C–522

M. A./M. Sc. (Second Semester) (Main/ATKT) EXAMINATION, May-June, 2019

MATHEMATICS

Paper Fourth

(Advanced Complex Analysis—II)

Time : Three Hours] *[Maximum Marks : 80*

[Minimum Pass Marks : 16

Note : Attempt all Sections as directed.

Section—A 1 each

(Objective/Multiple Choice Questions)

Note : Attempt all questions.

Choose one correct answer out of four alternative answers :

1. For Weierstrass primary factors :

   (a) $E(z,0)=z-1, z\in C$

   (b) $E(z,0)=z^2-1, z\in C$

   (c) $E(z,0)=1-z, z\in C$

   (d) $E(z,0)=1-z^2, z\in C$

2. Euler's Gamma function is meromorphic with poles at :

   (a) non-negative integers

   (b) non-positive integers

   (c) negative integers

   (d) positive integers

3. Riemann-Zeta function is defined by :

   (a) $\zeta(s)=\sum_{n=1}^{\infty} n^{-s}, \operatorname{Re} s>1$

   (b) $\zeta(s)=\sum_{n=1}^{\infty} n^{s}, \operatorname{Re} s>1$

   (c) $\zeta(s)=\sum_{n=1}^{\infty} n^{-s}, \operatorname{Re} s<1$

   (d) $\zeta(s)=\sum_{n=1}^{\infty} n^{s}, \operatorname{Re} s<1$

4. Residue at the poles of $\Gamma(z)$ is given by :

   (a) $\frac{(-1)^n}{n!}$

   (b) $\frac{(-1)^{(n+1)}}{n!}$

   (c) $\frac{(-1)^n n}{(n+1)!}$

   (d) $\frac{(-1)^n (n+1)}{(n+1)!}$

5. If $|z|<\frac{1}{2}$, then :

   (a) $|E(z,p)|\geq 2e|z|^{p+1}$

(b) $|E(z, p)| \le 2e|z|^{p+1}$

(c) $|E(z, p) - 1| \le 2e|z|^{p+1}$

(d) $|E(z, p) - 1| \ge 2e|z|^{p+1}$

6. The germ of f at a is the collection of :
   (a) univalent
   (b) function elements
   (c) genus
   (d) None of the above

7. The purpose of analytic continuation is to :
   (a) enlarge the domain
   (b) shrink the domain
   (c) restrict the domain
   (d) None of the above

8. The function $P_r(\theta) = \sum_{n=-\infty}^{\infty} r^{|n|} e^{in\theta}$, $0 \le r < 1$, $-\infty < \theta < \infty$,

   is called :
   (a) Harmonic conjugate
   (b) Mean value property
   (c) Maximum Principle
   (d) Poisson Kernel

9. Which of the following statements is false ?
   (a) φ is super-harmonic iff $-\varphi$ is sub-harmonic
   (b) every harmonic function is sub-harmonic
   (c) every harmonic function is super-harmonic
   (d) None of the above

Section—C

3 each

(Short Answer Type Questions)

Note : Attempt all questions.

1. If p is a positive integer, then show that there exists $a, b > 0$ such that :
$$|E(z, p)| \le b \exp(a|z|^p).$$

2. Define meromorphic function and state Mittag-Leffler's theroem.

3. Show that when $0 < b < 1$ the series :
$$\frac{1}{2}\log(1+b^2) + i\tan^{-1} b + \frac{z-ib}{1+ib} - \frac{1}{2}\frac{(z-ib)^2}{(1+ib)^2} + \ldots,$$
is analytic continuation of the function defined by the series
$$z - \frac{1}{2}z^2 + \frac{1}{3}z^3 - \ldots\ldots$$

4. State and prove mean value theorem for harmonic functions.

5. Let G be a region. Show that the metric space Har (G) is complete.

6. Let G be a simply connected region and let f be an analytic function defined in G that does not assume the values 0 or 1. Then show that there is an analytic function g defined in G such that :
$$f(z) = -\exp\{i\pi \cos h[2g(z)]\}$$
for all z in G.

7. Let f be analytic in $D = \{z : |z| < 1\}$ and let $f(0) = f'(0) = 1$ and $|f(z)| \le M$ for all z in D. Then show that $M \ge 1$ and
$$f(D) \supset B\left(0, \frac{1}{6M}\right).$$

8. Use Hadamard's Factorization theorem to show that :

$$\sin \pi z = \pi z \sum_{n=1}^{\infty} \left(1 - \frac{z^2}{n^2}\right).$$

Section—D 5 each

(Long Answer Type Questions)

Note : Attempt all questions.

1. State and prove Schwartz reflection principle.

Or

State and prove Riemann functional equation.

2. Let G and Ω be regions such that there is a one-one analytic function f of G onto Ω; let $a \in G$ and $\alpha = f(a)$. If g_a and γ_α are the Green functions for G and Ω with singularities a and α respectively, then show that :

$$g_a(z) = \gamma_\alpha(f(z)).$$

Or

If the real part of an entire function $g(z)$ satisfies the inequality :

$$\text{Re}\, g(z) < r^{\rho+\epsilon},$$

for every $\epsilon > 0$ and all sufficiently large r, then show that $g(z)$ is a polynomial of degree not exceeding ρ.

3. State and prove Jensen's formula.

Or

State and prove Hadamard's three circles theorem.

4. State and prove Schottky's theorem.

Or

State and prove Montel-Caratheodory theorem.