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Вариант 18

N	Ответ
1	$\begin{cases} x = -3 - 108k \\ y = -3 - 107k \end{cases}, k \in Z$
2	$[15, \overline{1, 6, 1, 30}]$
3	$x \equiv 31131 \bmod 193732$
4	$c \equiv 1 \bmod 84$
5	$P(x) = -x^4 - 4x^3 - x^2 - 2x + 1$
6	рациональных корней нет
7	$x = 43_5 = 23_{10}$
8	$34 \bmod 73$
9	$[3, 3, 6, 2, 1, 3]$
10	$x^2 + 2x + 2$

ИДЗ №1.

№1

$1819x - 1836y = 51$

① Уравнение решаемо если 51 делится на НОД(1819; 1836)

НОД(1819, 1836) = 17

$1836 = 1819 \cdot 1 + 17$

$1819 = 17 \cdot 107$

$51 : 17 = 3 \Rightarrow$ уравнение решаемо

② $1819x - 1836y = 51$

$107x - 108y = 3 \Rightarrow$ $a = 107$
$b = -108$
$c = 3$
$d = 17$

③ $107x_0 - 108y_0 = 1$

$107x_0 = 1 + 108y_0$

$x_0 = -1 \quad y_0 = -1$

$-107 = 1 - 108$

$107x_1 - 108y_1 = 3$

$x_1 = x_0 \frac{c}{d} \Rightarrow x_1 = (-1)\,3 = -3$

$y_1 = y_0 \frac{c}{d} \Rightarrow y_1 = (-1)\,3 = -3$

$(-3) \cdot 107 - (-3)\,108 = 3$

④ $x = x_1 + \frac{b}{d} \cdot k \Rightarrow x = -3 + (-108)k = -3 - 108k;\ k \in Z$

$y = y_1 - \frac{a}{d} \cdot k \Rightarrow y = -3 - 107k;\ k \in Z$

⑤ Проверка:

$k = 0 \Rightarrow x = -3;\ y = -3;\ -3 \cdot 107 - 108 \cdot (-3) = 3$

$k = 1 \Rightarrow x = -111;\ y = -110;\ -111 \cdot 107 - 108(-110) = 3$

№2

$$\sqrt{252} = 15 + (\sqrt{252} - 15) = 15 + \frac{1}{\left(\frac{1}{\sqrt{252} - 15}\right)} =$$

$$= 15 + \frac{1}{\left(\frac{\sqrt{252} + 15}{252 - 225}\right)} = 15 + \frac{1}{\left(\frac{\sqrt{252} + 15}{27}\right)} =$$

$$= 15 + \frac{1}{1 + \frac{\sqrt{252} - 12}{27}} = 15 + \frac{1}{1 + \frac{1}{\left(\frac{27}{\sqrt{252} - 12}\right)}} =$$

$$= 15 + \frac{1}{1 + \frac{1}{\left(\frac{27(\sqrt{252} + 12)}{252 - 144}\right)}} = 15 + \frac{1}{1 + \frac{1}{\left(\frac{\sqrt{252} + 12}{4}\right)}} =$$

$$= 15 + \frac{1}{1 + \frac{1}{6 + \frac{\sqrt{252} - 12}{4}}} = 15 + \frac{1}{1 + \frac{1}{6 + \frac{1}{\left(\frac{4}{\sqrt{252} - 12}\right)}}} =$$

$$=15+\cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{\left(\frac{4(\sqrt{252}+12)}{252-144}\right)}}}=15+\cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{\left(\frac{\sqrt{252}+12}{27}\right)}}}=$$

$$=15+\cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{1+\frac{\sqrt{252}-15}{27}}}}=15+\cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{1+\cfrac{1}{\left(\frac{27}{\sqrt{252}-15}\right)}}}}=$$

$$=15+\cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{1+\cfrac{1}{\left(\frac{27(\sqrt{252}+15)}{252-225}\right)}}}}=15+\cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{1+\cfrac{1}{\sqrt{252}+15}}}}=$$

$$=15+\cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{1+\cfrac{1}{30+(\sqrt{252}-15)}}}}$$

$2\cdot 15=30 \Rightarrow$ решение верно.

