# C–523

## M. A./M. Sc. (Second Semester) (Main/ATKT) EXAMINATION, May-June, 2019

MATHEMATICS

Paper Fifth

(Advanced Discrete Mathematics—II)

*Time : Three Hours* ] [ *Maximum Marks : 80*

**Note :** Attempt all Sections as directed.

**Section—A** 1 each

**(Objective/Multiple Choice Questions)**

**Note :** Attempt all questions.

Choose the correct answer :

1. Two graphs are called isomorphic, if :
   (a) They have same number of edges
   (b) They have same number of vertices
   (c) They have equal number of vertices with given degree
   (d) All of these

2. Which of the following graphs are Eulerian ?
   (a) The complete graph $K_5$
   (b) The complete bipartite graph $K_{2,3}$
   (c) The Peterson graph
   (d) None of these





3. A vertex of degree one is called :
   (a) Isolated vertex
   (b) Pendant vertex
   (c) Even vertex
   (d) None of these

4. A connected planar graph with $n$ vertices and $e$ edges has $r$ regions given by :
   (a) $r = e - n + 1$
   (b) $r = e - n + k$
   (c) $r = e - n + 2$
   (d) $r = e - n + 3$

5. A finite state machine $M = < I, S, O, \delta, \lambda >$ is said to be reduced if and only if $S_i = S_j$ implies that :
   (a) $S_i \neq S_j$
   (b) $S_i = S_j$
   (c) $S_i \neq S_j = 0$
   (d) None of these

6. A turing machine contains everything included in finite state automata together with tape that is :
   (a) Finite in both directions
   (b) Infinite in both directions
   (c) Finite in left and infinite in right directions
   (d) None of these

7. The nullity of connected graph is :
   (a) $e - n + k$
   (b) $e - n + 1$
   (c) $n - 1$
   (d) None of these



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8. Let G be a single graph with 5 vertices. If G has 2 components then the maximum number of edges that G can have is :
   (a) 5
   (b) 6
   (c) 7
   (d) 2

9. A state of finite state machine M (with output alphabet O = {0, 1}) is said to be rejected state if its output is :
   (a) 0
   (b) 1
   (c) 2
   (d) All of these

10. Two states $S_i$ and $S_j$ of a finite state machine are said to be K-equivalent if their successors are :
   (a) K-equivalent
   (b) (K – 1) equivalent
   (c) K + 1 equivalent
   (d) K – 2 equivalent

11. In a Mealy machine the value of the output function depends on :
   (a) Present State
   (b) Present input
   (c) Present State and Present input both
   (d) None of these

12. Warshall's algorithm is used to find the :
   (a) Incidence matrix
   (b) Path matrix
   (c) Adjacency matrix
   (d) None of these







13. Ring sum of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a graph denoted by $G_1 \oplus G_2 =$ :
   (a) $= (V_1 \cup V_2, E_1 \cup E_2)$
   (b) $= (V_1 \cap V_2, E_1 \cap E_2)$
   (c) $G_1 \cup G_2 = G$ and $G_1 \cap G_2 =$ a null graph
   (d) None of these

14. A non-deterministic finite automaton (NFA) is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where δ is :
   (a) a finite non-empty internal state
   (b) a finite non-empty set inputs
   (c) Next state function
   (d) None of these

15. Two states $S_i$ and $S_j$ of a finite machine if the states $S_i$ and $S_j$ have the same output, said to be :
   (a) O-equivalent
   (b) K-equivalent
   (c) (K – 1) equivalent
   (d) (K + 1) equivalent

16. A strictly binary tree has an :
   (a) odd number of vertices
   (b) even number of vertices
   (c) odd and even number of vertices
   (d) None of these

17. All the vertices of an Eulerian graph are of :
   (a) odd degree
   (b) even degree
   (c) odd and even degree
   (d) None of these

18. A circuit is said to be simple, if :
   (a) it does not includes the same edge twice
   (b) it does not meet the same vertex twice
   (c) it includes the same edge twice
   (d) it meets the same vertex twice

19. If $H = (V, E)$ and $K = (V', E')$, then H and K are said to be vertex disjoint subgraphs if :
   (a) $V \cap V' \neq \phi$ then clearly $E' \cap E = \phi$
   (b) $V \cap V' = \phi$ then clearly $E \cap E' \neq \phi$
   (c) $V \cap V' \neq \phi$ then clearly $E \cap E' \neq \phi$
   (d) $V \cap V' = \phi$ then clearly $E \cap E' = \phi$.

