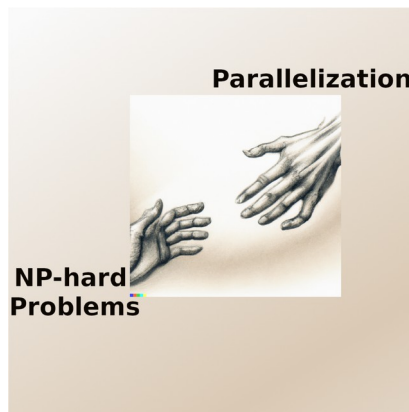




NP-Hard Problems Meet Parallelization



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Our Philosophy



... take a **fresh look** at some of the classic graph algorithms and devise **faster** and more parallel GPU and CPU implementations.

- Fallin et. al.

+
NP-hard

=

Our Philosophy

A High-Performance MST Implementation for GPUs

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ABSTRACT

Finding a minimum spanning tree (MST) is a fundamental graph algorithm with applications in many fields. This paper presents ECL-MST, a fast MST implementation designed specifically for GPUs. ECL-MST is based on a parallelization approach that unifies Kruskal's and Borůvka's algorithm and incorporates new and existing optimizations from the literature, including implicit path compression and edge-centric operation. On two test systems, it outperforms leading GPU and CPU codes from the literature on all of our 17 input graphs from various domains. On a Titan V GPU, ECL-MST is, on average, 4.6 times faster than the next fastest code, and on an RTX 3080 Ti GPU, it is 4.5 times faster. On both systems, ECL-MST running on the GPU is roughly 30 times faster than the fastest parallel CPU code.

CCS CONCEPTS

• Computing methodologies → Massively parallel algorithms.

KEYWORDS

Minimum spanning tree, minimum spanning forest, parallelism, performance optimization, GPU implementation

lines. In this example, the cheapest distribution grid that allows everyone to deliver or receive electricity is the MST shown.

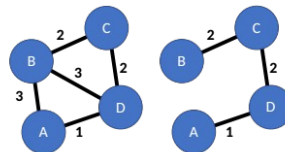


Figure 1: Example of a weighted graph on the left and the resulting MST on the right

Computing an MST (or MSF¹) is a fundamental graph algorithm with applications in many fields. For instance, it is a key building block in network analysis [12], chip design [1], eye tracking [17], route planning [13], and medical diagnostics like tumor recognition [4]. Since some of these applications repeatedly generate an MST, increasing the performance of this step is important and has the potential to speed up lifesaving computations.

There are three classic MST algorithms. Borůvka's algorithm [11]

SC'23

Outline



PhD Journey

- Motivation
 - Philosophy
 - Landscape
- Steiner Tree
 - Example
 - Algorithm
 - Halt-Optimization
 - GPU Optimization
 - Two-level parallelism
- Vehicle routing
 - Example
 - Local search algorithm
 - Experiments
- Future directions
- Summary

Current status



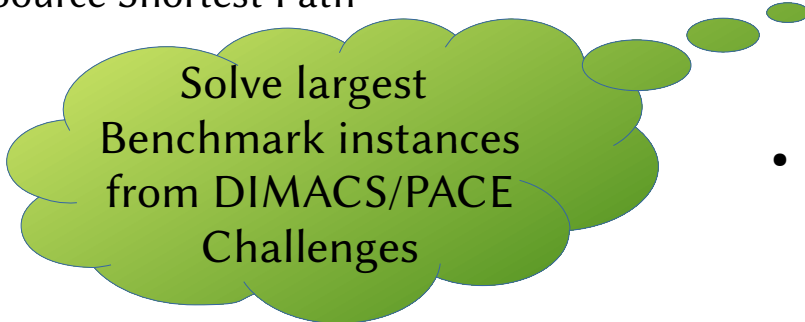
Irregular Mem. access

Poly-time Problems

- Parallelization is easier
- Algorithms are simpler
- Run few seconds on million/billion-scale
- Solution search space is small
- Exact solution

- **Examples**

- Minimum Spanning Tree
- Single Source Shortest Path



Solve largest
Benchmark instances
from DIMACS/PACE
Challenges

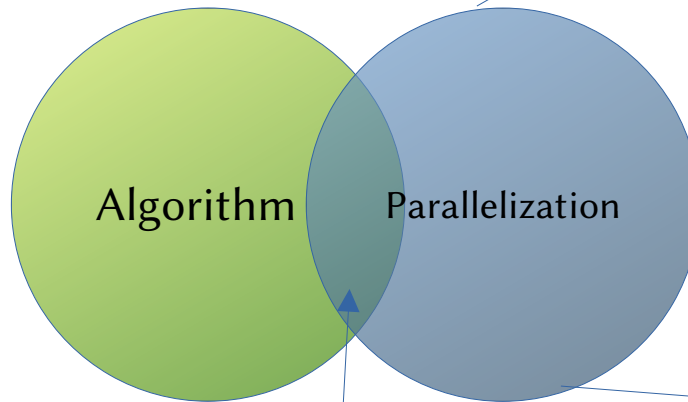
NP-Hard Problems

- Comparitively difficult.
 - Complicated algorithms
 - Few hours for thousand-sized instances
 - Solution search space is large
 - Approximate solution mostly - tradeoff
-
- Steiner Tree Problem
 - Travelling Salesman Problem
 - Vehicle routing problem
-
- More practical applications

Algorithms and Optimizations



- Parallel Implementations chosen from
 - Efficient serial algorithm
 - Algorithm amenable for parallelization
 - Design parallelism-friendly algorithms
- Optimizations



Data-driven versus Topology-driven Irregular Computations on GPUs

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Compiling Graph Applications for GPUs with GraphIt

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Abstract—main data trees, etc. A its operator, iterated app nodes, in the usually time overlapping drives impl the operat no work to mentations t might be w algorithms i only support important. (a topology counterhala In this p implementation goal is to um and how to generally fa a worklist, when certain implementat that combin
Keywords: i algorithmic

Abstract—The performance of graph programs depends highly on the algorithm, the size and structure of the input graphs, as well as the features of the underlying hardware. No single set of optimizations or one hardware platform works well across all settings. To achieve high performance, the programmer must carefully select which set of optimizations and hardware platforms to use. The GraphIt programming language makes it easy for the programmer to write the algorithm once and optimize it for different inputs using a scheduling language. However, GraphIt currently has no support for generating high-performance code for GPUs. Programmers must resort to re-implementing the entire algorithm from scratch in a low-level language with an entirely different set of abstractions and optimizations in order to achieve high performance on GPUs.

We propose G2, an extension to the GraphIt compiler framework, that achieves high performance on both CPUs and GPUs using the same algorithm specification. G2 significantly expands the optimization space of GPU graph processing frameworks with a novel GPU scheduling language and compiler that enables combining load balancing, edge traversal direction, active vertexset creation, active vertexset processing ordering, and kernel fusion optimizations. G2 also introduces two performance optimizations, Edge-based Thread Warps CTAs load balancing (ETWC) and EdgeBlocking, to expand the optimization space for GPUs. ETWC improves load balancing by dynamically partitioning the edges of each vertex into blocks that are assigned to threads, warps, and CTAs for execution. EdgeBlocking improves the locality of the program by reordering the edges and restricting random memory accesses to fit within the L2 cache. We evaluate G2 on 5 algorithms and 9 input graphs on both Pascal and Volta generation NVIDIA GPUs, and show that it achieves up to 5.11x speedup over state-of-the-art GPU graph processing frameworks, and is the fastest on 66 out of the 90 experiments.

Index Terms—Compiler Optimizations, Graph Processing, GPUs, Domain-Specific Languages

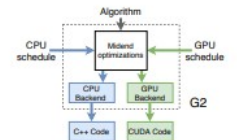


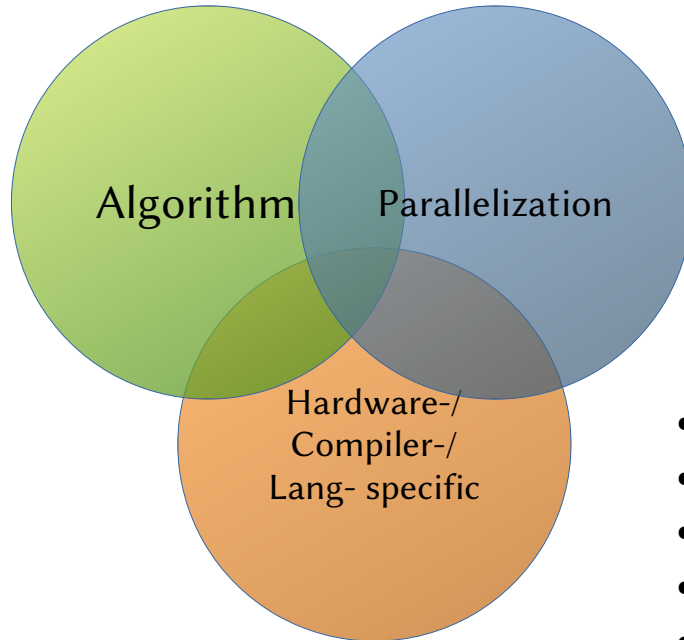
Fig. 1. G2 adds a new GPU backend to GraphIt that produces CUDA from the same GraphIt algorithm language and a scheduling language tail for GPU optimizations.

In prior work, we built the GraphIt DSL compiler [7] to generate high-performance CPU code from a high-level algorithm language. GraphIt achieves state-of-the performance on CPUs across different algorithms and inputs by introducing a scheduling language to tune optimizations. The algorithm language has primitives for topology-driven algorithms, data-driven algorithms, and priority-based algorithms. This algorithm and schedule representation makes it easy for the programmer to write an algorithm once generate different highly-optimized implementations by simply changing the schedule. We will discuss in detail the Graph algorithm and scheduling languages in Section IV.

Apart from a large body of work for optimizing algorithms on different inputs on CPUs [6], [8], [9], [10], [11], [13], [14], [15], researchers have also used different hardware platforms for high-performance graph processing, including GPUs [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32], [33], [34], [35], [36], [37], [38], [39], [40], [41], [42], [43], [44], [45], [46], [47], [48], [49], [50], [51], [52], [53], [54], [55], [56], [57], [58], [59], [60], [61], [62], [63], [64], [65], [66], [67], [68], [69], [70], [71], [72], [73], [74], [75], [76], [77], [78], [79], [80], [81], [82], [83], [84], [85], [86], [87], [88], [89], [90].

