

ANNUAL EXAMINATION 2020

(Only for Regular Students)

Centre No. 135

Centre Name- Disha College, Raipur (C.G.)

Class-B.Sc.-I

Subject- Mathematics

Paper No-III

Paper Name- Vector Analysis & Geometry

Time- 3 hrs.

M.M.-50

Note – Attempt all units. Solve any two from each units. Each question carries equal marks.

Unit-I

- Q1. ;fn $r^2 = x^2 + y^2 + z^2$ rc r^n dkr dlt, A
If $r^2 = x^2 + y^2 + z^2$ then find grad r^n
- Q2. ;fn $\vec{V} = e^{xyz}(-\hat{i} + \hat{j} + \hat{k})$ rks \vec{V} dlt, A
If $\vec{V} = e^{xyz}(-\hat{i} + \hat{j} + \hat{k})$ find curl \vec{V}
- Q3. ;fn $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ rfk $\vec{c} = 3\hat{i} + 2\hat{j} - 5\hat{k}$ rks I R; krr dlt,
fd $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$
If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + 2\hat{j} - 5\hat{k}$ then verify
that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

Unit-II

- Q1. Use green's theorem in plane to evaluate $I = \oint_C [(x + 2y)dx + (y + 3x)dy]$ where C is the circle $x^2 + y^2 = 1$.
- Q2. Evaluate $\int_C \vec{F} \cdot \vec{dr}$ where $\vec{F} = (x^2 + y^2)i - 2xyj$ and C is the rectangle in the xy plane bounded by $y=0, x=a, y=b, x=0$.

- Q3. Verify Gauss divergence theorem over the surface of a cube bounded by coordinate planes and the planes $x = y = z = a$, $\iint_S [(x^3 - yz)\hat{i} - 2x^2y\hat{j} + 2z\hat{k}] \cdot \hat{n} ds$

Unit-III

- Q1. Trace the parabola. $9x^2 + 24xy + 16y^2 - 2x + 14y + 1 = 0$ and find the coordinates of its focus and the equation to its directrix.

- Q2. $\frac{l}{r} = A \cos \theta + B \sin \theta$ 'kdo $\frac{l}{r} = 1 + e \cos \theta$ dks Li 'kz djsx ds fy, çfrclw/k
 $(A-e)^2 + B^2 = 1$ gA

Show that the condition that the line $\frac{l}{r} = A \cos \theta + B \sin \theta$ may touch the conic

$$\frac{l}{r} = 1 + e \cos \theta \text{ is } (A-e)^2 + B^2 = 1$$

- Q3. fl) dhft, fd nh?kbr $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ds fclnq l s [kps x, vfoijoy; dk l ehdj.k ftl dk mRdnz

$$\text{dsk ' } \propto \text{ ' gsvkj tks nh?kbr l s l ukfkk } \frac{x^2}{\cos^2 \alpha} - \frac{y^2}{\sin^2 \alpha} = a^2 - b^2 \text{ gA}$$

Prove that the equation to the hyperbola drawn though point on the

ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose eccentric angle is ' α ' and which is confocal with

$$\text{the ellipse is } \frac{x^2}{\cos^2 \alpha} - \frac{y^2}{\sin^2 \alpha} = a^2 - b^2$$

Unit-IV

- Q1. I jy j[kvka $\frac{x-3}{+3} = \frac{y-8}{-1} = \frac{z-3}{1}$ rFkk $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ ds chp dh U; ure njh dh eki rFkk U; ure njh dh I jy j[kk dk l ehdj.k Kkr dhft, A

Find length and equation to the shortest distance between the lines $\frac{x-3}{+3} = \frac{y-8}{-1} = \frac{z-3}{1}$

$$\text{and } \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$$

- Q2. ml 'kdk dk l ehdj.k Kkr dhft, ftl dk 'k'k' (α, β, γ) vjg vk/kj oØ $ax^2 + by^2 = 1, z = 0$ gA

Find the equation of the cone whose vertex is (α, β, γ) and base curve

$$ax^2 + by^2 = 1, z = 0$$

- Q3. ml cyu dk l ehdj.k Kkr dhft, ftl ds tud $x = \frac{y}{-2} = \frac{z}{3}$ ds l ekvj gS rFkk vk/kj oØ $x^2 + 2y^2 = 1, z = 3$ gA

Find the equation of the cylinder whose generators are parallel to the line $x = \frac{y}{-2} = \frac{z}{3}$

and whose guiding curve is the ellipse $x^2 + 2y^2 = 1, z = 3$

Unit-V

- Q1. Find the equation to the generators of the hyperboloid $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$

Which pass through the point ($a \cos \alpha, b \sin \alpha, 0$)

$$\text{vfrijoy; t } \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \text{ ds fclnq } (a \cos \alpha, b \sin \alpha, 0) \text{ l s tkus okys tudk ds l ehdj.k Kkr dhft, A}$$

- Q2. 'kdot $ax^2 + by^2 + cz^2 = 1$ ds fclnq (α, β, γ) ij Li 'k' r t dk l ehdj.k Kkr dhft, A

Find equation of tangent plane at (α, β, γ) to the conicoid $ax^2 + by^2 + cz^2 = 1$

- Q3. fl) dhft, fd fdl h fLFkj fclnq l s, d ijoy; t ij ikp vfikyEc [kps tk l drsgA

Prove that five normals can be drawn from a fixed point to the paraboloid.