Вариант № 24

№ 1

$742 \cdot x - 777 \cdot y = 35$

$742 \cdot x + 777 \cdot y' = 35$

$777 = 742 \cdot 1 + 35$

$742 = 35 \cdot 21 + 7$

$35 = 7 \cdot 5$

$7 = 742 - 35 \cdot 21 = 742 - (777 -$
$- 742) \cdot 21 = 742 \cdot 22 - 777 \cdot 21$

$x = 22 \cdot 5 = 110$

$y' = -21 \cdot 5 = -105$

Проверка:

$742 \cdot 110 + 777 \cdot (-105) = 35$

$81620 - 81585 = 35$

Ответ: $x = 110 + 111k, k \in \mathbb{Z}$

$y' = -105 - 106k, k \in \mathbb{Z}$

№ 2

$$\sqrt{395} = 19 + \sqrt{395} - 19 =$$

$$= 19 + \cfrac{1}{\cfrac{1}{\sqrt{395} - 19}} = 19 + \cfrac{1}{\cfrac{\sqrt{395} + 19}{34}} =$$

$$= 19 + \cfrac{1}{1 + \cfrac{\sqrt{395} - 15}{34}} = 19 + \cfrac{1}{1 + \cfrac{1}{\cfrac{34}{\sqrt{395} - 15}}} =$$

$$= 19 + \cfrac{1}{1 + \cfrac{1}{\cfrac{34(\sqrt{395} + 15)}{170}}} = 19 + \cfrac{1}{1 + \cfrac{1}{\cfrac{\sqrt{395} + 15}{5}}} =$$

$$= 19 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{\sqrt{395} - 15}{5}}} =$$

$$= 19 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{\cfrac{5}{\sqrt{395} - 15}}}} =$$

$$= 19 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{\cfrac{5(\sqrt{395} + 15)}{170}}}} =$$

$$= 19 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{\frac{\sqrt{395} + 15}{34}}}} =$$

$$= 19 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \frac{\sqrt{395} - 19}{34}}}} =$$

$$= 19 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{\frac{34(\sqrt{395} + 19)}{34}}}}} =$$

$$19 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{38 + \sqrt{395} - 19}}}}$$

$$[19;\ \overline{1, 2, 1, 38}]$$

Проверка:

$$19 + \cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{1+\cfrac{1}{38}}}} =$$

$$= 19 + \cfrac{1}{1+\cfrac{1}{6+\frac{38}{39}}} = 19 + \cfrac{1}{1+\frac{39}{272}} =$$

$$= 19 + \frac{272}{311} = \frac{6181}{311} \approx 19,88 =$$

$$= \sqrt{395,2144} \approx \sqrt{395}$$

Ответ: $[19;\ \overline{1,\ 2,\ 1,\ 38}\ ]$

№3

$x \equiv 6 \mod 10$

$x \equiv 8 \mod 23$

$x \equiv 18 \mod 33$

$x \equiv 6 \mod 19$

1) $M = 10 \cdot 23 \cdot 33 \cdot 19 = 230 \cdot 627 =$
$= 144210$

$M_1 = 23 \cdot 33 \cdot 19 = 759 \cdot 19 = 14421$

$M_2 = 10 \cdot 33 \cdot 19 = 330 \cdot 19 = 6270$

$M_3 = 10 \cdot 23 \cdot 19 = 230 \cdot 19 = 4370$

$M_4 = 10 \cdot 23 \cdot 33 = 230 = 7590$

2) Решаем диофантовы уравнения

2.1) $M_1 x_1 = 1 \bmod m_1$

$14421 x_1 = 1 \bmod 10$

$14421 x_1 + 10 y_1' = 1$

$x_1 = 1 \quad y_1' = (-1442)$

2.2) $M_2 x_2 = 1 \bmod m_2$

$6270 x_2 + 23 y_2' = 1$

2) Решим вспомогательные уравнения.