For peek performance

- Input Characteristics
 - Diameter
 - Max degree
 - Road/social network
- Properties of substeps

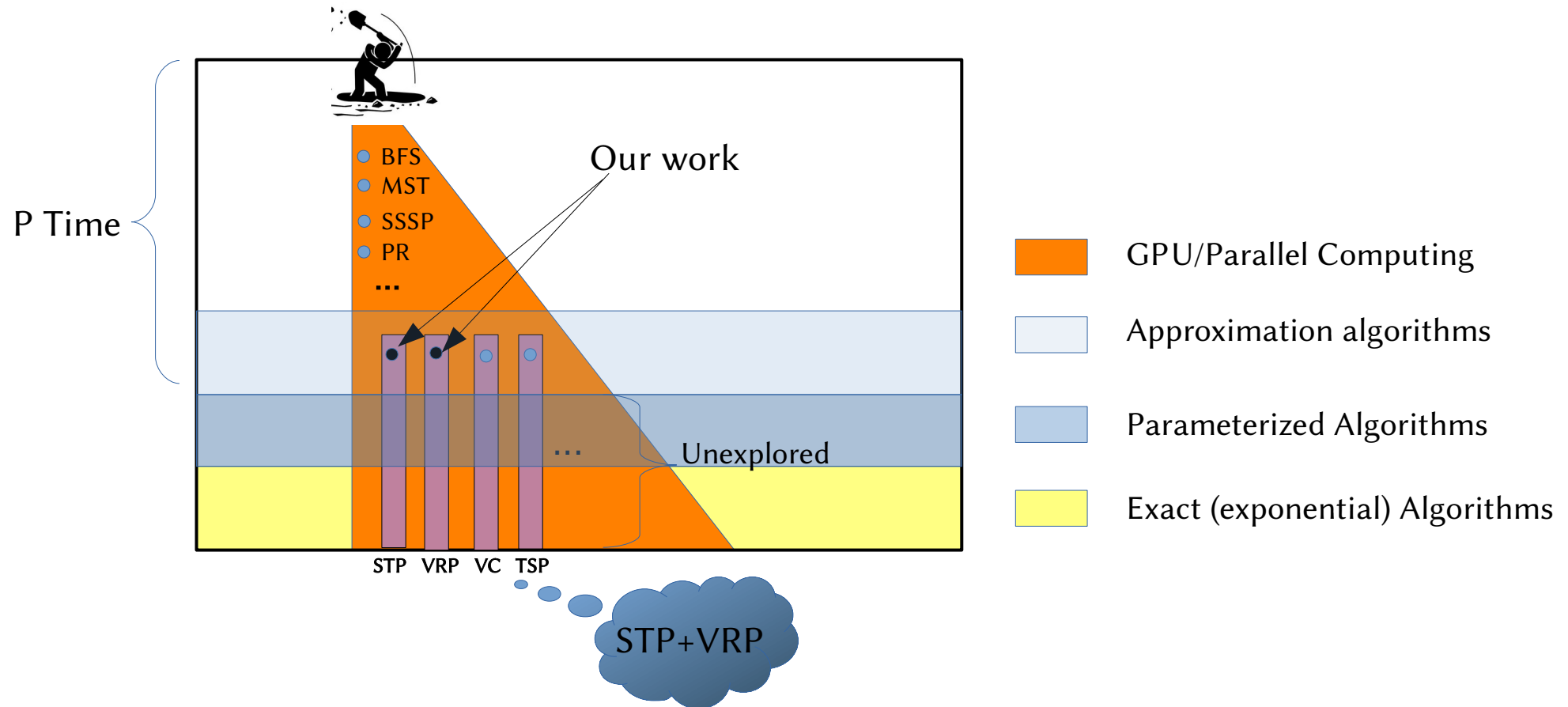


- Vertex-/Edge -centric
- Data-/Topology- driven
- Push/Pull-based
- Load-balancing
- ...

- Shared memory
- Warp-Intrinsics
- Data accesses within reg/caches
- Vectorization loads/adds
- Language-specific 128b CAS CC9+
- Architecture-specific #L1-3 Caches
- Use profilers and <https://godbolt.org>

Is that all?

Landscape of Parallelization



Steiner Tree Problem (STP) – Example

Input : Graph $G(V, E, W)$ $W:E \rightarrow \mathbb{Z}^+$ and $L \subseteq V$ terminals.

Output: A tree $T'(V' \supseteq L, E' \subseteq E)$ of G such that minimize $W(E')$.

// Minimum weighted tree with all terminals.

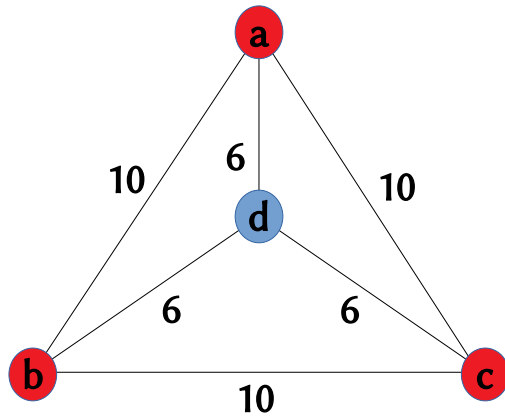


Fig 1 (a)

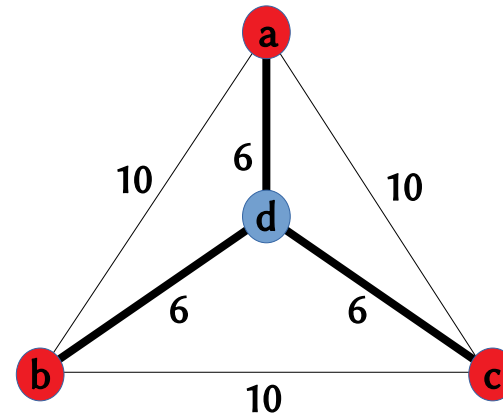


Fig 1 (b)



● Terminals
● Non-terminals

Steiner Tree Problem (STP) – Example

Input : Graph $G(V, E, W)$ $W:E \rightarrow \mathbb{Z}^+$ and $L \subseteq V$ terminals

Output : Minimum weighted tree with all terminals

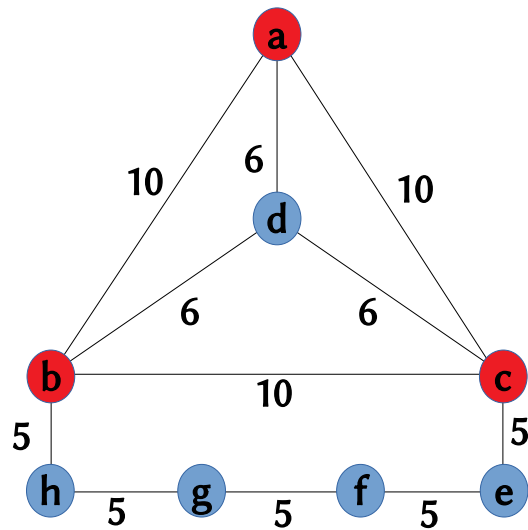


Fig 2 (a)

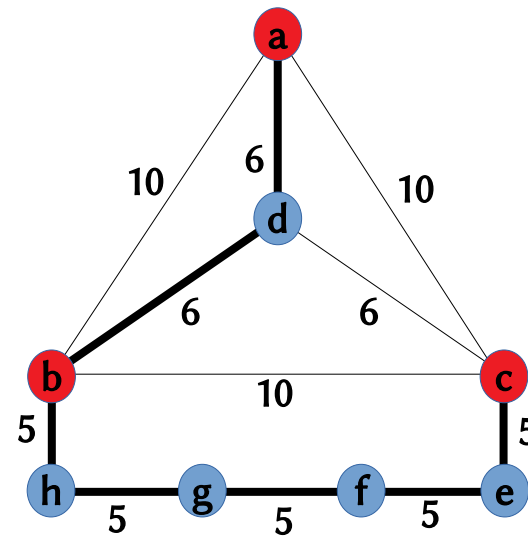


Fig 2 (b)

$|MST| = 37$
 $|OPT| = 18$

● Terminals
● Non-terminals

Steiner Tree Problem (STP) - Hardness

Input : Graph $G(V, E, W)$ $W:E \rightarrow \mathbb{Z}^+$ and $L \subseteq V$ terminals

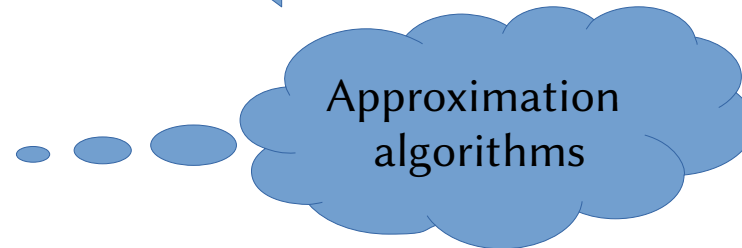
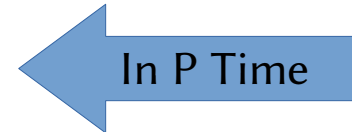
Output : Minimum weighted tree with all terminals

Take away

- MST solution is a valid feasible Steiner Tree solution
- However, solution can be arbitrarily bad w.r.t OPT.

Special cases

- $L = \{u, v\}$ or $k = 2$ STP=ShortestPath_In_G(u, v)
- $L = V$ or $k = n$ STP=MST(G)
- In general STP is NP-Hard



How to deal with NP-Hardness

- What could be naive solutions? Enumerate all Spanning trees.

Approximation algorithm

- Runs in Polynomial time.
- Outputs an approximate solution with some guarantee.
 - e.g 2 or some constant, log n, etc.
- There are several algorithms
 - Kou, Markowsky and Berman[KMB81]
 - Mehlhorn [M88]
 - Robins and Zelikovsky [RZ2000]

$$|ALG| \leq 2 |OPT|$$



KMB

L. Kou, G. Markowsky, and L. Berman. A fast algorithm for Steiner trees. *Acta Informatica*, 1981.

KMB Algorithm $G(V,E,W,L)$

Phase 1

// Input G

Computes the shortest distance between every **pair** of terminals

Phase 2

// Construct $G' = K_L$

Build a graph G' over terminals, having edge-weights corresponding to the shortest distances computed in Phase 1

// Every edge in G' corresponds to a path in G

$MST(G')$

Phase 3

// Construct G''

