Вариант 23 Аничков

1	$\begin{cases} x = 180 + 181k \\ y' = -186 - 187k \end{cases}$
2	$[14 : \overline{1, 4, 1, 28}]$
3	329046 mod 881790
4	44
5	$p(x) = x^4 + 4x^3 + 2x^2 + 5x + 5$
6	рациональные корней нет
7	$23_{10} = 35_6$
8	$473 \bmod 94 \equiv \frac{61}{65} \bmod 94$
9	[4, 4, 1, 1, 4, 5]
10	$x^2 + 2x + 2$

№3

$x_1 \equiv 11 \bmod 35$; $x_2 \equiv 28 \bmod 34$; $x_3 \equiv 3 \bmod 39$; $x_4 \equiv 4 \bmod 19$

1) $M = 35 \cdot 34 \cdot 39 \cdot 19 = 881790$

$M_1 = 25194$ $\quad M_3 = 22610$

$M_2 = 25935$ $\quad M_4 = 46410$

2) $25194x_1 - 35y = 1$

$x_1 = -6$

r	25194	35	29	6	5	1
q		719	1	4	1	5
x	1	0	1	-1	5	-6

$25935x_2 - 34y = 1$

$x_2 = -5$

r	25935	34	27	7	6	1
q		762	1	3	1	6
x	1	0	1	-1	4	-5

$22610x_3 - 39y = 1$

$x_3 = -4$

r	22610	39	29	10	9	1
q		579	1	2	1	9
x	1	0	1	-1	3	-4

$46410x_4 - 19y = 1$

$x_4 = 8$

r	46410	19	12	7	5	2	1
q		2442	1	1	1	2	2
x	1	0	1	-1	2	-3	8

3) $x \equiv (25194(-11) \cdot 11 + 25935 \cdot (-5) \cdot 28 + 22610(-4) \cdot 3 + 46410 \cdot 8 \cdot 4) \bmod 881790$

$x \equiv -4079904 \bmod 881790$

$-4079904 + 881790 \cdot 5 = 329046 \Rightarrow x \equiv 329046 \bmod 881790$

Ответ: $329046 \bmod 881790$

№ 8

$\frac{61}{65} \mod 94 \Rightarrow 65x \equiv 61 \mod 94$

$65x - 94y = 61$

$y' = -y$

НОД$(65; 94) = 1 \Rightarrow$ $65x + 94y' = 1$

$94 = 1 \cdot 65 + 29$

$65 = 29 \cdot 2 + 7$

$29 = 7 \cdot 4 + 1$

$7 = 7 \cdot 1$

r	65	94	65	29	7	1
q		0	1	2	4	7
x	1	0	1	-1	3	-13

$x = -13$, но $-13 \notin \mathbb{Z}_{94} = 94 - 13 = 81$

$x = 81 \cdot 7 = 567$

$x = 567 + 94k, k \in \mathbb{Z}$

$x \equiv 567 \mod 94 \equiv 473 \mod 94$

Ответ: $473 \mod 94 \equiv \frac{61}{65} \mod 94$

№1

$3179x - 3077y = -102$

1) Пусть $y' = -y \Rightarrow 3179x + 3077y = -102$

НОД$(3179, 3077) = 17$

$187x + 181y' = -6$

~~187 = 1·181 + 6~~

$3179 = 3077 \cdot 1 + 102$

$3077 = 30 \cdot 102 + 17$

$102 = 6 \cdot 17$

i	-1	0	1	2
r	187	181	6	1
q		1	30	6
x	1	0	1	-30
y	0	1	-1	31

$x_0 = -30,\ y_0' = 31$

$x_1 = x_0 \cdot \frac{c}{d} = -30 \cdot (-6) = 180$

$y_1 = y_0 \cdot \frac{c}{d} = 31 \cdot (-6) = -186$

$$\begin{cases} x = 180 + 181k \\ y' = -186 - 187k \end{cases}$$

Проверка

$3179(180 + 181k) + (3077)(-186 - 187k) = -102$

Ответ: $\begin{cases} x = 180 + 181k \\ y' = -186 - 187k \end{cases},\ k \in Z$

№5

$p(-1)=-1;\ p(-3)=-19;\ p(1)=17;\ p(-4)=17;\ p(-2)=-13$

$$p(x)=\frac{(x+3)(x-1)(x+4)(x+2)}{2\cdot(-2)\cdot3\cdot1}(-1)+\frac{(x+1)(x-1)(x+4)(x+2)}{(-2)(-4)(1)(-1)}(-19)+$$

$$+\frac{(x+1)(x+3)(x+4)(x+2)}{2\cdot4\cdot5\cdot3}17+\frac{(x+1)(x+3)(x-1)(x+2)}{(-3)(-1)(-5)(-2)}17+\frac{(x+1)(x+3)(x-1)(x+4)}{(-1)\,1\,(-3)\,2}(-13)=$$

