EEE4118F - GA Cascade Control Design Report

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Contents

Abstract

This study aimed to construct a functioning robust digital controller to monitor and manage the speed and position of a servo motor system in three stages: System identification, Controller Design, and Practical Implementation. The System Identification stage involved simulating the system to determine a mathematical representation of the servo motor system, which was used to design and test the controller. The Controller Design stage constructed a cascade controller model, which was used to control both the speed and position of the output signal. The Practical Implementation stage implemented the controller with the physical servo motor system and tested the validity of the simulink system designed in the previous stages. The final controllers after the practical implementation were calculated to be : $G_s(z) = 0.1$ and $G_p(z) = 0.33$. The uncertainty in this experiment resulted from the tools used to take the measurements of the data and the components involved in the servo motor system. To mitigate these uncertainties, it is recommended to perform multiple tests and sensitivity analyses. The study concluded that the implementation of the cascade controller is effective in improving the performance of the servo motor system.

I. Introduction

This project is based on the construction and implementation of a robust digital position controller for a servo motor system. The controller model required for this system, based on its capabilities, is a cascade controller model. The controller's purpose will be to monitor and correct the angle of rotation of the output potentiometer in order to minimise the overshoot, settling time and the error within the output.

This project was split into three investigations, the first of which was conducted to determine the parameters of the plant transfer function as well as the gear reduction ratio. Using these parameters, the second investigation produced a design for a cascade controller model made up of speed and position controllers for use in ideal conditions in order to meet the specifications provided. The third investigation was conducted in order to physically implement the cascade controller model designed based on ideal conditions. The implementation was done through the use of digitised versions of the speed and position controllers in the servo motor software.

The performance of the cascade controller model was then compared to the desired performance provided in the form of the requirements from the user.

II. Model Summary

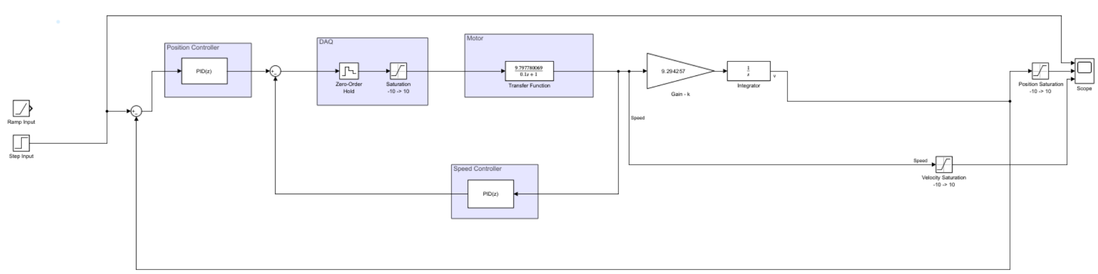


Fig. 1: Simulink block diagram illustrating the system.

The model designed on Simulink changed over the course of the project, starting with the Simulink model used in System Identification which was used to test the validity of the calculations for the plant transfer function by comparing the resulting ideal case plots with the data-based plots based on the values measured from the simulation software. The changes made to the model are due to the introduction of controllers which affect the speed - either speeding it up or slowing it down to decrease settling time or decrease the oscillations of the system - and the position of the system - to reduce the overshoot.

Figure 1 shows the final model of the system which includes two controllers (in feedback loops to create a cascade controller), a zero order hold and saturation block (to illustrate the effects of the DAQ in the physical system), the plant, a gain block (accounting for the gear reduction ratio of the servo motor system), an integrator (which accounts for and reduces the steady state error at the output) and two saturation blocks at the output (which restrict the output voltage to a value between 10V and -10V in order to only work within the restraints of the physical system).

The input into the system is a unit step which is used to signify the change of voltage that would occur when a desired position - or angle of rotation - is chosen when using the servo motor system. The change in the angle relative to the origin at the output is measured in volts and this is the reasoning behind the type of input supplied to the system. The input signal provides the system with enough information of the position it wishes the motor to move to. The two controllers are both digital controllers - allowing for the implementation of the controller to take place digitally through the simulation software used to run the servo motor system. The zero order hold was present in the system identification because the continuous time signal supplied at the input needs to be converted into a discrete signal before it can be processed by the rest of the system. The zero-order hold holds the continuous-time signal at a constant value for a brief period, effectively converting the signal to a piecewise constant signal that can be processed by an analog controller. With the conversion of the controllers to digital controllers in the z domain, it is expected that the zero order hold can be removed but since in the case of this specific servo motor system, both controllers turned out to be proportional in nature therefore without the factor of z present resulting in a need for the zero order hold. The zero order hold is required in this case because the controllers cannot sample the input signal for themselves without the presence of a z term anywhere in either of them. The integrator is present in the system to allow for the conversion of the speed at the output of the plant to be converted back into a position value which can be processed as the output of the system - resulting in the angle change by the specified voltage value.