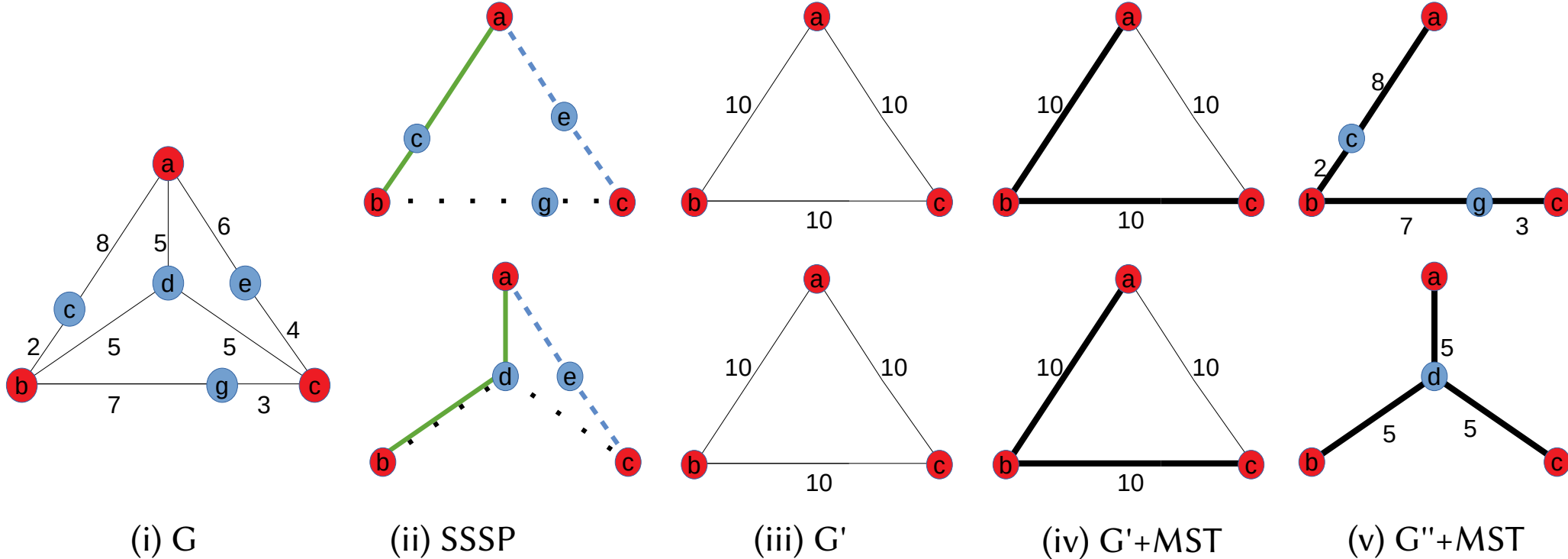
For every edge in $MST(G')$ substitute the edges with the corresponding shortest path in G

// Collect all the edges & vertices of the corresponding path to construct G''

$MST(G'')$

Takeaway: One more invocation for SSSP/MST algorithm. $G \rightarrow G' \rightarrow G''$

KMB Algorithm – Running example



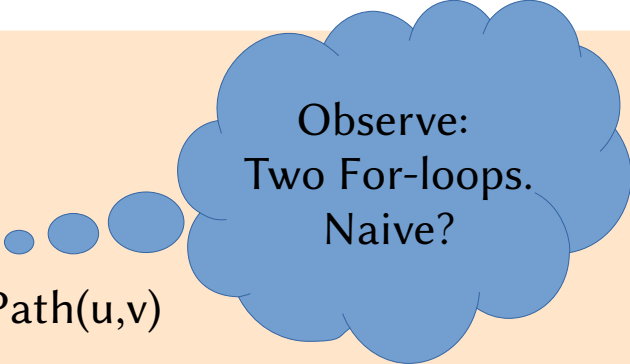
1) A parallel KMB may output a different answer. (2) Last MST may be avoided

Fig. 3 Execution steps of KMB algorithm, where ● are terminals and ● are non-terminals.

KMB Algorithm $G(V, E, W), L$

Phases 1 & 2

```
For u in L {  
  For v in L {  
     $P_{uv} = \text{ShortestPath}(u, v)$   
     $W'(u, v) = |P_{uv}|$   
  }  
}  
T' = \text{MST}(G', W')
```



Observe:
Two For-loops.
Naive?

Phase 3

```
For (u,v) in edges of T' {  
   $G'' = G'' \cup P_{uv}$   
  //Add vertices & edges of  $P_{uv}$   
}  
  
T'' = \text{MST}(G'', W)
```

KMB Algorithm $G(V, E, W, L)$

Input: Graph $G(V, E, W, L)$

Output: 2-approx Steiner Tree $T(V_T, E_T)$ $V_T \supseteq L$

For $u \in L$ {

SSSP(G, W, L, u) with Halt

Compute W' incrementally

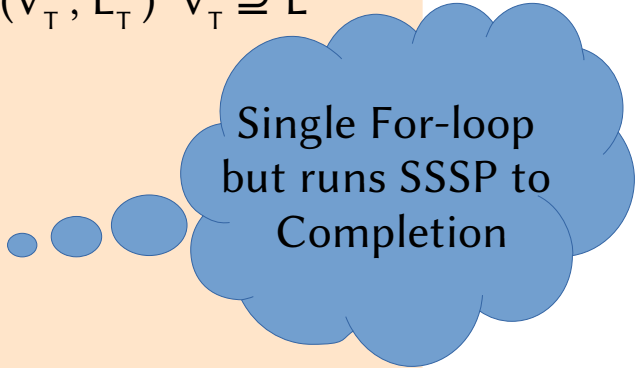
}

$T' = \text{MST}(G', W')$

Compute G'' and its vertices, adjList using T'

$T'' = \text{MST}(G'', W)$

return T''


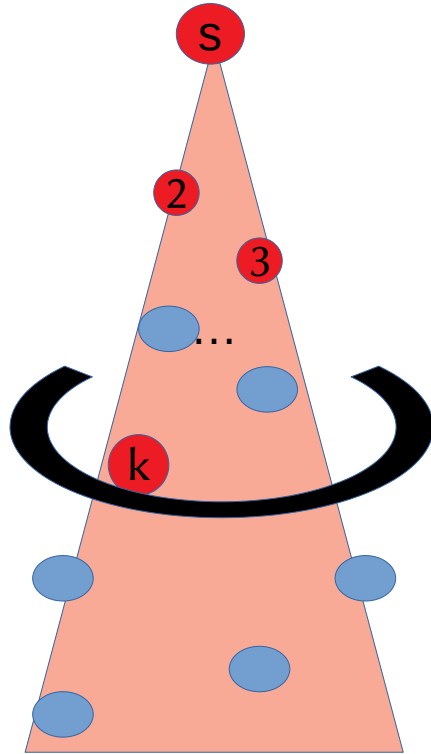


Single For-loop
but runs SSSP to
Completion

CPU Implementation - Optimization

- SSSP-halt optimization

Steps
of
SSSP
execution

Dijkstra Property: when a node u is picked from Q for processing then the $\text{distance}[u]$ is saturated using all the visited nodes.

Halt SSSP when all terminals are visited

Fig. 4 SSSP-halt visualization

KMB Algorithm $G(V, E, W, L)$

Input: Graph $G(V, E, W, L)$

Output: 2-approx Steiner Tree $T(V_T, E_T) \ V_T \supseteq L$

For $u \in L$ {

parallel SSSP(G, W, L, u);

Compute W' incrementally;

}

$T' =$ **parallel** MST(G', W');

Compute G'' and its vertices, adjList ;

$T'' =$ **parallel** MST(G'', W);

return T''

A novel aspect of our work is to run multiple parallel-SSSPs in parallel.

Subroutines?
Gunrock

Design choice for parallelization

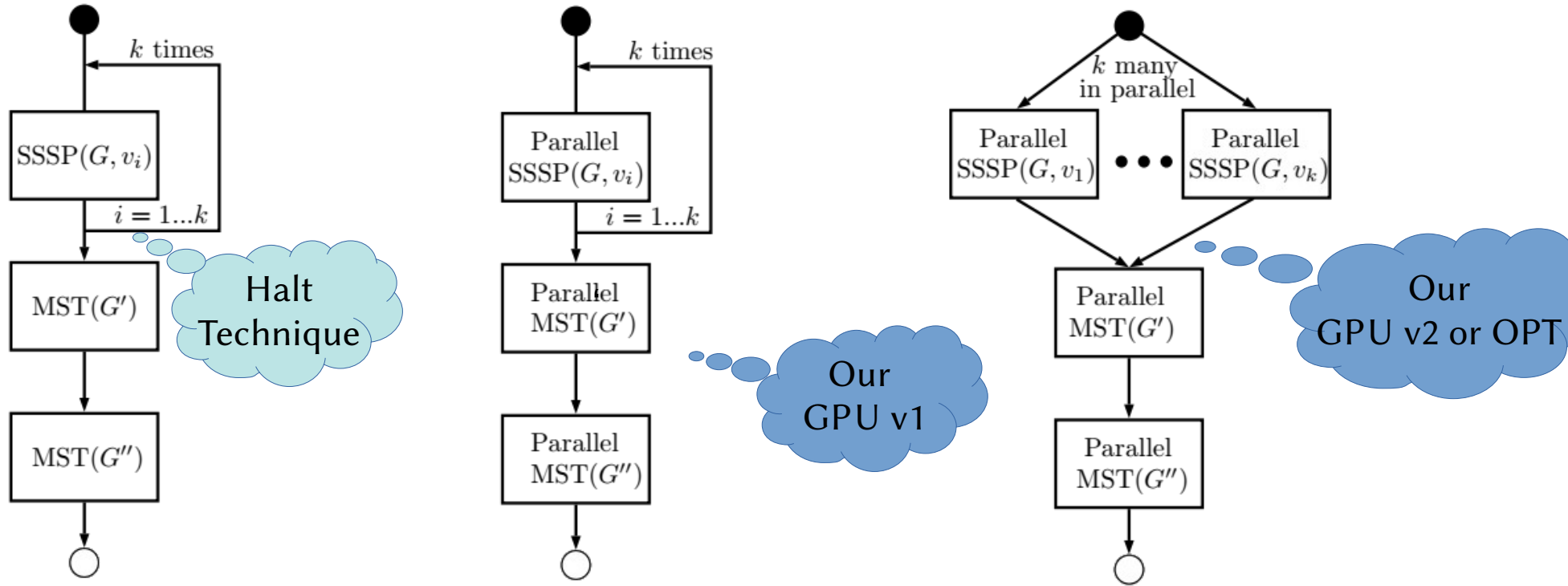
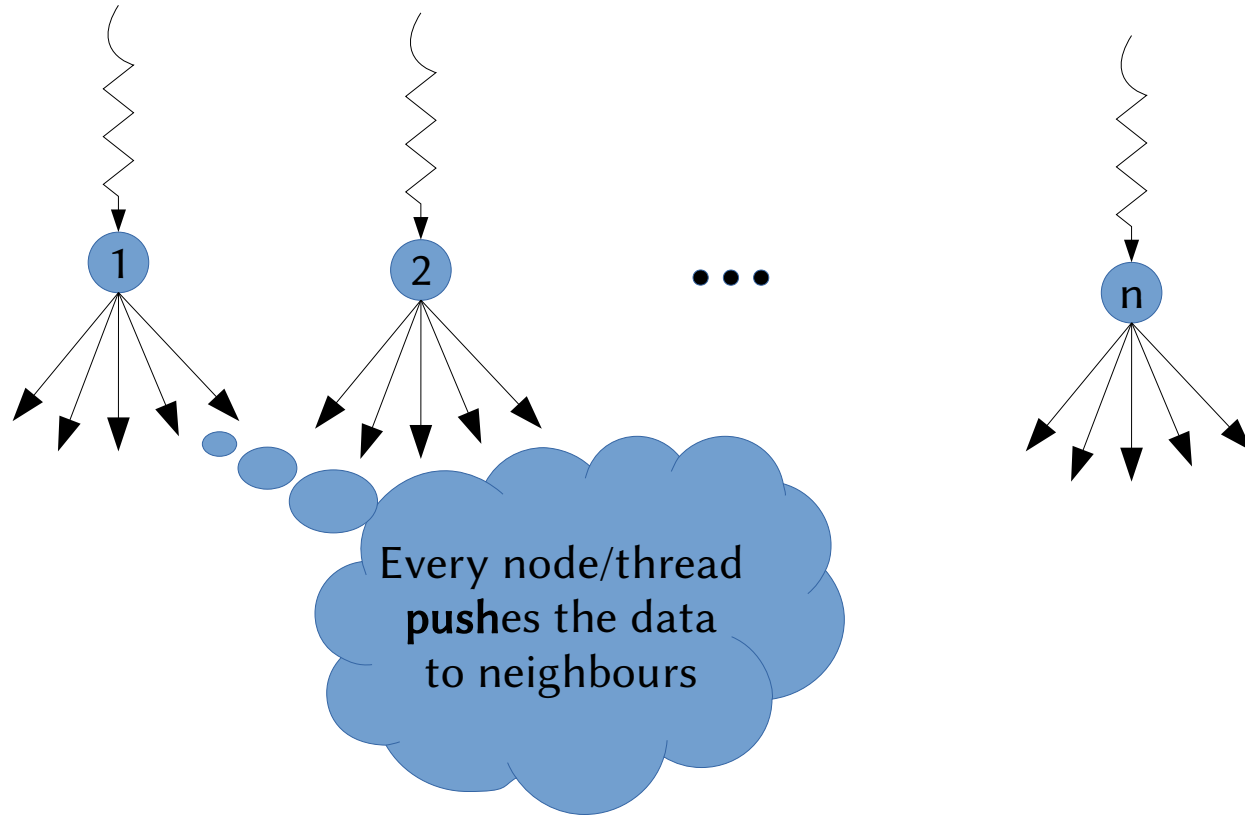


