Principle of Least Action (Lagrangian) Describes system
What is in it.

$L = T - V$ = Kinetic Energy − Potential Energy (Euler (Balance Beam)

Klein-Gordon

$\Box$: ripples movement (x, y, z, t) space + time 4D

ϕ : some sort of field

m^2 : resistance of field to move

$$\Box\phi + m^2\phi = 0$$

$$\mathcal{L}_{KG} = \frac{1}{2}\partial^{\mu}\phi\,\partial_{\mu}\phi - \frac{1}{2}m^2\phi^2$$

Euler (Fancy shortcut to explain a 3D effect in 2D space

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

$\cos\theta$ = real axis

$i\sin\theta$ = imaginary axis

Spinning circle / helix

i = rotation

Goal: Take Schrödinger → GPE make it physical not probabalistic.

Need to use Madelung to turn GPE into fluid dynamics

Let's treat this as real not abstract. Ψ is a blurry probabalistic state.

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{ext} + g|\Psi|^2\right)\Psi(x,t)$$

Madelung $\Psi = \sqrt{\rho}\cdot e^{i\varphi}$

GPE

GPE

$$i\hbar\frac{\partial \Psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2\Psi + V_{ext} + g|\Psi|^2\Psi\right)\Psi$$

1.) Field Change over Time

$i\hbar\frac{\partial \Psi}{\partial t}$

i: imaginary unit, (rotation phase changes like gradient clocks tiny little clocks)

$\hbar$: plank constant (smallest ripple or wave bend) scales phase or i

Wave equation $\frac{\partial^2 y}{\partial x^2} = k^2\frac{\partial^2 y}{\partial t^2}$ concavity = acceleration

$\frac{\partial}{\partial t}$: Partial derivative time. Not 2nd derivative. Probability cloud.

$\Psi(\vec{r}\,x,t)$ = Slope on x-axis is evolving with respect to time. $x = (x,y,z)$

i = can be thought of a proxy for time Paper II or phase. Grow and decay is prevented and it allows for interaction.

2.) Kintetic Energy

$-\frac{\hbar^2}{2m}\nabla^2\Psi$

probability

$\nabla^2\Psi : \frac{\partial^2\Psi}{\partial^2 x^2} + \frac{\partial^2\Psi}{\partial^2 y^2} + \frac{\partial^2\Psi}{\partial z^2}$

Laplacian

Curvature of the field in 3D informs the field how it will evolve if time is applied like on the left side of the equation. This is its concavity. Big bend ∩ or ∪ in wave.

$-\hbar$ = wave bend on quantum scale

m = mass (stiffness of material more or less resistance to inertia

$$\text{Kinetic energy} = \frac{p^2}{2m}$$

3.) $g|\Psi|^2\Psi$

Self-interaction term. Differentiates from Schrödinger because BEC involves a group of atoms. $|\Psi|^2$ is a probability of outcomes. Likelihood of outcomes. We know water waves energy amplitude^2 or sound pressure^2. Let's make this real! Tells if field is crowded.

g repulse of self-interaction or attraction if $g>0$ or $g<0$. $g = \frac{4\pi\hbar^2 a}{m}$ in BEC or GPE

$\Psi = (\vec{r}\,x,t)$ where x is really x,y,z.

GPE is like a heat equation 1st derivative of time based on probability cloud.

Flat Space GPE Lagrangian

$$\mathcal{L}_{flat} = \underbrace{\frac{i\hbar}{2}\left(\Psi^* \partial_t \Psi - \Psi \partial_t \Psi^*\right)}_{\text{moves in time}} - \underbrace{\frac{\hbar^2}{2m}|\nabla \Psi|^2}_{\text{moves in space}} - \underbrace{V(|\Psi|^2)}_{\text{potential = thickness of fluid}}$$

Relativistic Space

$$\mathcal{L}_{rel} = \frac{\hbar^2}{2m}\eta^{\mu\nu}\partial_\mu \Psi^* \partial_\nu \Psi - V(\Psi^* \Psi)$$

Minkowski metric $\eta^{\mu\nu}$ time + space 4D equation

We treat time as moving forward. Time flowing. <u>I don't believe time is a coordinate</u> it is phase change. We will deal with this later.

Covariant

$$\mathcal{L}_{cov} = \frac{\hbar^2}{2m} g^{\mu\nu} \partial_\mu \Psi^* \partial_\nu \Psi - V$$

Action

space is warped

$$S = \int d^4x \sqrt{-g}\, \mathcal{L}$$

Add up all field behavior over the entire universe.

every 4d box in space

dx, dy, dz, dt

Euler - Lagrange Equation for Fields

$$\frac{\partial \mathcal{L}}{\partial \Psi^*} - \nabla_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \Psi^*)}\right) = 0$$

1.) $$\frac{\partial \mathcal{L}}{\partial \Psi^*} = \frac{dV}{d\Psi^*}$$

2.) $$\frac{\partial \mathcal{L}}{\partial(\partial_\mu \Psi^*)} = \frac{\hbar^2}{2m} g^{\mu\nu}\partial_\nu \Psi$$

3.) $$\nabla_\mu\left(\frac{\hbar^2}{2m} g^{\mu\nu}\partial_\nu \Psi\right) = \frac{\hbar^2}{2m}\Box \Psi$$

Madelung Transformation (Fluid Dynamics) $\Psi = \sqrt{\rho}\, e^{i\phi}$

1.) $\partial_\mu \Psi = \left(\frac{1}{2\sqrt{\rho}} \partial_\mu \rho + i\sqrt{\rho}\, \partial_\mu \phi\right) e^{i\phi}$

Imaginary

$\partial_\mu (\rho \partial^\nu \phi) = 0$

Real

$\frac{\hbar^2}{2m} \frac{\Box\sqrt{\rho}}{\sqrt{\rho}} - \frac{\hbar^2}{2m} (\partial_\mu \phi)(\partial^\mu \phi) + V(\rho) = 0$

Ripple Equation

$\rho \approx \text{const} \Rightarrow \partial_\mu \rho \approx 0$

$\frac{d}{dt}(\rho\dot{\phi}) + 3H\rho\dot{\phi} = 0 \Rightarrow \ddot{\phi} + 3H\dot{\phi} = 0$

Hamilton - Jacobi

$\frac{\hbar^2}{2m} (\partial^u \phi)(\partial_u \phi) \approx V(\rho)$

$\phi(t) = w_c t + \epsilon e^{-\gamma t} \cos(w_m t + \phi_0)$

$\dot{\phi}(t) = w_c - \epsilon\gamma e^{-\gamma t} \cos(w_m t + \phi_0) - \epsilon w_m e^{-\gamma t} \sin(w_m t + \phi_0)$

$H(t) \propto \dot{\phi}(t) \Rightarrow$ phase modulation ripples Hubble expansion

$dt = -\frac{dz}{(1+z)\, H(z)}$

$H(z) \approx H_0 \left[1 + \epsilon' e^{-\gamma' z} \sin(w'_m z + \phi'_0)\right]$