Константинов

Иван

0362 Гр. 26 ВАР.

1	$x = 105 - 134 \cdot k \,;\, y = 112 - 143 \cdot k$
2	$[18, \overline{3, 3, 3, 36}]$
3	219081
4	$19^{7^{37}} \bmod 52 \equiv 19$
5	$2x^4 - 3x^3 - 5x^2 + 2x - 1$
6	НЕТ РАЦИОН. корней
7	$36_9 \,;\, 33_{10}$
8	$11 \bmod 61$
9	$[4, 3, 4, 1, 1, 4]$
10	$1\frac{5}{8}x^2 + 3\frac{5}{8}x + 2{,}75$

№1

№1 $1859x - 1742y = 91$

$\gcd(1859, -1742) = 13$

$143x - 134y - 7 = 0$

$143x_1 - 134y_1 = 1$

$$\begin{cases} x_1 = 15 \\ y_1 = 16 \end{cases} \Leftrightarrow \begin{cases} x_0 = 105 \\ y_0 = 112 \end{cases} \Leftrightarrow \begin{cases} x = 105 - 134 \cdot k \\ y = 112 - 143 \cdot k \end{cases}$$

Проверка:

$k=0: \quad 1859 \cdot (105) - 1742 \cdot (112) = 91$

$195195 - 195104 = 91$

$91 = 91$

$k=1: \quad 1859 \cdot (105 - 134) - 1742 \cdot (112 - 143) = 91$

$-53911 + 54002 = 91$

$91 = 91$

Ответ: $\begin{cases} x = 105 - 134 \cdot k \\ y = 112 - 143 \cdot k \end{cases}$

№2

№2 $\sqrt{335} = 18 + \sqrt{335} - 18 = 18 + \cfrac{1}{\left(\cfrac{(\sqrt{335}+18)}{(\sqrt{335}-18)(\sqrt{335}+18)}\right)} =$

$= 18 + \cfrac{1}{3 + \cfrac{\sqrt{335}+18}{11}} = 18 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{\sqrt{335}+18}{11}}} =$

$= 18 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{\sqrt{335}+18}{11}}} = 18 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{3\ \cfrac{\sqrt{335}+18}{11}}}} =$

$= 18 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{36 + \cfrac{\sqrt{335}+18}{11}}}}}$

Проверка:

$\sqrt{335} \approx 18,30300521\cdot$

$18,30300521 \approx [18, \overline{3,3,3,36}]$

$[a; \overline{\ldots 2a}]$

$18 \cdot 2 = 36$

Ответ: $[18, \overline{3,3,3,36}]$

№3

№3

$x \equiv 6 \bmod 25$ $\quad M = 342\,550$

$x \equiv 5 \bmod 13$ $\quad M_1 = 13\,702$

$x \equiv 4 \bmod 31$ $\quad M_2 = 26\,350$

$x \equiv 19 \bmod 34$ $\quad M_3 = 11\,050$

$M_4 = 10075$

$13702 \cdot x_1 \equiv 6 \bmod 25$ $\quad x_1 = 3$

$26350 \cdot x_2 \equiv 5 \bmod 13$ $\quad x_2 = 8$

$11050 \cdot x_3 \equiv 4 \bmod 31$ $\quad x_3 = 18$

$10075 \cdot x_4 \equiv 19 \bmod 34$ $\quad x_4 = 11$

~~x = (13702 · 6 · 12 + 26350 · 1 · 5 + 11050 · 11 · 4 + 10075 · 19 · 3) · (−1)) mod 342550~~

~~x ≡ 2138769 mod 342550~~

~~x ≡ 123469 mod 342550~~

Проверка:

$x \equiv (13702 \cdot 6 + 26350 \cdot 5 \cdot 8 + 11050 \cdot 4 \cdot 18 + 10075 \cdot 19 \cdot 11) \bmod 342550$

