ANNUAL EXAMINATION 2020

(Only for Regular Students)

Centre No. 135	***Centre Name- Disha College, Raipur (C.G.)***
Class-B.Sc.-I	**Subject- Mathematics**
Paper No- I	**Paper Name- Algebra & Trignometry**
Time- 3 hrs.	**M.M.-50**

Note:-Attempt any one part from each unit.

UNIT-I

Q1(a) Show that the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

satisfies cayley Hamilton theorem and hence find A^{-1}.

n'kkb;s fd vkO;wg $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ **dsyh gSfeYVu çes; dks lUrq"V djrk gS rFkk** A^{-1} **Kkr dhft,A**

(b) Prove that Eigen values of a unitary matrix are of unit modulus.

fl) dhft, fd fdlh ,sfdd vkO;wg ds vkbxsu eku bdkbZ ekikad ds gksrs gSA

Q2(a) Reduce the following matrix in the normal form and find its rank.

$$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$$

fuEu vkO;wg dk çlkekU; #i esa cnfy, vkSj mldh tkfr Kkr dhft,A

(b) Define Linearly Dependence and Independence of vectors and show that the vector R_1, R_2 and R_3 are linearly independence.

jSf[kd rk Lora= dk le>kb,A crkb, fd D;k fuEufyf[kr vkO;wg dh ifDr vkO;wg R_1, R_2 **vkSj** R_3 **jSf[kd rk Lora= gSA**

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \begin{bmatrix} 3 & 1 & -4 \\ 2 & 2 & -3 \\ 0 & -4 & 1 \end{bmatrix}$$

UNIT-II

Q3(a) Show that the following equations are inconsistent (using matrix method).

fn[kkb, fd fuEufyf[kr lehdj.k vlaxr gS&

$x + y + z = -3$

$3x + y - 2z = -2$

$2x + 4y + 7z = 7$

(b) If r_1, r_2, r_3 are the roots of the equation $2x^3 - 3x^2 + kx - 1 = 0$ find constant K if sum of two roots is 1 and then find the roots of the equation thus obtained.

;fn r_1, r_2, r_3 **cgqin** $2x^3 - 3x^2 + kx - 1 = 0$ **ds 'kwU;d gSA vpj** K **dk fu/kkZj.k dhft,] ;fn nks 'kwU;kdks dk ;ksx 1 gSA ifj.kkeh cgqin ds 'kwU;dks dk Kkr dhft,A**

Q4(a) If $\propto, \beta, \gamma$ are the roots of the cubic $x^3 - px^2 + qx - r = 0$ Find the equation whose roots are

$$\beta\gamma + \frac{1}{\propto}, \gamma \propto + \frac{1}{\beta}, \propto \beta + \frac{1}{\gamma}$$

;fn $\propto, \beta, \gamma$ **f=?kkr lehdj.k** $x^3 - px^2 + qx - r = 0$ **ds ewy gS] rks og lehdj.k Kkr dhft, ftlds ewy** $\beta\gamma + \frac{1}{\propto}, \gamma \propto + \frac{1}{\beta}, \propto \beta + \frac{1}{\gamma}$ **gSA**

(b) Solve by Cardon's Method $9x^3 + 6x^2 - 1 = 0$

dkMZu fof/k ls gy dhft, $9x^3 + 6x^2 - 1 = 0$

UNIT-III

Q5(a) Define Equivalence relation and if I is the set of non zero integers and a relation R is defined by xRy if $x^y = y^x$ where $x, y \in I$ then. Is the relation R on equivalence relation?

;fn I **'kwU; jfgr iw.kkZadks dk leqPp; gS vkSj lEcU/k** R **bl çdkj ifjHkkf"kr gS fd** $xRy \Leftrightarrow x^y = y^x$ **rks fl) dhft, fd** R, I **esa rqY;rk lEcU/k gSA**

(b) Show that the set of fourth roots of unity forms an obelian group with respect to multiplication.

fl) dhft, dh bdkbZ ds prqFkZ ewyksa dk leqPp; xq.ku lafØ;k ds vUrxZr ,d ifjfer vkcsyh lewg gSA

Q6(a) State and prove Lagranges theorem.

ysxzkat çes; dk dFku fyf[k, rFkk fl) dhft,A

(b) If G is a group and H be a non empty subset of G, then H is a subgroup of G if and only if $a \in H, b \in H \Rightarrow ab^{-1} \in H$ where b^{-1} is the inverse of b in G.

,d lewg G **ds ,d vfjDr mileqPp;** H **ds mi lewg gksus ds fy, vko';d ,oa i;kZIr çfrcU/k ;g gS fd** $a \in H, b \in H \Rightarrow ab^{-1} \in H$ **tgk¡** b^{-1} $b \in H$ **dk çfrykse gSA**

UNIT-IV

Q7(a) If $f: G \to G^1$ is any group homomorphism then f is one-one if and only if $Kerf = \{e\}$ where $Kerf$ is the kernel of f.

,d lekdkfjrk] rqY;dkfjrk gS] ;fn vkSj dsoy ;fn mldh vf"V rPN gSA $Kerf = \{e\}$

(b) The relation of isomorphism in the set of all groups is an equivalence relation.

lHkh lewgksa ds leqPp; esa rqY;dkfjrk dk lEcU/k ,d rqY;rk lEcU/k gksrk gSA

Q8(a) If f is a homomorphism from a ring (R,+,.) onto a ring $(R^1,+^1,.^1)$ then prove that

$$R/Kerf \cong R^1$$

;fn f **oy;** (R, +,.) **ls vkPNknd** onto **oy;** $(R^1, +^1,.^1)$ **ij ,d lekdkfjrk gS rks fl) dhft,A**

$$R/Kerf \cong R^1$$

(b) Every finite Integral domain is a field.

fl) dhft, fd çR;sd ifjfer iw.kkZadh; Mksesu ,d QhYM gksrk gSA

UNIT-V

Q9(a) State and prove Demoivres theorem.

Mh&eksvk;oj çes; fyf[k, rFkk fl) dhft,A

(b) Prove that $tanh^{-1}x = sinh^{-1}\frac{x}{\sqrt{1-x^2}}$

fl) dhft, fd $tanh^{-1}x = sinh^{-1}\frac{x}{\sqrt{1-x^2}}$

Q10(a) If n is any positive integer, then prove that

$\left(\sqrt{3} + i\right)^n + \left(\sqrt{3} - i\right)^n = 2^{n+1}Cos\frac{n\pi}{6}$

;fn n **dksbZ /ku iw.kkZad gS rks fl) dhft, fd** $\left(\sqrt{3} + i\right)^n + \left(\sqrt{3} - i\right)^n = 2^{n+1}Cos\frac{n\pi}{6}$

(b) Prove that $\frac{\pi}{4} = \left(\frac{2}{3} + \frac{1}{7}\right) - \frac{1}{3}\left(\frac{2}{3^2} + \frac{1}{7^3}\right) + \frac{1}{5}\left(\frac{2}{3^5} + \frac{1}{7^5}\right) - \cdots$

fl) dhft, fd $\frac{\pi}{4} = \left(\frac{2}{3} + \frac{1}{7}\right) - \frac{1}{3}\left(\frac{2}{3^2} + \frac{1}{7^3}\right) + \frac{1}{5}\left(\frac{2}{3^5} + \frac{1}{7^5}\right) - \cdots$

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