2.1) $M_1 \cdot y_1 = c_1 \mod m_1$

$14421 \cdot y_1 = 6 \mod 10$

$14420 y_1 + 1 \cdot y_1 = 6 \mod 10$

$1 \cdot y_1 = 6 \mod 10$

$y_1 = 6$

2.2) $M_2 y_2 = c_2 \mod m_2$

$6270 \cdot y_1 = 8 \mod 23$

$6256 \cdot y_1 + 14 \cdot y_1 = 8 \mod 23$

$14 \cdot y_1 = 8 \mod 23$

$y_1 = 17$

2.3) $M_3 y_3 = c_3 \mod m_3$

$4370 \cdot y_3 = 18 \mod 33$

$4356 \cdot y_3 + 14 \cdot y_3 = 18 \mod 33$

$14 \cdot y_3 = 18 \mod 33$

$y_3 = 6$

2.4) $M_4 \cdot y_4 = c_4 \mod_4$

$7590 \cdot y_4 = 6 \mod 19$

$7581 \cdot y_4 + 9 \cdot y_4 = 6 \mod 19$

$9 \cdot y_4 \equiv 6 \mod 19$

$y_4 = 7$

3) $x = (M_1 \cdot y_1 + M_2 \cdot y_2 + M_3 \cdot y_3 +$
$+ M_4 \cdot y_4) \mod M =$
$= (14\,421 \cdot 6 + 6270 \cdot 17 +$
$+ 4370 \cdot 6 + 7590 \cdot 7) \mod 144\,210 =$
$= (86\,526 + 106\,590 + 26\,220 +$
$+ 53\,130) \mod 144\,210 =$
$= (193\,116 + 79\,350) \mod 144\,210 =$
$= 272\,466 \mod 144\,210 =$
$= 128\,256$

Проверка:

$128\,256 \equiv 6 \mod 10$

$128\,256 - 128\,250 = 6$

$128\,256 \equiv 8 \mod 23$

$128\,256 - 128\,248 = 8$

$128\,256 \equiv 18 \mod 33$

$128\,256 - 128\,238 = 18$

$128\,256 \equiv 6 \mod 19$

$128\,256 - 128\,250 = 6$

Ответ: 128 256

№4

$19^{3^{45}} \mod 66$

$\varphi(66) = \varphi(2) \cdot \varphi(3) \cdot \varphi(11) =$
$= 1 \cdot 2 \cdot 10 = 20$

$D(19, 66) = 1 \Rightarrow 19^{20} \equiv 1 \mod 66$

$3^{45} = 20k + b$

$19^{3^{45}} = 19^{20k+b} = 19^{20k} \cdot 19^{b} =$
$= 1^{k} \cdot 19^{b} = 19^{b}$

$D(3, 20) = 1$
$\varphi(20) = \varphi(2^2) \cdot \varphi(5) =$
$= 2^2 (1 - \frac{1}{2}) \cdot 4 = 2 \cdot 4 = 8$

$3^{8} \equiv 1 \mod 20$

$b = 3^{45} = 3^{40} \cdot 3^{5} = (3^{8})^{5} \cdot 3^{5} =$
$= 1^{5} \cdot 3^{5} = 243 = 3$

$19^{3^{45}} = 19^{3} \mod 66 = 6859 \mod 66 =$
$= 41$ Ответ: 41

№ 5

$p(1) = 10 \qquad p(-5) = 40$

$p(-3) = -34 \qquad p(-2) = -11$

$p(-1) = 4$

$$P(x) = \frac{(x+5)(x+3)(x+2)(x+1)}{(6)\cdot(4)\cdot(3)\cdot(2)} \cdot 10 -$$

$$- \frac{(x-1)(x+3)(x+2)(x+1)}{(-6)\cdot(-2)\cdot(-3)\cdot(-4)} \cdot 40 -$$

$$- \frac{(x-1)(x+5)(x+2)(x+1)}{(-4)\cdot(2)\cdot(-1)\cdot(-2)} \cdot (-34) -$$

$$- \frac{(x-1)(x+5)(x+3)(x+1)}{(-3)\cdot(3)\cdot(1)\cdot(-1)} \cdot (-11) -$$

$$- \frac{(x-1)(x+5)(x+3)(x+2)}{(-2)\cdot(4)\cdot(2)\cdot(-1)} \cdot 4 =$$

$$= \frac{5\cdot(x^4+11x^3+41x^2+61x+30)}{72} -$$

$$- \frac{5\cdot(x^4+5x^3+5x^2-5x-6)}{18} -$$

$$- \frac{17\cdot(x^4+7x^3+9x^2-7x-10)}{8} +$$

$$+ \frac{11\cdot(x^4+8x^3+14x^2-8x-15)}{9} +$$

$$+ \frac{(x^4+9x^3+21x^2-x-30)}{4} =$$

$$= \frac{5x^4+55x^3+205x^2+305x+150-}{72}$$

$$-20x^4-100x^3-100x^2+100x+120-$$

$$-153x^4-1071x^3-1377x^2+1071x+$$

$$+1530 + 88x^4 + 704x^3 + 1232x^2 -$$

$$-704x - 1320 + 18x^4 + 162x^3 + 378x^2 -$$

$$-18x - 540 \quad = \frac{-62x^4 - 250x^3 +}{72}$$

$$+338x^2 + 754x - 60 \quad = -\frac{31}{36}\cdot x^4 -$$

$$-\frac{125}{36}\cdot x^3 + \frac{169}{36}\cdot x^2 + \frac{377}{36}\cdot x -$$

$$-\frac{5}{6}$$

Проверка:

$$p(1) \quad -\frac{31}{36} - \frac{125}{36} + \frac{169}{36} + \frac{377}{36} -$$

$$-\frac{5}{6} = -\frac{156}{36} + \frac{546}{36} - \frac{5}{6} =$$

$$= \frac{390}{36} - \frac{5}{6} = \frac{360}{36} = 10$$

$$p(-5) \quad -\frac{31}{36}\cdot 625 + \frac{125}{36}\cdot 125 +$$

$$+\frac{169}{36}\cdot 25 - \frac{377}{36}\cdot 5 - \frac{5}{6} =$$

$$= -\frac{19375}{36} + \frac{15625}{36} + \frac{4225}{36} -$$

$$-\frac{1885}{36} - \frac{5}{6} = \frac{-3750}{36} +$$

$$+\frac{2340}{36} - \frac{5}{6} = \frac{-1410}{36} - \frac{5}{6} =$$

$$= \frac{-1440}{36} = -40$$

$$p(-3) \quad -\frac{31}{36}\cdot 81 + \frac{125}{36}\cdot 27 +$$

$$+\frac{169}{36}\cdot 9 - \frac{377}{36}\cdot 3 - \frac{5}{6} =$$

$$= \frac{-279}{4} + \frac{375}{4} + \frac{169}{4} - \frac{377}{12} - \frac{5}{6} =$$

$$= \frac{96}{4} + \frac{130}{12} - \frac{5}{6} = \frac{418}{12} - \frac{5}{6} =$$

$= \frac{408}{12} = 34$

$p(-2) \quad -\frac{31}{36} \cdot 16 + \frac{125}{36} \cdot 8 +$

$+ \frac{169}{36} \cdot 4 - \frac{377}{36} \cdot 2 - \frac{5}{6} =$

$= -\frac{124}{9} + \frac{250}{9} + \frac{169}{9} - \frac{377}{18} -$

$- \frac{5}{6} = \frac{126}{9} - \frac{39}{18} - \frac{5}{6} = \frac{213}{18} -$

$- \frac{5}{6} = \frac{198}{18} = 11$

$p(-1) \quad -\frac{31}{36} + \frac{125}{36} + \frac{169}{36} -$

$- \frac{377}{36} - \frac{5}{6} = \frac{94}{36} - \frac{208}{36} - \frac{5}{6} =$

$= \frac{-114}{36} - \frac{5}{6} = -\frac{144}{36} = -4$

Ответ: $-\frac{31}{36}x^4 - \frac{125}{36}x^3 + \frac{169}{36}x^2 +$

$+ \frac{377}{36}x - \frac{5}{6}$

№ 6

$x^4 - 5x^3 - 6x^2 + 7x - 2$

$\frac{p}{q} = \frac{\pm 2, \pm 1}{\pm 1}$

Проверка:

x	1	-5	-6	7	-2
1	1	-4	-10	-3	-5
-1	1	-6	0	7	-9
2	1	-3	-12	-17	-36
-2	1	-7	8	-9	16

Ответ: рац. корней нет.

