E-766

M.A./M.Sc. (Third Semester)
EXAMINATION, Dec.-Jan., 2020-21

MATHEMATICS

Paper Third (C)

(Mathematical Biology-I)

Time : Three Hours

Max.Marks: 80
Min. Marks: 16

Note: Attempt all Parts as directed.

Section-A

1 each

(Objective/Multiple Choice Questions)

Note: Choose one correct answer out of four alternative answers (A) through (D).

1. The differential equation $\frac{dy}{dt} = \frac{3y}{10}\left(1 - \frac{y}{200}\right)$ has :

(A) one stable and one unstable equilibrium point

(B) two stable equilibrium points

(C) two unstable equilibrium points

(D) none of the above

2. In which of the following model, solution to the population equation is $N(t) = \frac{N_0 K e^{rt}}{K + N_0(e^{rt} - 1)}$:

(A) discrete logistic model

(B) logistic model

(C) Malthus model

(D) none of the above

3. Which of the following is in-correct about phase diagram?

(A) It help in analyzing qualitative nature of solution of differential equation

(B) For 1D differential equations, the stability of steady state can be found by finding the slope of curve at these points

(C) It gives exact solution of differential equation

(D) none of the above

4. If $x_{n+1} = 9x_n(1 - x_n)$ and $x_0 = \frac{1}{3}$, then :

(A) $x_1 = -18$

(B) $x_1 = \frac{1}{9}$

(C) $x_1 = 2$

(D) none of the above

5. If the Lotka-Volterra competition model is given belby :

$$x' = x(1 - x - y)$$
$$y' = y(2 - x - y)$$

then :

(A) x species is more aggressive and make y species to extinct

(B) both species have same intensity of competition and hence will exist together

(C) y species is more aggressive and make x species to extinct

(D) growth rate of y species is higher than that of x species, so x species will extinct.

6. In the difference equation $x_{n+1} = rx_n$, if $r < -1$, then x_n

(A) tends to zero

(B) oscillates unboundedly

(C) decreases monotonically to zero

(D) none of the above

7. What is/are the equilibrium points of the difference equation

$$x_{n+1} = \frac{Rx_n}{(R-1)x_n + 1}$$

(A) 1 and R

(B) 0 and 1

(C) only 1

(D) none of the above

8. In many fish populations there is an annual birth process with the number of births depending on the adult population size at the time, followed by a continuous death rate until the next birth cycle at which time the survivors make up the adult population. Such population is an example of :

(A) discrete model

(B) discrete time metered model

(C) continuous model

(D) none of the above

Read the following carefully and answer the questions (9)-(12).

Let a 2×2 matrix A be the Jacobian of an autonomous system at an equilibrium point and λ, μ be the eigenvalues of A.

9. If λ, μ both are real and negative then the equilibrium point is :

(A) a focus

(B) a saddle point

(C) a centre

(D) a node

10. If λ, μ both are pure imaginary then the equilibrium point is :

(A) a centre
(B) a spiral
(C) a saddle point
(D) a node

11. If λ, μ are real but with different signs then the equilibrium point is :

(A) a spiral
(B) a centre
(C) a saddle point
(D) a node

12. In which of the following situations the trajectory is spiral :

(A) If both eigenvalues have real part zero
(B) If both eigenvalues have nonzero real part
(C) If both eigenvalues have real parts only
(D) none of the above

13. Which one of the following is not an assumption of classical Predator-Prey model :

(A) food supply for the fish is unlimited
(B) per capita growth rate of the fish population in the absence of sharks would be constant
(C) if there were no fish, the sharks would have a constant per capita death rate
(D) none of the above

14. Which of the following is a mathematical model of predator-prey interactions :

(A) The logistic model
(B) The Lotka-Volterra model
(C) Catrophist theory
(D) none of the above

15. Interspecific interactions in which one get benefit and the other is unaffected is called :

(A) Competition
(B) Predation
(C) Mutualism
(D) Commensalism

16. Which of the following statement is true :

(A) Rosenzweig-MacArthur is a special case of Kolmogorov model
(B) Kolmogorov model is a special case of Rosenzweig-MacArthur model
(C) Every Kolmogorov model satisfies Rosenzweig-MacArthur equations
(D) none of the above

17. Assume that two species x, y are in coexistence in the competitive interaction. If constant effort harvesting will be introduced in species x then which of the following change will occur :

(A) $x-$ extinction
(B) $y-$ extinction
(C) extinction of both species
(D) none of the above

18. In the Rosenzweig-MacArthur model with contant yield harvesting of $y-$ species, which of the following represents vertical asymptote?