$$=\frac{(x^2-x+3x+2)(x^2+2x+4x+8)}{-12}(-1)+\frac{(x^2-1)(x^2+2x+4x+8)}{-8}(-19)+\frac{(x^2+3x+x+3)(x^2+6x+8)}{120}\times$$

$$\times17+\frac{(x^2-1)(x^2+5x+6)}{30}17+\frac{(x^2-1)(x^2+7x+12)}{6}(-13)=$$

$$=\frac{1}{12}(x^4+8x^3+17x^2+28x+24)+\frac{19}{8}(x^4+6x^3+7x^2-6x-8)+\frac{17}{120}(x^4+10x^3+35x^2+50x+24)+$$

$$+\frac{17}{30}(x^4+5x^3+5x^2-5x-6)-\frac{13}{6}(x^4+7x^3+11x^2-7x-12)=$$

$$=x^4\left(\frac{1}{12}+\frac{19}{8}+\frac{17}{120}+\frac{17}{30}-\frac{13}{6}\right)+x^3\left(\frac{114}{8}+\frac{8}{12}+\frac{170}{120}+\frac{85}{30}-\frac{91}{6}\right)$$

$$+x^2\left(\frac{17}{12}+\frac{133}{8}+\frac{595}{120}+\frac{85}{30}-\frac{143}{6}\right)+x\left(\frac{28}{12}-\frac{114}{8}+\frac{850}{120}-\frac{85}{30}\right.$$

$$\left.+\frac{91}{6}\right)+\left(\frac{24}{12}-19+\frac{408}{120}-\frac{102}{30}+26\right)=x^4+4x^3+2x^2+5x+5$$

Проверку

$p(1)=1+4+2+5+5=17$

$p(-1)=(-1)^4+4(-1)^3+2(-1)^2+5(-1)+5=-1$

$p(-2)=(-2)^4+4(-2)^3+2(-2)^2+5(-2)+5=-13$

$p(-3)=(-3)^4+4(-3)^3+2(-3)^2+5(-3)+5=-19$

$p(-4)=(-4)^4+4(-4)^3+2(-4)^2+5(-4)+5=17$ Ответ: $p(x)=x^4+4x^3+2x^2+5x+5$

№7

$4x_6 + 142_6 = 414_6$

1 способ

$4_6 = 4_{10}$

$142_6 = 62_{10}$

$414_6 = 154_{10}$

$4x + 62 = 154$

$4x = 154 - 62$

$4x = 92$

$x = 23$

$23_{10} = 35_6$

Ответ: 35

2 способ

$$\cancel{\begin{array}{r} 414 \\ -\ 142 \\ \hline 224 \end{array}} \qquad \cancel{\begin{array}{r|l} 224 & 4 \\ -\ 20 & \hline 24 & 34 \end{array}}$$

$$\begin{array}{r} {}_6 \\ 414 \\ -\ 142 \\ \hline 232 \end{array} \qquad \begin{array}{r|l} 232 & 4 \\ -\ 20 & \hline 32 & 35 \end{array}$$

Проверка.

$4 \cdot 23 + 62 = 154$ → Корни найдены верно.

№10

$$
\begin{array}{r|l}
4x^5+3x^4+3x^3+4x^2+x+4 & 2x^3+x^2+3x+1 \\
-\ 4x^5+2x^4+x^3+2x^2 & \overline{2x^2+3x+2} \\
\hline
x^4+2x^3+2x^2+x+4 & \\
-\ x^4+3x^3+4x^2+3x & \\
\hline
4x^3+3x^2+3x+4 & \\
-\ 4x^3+2x^2+x+2 & \\
\hline
x^2+2x+2 &
\end{array}
$$

ответ: x^2+2x+2

1	$x=-3077k-1$ $y=-3179k-1$
2	[4; 1; 4; 1; 4]
3	644046
4	44
5	—
6	рациональных корней нет
7	414_6 или 154_{10}
8	41
9	[4, 4, 1, 1, 4, 5]
10	x^2+2x+2

Вариант 23

№1

$$3179x - 3077y = -102$$

$$3179\cdot(-1) - 3077\cdot(-1) = -102$$

$$3179\cdot(-1) - 3077\cdot(-1) = 3179x - 3077y$$

$$3179(-1-x) = 3077(-1-y)$$

$$(-1-y) = 3179k$$

$$3179(-1-x) = 3077\cdot 3179k$$

$$-1-x = 3077k$$

$$1+x = -3077k$$

$$x = -3077k - 1$$

$$y = -3179k - 1$$

№2

$$\sqrt{220} = 14 + \sqrt{220} - 14 = 14 + \frac{1}{\frac{1}{\sqrt{220}+14}}$$

r	22610	39	29	10	9	1	
q		579	1	2	1	9	
x	1	0	1	−1	3	−4	

$x_3 = -4 \quad -4 \bmod 39 \equiv 35 \bmod 39$

r	46410	19	12	7	5	2	1
q		2442	1	1	1	2	2
x	1	0	1	−1	2	−3	8

$x_4 = 8$

$x = 25194 \cdot (29) \cdot 11 + 25935 \cdot (29) \cdot 28 + 25810 \cdot (35) \cdot 34 +$