~~x ≡ 4201971 mod 342550~~

~~x ≡ 913114 mod 342550~~

~~x ≡ 260187 mod 342550~~

~~Проверки: 260187 ≡ 6 mod 25~~

~~260187 ≡ 5 mod 13~~

~~260187 ≡ 4 mod 31~~

~~260187 ≡ 19 mod 34~~

$x \equiv 561631 \bmod 342550$

$x \equiv 219081 \bmod 342550$

Проверки: $219081 \equiv 6 \bmod 25$

$219081 \equiv 5 \bmod 13$

$219081 \equiv 4 \bmod 31$

$219081 \equiv 19 \bmod 34$

~~Ответ: 260187~~ Ответ: 219081

№4

№4

$19^{7^{37}} \underset{52}{\equiv} ?$; $A = 7^{37} \Rightarrow 19^{A} \underset{52}{\equiv} ?$

$A \underset{12}{\equiv} ? \Rightarrow 7^{37} \underset{12}{\equiv} ? \Rightarrow 1^{37} \equiv 1$

$19^{A} \underset{52}{\equiv} 19^{1}$

k	$19^k \underset{52}{\equiv} ?$
1	19
2	49
...	...
12	1

Ответ: $19^{7^{37}} \bmod 52 \equiv 19$

№5

№5

$x \quad y$

$p(-2) = 31 \quad 1$

$p(2) = -9 \quad 2$

$p(1) = -5 \quad 3$

$p(3) = 41 \quad 4$

$p(-1) = -3 \quad 5$

$y_1 = 31 \quad y_2 = -9 \quad y_3 = -5 \quad y_4 = 41 \quad y_5 = -3$

$x_1 = -2 \quad x_2 = 2 \quad x_3 = 1 \quad x_4 = 3 \quad x_5 = -1$

$$P_4(x) = 31 \cdot \frac{(x-2)(x-1)(x-3)(x+1)}{(-2-2)(-2-1)(-2-3)(-2+1)} + (-9)\frac{(x+2)(x-1)(x-3)(x+1)}{(2+2)(2-1)(2-3)(2+1)} +$$

$$+(-5) \cdot \frac{(x+2)(x-2)(x-3)(x+1)}{(1+2)(1-2)(1-3)(1+1)} + 41 \cdot \frac{(x+2)(x-2)(x-1)(x+1)}{(3+2)(3-2)(3-1)(3+1)} +$$

$$+(-3) \cdot \frac{(x+2)(x-2)(x-1)(x-3)}{(-1+2)(-1-2)(-1-1)(-1-3)} =$$

$$= 2x^4 - 3x^3 - 5x^2 + 2x - 1$$

Проверка: $p_4(-2) = 31 = 2 \cdot (-2)^4 - 3(-2)^3 - 5(-2)^2 + 2(-2) - 1$

$p_4(2) = -9 = 2(2)^4 - 3(2)^3 - 5(2)^2 + 2 \cdot 2 - 1$

$p_4(1) = -5 = 2 \cdot (1)^4 - 3(1)^3 - 5(1)^2 + 2 \cdot 1 - 1$

$p_4(3) = 41 = 2 \cdot (3)^4 - 3(3)^3 - 5(3)^2 + 2 \cdot 3 - 1$

$p_4(-1) = -3 = 2 \cdot (-1)^4 - 3(-1)^3 - 5(-1)^2 + 2 \cdot (-1) - 1$

Ответ: $2x^4 - 3x^3 - 5x^2 + 2x - 1$

№6

№6 $x^4-5x^3-6x^2+7x-2$

	1	-5	-6	7	-2
1	1	-4	-10	-3	-5
-1	1	-6	0	7	-9
2	1	-3	-12	-17	-36
-2	1	-7	8	-9	10

$d(1)=\{\pm 1\}$

$d(-2)=\{\pm 1;\pm 2\}$

$\frac{p}{q}=\{\pm 2;\pm 1\}$

нет рац. корней

$x^4-5x^3-6x^2+7x-2=x((x-6)(x+1)+7)-2$

Проверка:

$f(1)=\underbrace{1-5-6+7-2}_{=-5}\neq 0$ $f(-1)=1+5-6-7-2=\underbrace{-9}_{=-9}\neq 0$

$f(2)=\underbrace{16-40-24+14-2}_{=-36}\neq 0$ $f(-2)=\underbrace{16+40-24-14-2}_{=16}\neq 0$

Ответ: нет корней рац.

