

Exercise 3

① $21K = 36210$

$$\begin{aligned}\varphi(36210) &= \varphi(3621) \cdot \varphi(10) \\ &= \varphi(1207) \cdot \varphi(2) \cdot \varphi(5) \cdot \varphi(2) \\ &= \varphi(17) \cdot \varphi(71) \cdot \varphi(5) \cdot \varphi(5) \cdot \varphi(2) \\ &= 16 \cdot 70 \cdot 2 \cdot 4 \cdot 1 = 8960\end{aligned}$$

2) $C = 2002$

$$\begin{aligned}\varphi(2002) &= \varphi(1001) \cdot \varphi(2) \\ &= \varphi(143) \cdot \varphi(7) \cdot \varphi(2) \\ &= \varphi(13) \cdot \varphi(11) \cdot \varphi(7) \cdot \varphi(2) \\ &= 12 \cdot 10 \cdot 6 \cdot 1 \\ &= 720\end{aligned}$$

3) $d = 21$

$$\varphi(21) = \varphi(7) \cdot \varphi(3) = 6 \cdot 2 = 12$$

4) $m = 3$

$$\varphi(3) = 2$$

5) $x = 45$

$$\begin{aligned}\varphi(45) &= \varphi(5) \cdot \varphi(9) \\ &= \varphi(5) \cdot \varphi(3^2) \\ &= 4 \cdot 3^2 \left(1 - \frac{1}{3}\right) = 4 \cdot 9 \cdot \frac{2}{3} \\ &= 24\end{aligned}$$