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M.A./M.Sc. (First Semester)
EXAMINATION, Dec. - Jan., 2021-22
Statistics
(Real Analysis)

Time: Three Hours] *[Maximum Marks : 80*

Note: Attempt all sections as directed.

Section - A
(Objective/Multiple Choice Questions)

(2 marks each)

Note : Attempt any 10 (Ten) questions.

1. Statement "Every infinite bounded set has a limit point" is of
   (A) Dirichlet's & Theorem
   (B) Liouville's
   (C) Bolzano - Weirstrass Theorem
   (D) None of these

2. If $(x, y) \in R$ then $(y, x) \in R$ represents the relation
   (A) Transitive
   (B) Symmetric
   (C) Reflexive
   (D) Equivalence

P.T.O.

3. For the formula $\Gamma n \, \Gamma 1-n = \dfrac{\pi}{\sin(n\pi)}$ the value of 'n' for existance is -
   (A) n = 1 (B) n > 1
   (C) 0 < n < 1 (D) None of these

4. Integral $\int_0^\infty e^{-x} x^{\lambda-1} dx$ is a
   (A) Beta Function
   (B) Gamma Function
   (C) Gamma function and equal to $\Gamma \lambda$
   (D) Both (B) & (C)

5. If a function f is continuous in closed interval [a, b] and differentiable in (a,b) then there exists at least one point $c \in (a,b) \, s.t.$

   $f'(c) = \dfrac{f(b) - f(a)}{(b-a)}$ is an statement of
   (A) Rolle's Theorem
   (B) Cauchy's Mean value theorem
   (C) Lagrange's Mean Value theorem
   (D) None of these

6. The power series $\sum_{n=1}^{\infty} \dfrac{z^n}{n^p}$ is convergent if
   (A) p = 1 (B) p < 1
   (C) p > 1 (D) None of these

7. For the power series $\sum_{n=0}^{\infty} 2^n . z^n$ the value of radius of convergence will be -

(A) 1 (B) 2

(C) $\frac{1}{2}$ (D) 3

8. The value of $\frac{d}{dx}\log[\cos(x^n)]$ is

(A) $\frac{1}{\cos x^n}$ (B) $\frac{-\sin(x^n)}{\cos(x^n)}$

(C) $\frac{-\sin x^n \times nx^{n-1}}{\cos(x^n)}$ (D) $\frac{\text{n}\, x^{n-1} \times \sin x^n}{\cos x^n}$

9. The condition $\int_a^b f\,dx = \int_a^{-b} f\,dx = \int_a^b f\,dx$ then function 'f' is said to be

(A) Riemann integrable

(B) Darboux's condition of integrability

(C) Both (A) and (B)

(D) None of these

10. If $x \geq 0, y \geq 0, 3 \geq 0$ such that $x + y + z \leq 1$, then

$$\iiint_v x^{R-1} y^{m-1} z^{n-1} dx\,dy\,dz = \frac{\overline{|l}\,\overline{|m}\,\overline{|n}}{\overline{|(l+m+n+1)}}$$

is given by

(A) Liouville

(B) Dirichlet

(C) Binomial

(D) None of these

11. For real numbers a and b the relation a + b = b + a is called

(A) Associative

(B) Distributive

(C) Commutative

(D) Both (A) & (B)

12. For real numbers x and y the triangle inequalities will be-

(A) $|x+y| \leq |x| + |y|$

(B) $|x-y| \geq |x| - |y|$

(C) Both (A) and (B)

(D) None of these

13. A function is said to inverse function (or inverse exists of a function) if it is -

(A) One - one

(B) Onto

(C) Many one

(D) Both (A) and (B)

14. A sequence is bounded if it is
    (A) Bounded above
    (B) Bounded below
    (C) Both (A) and (B)
    (D) Neither (A) nor (B)
15. For a function to be maximum the value of its second order derivative should be
    (A) Positive
    (B) Negative
    (C) Both (A) and (B)
    (D) Zero

Section - B

(Very Short Answer Type Question)

(2 marks each)

1. Define bounded set.
2. Describe one- one and onto mapping.
3. Describe uniform convergence.
4. What is radius of convergence?
5. State Liouville's theorem.
6. Define maxima of a function.
7. Define derivability of a function.
8. What do you understand by proper integral?

Section - C

(Short Answer Type Questions)

(3 marks each)

1. Define change of variables in multiple integral.

2. Explain Heine - Borel theorem in detail.
3. What do you understand by absolute and conditionally convergent?
4. Describe Eucledian space IR^n also write down its properties.
5. Define power series and how you can obtain the value of its radius of convergence.
6. What do you understand by multiple integral?
7. What is maxima and minima of a function?
8. Prove that $f(x) = |x|$ is not differentiable at $x = 0.$

Section - D

(Long Answer Type Questions)

(5 marks each)

1. State and prove Bolzano - Weirstrass theorem.

   OR

   Discuss set theory, number system with its useful axioms.
2. Prove that continuity is necessary but not sufficient condition for differentiability.

   OR

   Discuss uniform continuity and uniform convergence with their properties.
3. State and prove Liouville's theorem.

   OR

   Define differentiation & integration under the integral sign. Also discuss the Leibnitz rule.
4. State and prove Cauchy's mean value theorem.

   OR

   Discuss proper and improper integrals and also define μ - test to solve them.