Fig. 5 Sequential vs GPU v1 vs GPU v2

KMBCPU
KMBGPU

GPU Implementation - SSSP



- n-threads
- One thread for each node
- Performs RELAX in parallel
- RELAXes its neighbours
- Till there is no change

Fig. 6 push SSSP

KMB Algorithm $G(V,E,W,L)$

MAIN

For s in L {

ThdsPerBlk = 512; // or 1024

Blks = $\lceil n / \text{ThdsPerBlk} \rceil$;

do {

INIT-KERNEL<Blks,ThdsPerBlk>(s, d_s , p_s , n);

SSSP-KERNEL<Blks,ThdsPerBlk>(., s, d_s , p_s , changed, n);

CopyTo(DArray, d_s); // = = = =

CopyTo(PArray, p_s); // From Device to Host

CopyTo(hChanged, changed); // = = = =

} while (hChanged);

}

- Note we reuse $d[]$ $p[]$ across iterations
- We need the $p[]$ for knowing the intermediate vertices in the shortest path

KMB Algorithm $G(V,E,W,L)$

```
SSSP-KERNEL(...,s, ds, ps, changed, n) {
```

```
u = tid // compute tid;
```

```
if tid < n {
```

```
  For v ∈ adjacent[u] { // Using CSR arrays
```

```
    // Relax Operation (u, v, W(u,v))
```

```
    newCost = ds[u] + W(u, v);
```

```
    old = ds[v];
```

```
    If newCost < old
```

```
      Atomic-MIN(ds[v], newCost);
```

```
    // Updates Parent array
```

```
    If Atomic-MIN is success {
```

```
      ps[v] = u;
```

```
      changed = true;
```

```
    }
```

```
  }
```

```
}
```

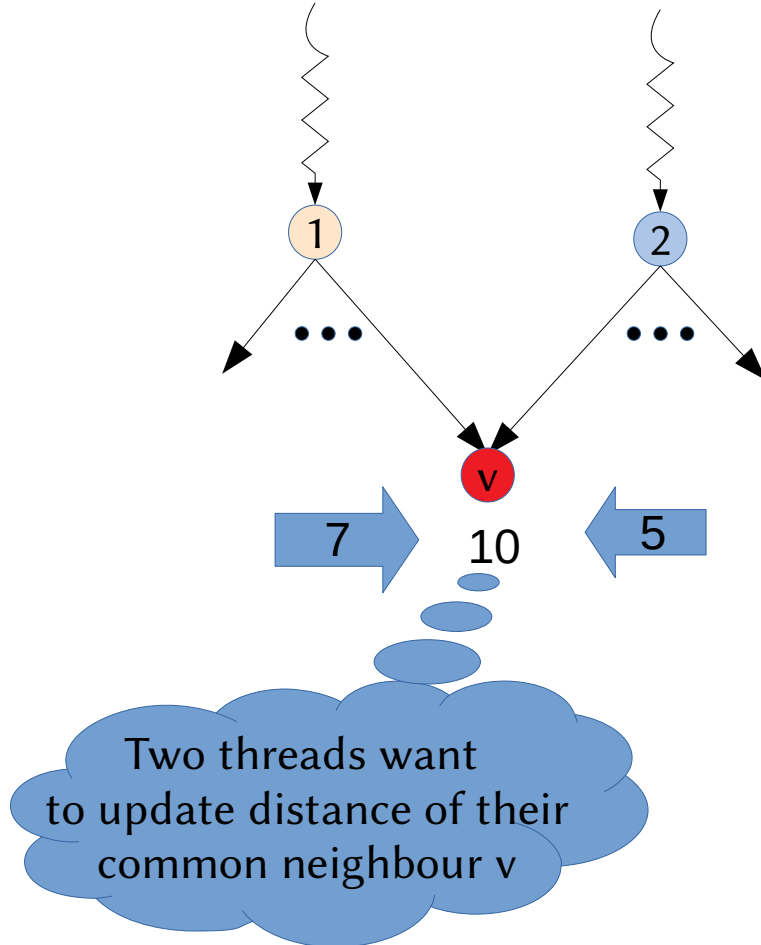
Note :

- Parent of v should be updated if the Atomic-MIN is success



Is it enough?

Parent update - Challenge



```
<snip>
```

```
..
```

```
newCost =  $d_s[u] + W(u, v)$  ;
```

```
old =  $d_s[v]$ ;
```

```
If newCost < old
```

```
    Atomic-MIN( $d_s[v]$ , newCost);
```

```
// Updates Parent array
```

```
If Atomic-MIN is success {
```

```
     $p_s[v] = u$ ;
```

```
    changed = true;
```

```
}
```

```
</snip>
```

Fig. 7 Challenges in parent update

Parent update - Challenge

Shared
d[], p[]

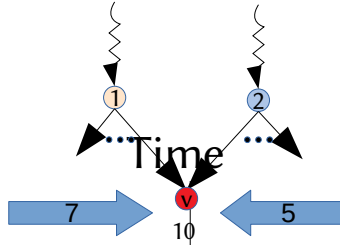
Thread 1

newCost=7
old=10

d[v]=7 //oldA=10

p[v] = 1.

Wrong!



Thread 2

newCost=5
old=10

d[v]=5 //oldA=7

p[v]=2

<snip>

```
newCost = d[u] + W (u, v) ;
old = d[v];
```

```
If newCost < old
    oldA=Atomic-MIN(d[v], newCost);
```

// Atomic-MIN is Success

```
If oldA != old {
    // Update's Parent array
    p[v] = u;
    changed = true;
}
```

It is a challenge to find which “thread” updated d[v] to the minimum

How to update both distance and parent at the same time? Locks?

Synchronization optimization • Pull

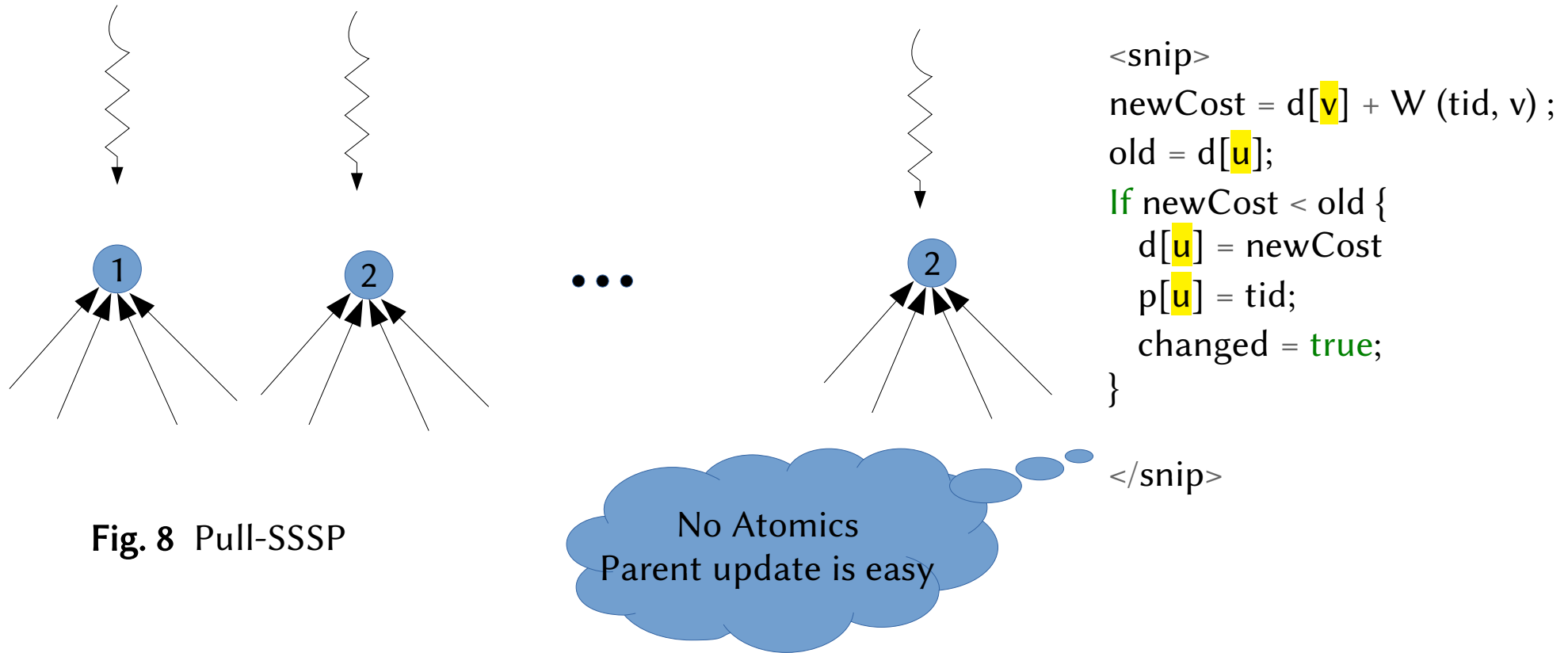


Fig. 8 Pull-SSSP

Because, one thread is writing to an index

GPU Optimizations

- Synchronization
 - Push
 - Pull
- Computation
 - Data-driven
 - Edge-based
 - Controlled Computation unrolling
 - Δ^2
 - 2Δ
 - $t\Delta$
- Memory
 - Shared memory