III. System Identification

A. Methodology

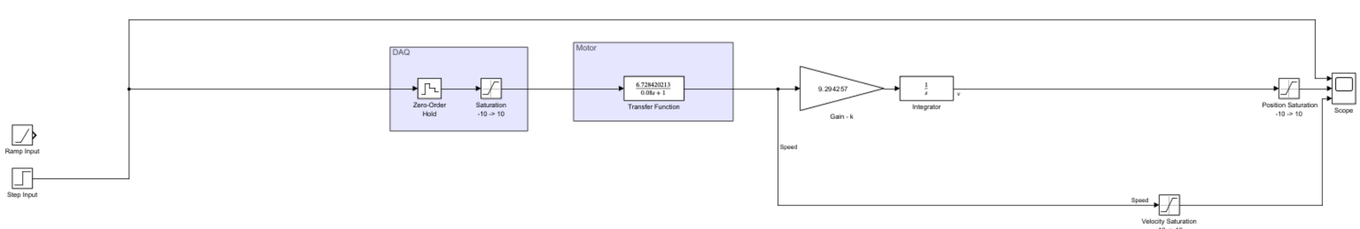
The software in use for the simulation of the servo motor and its controller is coded in CSharp and allows for the testing of different inputs to the system. Differing input types are on offer within the software - of which the ramp input and step input where used in this experiment. The input and output values are voltage values which is the conversion done by the system to represent a change in the angular distance from the origin as a change in voltage at the output.

The plant's transfer function was determined using six tests - which make up three test cases - allowing for the calculation of the open loop gain (A) and time constant (τ). The tests will determine a range of A values and a correlating range of τ values.

The first test was conducted using a ramp input in order to determine the saturation point of the system to determine the largest testable step. The second test made use of a step input from 0.3V to 0.55V - a step of 0.25V. This test was conducted with three stages of varying the load, first with no load applied, then with the effect of one pair of magnets affecting the system and the last stage involved the use of two pairs of magnets in order to measure the difference the load has on the output. These tests were conducted to measure three sets of values for the A and τ values in order to create a range for the two uncertainties involved in the plant transfer function. The gear reduction ratio (k) was also measured across the three step tests and averaged in order to determine the effective constant value throughout the testing. The three loading stages allowed for the bounds of the range to be determined as well as the nominal plant equation using the measurements from the second test. The values for A and τ were determined from the data logs of the simulations by constructing the required plot based on the step input and the motor velocity step response.

The value for k was calculated using the slope of the output plots for each of the step tests when the system was not in voltage saturation (dv/dt). The slope of each of the output plots was then divided by the motor velocity before the step is applied.

Using MATLAB Simulink, the A and τ ranges were implemented along with the constant k value. The simulation was conducted in order to reaffirm the accuracy of the calculations as well as the Simulink system design for the next stages of the controller design. The accuracy was tested by comparing the plotted results from the simulation software data logs with the plots produced on Simulink. The Simulink diagram within the testing is as follows:



B. Results

1) A & τ Calculations

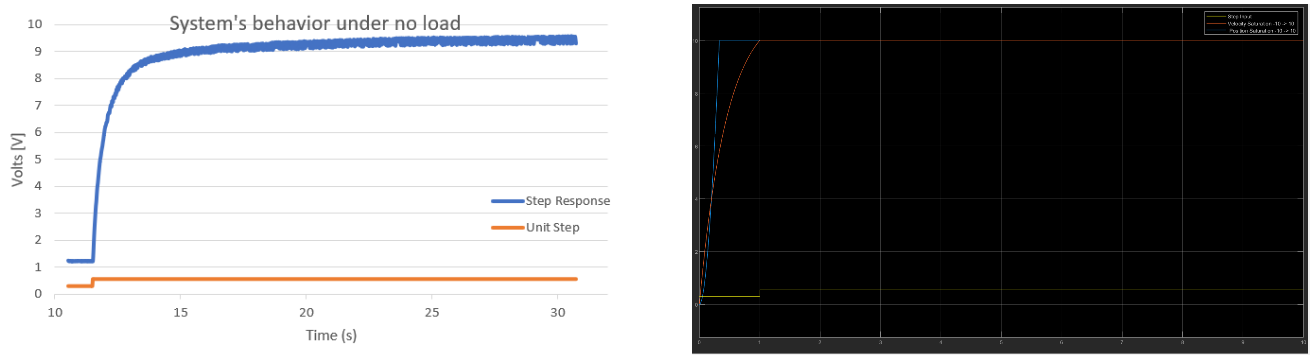


Fig. 2: Graphs showing the data gathered and the simulations of the relationship between the motor velocity and time when no load is applied

