Roll No. .....

D–987

M. A./M. Sc. (Fourth Semester) (Main/ATKT) EXAMINATION, May-June, 2020

MATHEMATICS

(Optional)

Paper Third (B)

(Fuzzy Sets Theory and its Application—II)

Time : Three Hours] [*Maximum Marks : 80*

Note : Attempt all Sections as directed.

Section—A 1 each

(Objective/Multiple Choice Questions)

Note : Attempt all questions.

Choose the correct answer :

1. The compound statement "If x is A then y is B else y is C" is equivalent to :
   (a) $(A \times B) \cup (A' \times C)$
   (b) $(A' \times B) \cup (A \times C)$
   (c) Both (a) and (b)
   (d) None of these

2. Relative quantifier defined on [0, 1] and which characterized linguistic term such as almost all, about half, must and so on is called Fuzzy quantifiers of :
   (a) First kind
   (b) Second kind
   (c) Third kind
   (d) None of these
3. Let a modifier h be an increasing bijective, then it is called a strong modifier if $\forall a \in [0,1]$:
   (a) $h(a) = a$
   (b) $h(a) < a$
   (c) $h(a) > a$
   (d) None of these
4. What is the value of Lukasiewic implication $((a \Rightarrow b)$ in three valued logic if $a = \frac{1}{2}$ and $b = 1$?
   (a) 0
   (b) $\frac{1}{2}$
   (c) 1
   (d) None of these
5. The data driven method proceeds from IF clauses to THEN clauses in the chain through the production rules it is commonly called :
   (a) Forward chain
   (b) Backward chain
   (c) Metaknowledge base
   (d) None of these

6. A Fuzzy implication J is a function of the form :

   (a) $J : [0,1] \to [0,1]$

   (b) $J : [0,1] \times [0,1] \to R$

   (c) $J : [0,1] \to R$

   (d) $J : [0,1] \times [0,1] \to [0,1]$

7. Consider the function $f(a) = \log(1+a) \quad \forall a \in [0,1]$ its Pseudoinverse is :

$$f^{-1}(a) = \begin{cases} e^a - 1, & \text{when } 0 \le a \le \log 2 \\ 1 \quad , & \text{otherwise} \end{cases}$$

   then Fuzzy complement generated by f is given by :

   (a) $C(a) = \dfrac{1+a}{1-a}$

   (b) $C(a) = 1 - a$

   (c) $C(a) = \dfrac{1-a}{1+a}$

   (d) None of these

8. Any fuzzy implication suitable for approximate reasoning based on the generalized modus ponens should satisfy ..... for arbitrary Fuzzy sets A and B.

   (a) $B(y) = \sup_{x \in X} i\left[A(x), J(A(x), B(y))\right]$

   (b) $C[A(x)] = \sup_{y \in Y} i\left[C(B(y) J(A, (x), B(y))\right]$

   (c) $J(A(x), C(z) = \sup_{y \in Y} i$

$$\left[J(A(x), B(y)), J(B(y), C(z))\right]$$

   (d) All of the above

9. A collection of rules referring to a particular system is called :

   (a) Fuzzy system

   (b) Fuzzy expert system

   (c) Fuzzy rule base

   (d) None of these

10. The purpose of Defuzzification is :

   (a) Conversion of Fuzzy set into a single real number

   (b) Conversion of Fuzzy set into a real interval

   (c) Both (a) and (b) are true

   (d) None of these

11. Neural networks are :

   (a) Fuzzy system

   (b) Fuzzy rules base

   (c) Fuzzy expert system

   (d) None of these

12. Neural networks have been proven very useful in numerous applications of Fuzzy set theory :

   (a) for constricting membership function

   (b) for solving fuzzy relation equation

   (c) for modifying fuzzy inference rules

   (d) All of the above

13. Assume that :

$$X = \{x_1, x_2, ..., x_n\}$$

$$Y = \{y_1, y_2, ..., y_m\}$$

$$Z = \{z_1, z_2, ..., z_q\}$$

and let A^t, B^t, C^t and E^t denotes the Fuzzy sets that characterize respectively, the stimulus, response, current internal state and emerging internal state of the automaton at time *t*, what is relation between C^t and E^t ?

