Осенко Александр 0362 Вариант 11

№ 1	$\begin{cases} x = -483 + 160k, k \in Z \\ y = 329 - 109k, k \in Z \end{cases}$
№ 2	$[11; \overline{1,1,1,22}]$
№ 3	472181 mod 580382
№ 4	22 mod 67
№ 5	$x^4 + 3x^3 + (-5)x^2 - x + 5$
№ 6	Рац. корней нет.
№ 7	72_{10}; 80_9
№ 8	x = 81 mod 86
№ 9	[2,5,1,4,4,2]
№ 10	$3x^2 + 2x + 3$

1. $763x + 1120y = -49$

1) НОД(1120; 763) = 7

$1120 = 763 \cdot 1 + 357$

$763 = 357 \cdot 2 + 49$

$357 = 49 \cdot 7 + 14$

$49 = 14 \cdot 3 + 7$

$14 = 7 \cdot 2 + 0$

2) $763/7 = 109$

$1120/7 = 160$

$-49/7 = -7$

$7/7 = 1$

3) $109x + 160y = -7$

$109x + 160y = 1$

	-1	0	1	2	3	4
r	160	109	51	7	2	1
q		1	2	7	3	
x	0	1	-1	3	-22	69
y	1	0	1	-2	15	-47

$x_1 = x_0 \cdot \frac{c}{d} = 69 \cdot (-7) = -483 \Rightarrow x = x_1 + \frac{b}{d}k = -483 + 160k, k \in Z$

$y_1 = y_0 \cdot \frac{c}{d} = -47 \cdot (-7) = 329 \Rightarrow y = y_1 - \frac{a}{d}k = 329 - 109k, k \in Z$

Проверка:

1) $k = 1$

$x = -483 + 160 = -323$

$y = 329 - 109 = 220$

$763 \cdot (-323) + 1120 \cdot 220 = 49$

$49 = 49$

2) $k = 0$

$x = -483$

$y = 329$

$763 \cdot (-483) + 1120 \cdot 329 = 49$

$49 = 49$

2. $$\sqrt{136} = 11 + \sqrt{136} - 11 = 11 + \cfrac{1}{\cfrac{1}{\sqrt{136}-11}} = 11 + \cfrac{1}{1 + \cfrac{\sqrt{136}-4}{15}} =$$

$$= 11 + \cfrac{1}{1 + \cfrac{1}{\cfrac{15}{\sqrt{136}-4}}} = 11 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{\sqrt{136}-4}{8}}} =$$

$$= 11 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{\cfrac{8}{\sqrt{136}-4}}}} = 11 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{\sqrt{136}-11}{15}}}} =$$

$$= 11 + \cfrac{1}{\underline{1} + \cfrac{1}{\underline{1} + \cfrac{1}{\underline{1} + \cfrac{1}{\underline{22} + \sqrt{136} - 11}}}}$$

Проверка:

$[a, \ldots, 2a]$

$11 \cdot 2 = 22$ – верно

Ответ: $[11, \overline{1, 1, 1, 22}]$

3. $X \equiv 17 \bmod 22$ $\quad M = 22\cdot37\cdot31\cdot23 = 580382$

$X \equiv 24 \bmod 37$ $\quad M_1 = 37\cdot31\cdot23 = 26381$

$X \equiv 20 \bmod 31$ $\quad M_2 = 22\cdot31\cdot23 = 15686$

$X \equiv 14 \bmod 23$ $\quad M_3 = 22\cdot37\cdot23 = 18722$

$\quad M_4 = 22\cdot37\cdot31 = 25234$

1) $M_1 x_1 = 1 \bmod 22$

$26381x + 22y' = 1$

	-1	0	1	2
r	26381	22	3	1
q		1199	7	
x	1	0	1	-7

$x_1 = -7$

2) $M_2 x_2 = 1 \bmod 37$

$15686x + 37y' = 1$

	-1	0	1	2	3
r	15686	37	35	2	1
q		423	1	17	
x	1	0	1	-1	18

$x_2 = 18$

3) $M_3 x_3 = 1 \bmod 31$

$18722x + 31y' = 1$

	-1	0	1	2	3
r	18722	31	29	2	1
q		603	1	14	
x	1	0	1	-1	15

$x = 15$

4) $M_4 x_4 = 1 \bmod 23$

$25234x + 23y' = 1$

	-1	0	1	2	3	~~4~~
r	25234	23	3	2	1	
q		1097	7	1	1	
x	1	0	1	-7	8	~~-15~~

$x_4 = +8$

$x_1 = -7 \bmod 22 = 15 \bmod 22$

~~$x_4 = -3 \bmod 23 = 20 \bmod 23$~~

$X = (M_1 x_1 c_1 + M_2 x_2 c_2 + M_3 x_3 c_3 + M_4 x_4 c_4) \bmod M =$

$= (26381\cdot17\cdot15 + 15686\cdot24\cdot18 + 18722\cdot20\cdot15 + 25234\cdot14\cdot8) \bmod M =$

$= \cancel{110437}\ 472181 \bmod 580382$

Проверка

1) ~~110437/22~~ $472181/22 = 21462(17)$

2) $472181/37 = 12761(24)$

3) $472181/31 = 15231(20)$

4) $472181/23 = 20529(14)$

верно

Ответ: $472181 \bmod 580382$

4 $14^{17^{177}} \bmod 67$

$\varphi(67)=66$

$14^{66}=1 \bmod 67$

$14^{17^{177}}=14^{66k+a}=(14^{66})^k \cdot 14^a=14^a \bmod 67$

$\varphi(66)=\varphi(3)\cdot\varphi(2)\cdot\varphi(11)=20$

$17^{20}=1 \bmod 66$

$17^{177}=(17^{20})^8\cdot 17^{17}\equiv 17^{17} \bmod 66$

$17^{17}\equiv 17\cdot 17^{16}\equiv 17\cdot(((17)^2)^2)^8\equiv 17\cdot 625^4\equiv 17\cdot 31^4\equiv 17\cdot(-29)^2\equiv -289\equiv$

$\equiv 41 \bmod 66$

$14^{17^{177}}\equiv 14^{41}\equiv 14\cdot(14^2)^{20}\equiv 14\cdot 25^{10}\equiv 14\cdot 22^5\equiv 14\cdot 22\cdot 22^4\equiv$

$\equiv 14\cdot 22\cdot 15^2\equiv 14\cdot 22\cdot 24\equiv 22 \bmod 67$

Ответ: $22 \bmod 67$

5. $p(2)=23;\ p(-1)=-1;\ p(-4)=-7;\ p(-3)=-37;\ p(-2)=-21$

$$p(x)=23\frac{(x+1)(x+4)(x+3)(x+2)}{(2+1)(2+4)(2+3)(2+2)}-1\frac{(x-2)(x+4)(x+3)(x+2)}{(-1-2)(-1+4)(-1+3)(-1+2)}-$$

$$-7\frac{(x-2)(x+1)(x+3)(x+2)}{(-4-2)(-4+1)(-4+3)(-4+2)}-37\frac{(x-2)(x+1)(x+4)(x+2)}{(-3-2)(-3+1)(-3+4)(-3+2)}-21\frac{(x-2)(x+1)(x+4)(x+3)}{(-2-2)(-2+1)(-2+4)\cdot 1}=$$

$$=\frac{23}{360}(x^4+10x^3+35x^2+50x+24)+\frac{1}{18}(x^4+7x^3+8x^2-28x-48)-$$

$$-\frac{7}{36}(x^4+4x^3-x^2-16x-12)+\frac{37}{10}(x^4+5x^3-20x-16)-$$

$$-\frac{21}{8}(x^4+6x^3+3x^2-26x-24)=x^4\left(\frac{23}{360}+\frac{1}{18}-\frac{7}{36}+\frac{37}{10}-\frac{21}{8}\right)+$$

$$+x^3\left(\frac{23}{36}+\frac{7}{18}-\frac{7}{9}+\frac{37}{2}-\frac{63}{4}\right)+x^2\left(\frac{161}{72}+\frac{4}{9}+\frac{7}{36}-\frac{63}{8}\right)+$$

$$+x\left(\frac{115}{36}-\frac{14}{9}+\frac{28}{9}-74+\frac{273}{4}\right)+\left(\frac{23}{15}-\frac{24}{9}+\frac{7}{3}-\frac{296}{5}+63\right)=x^4+3x^3-5x^2-x+5$$

Проверка

1) $p(2)=16+24-20-2+5=23$

2) $p(-1)=1-3-5+1+5=-1$

3) $p(-2)=16-24-20+2+5=-21$

верно

Ответ: $x^4+3x^3-5x^2-x+5$.