Δ – max degree of the graph

GPU Optimizations

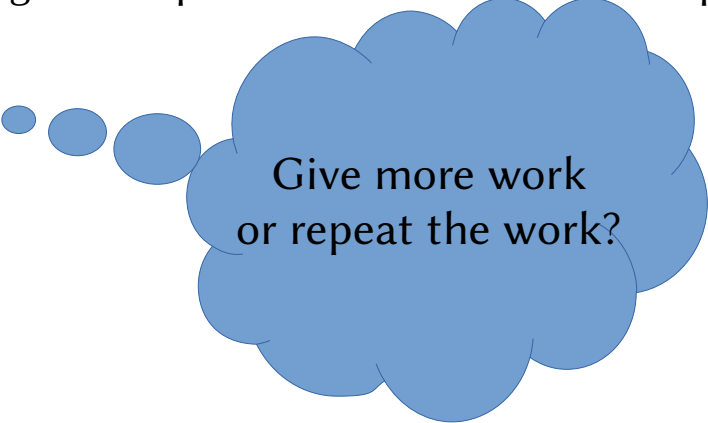
- Synchronization
 - Push
 - **Pull**
- Computation
 - Data-driven
 - Edge-based
 - **Controlled Computation unrolling**
 - Δ^2
 - 2Δ
 - **$t\Delta$**
- Memory
 - **Shared memory**



Δ – max degree of the graph

Compute optimization

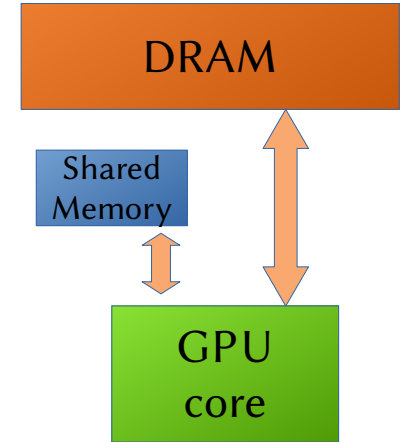
- Computation Unrolling
 - Instead of one thread doing Δ work, perform more work per thread
 - Update also neighbours of neighbours (Δ^2)
 - **Repeat the work**; Say 2 times or t times (2Δ or $t\Delta$); e.g. we do pull 3 times in the kernel – 3-pull
- Data-driven
 - Needs Worklist (WL)
 - Active/Change nodes are inserted into WL
- Edge-based optimization
 - m -threads are launched
 - RELAXes one edge or a group of edges



Give more work
or repeat the work?

Memory optimization

- Programmable shared memory can be useful
- When there are multiple reads to DRAM
- We can move data to shared memory
- For e.g. In 3-pull, we moved CSR AdjList to shared
- As the neighbours AdjList is accessed 3 times
- Of the total 48K per block
- when using 512 threadPerBlock we have 24 words to store per thread
- Hence, if $\text{degree}(\text{node}) < 25$ we use shared, we move $\text{CSR AdjList}[\text{node}]$ to Shared
- With shared memory we achieve 25% of improvement in 3-pull



Double-barrel approach

- SSSP happens in parallel
- To run two SSSP, we have to run one after the other
- Instead we use Double-barrel approach
- This can be generalized (p-SSSP)



In our Double-barrel approach, we run two individually parallel SSSPs also in parallel.

Image source: <https://stock.adobe.com/>

Double-barrel approach

Result Array: $d[n]$

Initialize($d = \text{INTMAX}$)

$d[\text{src}] = 0$

FixedPoint{

doRELAX($G, d, \text{changed} \dots$);

}



Result Array: $d[2n]$

Initialize($d = \text{INTMAX}$)

$d[\text{src1}] = 0$; $d[n + \text{src2}] = 0$

FixedPoint{

doRELAX($G, \text{dist}, \text{changed}, \dots$);

}

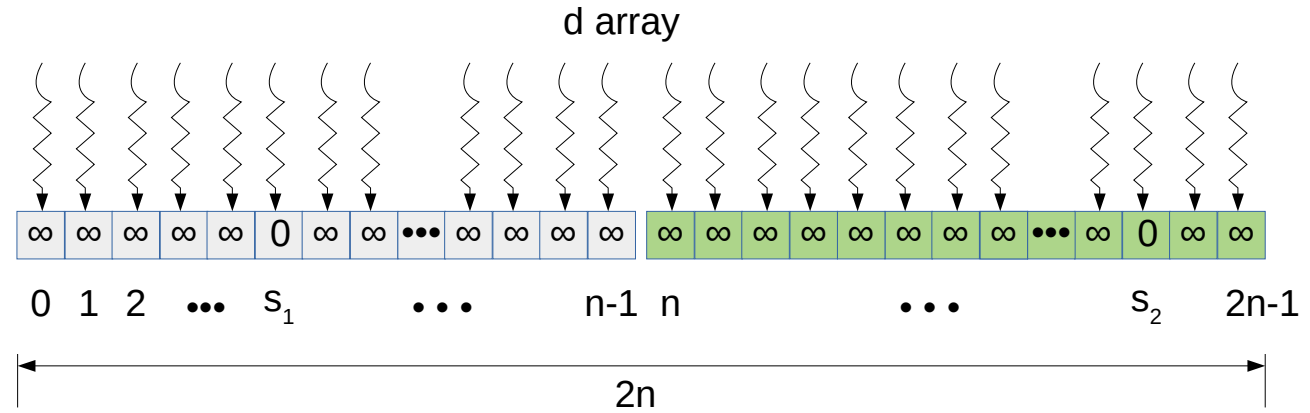


Fig. 9 Double-barrel approach.

Tunable
arbitrary value

Key takeaways so far

- Solving Steiner Tree Problem is NP-hard
- KMB Algorithm, a 2-approximation algorithm
- CPU implementation has SSSP-halt optimization
- SSSP with parent array update was challenging
- Pull-based SSSP is great for KMBGPU even without SSSP-halt
- Parallel-SSSPs in parallel (p-SSSP)

Experimental setup & Graphsuite

CPU

- Intel(R) Xeon(R) E5-2640 v4 @ 2.40GHz
- 64GB RAM

GPU

- Tesla P100 @ 1.33 GHz
- 12GB global memory
- CentOS Linux release 7.5
- GCC 7.3.1 with O3
- CUDA 10.2

Graphsuite

- Total 14 Graphs
 - 11 from PACE Challenge [PACE2018]
 - 2 from SteinLib
 - 1 from SNAP
- n : 17K – 235K
- m : 27K – 498K
- k : 0.1K – 6K

Baselines

- PACE'18 Winner – CIMAT [PACE2018]
- ODGF's KMB/JEA [BC19]

- PACE 2018 - <https://pacechallenge.org/2018/steiner-tree/>
- CIMAT Team - <https://github.com/HeathcliffAC/SteinerTreeProblem>
- S. Beyer and M. Chimani, Strong Steiner Tree Approximations in Practice, JEA 2019.

Challenges in parallelizing KMB

- Graph algorithms in general has an irregular access pattern.
 - Defies the scope of parallelizing
- Involvement of multiple primitive algorithms (such as SSSP and MST)
 - Dependence on an algorithm input from the output of previous algorithm
- Maintaining consistent parent information in SSSP along with distances.
 - Individual atomic instructions may not lead to atomic transactions.
- Parallel KMB may output different solutions during different invocations,
 - Makes it difficult to validate the solution,

Experiments - Speed-up

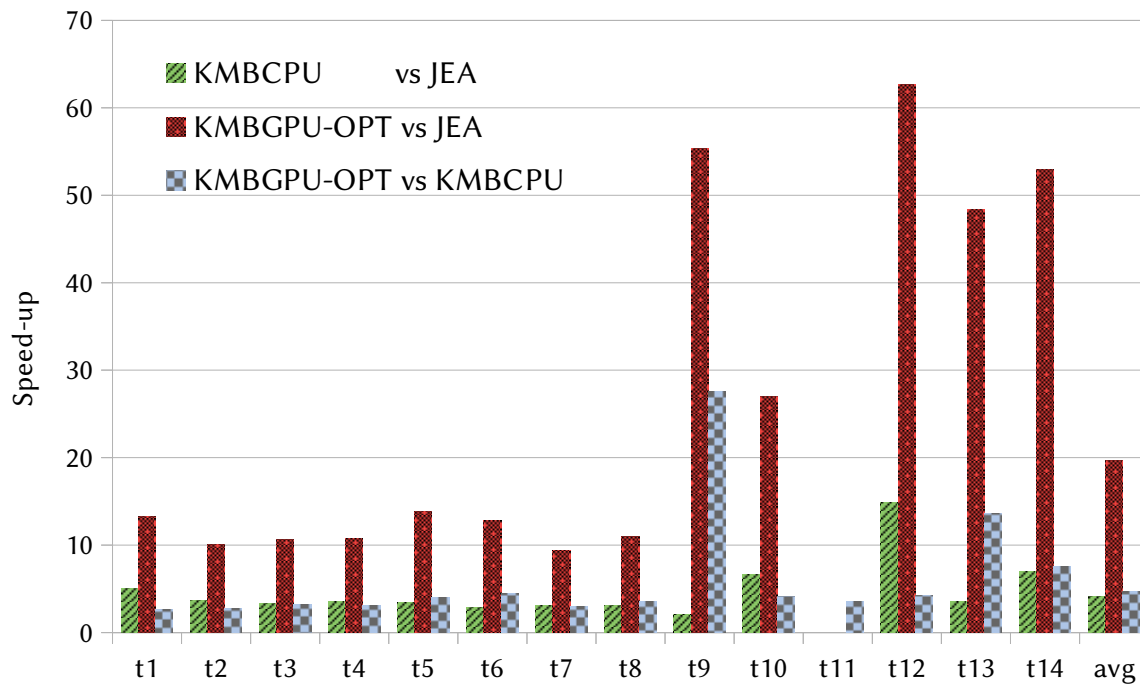


Fig. 10 Speed-up comparisons of the implementations (higher is better). JEA timed-out on t11

Takeaway: KMBCPU and KMBGPUOPT is better than JEA

Comparison of GPU time with Shared mem.