№

№7

№7 $7_9 \cdot x_9 + 14_9 = 301_9$

I сп.

$$\begin{array}{r} 301_9 \\ -\ 14_9 \\ \hline 276_9 \end{array}$$

$276_9 : 7_9 = 36_9$

$x_9 = 36_9 = 33_{10}$

II сп. $7_9 = 7_{10}$

$14_9 = 13_{10}$

$301_9 = 244_{10}$

$$x_{10} = \frac{244_{10} - 13_{10}}{7_{10}}$$

$x = 33_{10} = 36_9$

Ответ: $x_9 = 36_9$

$x_{10} = 33_{10}$

№8

№8 $\frac{2}{39} \bmod 61 \equiv x \Rightarrow 39x \equiv 2 \bmod 61$

$39x - 61y = 2$

$39x + 61y' = 2$

НОД $(61;39) = 1 \Rightarrow 39x_0 + 61y_0' = 1$

$x_1 = -50$

$x = -50 + 61k;\ k \in \mathbb{Z}$

$x \equiv -50 \bmod 61 \equiv 11 \bmod 61$

Ответ: $\frac{2}{39} \bmod 61 \equiv 11 \bmod 61$

№9

№9 $\frac{569}{132} = 4 + \cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{4}}}}} = [4,3,4,1,1,4]$

Проверка

$$4 + \cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{4}}}}} = 4 + \cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{1\frac{4}{5}}}} =$$

$$= 4 + \cfrac{1}{3+\cfrac{1}{4+\frac{5}{9}}} = 4 + \cfrac{1}{3+\frac{9}{41}} = 4 + \frac{41}{132} =$$

$$= \frac{569}{132}$$

$569 = 132 \cdot 4 + 41$

$132 = 41 \cdot 3 + 9$

$41 = 9 \cdot 4 + 5$

$9 = 5 \cdot 1 + 4$

$5 = 4 \cdot 1 + 1$

$4 = 1 \cdot 4 + 0$

Ответ: $[4,3,4,1,1,4]$

№7 $4_5 x_5 + 210_5 = 1011_5$

I сп.

$$\begin{array}{r} 1011_5 \\ -\ 210_5 \\ \hline 30?_5 \end{array}$$

$30?_5 : 4_5 = 34_5$

$34_5 = x_5$

$19_{10} = x_{10}$

II сп.

$4_5 = 4_{10}$

$210_5 = 55_{10}$

$1011_5 = 131_{10}$

$4x + 55 = 131$

$x = \frac{131-55}{4}$

$x_{10} = 19_{10}$

Ответ: $x_{10} = 19_{10}$
$x_5 = 34$

№10

№10

$$
\begin{array}{l|l}
3x^5+2x^4+2x^3+3x^2+4x+3 & 4x^3+2x^2+x+1 \\
\hline
3x^5+1{,}5x^4+0{,}75x^3+0{,}75x^2 & 0{,}75x^2+\frac{1}{8}x+0{,}25
\end{array}
$$

$0{,}5x^4+1{,}25x^3+2{,}25x^2+4x+3$

$0{,}5x^4+0{,}25x^3+\frac{1}{8}x^2+\frac{1}{8}x$

$x^3+2\frac{1}{8}x^2+\frac{3\cdot8+7}{8}x+3$

$x^3+0{,}5x^2+0{,}25x+0{,}25$

$1\frac{5}{8}x^2+3\frac{5}{8}x+2{,}75$

$\mathbb{Z}/5\mathbb{Z}[x]$

Проверка: $(0{,}75x^2+\frac{1}{8}\cdot x+0{,}25)\cdot(4x^3+2x^2+x+1)+$

$+1\frac{5}{8}x^2+3\frac{5}{8}x+2{,}75=$

$=3x^5+2x^4+2x^3+3x^2+4x+3$

Ответ: $1\frac{5}{8}x^2+3\frac{5}{8}x+2{,}75$