20. The maxium number of edges in a complete bipartite graph of $n$ vertices is :
   (a) $4n^2$
   (b) $n^2$
   (c) $\frac{n^2}{2}$
   (d) $\frac{n^2}{4}$

## Section—B 2 each

### (Very Short Answer Type Questions)

**Note :** Attempt all questions in 2—3 sentences.

1. Define Regular Graph.
2. Define fundamental cut sets.
3. Define Pendant Vertex.
4. Define Finite State Machine.







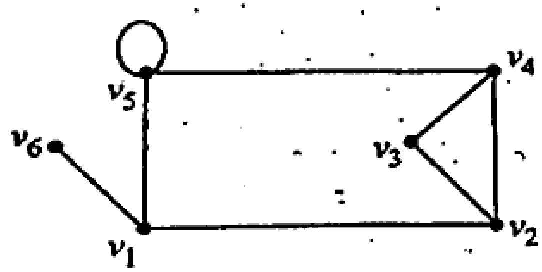
5. Define Binary tree.
6. Define Directed Graph.
7. Define Moore Machine.
8. Define Eulerian circuit.

## Section—C 3 each

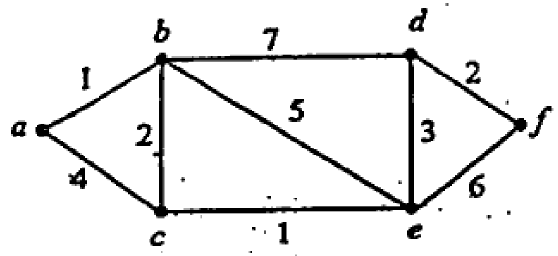
### (Short Answer Type Questions)

**Note :** Attempt all questions in less than 75 words

1. Prove that A tree T with $n$ vertices has $n - 1$ edge.
2. Define a Turing Machine.
3. Define Complete Bipartite Graphs with an example.
4. Find the adjacency matrix of the given graph.




5. Define Reduced machine.
6. Find the shortest path from $a$ to $f$ in the following graph using Dijkstra's algorithm :




7. Define Partial Recursive functions.
8. Define Planar graphs with an example.



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## Section—D

5 each

### (Long Answer Type Questions)

**Note :** Attempt all questions.

1. Let G be a simple graph with $n$ vertices. If G has $k$ components, then the maximum number of edges that G can have is $\frac{(n-k)(n-k+1)}{2}$.

*Or*

Define with example :

(i) Complete graph

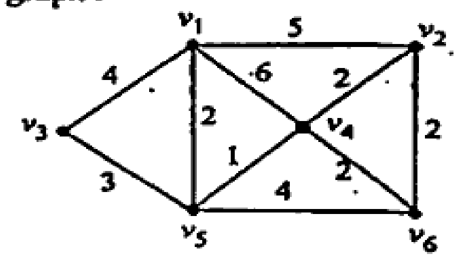
(ii) Degree of Vertex

(iii) Tree

(iv) Path

(v) Acyclic graph

2. Using Kruskal's algorithm, find minimum spanning tree of the given graph :




*Or*

Explain the incidence matrix and adjacency matrix.

3. Let the preorder and inorder search of a binary tree T yield the following sequence of vertices (nodes) :

Inorder : $d\ b\ p\ h\ q\ s\ e\ a\ c\ r\ k\ f\ l$

Preorder : $a\ b\ d\ e\ h\ p\ q\ s\ c\ f\ k\ r\ l$







*Or*

Consider the finite automation given by table convert this finite automation into a Moore machine :

| Present State | Next State | |
|---|---|---|
| | $a = 0$ | $a = 1$ |
| $S_0$ | $S_2$ | $S_1$ |
| $S_1$ | $S_3$ | $S_0$ |
| $S_2$ | $S_0$ | $S_3$ |
| $S_3$ | $S_1$ | $S_2$ |

4. Show that the following two machines $M_1$ and $M_2$ are equivalent :

| $M_1$ | | | |
|---|---|---|---|
| State | Input | | Output |
| | 1 | 2 | |
| ⇒A | B | C | 0 |
| B | F | D | 0 |
| C | G | E | 0 |
| D | H | B | 0 |
| E | B | F | 1 |
| F | D | H | 0 |
| G | E | B | 0 |
| H | B | C | 1 |

| $M_2$ | | | |
|---|---|---|---|
| State | Input | | Output |
| | 1 | 2 | |
| ⇒ A | B | C | 0 |
| B | C | D | 0 |
| C | D | E | 0 |
| D | E | B | 0 |
| E | B | C | 1 |

*Or*

Construct the state diagram for the finite state machine with the state table as given below :

| State | $f$ Input | | $g$ Output | |
|---|---|---|---|---|
| | 0 | 1 | 0 | 1 |
| $S_0$ | $S_1$ | $S_0$ | 1 | 0 |
| $S_1$ | $S_2$ | $S_1$ | 0 | 1 |
| $S_2$ | $S_3$ | $S_1$ | 1 | 1 |
| $S_3$ | $S_2$ | $S_1$ | 0 | 0 |