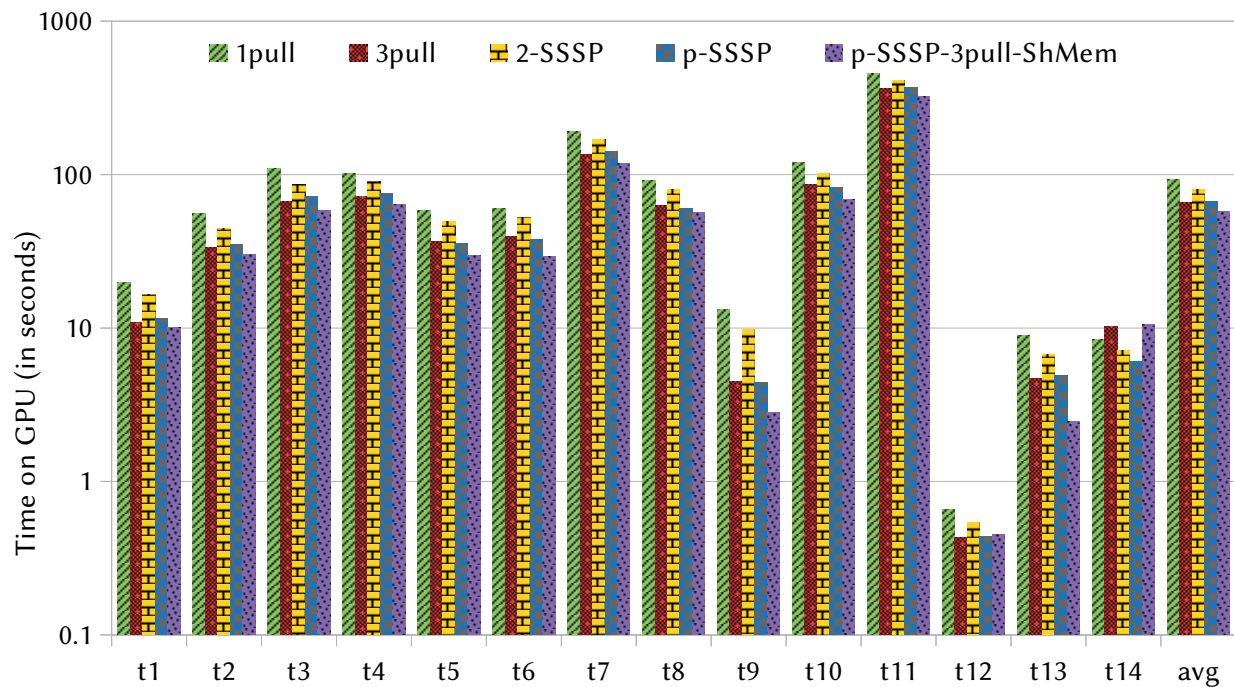


Fig. 11 Comparison of 1-Pull, 3-Pull, Double-barrel & p-SSSP+3-Pull+shared memory (smaller is better). Note: 1-Pull is KMBGPU whereas p-SSSP-3pull-ShMem is KMBGPU-OPT

Takeaway: Combining GPU optimizations p-SSSP, 3-Pull & Shared memory performs best.

Comparison of p-SSSP

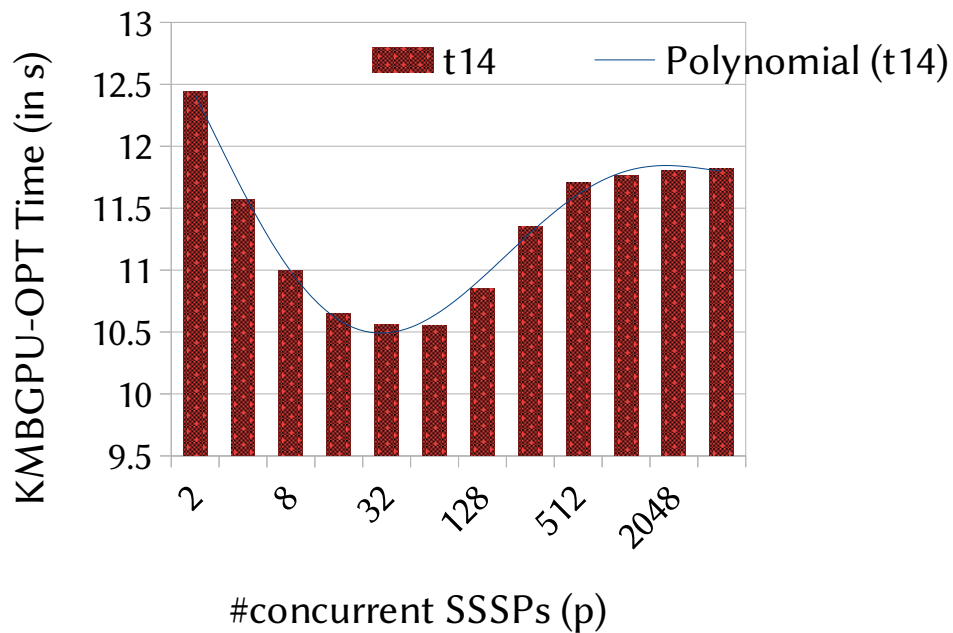


Fig. 12 KMBGPU with varying p-SSSP for the same graphs t14 (Smaller is better).

Takeaway: As we increase the #parallel SSSPs it reaches a point and then increases.

Experiments – Scalability of GPU and CPU

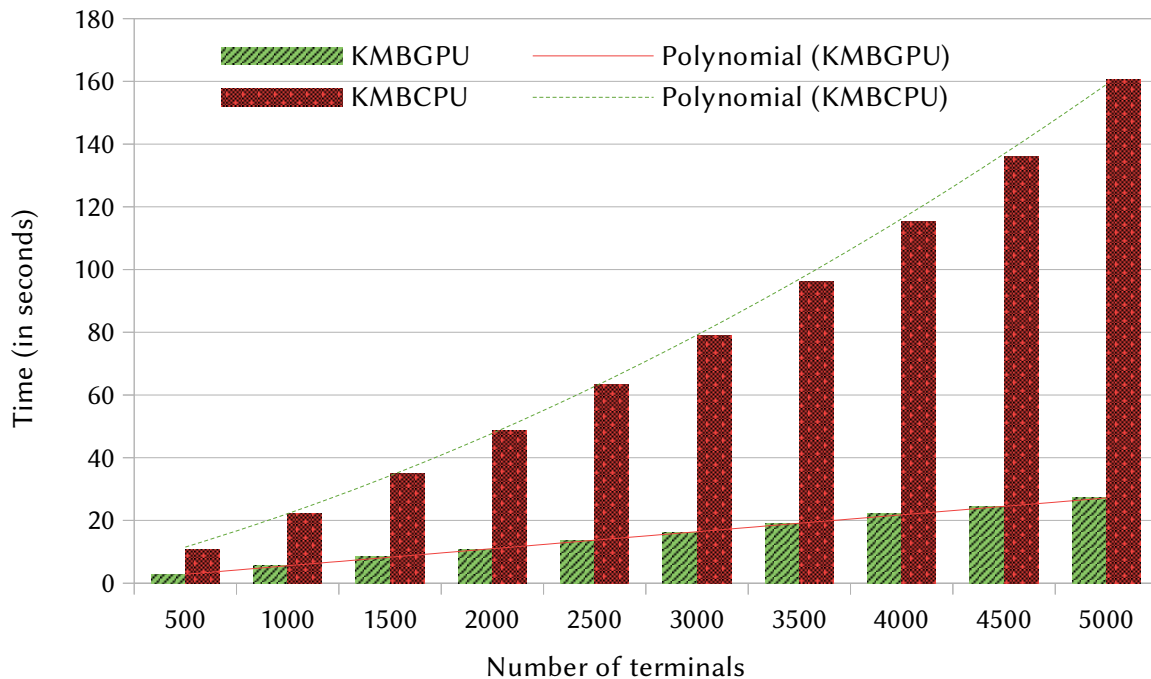


Fig. 13 Scalability plot on **t14** with increasing terminal size (lower is better)

Takeaway: KMBGPU-OPT scales better than KMBCPU

Summary - STP



- Optimized CPU implementation for KMB algorithm
 - Novel SSSP-halt technique
 - Speed-up upto 15x (average 4x) improvement over JEA/OGDF's KMB[BC19]
- Optimized GPU implementation for KMB algorithm
 - Novel p-SSSP technique (multiple parallel-SSSP in parallel)
 - Speed-up upto 27x (average 4x) over sequential CPU [MNN22]
 - Speed-up upto 62x (average 20x) over sequential JEA/OGDF's KMB [BC19]

</work>

S. Beyer and M. Chimani, Strong Steiner Tree Approximations in Practice, JEA 2019.

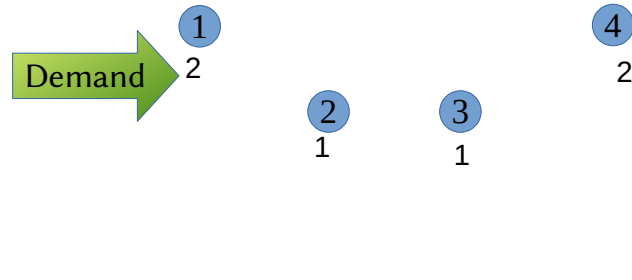
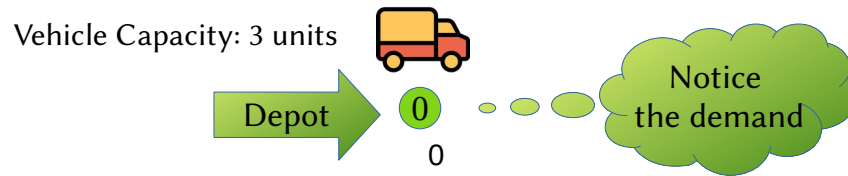
Capacitated Vehicle Routing Problem (CVRP)



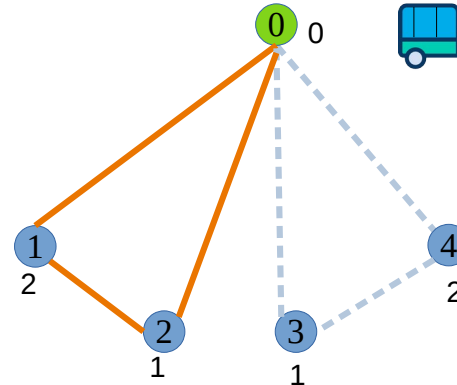
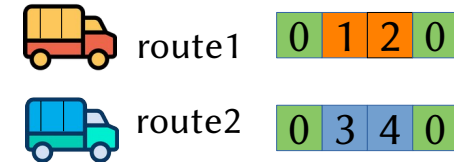
Input : Given n nodes (single Depot and customers) with their coordinates (x_i, y_i) and demands $d_i > 0$ for $i \in n$, Vehicle capacity C . Node 0 is Depot and has zero demand.

Output: Set of routes serving all the customers respecting the vehicle capacity from/to Depot.

Goal : Minimize total distance travelled.



If Capacity = sum of demands d_i ,
CVRP \rightarrow Travelling Salesman Problem



NP-Hard

CVRP Limitations



Current state-of-the-art

- work only on smaller instances
- has a large solution Gap
- takes a lot of time

Instance	Number of customers	Time (s)	
		Base2	Base1
Flanders2	30,000	8,355	2,534
Flanders1	20,000	7,768	2,031
Brussels1	15,000	7,164	871

Table 4: State-of-the-art GPU methods are time-consuming.