Ответ: $[15;\overline{1,6,1,30}]$

№3

x_{min} - ? $\quad x \equiv 23 \bmod 28$

$x \equiv 14 \bmod 37$

$x \equiv 4 \bmod 17$

$x \equiv 1 \bmod 11$

(1) $M = 28 \cdot 37 \cdot 17 \cdot 11 = 193732$

$M_1 = 37 \cdot 17 \cdot 11 = 6919$

$M_2 = 28 \cdot 17 \cdot 11 = 5236$

$M_3 = 28 \cdot 37 \cdot 11 = 11396$

$M_4 = 28 \cdot 37 \cdot 17 = 17612$

(2) $M_1 x_1 = 1 \bmod m_1 \Rightarrow 6919 x_1 = 1 \bmod 28$

$6919x - 28y = 1$

i	-1	0	1	2	
r	6919	28	3	1	0
q		247	9	3	
x	1	0	1	-9	
y	0	1	-247	2224	

$\Rightarrow x_1 = -9$

$x = x_1 \cdot 1 + 28k = -9 + 28k =$

$= (-9 + 28) \bmod 28 = 19 \bmod 28$

$M_2 x_2 = 1 \bmod m_2 \Rightarrow 5236 x_2 = 1 \bmod 37$

$5236x - 37y = 1$

i	-1	0	1	2	3	
r	5236	37	19	18	1	0
q		141	1	1	18	
x	1	0	1	-1	2	
y	0	1	-1	0	-1	

$\Rightarrow x_2 = 2 \quad x = x_2 + 37k = 2 + 37k =$

$= 2 \bmod 37$

$M_3 x_3 = 1 \bmod m_3 \Rightarrow 11396 \cdot x_3 = 1 \bmod 17$

$11396x - 17y = 1$

i	-1	0	1	2	3	
r	11396	17	6	5	1	0
q		670	2	1	5	
x	1	0	1	-2	3	
y	0	1	-670	671		

$\Rightarrow x_3 = 3 \quad x = x_3 + 17k = 3 \bmod 17$

$M_4 x_4 = 1 \mod M_4 \Rightarrow 17612 x_4 = 1 \mod 11$

$$17612x - 11y = 1$$

i	-1	0	1	2
r	17612	11	$\underline{1}$	0
q		1601	11	
x	1	0	$\underline{1}$	-11
y	0	1	-1601	17612

$x_4 = 1 \qquad x = x_4 + 11k = 1 \mod 11$

(3) $x = (M_1 x_1 c_1 + M_2 x_2 c_2 + M_3 x_3 c_3 + M_4 x_4 c_4) \mod M$

$$x = (6919 \cdot 19 \cdot 23 + 14 \cdot 5236 \cdot 2 + 4 \cdot 11396 \cdot 3 + 1 \cdot 17612 \cdot 1) \mod 193\,732$$

$x = 3324575 \mod 193\,732$

$x = 31\,131 \mod 193\,732$

Ответ: $x \equiv 31\,131 \mod 193\,732$

Проверка:

$31131 : 28 = 1111$ (ост. 23) $\Rightarrow 31131 = 23 \bmod 28$

$31131 : 37 = 841$ (ост 14) $\Rightarrow 31131 = 14 \bmod 37$

$31131 : 17 = 1831$ (ост 4) $\Rightarrow 31131 = 4 \bmod 17$

$31131 : 11 = 2830$ (ост 1) $\Rightarrow 31131 = 1 \bmod 11$

№4

$r - ?$ $\quad 41^{11^{45}} : 84$

$c = 41^{11^{45}} \bmod 84$

Пусть $k = 11^{45} \Rightarrow 41^k \bmod 84$

$\varphi(84) = \varphi(2^2) \cdot \varphi(3) \cdot \varphi(7) = 2^2\left(1 - \frac{1}{2}\right) \cdot 2 \cdot 6 =$

$= 4 \cdot \frac{1}{2} \cdot 12 = 24$

$k = 11^{45} = 24n + b$

$b = 11^{45} \bmod 24 \Rightarrow \varphi(24) = \varphi(2^3) \cdot \varphi(3) =$

$b = 11^{8 \cdot 5 + 5} \bmod 24 \qquad = 2^3\left(1 - \frac{1}{2}\right) \cdot 2 = 8$

$\cancel{c = 41^{24n+b} \bmod 84 \equiv 41^{24n} \cdot 41^{b} \bmod 84 \equiv}$
$\cancel{\equiv 41^{b} \bmod 84}$

$b \equiv 11^{8 \cdot 5 + 5} \bmod 24 \equiv 11^{8 \cdot 5} \bmod 24 \cdot 11^5 \bmod 24 \equiv 11^5 \bmod 2$

$m = 5 = 101_2$

$b = 11 \bmod 24$

a_i	c	c^2	$c^2 a^{a_i}$	$c \equiv c^2 \cdot a^{a_i} \bmod k$
1	1	1	11	11
0	11	121	121	1
1	1	1	11	11

a_i	c	c^2	$c^2 \cdot a^i$	$c \equiv c^2 a^{a_i} \bmod k$
1	1	1	41	41
0	41	1681	1681	1
1	1	1	41	41
1	41	1681	1681	1

$c \equiv 41^{11} \bmod 84$

$m = 11 = 1011_2$

$c \equiv 1 \bmod 84$

Ответ: $c \equiv 1 \bmod 84$.