From the data, the gain A and the time constant τ is gathered:

$A = 37.61322156$

$\tau = 0.46s$

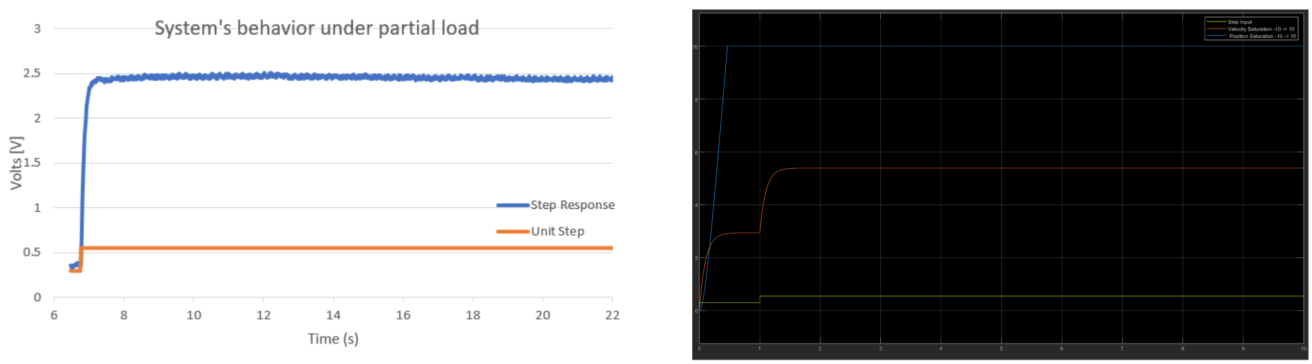


Fig. 3: Graphs showing the data gathered and the simulations of the relationship between the motor velocity and time when partial load is applied

From the data, the gain A and the time constant τ is gathered:

$A = 9.797780069$

$\tau = 0.1s$

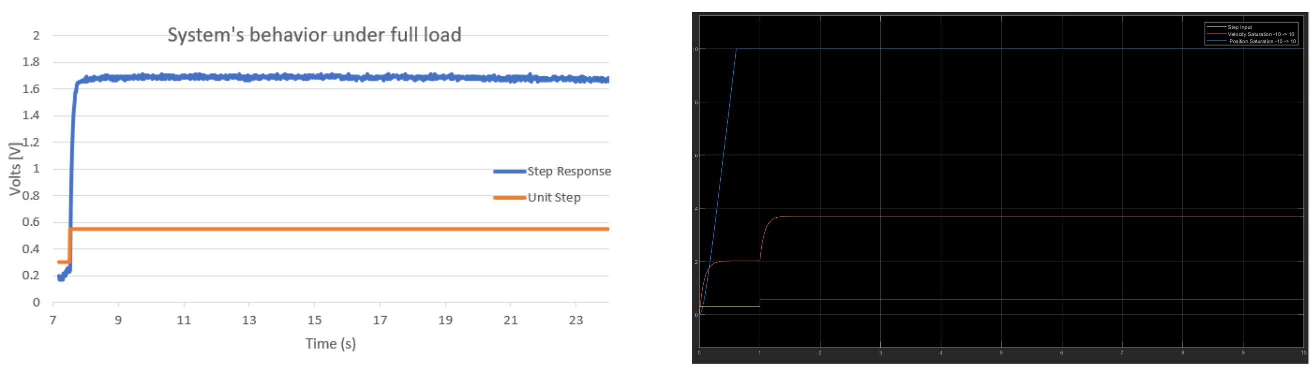


Fig. 4: Graphs showing the data gathered and the simulations of the relationship between the motor velocity and time when full load is applied

From the data, the gain A and the time constant τ is gathered:

$A = 6.728420213$

$\tau = 0.08s$

2) k Calculation

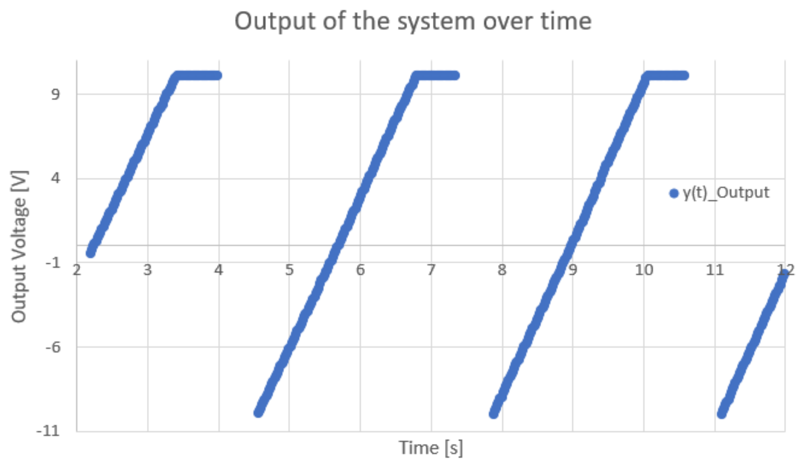


Fig. 5: Graph showing the output of the system over time

From the data, the gear reduction ratio k was determined: $k = 9.294257$

C. Analysis

The process of system identification was conducted with the servo motor system in order to construct a transfer function representation of the plant. The calculations for the open loop gain (A) and time constant (τ) were used to construct the transfer function of the plant.

The values for A and τ were calculated from the results of three unit step tests of varying loads. The range of values for A was calculated to be $[6.73, 37.61]$ and the range for τ was calculated to be $[0.08, 0.46]$. The gear reduction ratio (k) was calculated to be a constant value across the three step tests. The k value for the specific servo motor system at a specified attenuation level was determined to be 9.294257. The nominal plant was defined as the partially loaded system - with values for A and τ that fall within the specified ranges.

The insertion of these values yielded the step responses which can be seen in the figures on the right hand side of Figures 2, 3 and 4. The Simulink system created an ideal model for the output of the system to allow for the determination of the accuracy of the calculations within the system. This facilitated the validity testing of the data as well as well as the calculations by comparing the motor velocity step responses in the practical case and ideal case using the same data set.

The largest difference between the ideal case created on Simulink and the plot from the data set was noted to occur when no-load is applied to the system. This caused the motor velocity in the practical data to tend asymptotically towards 10V, but never reach the value. In the Simulink plot of the data as can be seen in Figure 2, the motor velocity saturates to 10V after a number of seconds. Neither of the other step tests showed a major difference between the practical and ideal plant systems. Therefore, from this point the Simulink diagram can be used to accurately model the plant of the servo motor system, allowing for the controller for the system to be designed remotely and implemented when complete.

IV. Specifications & Controller Design

A. Methodology

1) Requirements & Specifications

The client supplied a list of requirements which the system needs to adhere to in order to be considered successful.

a) Requirements

The client's requirements for the project include critical aspects such as the ability to track the set point accurately, limit overshoot, achieve settling time, reject output disturbance, and maintain stability during plant alterations.

To meet the set point tracking requirement, the control system must have zero steady-state error. The client specified that the overshoot should not exceed 20% of the final value. The settling time refers to how quickly the system reacts to a change in input and reaches a steady state, and the requirement is that it remains below 1 second. Output disturbance rejection refers to the controller's ability to recover from external disturbances and return to the previous final value, and the recovery time should be within 1 second.

b) Specifications

Using the requirements for the system relating to the overshoot, settling time and steady state error the ideal plant case was designed using a model of a second order system according to the equation: $\frac{A{\omega_n}^2}{s^2+2\zeta\omega_n s+{\omega_n}^2}$. The overshoot from the desired step was used to calculated the damping factor, denoted by ζ, of the plant. The settling time - which is the time taken to reach steady state after the input of the step - was used to calculate the natural frequency (ω_n) of the ideal plant system.

The sensitivity bode plot was constructed, from which the low frequency and high frequency bounds were read. The low frequency bound was taken as the frequency correlating to a closed loop gain of $-20dB$. The robust stability margin was calculated from the peak of the bode plot which shows the highest closed loop dB value reached and the frequency at which this gain occurs. The bounds were then programmed into the setup of the QFT tool used to design the controller to manipulate the response to satisfy the bounds calculated.

2) Controller

The design of the controller was done using the QFT controller design tool. The bounds for the controller design were implemented in the command which runs the QFT design tool. The nominal plant transfer function was used for the design of the controller. First, block diagram simplification was computed for the innermost feedback loop - containing the plant and the speed controller. The speed within the system was very fast to begin with - leading to a settling time within a second with minimal change required. The speed controller was set to a proportional controller with a value less than one in order to increase the speed. The value was less than one due to the positioning of the controller relative to the plant.