(a) $C^t = E^{t+1}$

(b) $E^t = C^{t-1}$

(c) $C^t = E^{t-1}$

(d) None of these

14. In any Fuzzy controller first, measurement are taken of all variable that represent relevant conditions of the controlled process next these measurement are converted into appropriate Fuzzy sets to express measurement uncertainties. This step is called :

(a) A Fuzzy rule base

(b) Fuzzy inference engine

(c) Fuzzification

(d) Defuzzification

15. Which one is not a defuzzification method ?

(a) Center of gravity method

(b) Center of maxima method

(c) Center of range method

(d) Mean of maxima method

16. The method :

$$x^* = \frac{\sum_{x_i \in M} x_i}{|M|}$$

where M = $\{x_i : A(x_i)$ is equal to height of Fuzzy set$\}$ is known as :

(a) Center of sum method

(b) Mean of maximum method

(c) Center of gravity method

(d) None of these

17. Assume that each individual of a group of four judges has a total preference ordering Pi ($i \in N$) on four figure skaters *a*, *b*, *c*, *d*. The ordering are $P_1 = (a, b, c, d)$, $P_2 = (a, c, d, b)$, $P_3 = (b, a, c, d)$, $P_4 = (a, d, b, c)$. Use multi-person decision making to determine the group decision the fuzzy social preference relation is given by :

$$S = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & .6 & .8 & .8 \\ .2 & 0 & .6 & .4 \\ 0 & .2 & 0 & .6 \\ 0 & 4 & .2 & 0 \end{bmatrix} \end{array}$$

What is the value of ${}^{.8}S$ (i. e. $\alpha = .8$ cuts of above Fuzzy relation) ?

(a) ϕ

(b) $\{(a, c)\}, (a, d), (a, b)$

(c) $\{(a, c), (a, d)\}$

(d) None of these

18. If the priority Fuzzy set P on {A, B} is P (A) = 0.75 and P (B) = 0.5, then what is ranking relation ?

(a) A ≤ B

(b) B ≤ A

(c) A = B

(d) None of these

19. If Z_l and Z_u are lower bound and upper bound of the optimal values in Fuzzy linear programming problem :

$$\text{Max} \sum_{j=1}^{n} C_i x_j$$

S. t. $$\sum_{j=1}^{n} A_{ij} x_j \leq B_i \, (i \in Nm)$$

$$x_i \geq 0 \ (j \in Nn)$$

then the Fuzzy set of optimal values G which is a Fuzzy subset of R^n, is defined by :

(a) $$G(x) = \begin{cases} 1 & , \text{ when } \quad z_u \leq cx \\ \dfrac{cx - z_l}{z_u - zl}, & \text{when } \quad z_l \leq cx \leq z_u \\ 0 & , \text{ when } \quad cx \leq z_l \end{cases}$$

(b) $$G(x) = \begin{cases} 0 & , \text{ when } \quad z_u \leq cx \\ \dfrac{cx - z_l}{z_u - zl}, & \text{when } \quad z_l \leq cx \leq z_u \\ 1 & , \text{ when } \quad cx \leq z_l \end{cases}$$

(c) $$G(x) = \begin{cases} 1 & , \text{ when } \quad z_u \leq cx \\ \dfrac{cx - z_l}{z_u}, & \text{when } \quad z_l \leq cx \leq z_u \\ 0 & , \text{ when } \quad \text{otherwise} \end{cases}$$

(d) None of these

20. If goal $G = \frac{.11}{a_1} + \frac{.3}{a_2} + \frac{.48}{a_3} + \frac{.8}{a_4}$ and subject to constraints :

$$C_1 = \frac{.4}{a_1} + \frac{.6}{a_2} + \frac{.2}{a_3} + \frac{.2}{a_4}$$

$$C_2 = \frac{.1}{a_1} + \frac{.9}{a_2} + \frac{.7}{a_3} + \frac{1}{a_4},$$

then the Fuzzy decision is :

(a) $D = \frac{.11}{a_1} + \frac{.9}{a_2} + \frac{.48}{a_3} + \frac{.8}{a_4}$

(b) $D = \frac{.4}{a_1} + \frac{.9}{a_2} + \frac{.7}{a_3} + \frac{.8}{a_4}$

(c) $D = \frac{.1}{a_1} + \frac{.3}{a_2} + \frac{.2}{a_3} + \frac{.2}{a_4}$

(d) None of these

Section—B 2 each

(Very Short Answer Type Questions)

Note : Attempt all questions.