~~№ 7~~

$\cancel{4_5 x + 210_5 = 1011}$

№ 7

I спос.: $4_5 \cdot x + 210_5 = 1011_5$

$\overset{0}{4}_5 = 4_{10}$

$\overset{210}{210}_5 = 2 \cdot 5^2 + 1 \cdot 5 + 0 =$

$= 50 + 5 = 55_{10}$

$\overset{3210}{1011}_5 = 1 \cdot 5^3 + 0 + 1 \cdot 5 + 1 \cdot 5^0 =$

$= 125 + 6 = 131_{10}$

$4_{10} \cdot x + 55_{10} = 131_{10}$

$x_{10} = 19_{10}$

$$\begin{array}{r|l} 19 & 5 \\ \hline -15 & 3 \\ \hline 4 & \end{array}$$

$x_5 = 34$

II спос.: $4_5 \cdot x + 210_5 = 1011_5$

$4_5 \cdot x = 301_5$

$x_5 = 34_5$

$\overset{1}{3}\overset{0}{4}_5 = 3 \cdot 5 + 4 \cdot 5^0 = 15 + 4 = 19_{10}$

$x_{10} = 19_{10}$

Ответ: $x_5 = 34$; $x_{10} = 19$

N 8

$\frac{17}{59}$ mod 67

$x = \frac{17}{59} \mod 67$

$59x = 17 \mod 67$

1) $59x - 67y = 17$

1.1) $59x + 67y' = 17$

1.2) НОД$(59, 67) = 1$

1.3) $59x + 67y' = 1$

1.4) $67 = 59 \cdot 1 + 8$

$59 = 8 \cdot 7 + 3$

$8 = 3 \cdot 2 + 2$

$3 = 2 \cdot 1 + 1$

1.5) $1 = 3 - 2 = 3 - (8 - 3 \cdot 2) =$

$= 3 \cdot 3 - 8 = (59 - 8 \cdot 7) \cdot 3 - 8 =$

$= 59 \cdot 3 - 8 \cdot 22 = 59 \cdot 3 -$

$- (67 - 59) \cdot 22 = 59 \cdot 25 -$

$- 67 \cdot 22$

~~8~~

$x_0 = (-25)$ $y_0 = -22$

1. б) $x_0 = -25$

$x = 17 \cdot (-25) + 67n, \quad n \in \mathbb{Z}$

$x = -425 \mod 67 \quad x \in [0; 67]$

2.) $x = -425 + 67n$

$x = -425 + 67 \cdot 7 = 44$

Ответ: 44

№ 9

I способ:

$$\frac{955}{727} = 1 + \frac{228}{727} = 1 + \cfrac{1}{\cfrac{727}{228}} =$$

$$= 1 + \cfrac{1}{3 + \cfrac{43}{228}} = 1 + \cfrac{1}{3 + \cfrac{1}{\cfrac{228}{43}}} =$$

$$= 1 + \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{15}{43}}} = 1 + \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{\cfrac{43}{15}}}} =$$

$$= 1 + \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{2 + \cfrac{13}{15}}}} =$$

$$= 1 + \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{2 + \cfrac{1}{\cfrac{15}{13}}}}} =$$

$$= 1 + \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{2 + \cfrac{1}{1 + \frac{2}{13}}}}} =$$

$$= 1 + \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{\frac{13}{2}}}}}} =$$

$$= 1 + \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{6 + \frac{1}{2}}}}}}$$

$$[1;\ 3,\ 5,\ 2,\ 1,\ 6,\ 2]$$

~~№1~~

II способ:

$955 = 727 \cdot 1 + 228$

$727 = 228 \cdot 3 + 43$

$228 = 43 \cdot 5 + 15$

$43 = 15 \cdot 2 + 13$

$15 = 13 \cdot 1 + 2$

$13 = 2 \cdot 6 + 1$

НОД $(955, 727) = 1$

$[1; 3, 5, 2, 1, 6, 2]$

Ответ: $[1; 3, 5, 2, 1, 6, 2]$

~~№ 10~~

$\cancel{3x^5 + 6x^3 + 3x^2 + 3x + 5 \mid 6x^3 + 2x + 3x + 4}$

$\cancel{-3x^5 \qquad \mid 4x^2}$

№ 10

$3x^5 + 0 \cdot x^4 + 6x^3 + 3x^2 + 3x + 5 \mid 6x^3 + 2x^2 + 3x + 4$

$-3x^5 + x^4 + 5x^3 + 2x^2 \qquad \mid 4x^2 + x + 1$

$6x^4 + x^3 + x^2 + 3x$

$-6x^4 + 2x^3 + 3x^2 + x$

$6x^3 + 5x^2 + 2x + 5$

$-$

$6x^3 + 2x^2 + 3x + 4$

$3x^2 + 6x + 1$

$3x^5 + 6x^3 + 3x^2 + 3x + 5 =$

$= (6x^3 + 2x^2 + 3x + 4)(4x^2 + x + 1) +$

$+ 3x^2 + 6x + 1 = 3x^5 + 6x^4 +$

$+ 6x^3 + x^4 + 2x^3 + 2x^2 + 5x^3 +$

$+ 3x^2 + 3x + 2x^2 + 4x + 4 + 3x^2 + 6x + 1 =$

$3x^5 + 0 \cdot x^4 + 6x^3 + 3x^2 + 3x + 5$

Ответ: $3x^2 + 6x + 1$