After the simplification of the innermost loop, the new block representing that loop was multiplied by the gear reduction ratio (k) and the integrator ($\frac{1}{s}$). This new transfer function was stored and represented the whole system bar the position controller. The feedback loop containing the new transfer function and the position controller is all that is left in the system. From this, the position controller was designed in order to assist the new transfer function in meeting the bounds on the Inverse Nichols Chart (INC) and in meeting the specifications at the output of the system.

3) Simulink System

The two controllers calculated were implemented in a Simulink model of the system which displayed the step responses of the speed and position. The three different plant models - from the range of A and τ values - were all tested for their position and speed step responses to ensure they met the specifications relating to the overshoot and settling time. The two controllers were introduced into the system through the inclusion of two feedback loops which forms a cascade controller. The use of two separate controllers allows for either the speed or position to be adapted through changing the separate controllers based on the requirements.

B. Results

1) Specifications

The specifications were calculated to allow for the determination of the bounds for the QFT design of the controller. The calculations used to determine the specifications are shown below:

a) Overshoot

$M_{pp} = e^{-\frac{\zeta * \pi}{\sqrt{1-\zeta^2}}}$

$M_{pp} = 0.2$

$\frac{\zeta * \pi}{\sqrt{1-\zeta^2}} = -\ln 0.2$

$\zeta^2 \pi^2 = ln(0.2)^2(1-\zeta^2)$

$\zeta^2 = \frac{\ln(0.2)^2}{\pi^2 + \ln(0.2)^2}$

$\zeta = \sqrt{\frac{\ln(0.2)^2}{\pi^2 + \ln(0.2)^2}}$

$\therefore \zeta = 0.456$

The damping factor for the position controller is $\zeta = 0.456$.

The speed controller was specified to a proportional value and therefore the damping factor was used in the determination of the position controller only.

b) Settling Time

$t_{2\%} = \frac{\ln \frac{2}{100} * \sqrt{1-\zeta^2}}{-\zeta * \omega_n}$

$\omega_n = \frac{\ln \frac{2}{100} * \sqrt{1-\zeta^2}}{-\zeta * t_{2\%}}$

$\omega_n = \frac{\ln \frac{2}{100} * \sqrt{1-(0.456)^2}}{-0.456 * t_{2\%}}$

$\therefore \omega_n = 8.836 rad$

The natural frequency for the ideal position controller is $\omega_n = 8.836 rad$.

This value along with the damping factor determined above allowed for the use of the second order system equation to determine the ideal case for the plant which in turn allowed for the determination of the bounds for the controller design.

c) Steady State Error

The steady state error specification was met by implementing an integrator with its primary purpose to convert velocity into position. It also acted as a factor which allowed for a reduction in the steady state error. This component of the system was placed at the end of the model to give $k * 1/s$ where k is the gear reduction ratio. In the Simulink simulations, there is zero steady state error, which follows with the inclusion of the integrator.

d) Final Specifications

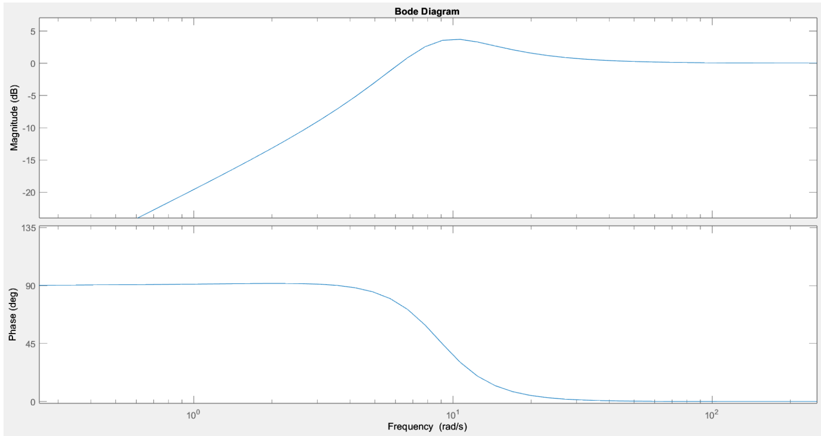


Fig. 6: Sensitivity Bode plot which illustrates the low and high frequency bounds for the QFT design.