№5

p - ? < степени 4

$p(-2) = 17$

$p(1) = -7$

$p(-4) = -7$

$p(-3) = 25$

$p(-1) = 5$

① $P(x) = \frac{(x-1)(x+4)(x+3)(x+1)}{(-2-1)(-2+4)(-2+3)(-2+1)}(17) +$

$+ \frac{(x+2)(x+4)(x+3)(x+1)}{(1+2)(1+4)(1+3)(1+1)}(-7) +$

$+ \frac{(x+2)(x-1)(x+3)(x+1)}{(-4+2)(-4-1)(-4+3)(-4+1)}(-7) +$

$+ \frac{(x+2)(x-1)(x+4)(x+1)}{(-3+2)(-3-1)(-3+4)(-3+1)}(25) +$

$+ \frac{(x+2)(x-1)(x+4)(x+3)}{(-1+2)(-1-1)(-1+4)(-1+3)}(5) =$

$$= \frac{17}{6}(x^4+7x^3+11x^2-7x-12) + \frac{7}{120}(x^4+10x^3+35x^2+50x+24) -$$

$$- \frac{7}{30}(x^4+5x^3+5x^2-5x-6) - \frac{25}{8}(x^4+6x^3+7x^2-6x-8) +$$

$$+ \frac{5}{12}(x^4+8x^3+17x^2-2x+24) =$$

$$= x^4\left(\frac{17}{6} - \frac{7}{120} - \frac{7}{30} - \frac{25}{8} - \frac{5}{12}\right) + x^3\left(\frac{17\cdot 7}{6} + \frac{7\cdot 10}{120} + \frac{7\cdot 5}{30} -\right.$$

$$\left.- \frac{25\cdot 6}{8} - \frac{5\cdot 8}{12}\right) + x^2\left(\frac{17\cdot 11}{6} - \frac{7\cdot 35}{120} - \frac{7\cdot 5}{30} - \frac{25\cdot 7}{8} - \frac{5\cdot 17}{12}\right) +$$

$$+ x\left(\frac{17\cdot(-7)}{6} - \frac{7\cdot 50}{120} + \frac{7\cdot 5}{30} + \frac{25\cdot 6}{8} + \frac{5\cdot 2}{12}\right) +$$

$$+ \left(\frac{17(-12)}{6} - \frac{7\cdot 24}{120} + \frac{7\cdot 6}{30} + \frac{25\cdot 8}{8} + \frac{5\cdot 24}{12}\right) =$$

$$= -x^4 - 4x^3 - x^2 - 2x + 1$$

Проверка:

$P(-2) = -(-2)^4 - 4(-2)^3 - (-2)^2 - 2(-2) + 1 = 17$

$P(1) = -1^4 - 4\cdot 1^3 - 1^2 - 2\cdot 1 + 1 = -7$

$P(-4) = -(-4)^4 - 4(-4)^3 - (-4)^2 - 2(-4) + 1 = -7$

$P(-3) = -(-3)^4 - 4(-3)^3 - (-3)^2 - 2(-3) + 1 = 25$

$P(-1) = -(-1)^4 - 4(-1)^3 - (-1)^2 - 2(-1) + 1 = 5$

чтд

Ответ: $P(x) = -x^4 - 4x^3 - x^2 - 2x + 1$

№6

$x^4 - 5x^3 - 6x^2 + 7x - 2 = 0$

$\frac{p}{q} = \frac{\pm 2;\ \pm 1}{\pm 1}$

x	1	-5	-6	7	-2
1	1	-4	-10	-3	-5
-1	1	-6	0	7	-9
2	1	-3	-12	-17	-36
-2	1	-7	8	-9	16

$\Rightarrow f(1) = -5$
$\Rightarrow f(-1) = -9$
$\Rightarrow f(2) = -36$
$\Rightarrow f(-2) = 16$

Проверка:

$f(1) = 1 - 5 - 6 + 7 - 2 = -5$

$f(-1) = 1 + 5 - 6 - 7 - 2 = -9$

$f(2) = 2^4 - 5\cdot 2^3 - 6\cdot 2^2 + 7\cdot 2 - 2 = -36$

$f(-2) = 16 + 5\cdot 8 - 6\cdot 4 - 7\cdot 2 - 2 = 16$

Ответ: рациональных корней нет

№7

$2_5 x + 222_5 = 413_5$ (в 5-ичной системе)