6. $4x^4 + 9x^3 + 2x^2 - 24x - 9$

$$\frac{p}{q} = \frac{\pm 9, \pm 3, \pm 1}{\pm 4, \pm 2}$$

	4	9	2	-24	-9
1	4	13	15	-9	...
-1	4	5	-3	-21	...
3	4	21	65	171	...
-3	4	-3	11	-57	162
9	4	45	407	3639	...
-9	4	-2	245	-2229	20052

=> рациональных корней нет

Ответ: рациональных корней нет.

7. $2x + 68 = 215_9$

I Способ:

$2x + 68 = 215_9$

$2x = 136_9$

$x = 80_9$

II Способ:

$$\left.\begin{aligned} 2_9 &= 2_{10} \\ 68_9 &= 62_{10} \\ 215_9 &= 176_{10} \end{aligned}\right\} \Rightarrow \begin{aligned} 2x + 62 &= 176_{10} \\ 2x &= 114_{10} \\ x &= 72_{10} \end{aligned}$$

Проверка:

$\overset{1}{8}\overset{0}{0}_9 = 8 \cdot 9 + 0 \cdot 9^0 = 72 + 0 = 72_{10}$ - верно

Ответ: 72_{10}; 80_9.

1. 8 $x = \frac{29}{63} \bmod 86$

$63x = 29 \bmod 86$

$63x + 86y' = 29$

НОД$(63, 86) = 1$

$63x + 86y' = 1$

	-1	0	1	2	3	4	5
r	86	63	23	17	6	5	1
q		1	2	1	2	1	
x	0	1	-1	3	-4	11	-15

$x_0 = -15$

$x = x_0 \cdot c = -15 \cdot 29 = -435$

$x = -435 \bmod 86 = 81 \bmod 86$

Ответ: $x = 81 \bmod 86$

9 $\frac{593}{273}$

I способ:

$593 = 273 \cdot \underline{2} + 47$
$273 = 47 \cdot \underline{5} + 38$
$47 = 38 \cdot \underline{1} + 9$
$38 = 9 \cdot \underline{4} + 2$
$9 = 2 \cdot \underline{4} + 1$
$2 = 1 \cdot \underline{2} + 0.$

$[2; 5, 1, 4, 4, 2].$

II способ:

$$\frac{593}{273} = 2 + \frac{47}{273} = 2 + \cfrac{1}{5 + \cfrac{38}{47}} = 2 + \cfrac{1}{5 + \cfrac{1}{1 + \cfrac{9}{38}}} =$$

$$= 2 + \cfrac{1}{5 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{2}{9}}}} = \underline{2} + \cfrac{1}{\underline{5} + \cfrac{1}{\underline{1} + \cfrac{1}{\underline{4} + \cfrac{1}{\underline{4} + \cfrac{1}{\underline{2}}}}}} = [2; 5, 1, 4, 4, 2]$$

Ответ: $[2; 5, 1, 4, 4, 2].$

10.

$$
\begin{array}{l|l}
-\,3x^5+3x^4+0x^3+0x^2+3x+2 & 2x^3+x^2+x+2 \\
\underline{\;\;3x^5+4x^4+4x^3+3x^2\qquad\qquad} & \overline{4x^2+2x+2}
\end{array}
\qquad \frac{\mathbb{Z}}{5\mathbb{Z}}
$$

$$
\begin{array}{r}
-\,4x^4+x^3+2x^2+3x+2 \\
\underline{4x^4+2x^3+2x^2+4x+0} \\
-\,4x^3+4x+2 \\
\underline{4x^3+2x^2+2x+4} \\
3x^2+2x+3
\end{array}
$$

Проверка: $(4x^2+2x+2)\cdot(2x^3+x^2+x+2)+3x^2+2x+3=$

$=[3x^5+4x^4+\underline{4x^3}+\underline{3x^2}]+[4x^4+\underline{2x^3}+\underline{2x^2}+\underline{4x}]+[\underline{4x^3}+\underline{2x^2}+\underline{2x}+4+\underline{3x^2}+$

$+\underline{2x}+3=3x^5+3x^4+0\cdot x^3+0\cdot x^2+3x+2=3x^5+3x^4+3x+2$ — верно

Ответ: $3x^2+2x+3$