From the figure above, the following were calculated as the final specifications for the QFT design:

$i)$ *Zero steady state error*

$ii)$ $|\frac{1}{1+L}| \leq -20dB$ *when* $\omega \leq 0.95$ *rad*

$iii)$ $|\frac{1}{1+L}| \leq 3.7dB\ \forall\ \omega$

2) Controller Design

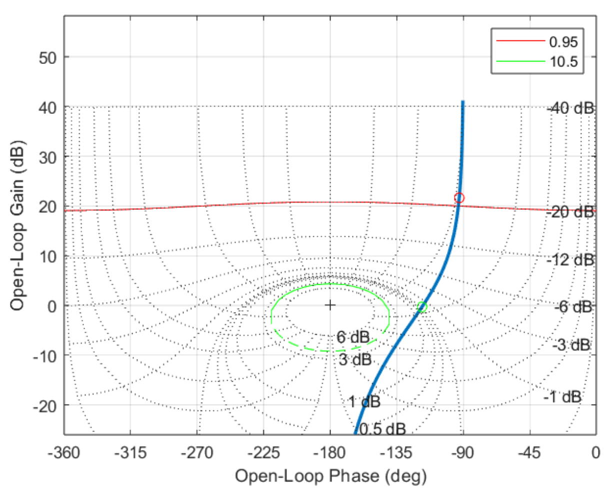


Fig. 7: QFT design tool showing the implementation of the position controller and the effect it has on the system relative to the bounds.

Using the above values for the system - namely the nominal plant and the gear reduction ratio - the speed and velocity controllers were calculated to be:

Speed - $G_s(z) = 0.1$

Position - $G_p(z) = 0.25$

3) Simulink Implementation

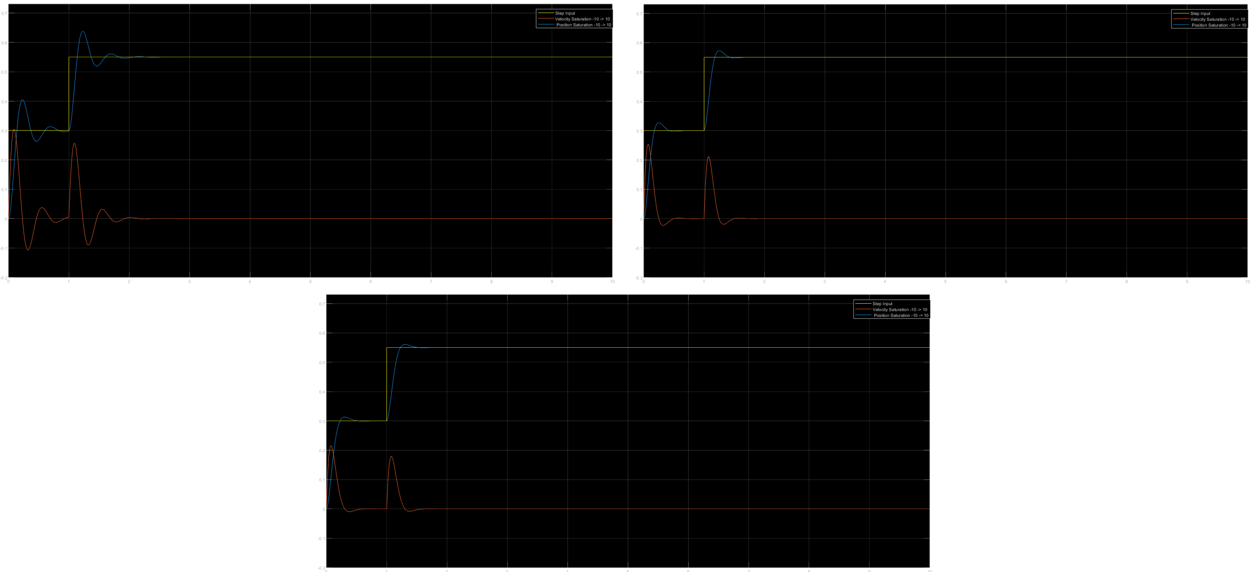


Fig. 8: Graphs showing the simulations of the relationship between the position, motor velocity and time for all three load cases.

C. Analysis

The calculated specifications allowed for the calculation of the bounds for the QFT design - one being the closed loop gain at low frequency and the other being the robust stability margin. The low frequency bound with a closed loop gain at $-20dB$ was calculated to occur at a frequency less than or equal to 0.95 radians. Therefore, at this frequency and the values less than it need to be above the line which indicates the closed loop gain at $-20dB$. The robust stability margin was calculated to occur at at a closed loop gain of $3.7dB$. Therefore, no frequency on the curve which indicates the QFT design of the system should breach the barrier of the circle formed by the curve marking a closed loop gain of $3.7dB$.