(A) $f(x) = y\phi(x)$
(B) $cx\phi(x) = e$
(C) $c \lim_{x\to\infty} x\phi(x) = e$
(D) none of the above

19. For fish population y subject to constant-effort harvesting E, the yield per unit time is given by :

(A) E
(B) $F(y)$
(C) Ey
(D) none of the above

20. At the bionomic equilibrium effort E_*, the sustainable economic rent is :

(A) positive
(B) negative
(C) zero
(D) none of the above

Section - B
(Very Short Answer Type Questions)

2 each

1. Draw the phase line diagram for the differential equation $y' = y^3 - y$.

2. Define potential energy function.

3. Write down the condition for asymptotic stability of an equalibrium point x_* of discrete population model $x_{n+1} = f(x_n)$.

4. What are the basic assumptions of Lotka-Volterra predator prey equations.

5. Give schematic representation of chemostat.

6. Write down the assumptions on f and g, so that following system represents model equations for species in competition

$$x' = xf(x, y)$$
$$y' = yg(x, y)$$

7. Write down equations for Rosenzweig-MacArthur model.

8. Write down model equations for constant yield harvesting of species in competition.

Section - C
(Short Answer Type Questions)

3 each

Note: Attempt all questions.

1. Find all equilibrium points and classify their stability for the model

$$\frac{dy}{dt} = \frac{4y}{10}\left(1 - \frac{y}{362}\right)\left(\frac{P}{23} - 1\right)$$

2. Consider a discrete population model with initial population $P_0 = 0$ with transition rule $P_N = p_{N-1} + 11$. Assume that population is counted every month then when will the population be 207?

3. Using graphical method solve the difference equation

$$x_{n+1} = \frac{rx_n}{x_n + A}.$$

4. A forest can support a maximum herd of 500 deers whose population has a linear growth rate of 20% per year. How may deer can be killed by hunting each year to maintain the herd size at 300, and what is the minimum initial herd size that can permit this?

5. Describe asymptotic stability of all the equilibrium points of the system

$$x' = y\,, \quad y' = 2(x^2 - 1)y - x\,.$$

6. Determine the response of the system

$$x' = x(100 - 4x - y),$$
$$y' = y(60 - x - 2y)\,.$$

to constant-yield harvesting of the $x-$ species.

7. Determine the behaviour as $t \to \infty$ in the first quadrant of the system

$$x' = x\,(100 - 4x - 2y)$$
$$y' = y\,(60 - x - y)\,.$$

8. Determine the equilibrium behavior of a predator-pre system modeled by

$$x' = x\left(1 - \frac{x}{30}\right) - \frac{xy}{x + 10}$$
$$y' = y\left(\frac{x}{x + 10} - \frac{3}{5}\right).$$

Section - D

5 each

(Long Answer Type Questions)

Note: Attempt all questions.

1. Give qualitative analysis of logistic growth model of population.

OR

Derive the harvesting logistic model with constant yield and explain the behavior of critical point.

2. Discuss in detail linear discrete population model.

OR

Explain two age group discrete population model with delayed recruitment.

3. Describe the possible phase portraits of $x' = Ax$, if the matrix A is similar to one of the following:

(i) $\begin{bmatrix} \lambda & 0 \\ 0 & \mu \end{bmatrix}$, $\lambda > \mu > 0$ or $\lambda < \mu < 0$.

(ii) $\begin{bmatrix} \lambda & 0 \\ 0 & \mu \end{bmatrix}$, $\lambda > 0 > \mu$.

(iii) $\begin{bmatrix} 0 & \beta \\ -\beta & 0 \end{bmatrix}$, $\beta \neq 0$.

OR

Discuss in detail Lotka-Volterra model.

4. Discuss the mathematical model for constant yield harvesting of species in competition with linear per capita growth rate.

OR

Discuss the optimization of harvesting returns.

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