I способ

$2_5 x = 413_5 - 222_5$

$$\begin{array}{r} 413_5 \\ -\ 222_5 \\ \hline 141_5 \end{array}$$

$$\begin{array}{l|l} 141 & 2 \\ \underline{13} & 43 \\ \ \ 11 & \\ \ \ \underline{11} & \\ \ \ \ \ 0 & \end{array}$$

$2_5 x = 141_5$

$x = 43_5 = 23_{10}$

II способ

$2_5 = 2_{10}$; $\overset{2}{2}\overset{1}{2}\overset{0}{2}_5 = 25 \cdot 2 + 5 \cdot 2 + 2 = 62_{10}$

$\overset{2}{4}\overset{1}{1}\overset{0}{3}_5 = 25 \cdot 4 + 5 + 3 = 108_{10}$

$2x + 62 = 108$

$2x = 46$

$x = 23_{10}$;

23	4
3	4

$23_{10} = 43_5$

Ответ: $x = 43_5 = 23_{10}$

№8

$x = \frac{59}{64} \bmod 73 \Rightarrow 64x = 59 \bmod 73$

$64x - 73y = 59$

$64x + 73y' = 59$

$НОД(64, 73) = 1 \Rightarrow 64x_0 + 73y'_0 = 1$

$73 = 64 \cdot 1 + 9$

$64 = 9 \cdot 7 + 1$

$9 = 9 \cdot 1$

r	64	73	64	9	1
q		0	1	7	
x	1	0	1	-1	8

$x_1 = x_0 \cdot c = x_0 73 = 8 \cdot 59 = 472$

$x = 472 + 73k, \; k \in Z$

$x \equiv 472 \bmod 73 = 34 \bmod 73$

Ответ: $\frac{59}{64} \bmod 73 = 34 \bmod 73$

№9

I способ

$$\frac{733}{221} = 3 + \frac{70}{221} = 3 + \frac{1}{\left(\frac{221}{70}\right)} = 3 + \frac{1}{3 + \frac{11}{70}} =$$

$$= 3 + \frac{1}{3 + \frac{1}{\left(\frac{70}{11}\right)}} = 3 + \frac{1}{3 + \frac{1}{6 + \frac{4}{11}}} = 3 + \frac{1}{3 + \frac{1}{6 + \frac{1}{\left(\frac{11}{4}\right)}}} =$$

$$= 3 + \frac{1}{3 + \frac{1}{6 + \frac{1}{2 + \frac{3}{4}}}} = 3 + \frac{1}{3 + \frac{1}{6 + \frac{1}{2 + \frac{1}{\left(\frac{4}{3}\right)}}}} = 3 + \frac{1}{3 + \frac{1}{6 + \frac{1}{2 + \frac{1}{1 + \frac{1}{3}}}}} =$$

$$= [3, 3, 6, 2, 1, 3]$$

II способ

$733 = 221 \cdot 3 + 70$

$221 = 70 \cdot 3 + 11$

$70 = 11 \cdot 6 + 4$ $\Rightarrow \frac{733}{221} = [3; 3, 6, 2, 1, 3]$

$11 = 4 \cdot 2 + 3$

$4 = 3 \cdot 1 + 1$

$3 = 1 \cdot 3$

Ответ: $\frac{733}{221} = [3, 3, 6, 2, 1, 3]$

№10

$$\frac{3x^5+x^4+3x^3+3x+1}{2x^3+2x^2+3x+1}$$ в кольце $\mathbb{Z}/5\mathbb{Z}$

$$\begin{array}{r|l}
3x^5+x^4+3x^3+0x^2+3x+1 & 2x^3+2x^2+3x+1 \\
\underline{-\;3x^5+3x^4+2x^3+4x^2} & \overline{4x^2+4x+4} \\
3x^4+x^3+x^2+3x & \\
\underline{-\;3x^4+3x^3+2x^2+4x} & \\
3x^3+4x^2+4x+1 & \\
\underline{-\;3x^3+3x^2+2x+4} & \\
x^2+2x+2 &
\end{array}$$

Проверка:

$(2x^3+2x^2+3x+1)(4x^2+4x+4)+(x^2+2x+2)=$

$=3x^5+3x^4+2x^3+4x^2+3x^4+3x^3+2x^2+4x+3x^3+3x^2+$

$+2x+4+x^2+2x+2=3x^5+x^4+3x^3+0\cdot x^2+3x+1=$

$=3x^5+x^4+3x^3+3x+1$ чтд

Ответ: x^2+2x+2