RQ1. Can we invent a simpler algorithm?



RQ2. Can we reduce Gap on large instances?



RQ3. Design Parallelization friendly algorithms?

Our ParMDS

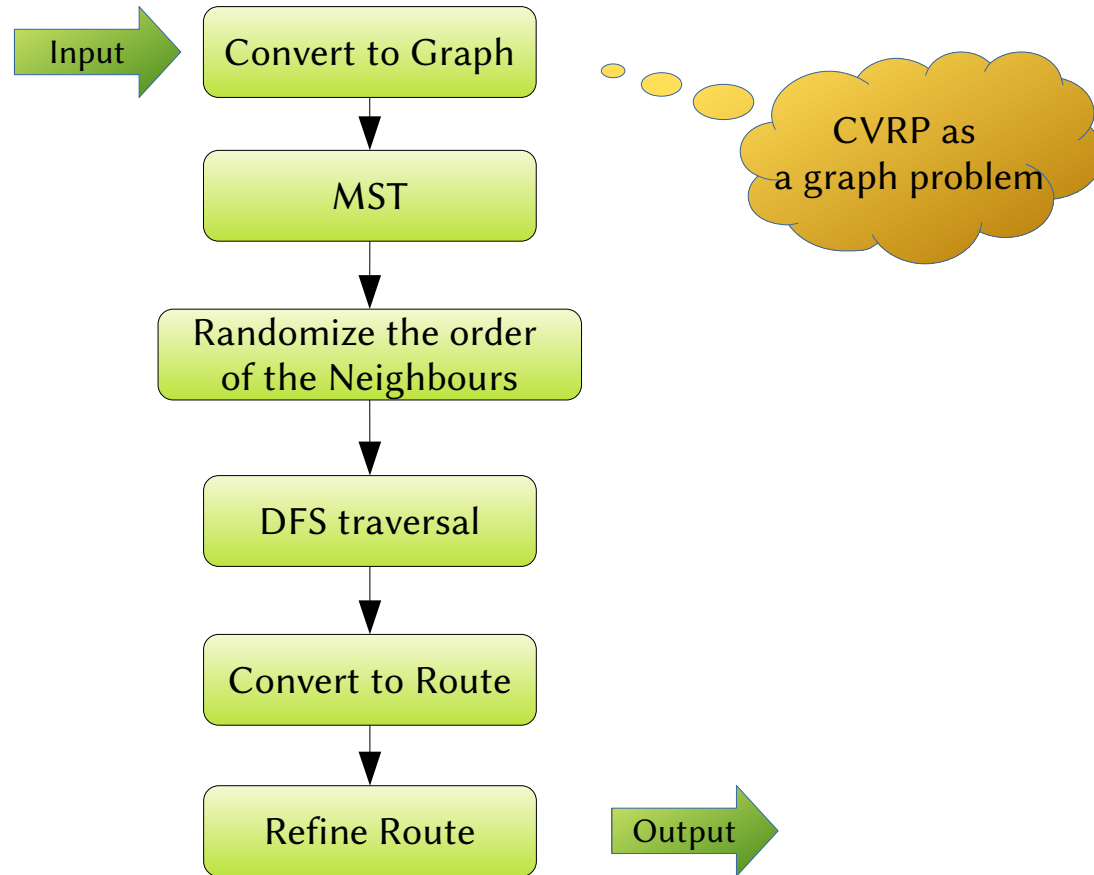
- Serial and **Par**allel implementation
- Combining **M**ST and **D**FS
- Uses Local-search approach
- Uses Randomization approach

$$Gap = \frac{Z_S - Z_{BKS}}{Z_{BKS}} \times 100$$

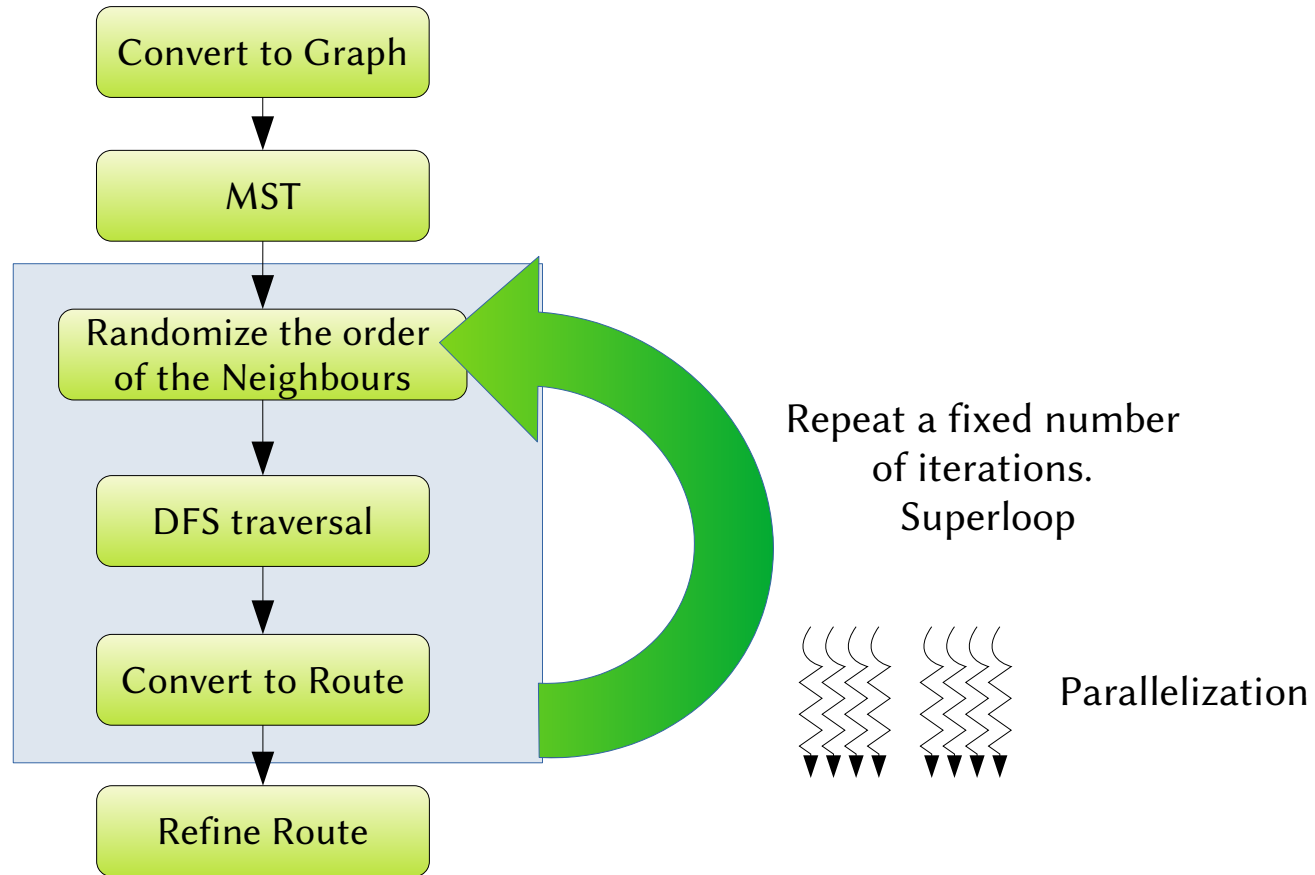
Baseline1: P. Yelmewad and B. Talawar. Parallel Version of Local Search Heuristic Algorithm to Solve Capacitated Vehicle Routing Problem, Cluster Computing, 2021.

Baseline2: M. Abdelatti and M. Sodhi. An improved GPU-accelerated heuristic technique applied to the Capacitated Vehicle Routing Problem, GECCO, 2020.