The reduction of the gain from the default setup of 1 to a value of 0.25 allowed for the system plot on the INC to meet the final specifications with regards to the low frequency bound and robust stability margin. The resulting controllers for speed and position were gain only controllers. The specifications regarding the overshoot and settling time were used to determine the ideal plant case for the system. This ideal plant case was used to develop a sensitivity bode plot which produced the low frequency and robust stability margin bounds for the QFT design.

The implementation of the two controllers in the Simulink model resulted in step responses for the system for the three plant cases. The plots of the ideal results show that the effect of the controllers on the system effectively controls the servo motor system according to the overshoot, settling time and steady state error specifications. From Figure 8, it can be seen that the full-load plant produces the best case results with a lower overshoot. On the other end of the spectrum, the no-load plant case produces a plot which has the highest amount of overshoot. The nominal case shows an amount of overshoot within the specifications but takes the shortest time to settle to the steady state value.

The location within the feedback loop of the speed controller was not taken into consideration, because it is strictly a proportional controller, the changes in the equations required to move it would not result in a different outcome. The position of the position controller allowed for the calculation to be simpler in its implementation based on its required inputs and outputs.

V. Practical Implementation

A. Methodology

The controllers designed in the previous section can be implemented into the simulation software used to run the servo motor system in order to effectively control the speed and position of the servo motor output.

The speed and position controllers each need to be converted into their difference equation equivalent. These difference equations can be introduced into the simulation software in the controller section of the code. The use of two gain controllers means that the elements of the code which require change can be multiplied strictly by a factor equivalent to the gain value measured. From this, tests were conducted on the system in order to test that the specifications were met by the controllers when implementing them on the physical servo motor system. The tests conducted include the following: unit step response, output disturbance and motor load variation.

Alterations can then be made by altering the gain of the position controller to better account for some of the uncertainties introduced in the implementation with the physical servo motor system.

B. Results

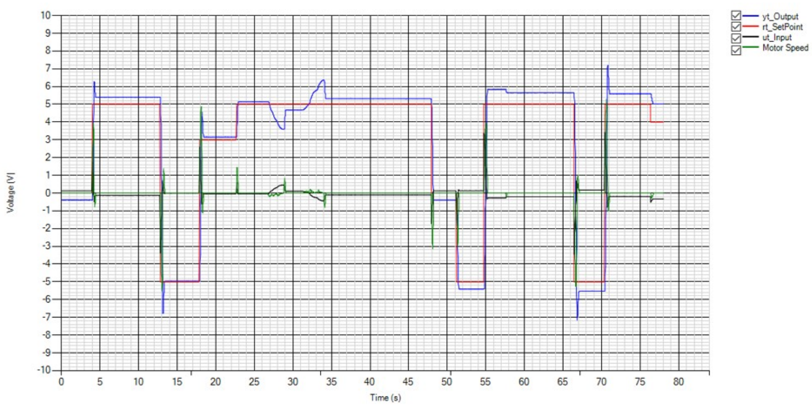


Fig. 9: Graph showing the results from the implementation testing being conducted on the servo motor system.

C. Analysis

The effectiveness of the controller in its implementation was judged on the following factors: Set point tracking, Over/undershoot, Settling time, Output disturbance rejection and Stability.

The set point tracking of the servo motor system with the inclusion of the cascade controller was within 0.4V of the final value, meaning that the overall specification was not met by the practical system, but it was within a small enough range to not be considered incorrect. The ideal plots on Simulink produced an output which had perfect set point tracking which indicates that the error in the practical implementation results from uncertainties in the physical servo motor system rather than issues with the implementation of the controller.

The overshoot in the practical system was measured as the difference between the peak of the signal and the steady state. The overshoot in the system met the specifications perfectly, resulting in an overshoot that is within 20% of the steady state value for all tests conducted.

The system's settling time was measured as the time from the input of the unit step to the first instance of settling. The specification required a settling time of 1 second which was met by all tests conducted on the practical implementation of the system.

The output disturbance rejection was tested on the system by physically disturbing the axle of the motor and monitoring the response of the system. The response when the controller was implemented was a return to the steady state value before the disturbance was introduced. The specification relating to this was met to a satisfactory standard.

The stability of the system was tested through the presence of oscillations in the step response of the servo motor system. From the results of practical implementation of the cascade controller it can be seen that there are no oscillations anywhere within the output signal. There is a pulse past the steady state point which is a result of the presence of overshoot. It can therefore be stated that the servo motor system is stable and the produces a stable output for step inputs.