$+ 46410 \cdot 8 \cdot 4) \bmod 881790 = 33270276 \bmod 881790$

Проверка $644046 \equiv 11 \bmod 35$

$644046 \equiv 28 \bmod 34$

$644046 \equiv 3 \bmod 39$

$644046 \equiv 4 \bmod 19$

Ответ: 644046

$+5,75 = 4x^3+3x^2+3x+4x \ldots$

Ответ: $2,25x^2+4,75x+5,75$

№6

$x^4-5x^3-6x^2+7x-2$

	1	-5	-6	7	-2
1	1	-4	-10	-3	-5
-1	1	-6	0	7	-9
2	1	-3	-12	-17	-36
-2	1	-7	8	-9	16

Проверка $f(1)=-5$

$f(-1)=-9$

$f(2)=-36$

$f(-2)=16$

Ответ: рациональных корней нет.

$) + 2{,}25x^2 + 4{,}75x +$

$f(1) = -5$

$f(-1) = -3$

$f(2) = -36$

$f(-2) = 16$

$4x + 142 = 414$

$4x + 62 = 154$

$4x = 154 - 62$

$4x = 92$

$x_{10} = 23$

$x_6 = 35$

$\overset{210}{142} = 2 + 24 + 36 = 62$

$\overset{210}{414} = 4 + 6 + 144 = 154$

$$\begin{array}{r|l} 23 & 6 \\ -18 & 3 \\ \hline 5 & \end{array}$$

$4 \cdot 35 + 142 = 414$ – 6-ичной

$4 \cdot 23 + 62 = 154$ – 10-ичной

r	762	7	3	1	6
q	0	1	−1	4	−5
x	1				

$x_2 = -5$ $\quad -5 \bmod 34 \equiv 29 \bmod 34$

№8

$x = \frac{61}{65} \bmod 94$

$65x \equiv 61 \bmod 94$

$65x - 94y = 61 \qquad y' = -y$

$65x + 94y' = 61$

НОД$(65; 94) = 1$

r	65	94	65	29	7	1	0
q		0	1	2	4	7	
x	1	0	1	−1	3	−13	

$x = -13 \cdot \frac{61}{1} + \frac{94}{1}k = -793 + 94k$

$x = -793 \bmod 94 = 41$

Ответ: 41

№ 18

$$\begin{array}{l|l} 4x^5+3x^4+3x^3+4x^2+x+4 & 2x^3+x^2+3x+1 \\ \underline{4x^5+2x^4+6x^3+2x^2} & \overline{2x^2+\frac{1}{2}x-1\frac{3}{4}} \\ \quad x^4-3x^3+2x^2+x+4 & \\ \quad \underline{x^4+\frac{1}{2}x^3+1{,}5x^2+\frac{1}{2}x} & \\ \qquad -3{,}5x^3+0{,}5x^2+\frac{1}{2}x+4 & \\ \qquad \underline{-3{,}5x^3-1\frac{3}{4}x^2-5\frac{1}{4}x-1\frac{3}{4}} & \\ \qquad\quad 2{,}25x^2+4{,}75x+5{,}75 & \end{array}$$

Проверка: $\left(2x^2+\frac{1}{2}x-1\frac{1}{4}\right)\cdot\left(2x^3+x^2+3x+1\right)+2{,}25x^2+4{,}75x+$
$+5{,}75=4x^5+3x^4+3x^3+4x^2+x+4$

Ответ: $2{,}25x^2+4{,}75x+5{,}75$

№ 6

$x^4-5x^3-6x^2+7x-2$

	1	−5	−6	7	−2
1	1	−4	−10	−3	−5
−1	1	−6	0	7	−9
2	1	−3	−12	−17	−36

Проверка $f(1)=-5$

$f(-1)=-9$

$f(2)=-36$

$f(-2)=$ [illegible]

№9

$$\frac{903}{214} = 4 + \frac{47}{214} = 4 + \cfrac{1}{\frac{214}{47}} = 4 + \cfrac{1}{4 + \frac{26}{47}} = 4 + \cfrac{1}{4 + \cfrac{1}{1 + \frac{21}{26}}} =$$

$$= 4 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{1 + \frac{5}{21}}}} = 4 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{4 \frac{1}{5}}}}}$$

