hline
28 & 4 \;|\; 7 \\
3 & 0 \;|\; 0 \\
 & 4
\end{array}$$

2) $6X + 142_7 = 526_7$

$6X = 354_7$

$$\begin{array}{r}
526_7 \\
-\ 142_7 \\
\hline
354_7
\end{array}$$

$$\begin{array}{r|l}
354 & 6 \\
\hline
33 & 43_7 \\
24 & \\
24 & \\
\hline
0 &
\end{array}$$

Ответ: 43_7

8. $\frac{6}{62} \bmod 77 = \frac{3}{31} \bmod 77$

$X = \frac{3}{31} \bmod 77$

$31X = 3 \bmod 77$

$31X - 77y = 3$; $-y = y'$

$31X + 77y' = 3$

$77 = 31 \cdot 2 + 15$

$31 = 15 \cdot 2 + 1$ $\Rightarrow$ 1 решение

$31x_0 + 77y_0 = 1$

r	31	77	31	15	1
q		0	2	2	
x	1	0	1	-2	5

$x_0 = 5$

$x = 3 \cdot 5 + 77n, \ n \in \mathbb{Z}$

$x = 15 + 77n$

Ответ: 15

9. 1) $\frac{123}{26} = 4 + \frac{19}{26} = 4 + \cfrac{1}{\left(\frac{26}{19}\right)} = 4 + \cfrac{1}{1 + \cfrac{7}{19}} =$

$= 4 + \cfrac{1}{1 + \cfrac{1}{\left(\frac{19}{7}\right)}} = 4 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{5}{7}}} =$

$= 4 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{\left(\frac{7}{5}\right)}}} = 4 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{2}{5}}}} =$

$= 4 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{\left(\frac{5}{2}\right)}}}} = 4 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{2}}}}} = [4, 1, 2, 1, 2, 2]$

2) $123 = 26 \cdot 4 + 19$

$26 = 19 \cdot 1 + 7$

$19 = 7 \cdot 2 + 5$

$7 = 5 \cdot 1 + 2$

$5 = 2 \cdot 2 + 1$

$2 = 1 \cdot 2 + 0$

Ответ: [4, 1, 2, 1, 2, 2]

10. $3x^5 + 5x^4 + 0 \cdot x^3 + 5x^2 + 6x + 2 \;\big|\; 4x^3 + 6x^2 + 3x + 1$

$$
\begin{array}{l|l}
-\;3x^5 + 5x^4 + 0\cdot x^3 + 5x^2 + 6x + 2 & 4x^3 + 6x^2 + 3x + 1 \\
\underline{\;\;3x^5 + x^4 + 4x^3 + 6x^2\;\;} & \overline{6x^2 + x + 1} \\
\quad -\;4x^4 + 3x^3 + 6x^2 + 6x & \\
\quad \underline{\;\;4x^4 + 6x^3 + 3x^2 + x\;\;} & \\
\qquad -\;4x^3 + 3x^2 + 5x + 2 & \\
\qquad \underline{\;\;4x^3 + 6x^2 + 3x + 1\;\;} & \\
\qquad\quad 4x^2 + 2x + 1 &
\end{array}
$$

$\frac{3}{4} = x \bmod 7$

$3 = 4x \bmod 7$

$x = 6$

Проверка:

$(6x^2 + x + 1)(4x^3 + 6x^2 + 3x + 1) + 4x^2 + 2x + 1 =$

$= 3x^5 + x^4 + 4x^3 + 6x^2 + 4x^4 + 6x^3 + 3x^2 + x +$

$+ 4x^3 + 6x^2 + 3x + 1 + 4x^2 + 2x + 1 =$

$= 3x^5 + 5x^4 + 5x^2 + 6x + 2$ - верно

$-1 \bmod 7 = 6 \bmod 7$

$-2 \bmod 7 = 5 \bmod 7$

$\frac{1}{4} = x \bmod 7$

$1 = 4x \bmod 7$

$x = 2$

$\frac{5}{4} = x \bmod 7$

$5 = 4x \bmod 7$

$x = 3$

Ответ: $4x^2 + 2x + 1$