The set point tracking error when the controller was initially implemented was large enough to not meet the expectation set out by the specification. The gain of the position controller was adjusted to reduce the error in the tracking of the set point by the output. The gain value was changed from 0.25 to 0.33 which allowed for the set point tracking error to reduce closer to the desired value. The trade off with this increase in gain as an increase in the overshoot of the output signal. At the higher gain the overshoot was still within the specifications defined for the experiment.

VI. Conclusion

A functioning robust digital controller was constructed in order to monitor and manage the speed and position of the servo motor system. The was done in three stages, namely: System identification, Controller Design & Practical Implementation.

The System Identification was done to allow for the construction of a robust digital position controller. The process of system identification involved simulating the system to determine a mathematical representation of the servo motor system which can be used to test the design of possible controllers. The plant transfer function was tested using a series of ramp and step tests. The open loop gain (A) range calculated from these tests was $[6.73, 37.61]$ and the range for the time constant (τ) was calculated to be $[0.08, 0.46]$. The nominal values for the plant was determined based on the results from the partially loaded step test. The values calculated were $A = 9.797780069$ and $\tau = 0.1$. The gear reduction ratio (k) for the system was calculated to be 9.294257. These values were then used to generate an ideal plant model of the system in Simulink and simulations were run on the system which were the same as what was done on the physical system to generate the data and the Simulink output plots were compared to the plots produced from the data logs in order to confirm the accuracy of the calculated values for the A, τ and k.

The Controller Design of a controller for the servo motor system was constructed based on the specifications calculated. A cascade controller model was designed which was used to control both the speed and position of the output signal. Both the speed and position controllers were determined to be strictly proportional controllers in negative feedback loops. The controllers were then used alongside the plant model to implement and test the system with a step input in order to measure the step response to simulate the changing of the input value to the system as would be the case in the physical system. The controllers were determined to be $G_s(z) = 0.1$ and $G_p(z) = 0.25$. Using the controllers in the Simulink system, the specifications were met to a satisfactory standard for all tests conducted on the system. Overshoot was present in the system, but was kept within 20% of the steady state value. The settling time across the board was always within one second, sometimes the system would settle within half a second.

The Practical Implementation of the controller with the physical servo motor system allowed for the testing of the validity of the simulink system designed in the previous stages of the project as well as testing the validity of the calculations from the simulated data. The controller implementation took place in the code used to run the servo motor system, wherein the effects of the controllers on the different values and variables within the system were included. The effect of the speed controller on the motor velocity and the effect of the gain on the variable used to store the position from the position controller were the two results effects the implementation had on the simulation software. The system met the specifications for overshoot, settling time, output disturbance rejection, and stability. Although the set point tracking error was not completely eliminated, it was reduced to a level that was acceptable for the experiment by adjusting the gain of the position controller. The gain of the position controller was increased slightly to account for some of the error in the set point tracking and the final controllers were determined to be: $G_s(z) = 0.1$ and $G_p(z) = 0.33$.

The uncertainty in this experiment result from the tools used to take the measurements of the data and the components involved in the servo motor system. Uncertainty exists in the measurement devices used to capture the data logs which could lead to a misrepresentation of the data in areas such as the overshoot and settling time. The simulink model used to simulate the system may not represent with exact detail what processes are taking place within the servo motor system. This could lead to deviations in the simulated results from the true results. Factors such as electromagnetic interference and vibrations exist in the surrounding area of the servo motor system, as well as within the system itself. These factors could introduce a noise factor into the system which would affect the accuracy and performance of the tools used for measurement and the controller during the practical implementation stage. The performance of certain components within the servo motor system could change from what is expected due to a difference in temperature or humidity to what is considered working conditions. The difference would reflect as a deviation from what is expected from the simulated system. The implementation of the controller in the code of the simulation software could introduce errors in the code, calibration of the control parameters and improper tuning of the controller gain values.

To mitigate these uncertainties, it is recommended to perform multiple tests and take the average of the results to obtain a more reliable estimate of the system's performance. It is also important to perform sensitivity analyses to determine how changes in the parameters of the system or controller affect the performance. By carefully considering and addressing these sources of uncertainty, it is possible to increase the confidence in the results and ensure that the implementation of the cascade controller is effective in improving the performance of the servo motor system. Another recommendation that could be taken into account is the redesign of the position controller with the inclusion of an integrator to account for the steady state error present in the practical implementation.