1. Define Lukasiewicz primitives $\wedge, \vee$ $\Rightarrow$ and $\Leftrightarrow$ in three valued logic.
2. What do you mean by Modifier ?
3. Let u_1, u_2 be t-conorms such that $u_1(a,b) \le u_2(a,b)$ for all $a, \in [0,1]$ and let j_1, j_2 be S-implication based on the same Fuzzy complement C and u_1, u_2 respectively, then $j_1(a,b) \le j_2(a,b)$ for all $a, b \in [0, 1]$.
4. If :

$$f(a) = a^w \text{ ' } w > 0 \quad \forall a \in [0,1],$$

then find Fuzzy complement and Fuzzy implication.

5. Define Fuzzy neural networks.
6. What do you mean by states of variables in any Fuzzy system ?
7. Write centre of area method.
8. Consider a group of people involved in a business partnership who intend to buy a common car for business purposes. There are five car model as Acclaim, Accord, Camry, Cutlass and Sable. Assume further that using the number suggested in table (a) for specifying the attractiveness grades the evaluation prepared by one person in the group is given in table (b). Calculate the overall relative preference grades.

Table (a) Suggested number for attractiveness grading :

$f(x_i, x_j)$	Attractiveness of x_i with respect to x_j
1	Little attractive
3	Moderately attractive
5	Strongly attractive

7	Very strongly attractive
9	Extremely attractive
2, 4, 6, 8	Intermediate values between levels

Table (b) given attractiveness grades :

$f(x_i, x_j)$	Acclaim	Accord	Camry	Cutlass	Sable
Acclaim	1	7	9	3	8
Accord	3	1	3	2	4
Camry	1	1	1	3	5
Cutlass	2	7	7	1	7
Sable	2	6	8	3	1

Section—C

3 each

(Short Answer Type Questions)

Note : Attempt all questions.

1. Define Fuzzy quantifiers.
2. Define the truth value of the Fuzzy proposition p : there are Q i's in I such that $v_1(i)$ is F_1 and v_2 (i) is F_2, where v_1, v_2 are variables that take values from set v_1, v_2 respectively, I is an index set, Q is a Fuzzy number on **R** and F_1, F_2 are Fuzzy sets on v_1, v_2 respectively.
3. Define R-implication.
4. Let a proposition "If x is A, then y is B" be given where :

$$X = \{x_1, x_2, x_3\}, \quad Y = \{y_1, y_2\}$$

$$L_A = \frac{.5}{x_1} + \frac{.6}{x_2} + \frac{1}{x_3}$$

$$U_A = \frac{.6}{x_1} + \frac{.8}{x_2} + \frac{1}{x_3}$$

$$L_B = \frac{.4}{y_1} + \frac{1}{y_2}$$

$$U_B = \frac{.6}{y_1} + \frac{1}{y_2}.$$

Assume that Lukasiewicz implication $J(a, b) = \min(1, 1 - a + b)$ for all $a, b \in [0, 1]$. Derive the conclusion "y is B′".

5. What do you mean by Fuzzification ?
6. Explain mean of Maxima method.
7. Draw the block diagram of a general scheme of a Fuzzy controller.
8. Define Fuzzy linear programming.

Section—D 5 each

(Long Answer Type Questions)

Note : Attempt all questions.

1. Define inference from Quantified propositions.

Or

Let sets of values of variables x and y be $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2\}$ respectively. Assume that a proposition. "If x is A, then y is B" is given where :

$$A = \frac{.6}{x_1} + \frac{1}{x_2} + \frac{.9}{x_3} \text{ and } B = \frac{.6}{y_1} + \frac{1}{y_2},$$

then a fact expressed by the proposition "x is A′", where $A' = \frac{.5}{x_1} + \frac{.9}{x_2} + \frac{1}{x_3}$. Derive a conclusion in the form "y is B′" by use the generalized modus ponens for composition rule of inference.

2. What is a role of Fuzzy relation equation solving conditional Fuzzy proposition ?

Or

Explain Fuzzy Expert System.

3. Formulate reasonable Fuzzy inference rules for an air conditioning Fuzzy control system.

Or

Explain Fuzzy Automata.

4. Explain Multistage decision-making.

Or

Assume that each individual of a group of eight decision makers has a total preference ordering P_i ($i \in Nn$) on a set of alternatives $X = \{w, x, y, z\}$ as follows :

$$P_1 = (w, x, y, z)$$

$$P_2 = P_5 = (z, y, x, w)$$

$$P_3 = P_7 = (x, w, y, z)$$

$$P_4 = P_8 = (w, z, x, y)$$

$$P_6 = (z, w, x, y)$$

Use Fuzzy multiperson decision-making to determine the group decision.

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