Ответ: [4, 4, 1, 1, 4, 5]

№ 3

$\equiv 11 \bmod 35$

$\equiv 28 \bmod 34$

$3 \bmod 39$

$4 \bmod 19$

35 ...

r	$23^{-1}94$	35	29	6	5	1	0
q		7 19	1	4	1	5	
x	1	0	1	−1	5	−6	

$x_1 = -6$ $\quad -6 \bmod 35 \equiv 29 \bmod 35$

r	$2^5 935$	34	27	7	6	1
q		762	1	3	1	6
x	1	0	1	−1	4	−5

$x_2 = -5$ $\quad -5 \bmod 34 \equiv 29 \bmod 34$

№3

$x \equiv 11 \bmod 35$

$x \equiv 28 \bmod 34$

$x \equiv 3 \bmod 39$

$x \equiv 4 \bmod 19$

$M = 35 \cdot 34 \cdot 39 \cdot 19 = 46410 \cdot 19 = 881790$

$M_1 = 34 \cdot 39 \cdot 19 = 25194$

$M_2 = 35 \cdot 39 \cdot 19 = 25935$

$M_3 = 35 \cdot 34 \cdot 19 = 22610$

$M_4 = 35 \cdot 34 \cdot 39 = 46410$

$$14+\cfrac{1}{1+\cfrac{1}{4+\cfrac{4+\{\sqrt{220}\}}{5}}}=14+\cfrac{1}{1+\cfrac{1}{4+\cfrac{18(\sqrt{220}+10)}{24}}}=14+\cfrac{1}{1+\cfrac{1}{4+\cfrac{24}{\{\sqrt{220}\}+24}}}=$$

$$=14+\cfrac{1}{1+\cfrac{1}{4+\ldots}}=14+\cfrac{1}{1+\cfrac{1}{4+\cfrac{\frac{1}{5}}{4+\{\sqrt{220}\}}}}=14+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{4+\{\sqrt{220}\}}}}}=$$

Ответ: $[14;\overline{1,4,1}]$

№2

$$\sqrt{220} = 14 + \sqrt{220} - 14 = 14 + \frac{1}{\frac{\sqrt{220}+14}{(\sqrt{220}-14)(\sqrt{220}+14)}} =$$

$$= 14 + \frac{1}{\frac{\sqrt{220}+14}{220-196}} = 14 + \frac{1}{\frac{\sqrt{220}+14}{24}} = 14 + \frac{1}{\frac{28+\{\sqrt{220}\}}{24}} = 14 + \frac{1}{1+\frac{1}{\frac{24}{4+\{\sqrt{220}\}}}}$$

$$= 14 + \frac{1}{1+\frac{1}{\frac{24(\sqrt{220}+10)}{\cancel{120}\,5}}} = 14 + \frac{1}{1+\frac{1}{\frac{\sqrt{220}+10}{5}}} = 14 + \frac{1}{1+\frac{1}{\frac{24+\{\sqrt{220}\}}{5}}} =$$

$$= 14 + \frac{1}{1+\frac{1}{4+\frac{4+\{\sqrt{220}\}}{5}}} =$$

$$= 14 + \frac{1}{1+\frac{1}{4+\frac{1}{\frac{4+\{\sqrt{220}\}}{5}}}}$$

$$= 14 + \frac{\frac{}{\sqrt{220}+14}}{24} =$$

№ 24

$$11^{21^{79}} \bmod 59$$

$$k = 21^{79} \rightarrow 11^{k} \bmod 59$$

$$\varphi(59) = 58$$

$$k = 21^{79} = 58n + b$$

$$b \equiv 21^{79} \bmod 58$$

$$\varphi(58) = \varphi(2) \cdot \varphi(29) = 28$$

$$b \equiv 21^{28 \cdot 2 + 23} \bmod 58 \Rightarrow 21^{23} \bmod 58 \rightarrow$$

$a = 21$, $m = 23$, $k = 58$

$23_{10} = 10111$

a_i	c	c^2	$c^2 a^{a_i}$	$c^2 a^{a_i} \bmod k$
1	1	1	21	21
0	21	441	441	35
1	35	1225	25725	31
1	31	[illegible]	20181	55
1	55	3025	63525	15

$b \equiv 15 \cdot \bmod 58$

$11^{15} \bmod 59 \rightarrow a=11,\ m=15,\ k=59$

$15_{10} = 1111_2$

a_i	c	c^2	$c^2 a^{a_i}$	$c^2 a^{a_i} \bmod k$
1	1	1	11	11
1	11	121	1331	33
1	33	1089	11979	2
1	2	4	44	44

$11^{21,79^2} \equiv 44 \bmod 59$

Ответ:

№6

$x^4 - 5x^3 \ldots$