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M.A./M.Sc. (THIRD SEMESTER) EXAMINATION, Dec. - Jan., 2021-22

MATHEMATICS

(Optional - B)

PAPER FIFTH

(GRAPH THEORY-I)

Time : Three Hours] ***[Maximum Marks:80***

Note : Attempt all sections as directed.

Section - A

(1 mark each)

(Objective/Multiple Choice Questions)

Note : Attempt all questions. Choose the correct answer.

1. The maximum number of degree of any vertex in a simple graph with n-vertices is

   (A) $\frac{n(n-1)}{2}$

   (B) $\frac{n-1}{2}$

   (C) $n-1$

   (D) $\frac{n}{2}$

2. The size of a simple graph of order n cannot exceed.

   (A) n

   (B) $n - 1$

   (C) ${}^nC_1$

   (D) ${}^nC_2$

3. A vertex with zero in degree is called-

   (A) Source

   (B) Sink

   (C) Zero vertex

   (D) None of these

4. A vertex of degree one is called-

   (A) Isolated vertex

   (B) Pendent vertex

   (C) Zero vertex

   (D) None of these

5. The sum of degrees of the vertices is an undirected graph is-

   (A) 3

   (B) 1

   (C) even

   (D) odd

6. The necessary conditions for two graphs to be isomorphic both must have

   (A) The same number of vertices

   (B) The same number of edges

   (C) Equal number of vertices with the same degree

   (D) All of above

7. When n is odd and n > 1, the chromatic number of C_n is:

   (A) 3

   (B) 2

   (C) 1

   (D) 0

8. The chromatic number of K_n is

   (A) n - 1

   (B) n

   (C) n + 1

   (D) n^2

9. The graph C_6 is :

   (A) Planar graph

   (B) Subgraphs

   (C) Bipartite graph

   (D) None of these

10. A graph G is *n*-colourable but not (k - 1) colourable is called-

    (A) k - 1 colourable

    (B) n - 1 colourable

    (C) K_n colourable

    (D) K - chromatic graph

11. The chromatic numatic number of K_n is-

    (A) n

    (B) n - 1

    (C) n - 2

    (D) None of these

12. For a graph G which is true-

    (A) G is two chromatic

    (B) G is non- nuel and bipartite

    (C) G has no circuits of odd length

    (D) all of above

13. Which of the following statements is true for a graph G

    (A) G is a split graph

    (B) G and $\overline{G}$ are traingulated graphs

    (C) G has no induced subgraph isomorphic to $2K_2$, C_4 or C_5

    (D) All of above

14. Which of the following statements is true-
    (A) Every interval graph is traingulated
    (B) Every interval graph is perfect
    (C) Both (A) and (B)
    (D) None of these
15. For any graph G of order n -
    (A) $\Delta(G) \leq n-1$
    (B) $\Delta(G) \geq = n-1$
    (C) $\Delta(G) \leq n-2$
    (D) None of these
16. For any non-trivial connected graph G.
    (A) $\alpha_0 \oplus \beta_0 = \alpha_0 \oplus \beta_1$
    (B) $\alpha_0 + \beta_0 = \alpha_1 + \beta_1$
    (C) $\alpha_0 + \beta_1 = \beta_0 + \alpha_1$
    (D) $\alpha_0 + \alpha_1 = \beta_0 + \beta_1$
17. Which of the following statement is true-
    (A) A non zero element of $C \cap B$ is called a bicycle
    (B) A bicycle has an even number of edges
    (C) Both (A) and (B)
    (D) None of these

18. For any graph G which is not true-
    (A) $\alpha_1 \leq \beta_0$
    (B) $\alpha_0 \leq \beta_1$
    (C) $\alpha_0 + \beta_0 = n$
    (D) $\alpha_0 = \alpha_1$
19. For any graph G the following is not true-
    (A) $C_0(G) = \varphi$
    (B) $C_0(G) \neq \varphi$
    (C) $\alpha_0 = \beta_1$
    (D) $\alpha_1 = \beta_0$
20. For any graph G, which is not true-
    (A) $\alpha_0 \leq \theta_0 \leq \theta_1$
    (B) $\theta_1 \neq \theta_0$
    (C) $\alpha_0 = \theta_1$
    (D) $\theta_1 = \theta_0$

Section - B

(2 marks each)

(Very Short Answer type Questions)

Note- Attempt all questions.

1. Define Homomorphism.
2. Explain Binary Operations.
3. Define cycle space.

4. Define critical graphs.
5. Define edge colouring.
6. Define face colouring.
7. Define interval graphs.
8. Define split graphs.

Section - C

(3 marks each)

(Short Answer Type Questions)

Note- Attempt all questions.

1. Prove that the number of edges m' in L(G) when G has degree sequence $(d_i)^n$ is given by

$$m' = (1/2)\sum_{i=1}^{n} d_i^2 - m$$

2. Show that G is connected iff L(G) is connected.
3. Explain cycle pases and cycle graphs.
4. Prove that any k- chromatic graph has at least k-vertices of degree at least k-1 each.
5. Explain clique parameters.
6. Explain Rosenteld numbers.
7. Prove that every instant graph is traingulated.
8. Prove that every triangulated graph is perfect.

Section - D

(5 marks each)

(Long Answer Type questions)

Note- Attempt All questions.

1. Prove that if a graph G is contractible to a graph H and $\Delta(H) \leq 3$ then G has a subgraph homeomorphic to H.

   OR

   Prove that any uniquely k-colourable graph is (k - 1) connected.
2. Prove that the sum of any two cuts of a graph G is also a cut to G.

   OR

   Prove that for any graph G with $\delta > o$

   $\alpha_1 + \beta_1 = n$
3. Prove that for any graph G of order $n > 12$ without isolated vertices $\pi_1 \leq \lfloor n^2/4 \rfloor$ and the partitiones need use only edges and traingles.

   OR

   Prove that every graph on $\binom{k+\ell}{k}$ vertices contains either a complete subgraph on k + 1 vertices or an independent set of $\ell+1$ vertices.
4. Prove that for any $s \geq 2$

   $R(S,S) \geq 2^{s/2}$

   OR

   Prove that a graph G is permutation graph iff G and G are comparability graphs.