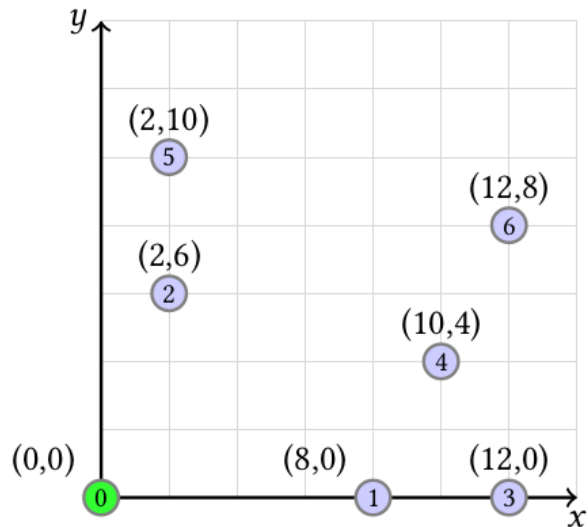
Overview - ParMDS



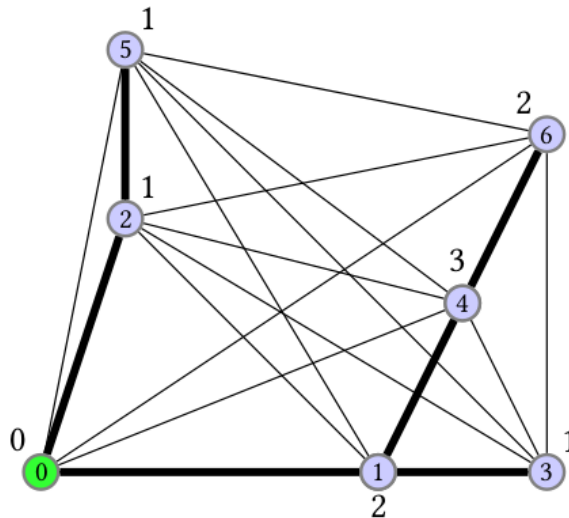
Overview - ParMDS



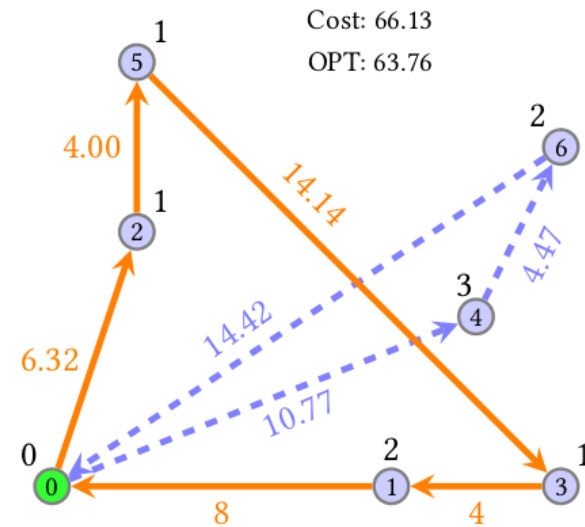
Example - Overview



(a) Input instance I



(b) Graph for I , along with node-demands

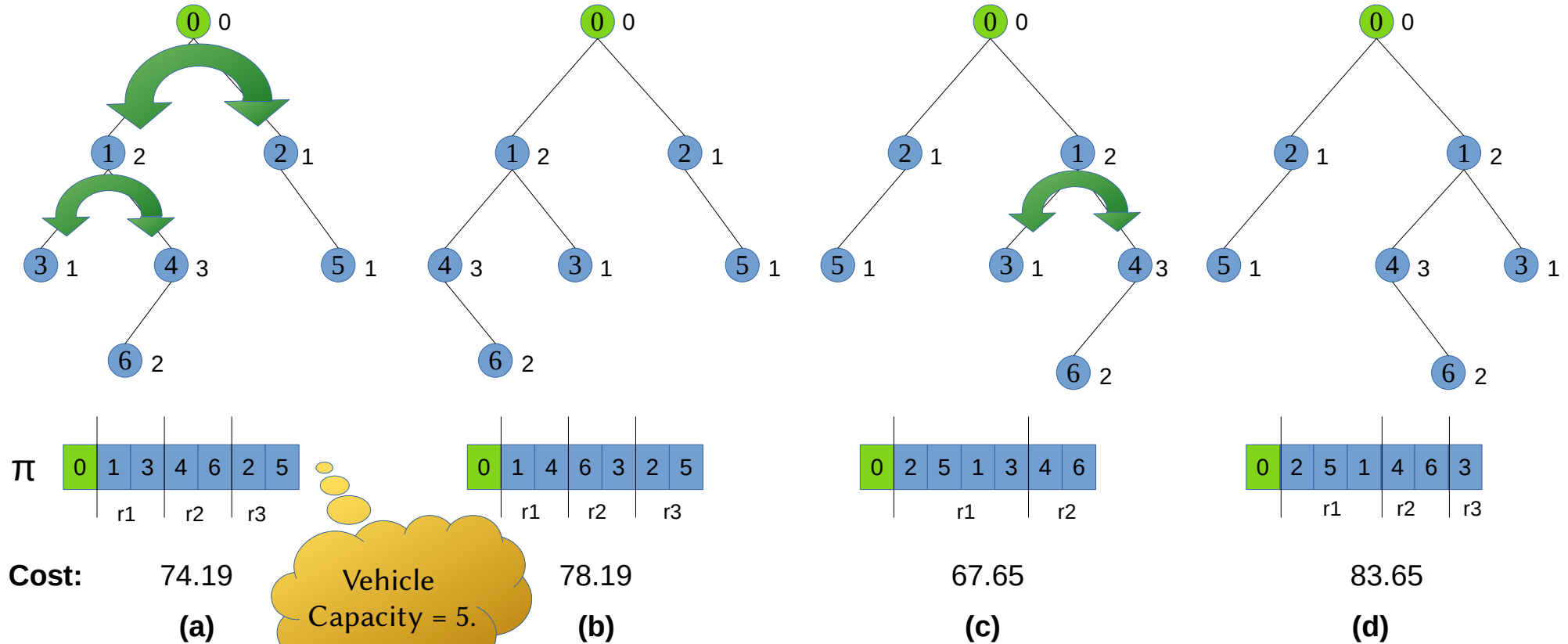


(c) Final routes generated by ParMDS

Cost: 66.13
OPT: 63.76

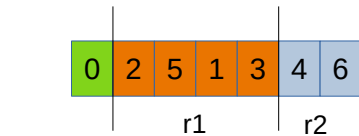
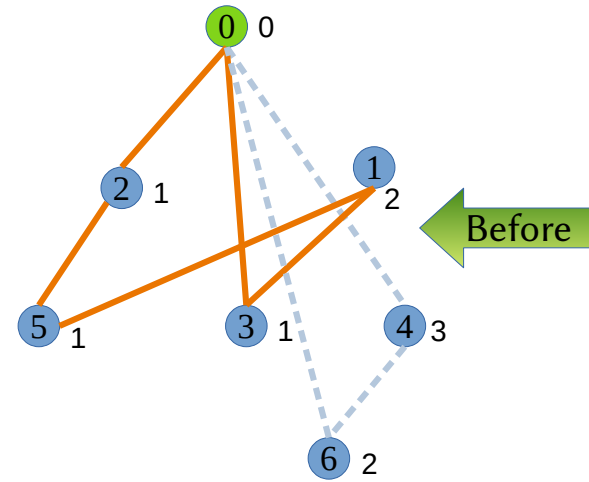
ParMDS on an example input instance with $n = 7$ and Vehicle Capacity = 5.

Example – DFS and Randomization



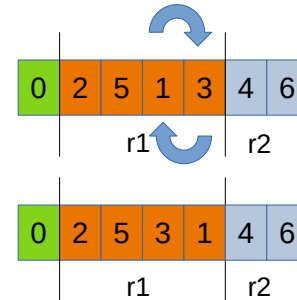
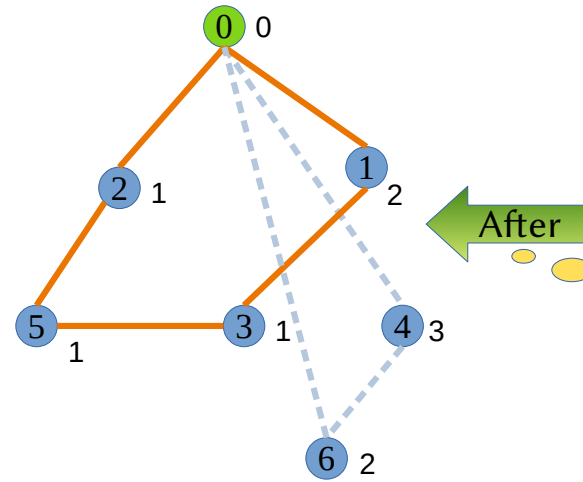
Takeaway: Randomizing neighbours of MST may yield a different DFS ordering. Hence, a different route!

Intra-route optimization - 2Opt



Cost: 67.65

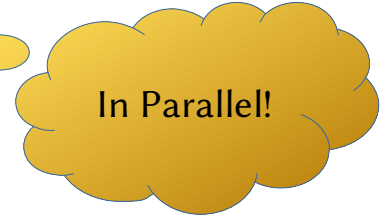
(a)



67.65

66.13

(b)



ParMDS Algorithm

Input: $G = (V, E)$, Demands $D := \bigcup_{i=1}^n d_i$, Capacity Q

Output: R , a collection of routes as a valid CVRP solution

C_R , the cost of R

```

1   $T \leftarrow \text{PRIMS\_MST}(G)$                                 /* Step 1 */
2   $C_R \leftarrow \infty$ 
3  for  $i \leftarrow 1$  to  $\rho$  do    /* Superloop */ /* Parallel */
4       $T_i \leftarrow \text{RANDOMIZE}(T)$  /* Shuffle Adjacency List */
5       $\pi_i \leftarrow \text{DFS\_VISIT}(T_i, \text{Depot})$           /* Step 2 */
6       $R_i \leftarrow \text{CONVERT\_TO\_ROUTES}(\pi_i, Q, D)$  /* Step 3 */
7       $C_{R_i} \leftarrow \text{CALCULATE\_COST}(R_i)$           /* Parallel */
8      if  $C_{R_i} < C_R$  then
9           $C_R \leftarrow C_{R_i}$           /* Current Min Cost */
10          $R' \leftarrow R_i$           /* Current Min Cost Route */
11     end
12 end
13  $R \leftarrow \text{REFINE\_ROUTES}(R')$                     /* Step 4 */
14 return  $R, C_R$ 

```

Zoom

```

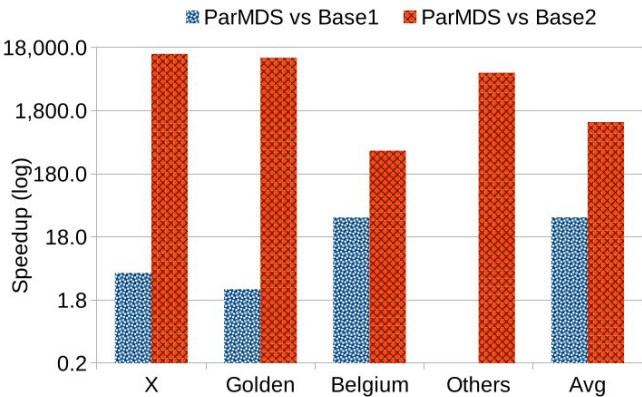
/* Standard: stride = 1; */
/* Strided : stride = #CPU cores */
/* Parallel for loop: Standard/Strided */
1 for  $i \leftarrow 1; i \leq \rho; i = i + \text{stride}$  do
2     for  $v \in V$  do
3         /* seed  $\leftarrow$  constant or  $i$  or  $\text{rand}()$  */
4          $\text{SHUFFLE\_NEIGHBORS}(\text{AdjList}(v), \text{seed});$ 
5     end
6 end
7 ...

```

Experiments

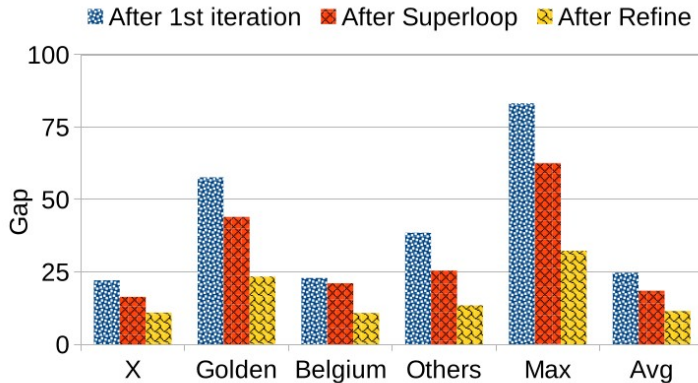
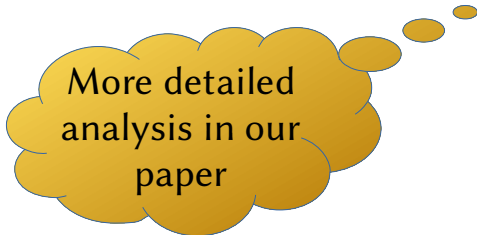


- 130 Instances of CVRPLIB
- Intel Xeon CPU E5-2640 v4
- Baselines on GPU
 - NVIDIA's Tesla P100
 - CUDA 11.5
- Our Code uses
 - **SeqMDS**: GCC 9.3.1
 - **ParMDS**: nvc++ compiler NVIDIA's HPC SDK 22.11



Speedup of ParMDS vs. baselines

Method	Execution Time (s) using Random
SeqMDS	1,722.44
ParMDS-Standard	1,522.26
ParMDS-Strided	186.50



Gap at the end of each step

Summary

- Fresh perspective of parallelism-friendly algorithms
- Performance: Algorithmic-, Parallelism- and Platform-Optimizations
- Our techniques are applicable
 - Two-level parallelism technique
 - Strided parallel Local-search
- Future directions
 - Paradigm specific parallelization: Greedy, Dynamic Programming
 - Most STL algorithms run parallel // My Prediction
 - Once source for muti-core and GPU

Thank you!

Questions?