

Roll No. ....

**D-988**

**M. A./M. Sc. (Fourth Semester) (Main/ATKT)  
EXAMINATION, May-June, 2020**

MATHEMATICS

Paper Third (C)

**(Mathematical Biology—II)***Time : Three Hours ]**[ Maximum Marks : 80**[ Minimum Pass Marks : 16***Note :** Attempt all Sections as directed.**Section—A**

1 each

**(Objective/Multiple Choice Questions)****Note :** Attempt all questions.

Choose the correct answer out of four alternative answers :

- In a linear structured discrete population :
  - the total population size approached to a limit
  - the proportion of the population in each age class tends to a limit
  - Both (a) and (b) are true
  - None of the above

- In a linear continuous population model  $\int_0^{\infty} \beta(a) \rho(a, t) da$  is called :
  - renewal condition
  - birth modulus
  - density function
  - None of the above
- Which of the following is known as Von Foerster equation ?
  - $\mu(a) \rho_a(a, t) + \rho_t(a, t) + \rho(a, t) = 0$
  - $\rho_a(a, t) + \mu(a) \rho_t(a, t) + \rho(a, t) = 0$
  - $\rho_a(a, t) + \rho_t(a, t) + \mu(a) \rho(a, t) = 0$
  - None of the above
- In a linear continuous population model, if the total population size is constant, then which of the following is true ?
  - the birth rate is also constant
  - $\rho(a, t)$  is independent of  $t$
  - Both (a) and (b) are true
  - None of the above
- Which of the following is a way to include spatial location in a population ?
  - by means of metapopulations
  - by allowing motion of individuals in the population
  - Both (a) and (b) are true
  - None of the above

**P. T. O.**

6. A partial differential equation requires :
- exactly one dependent and one independent variable
  - two or more independent variables
  - more than one dependent variable
  - None of the above
7. Which of the following is known as Heat equation ?
- $u_{tt}(x, t) = D u_{xx}(x, t)$
  - $u_x(x, t) = D u_{tt}(x, t)$
  - $u_t(x, t) = D u_{xx}(x, t)$
  - None of the above
8. While solving a partial differential equation using a variable separable method, we equate the ratio to a constant which :
- can be positive or negative integer or zero
  - can be positive or negative rational number or zero
  - must be a positive integer
  - None of the above
9. Which of the following models is used for Bacteria born diseases ?
- SIR model
  - SIS model
  - SEIR model
  - None of the above
10. In an SIR model  $\lim_{n \rightarrow \infty} (S + I)' =$
- 0
  - $I_\infty$
  - Both (a) and (b) are true
  - None of the above

P. T. O.

11. In an epidemic model  $\left(1 - \frac{S_\infty}{N}\right)$  can be called :
- attack rate
  - attack ratio
  - Both (a) and (b) are true
  - None of the above
12. Expression for reproduction number in SEIR model is given by :
- $\epsilon \frac{\beta N}{\alpha} + \frac{\beta N}{k}$
  - $\frac{\beta N}{\alpha} + \epsilon \frac{\beta N}{k}$
  - $\frac{\beta N}{\alpha} + \frac{\beta N}{\epsilon k}$
  - None of the above
13. The incubation period is the interval between :
- Appearance of clinical symptoms and death
  - The time of infection and appearance of clinical symptoms
  - Time of infection and appearance of antibodies
  - None of the above
14. Which of the following factors will reduce the rate of spread of an epidemic ?
- Contact rate
  - Transmission probability per contact
  - Duration of infectiousness
  - All of the above

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15. Diseases that are always present in a community, usually at a low, more or less constant, frequency are classified as having an ..... pattern.
- (a) endemic
  - (b) epidemic
  - (c) pandemic
  - (d) None of the above
16. A disease vector is a/an :
- (a) organism that transmits a disease
  - (b) environmental condition associated with a disease
  - (c) symptom of a disease
  - (d) None of the above
17. What is a virus pandemic ?
- (a) A rapid global outbreak starting from a single focus
  - (b) A sharp and rapid epidemic involving more than one country
  - (c) A characteristic of common cold virus
  - (d) None of the above
18. What does the reproductive number of a virus tell us ?
- (a) How virulent the infection will be
  - (b) How long the 'incubation period' will be
  - (c) How many contacts will be infected from one case
  - (d) None of the above
19. Which of the following is a virus borne disease ?
- (a) influenza
  - (b) meningitis
  - (c) tuberculosis
  - (d) None of the above

P. T. O.

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20. Of the following factors, which are the most important factors for virus spread and infection ?
- (a) Age
  - (b) Sex
  - (c) Genetic factors
  - (d) Travel

**Section—B**

2 each

**(Very Short Answer Type Questions)**

**Note :** Attempt all questions in **2-3** sentences.

1. Define stable age distribution.
2. Write down Lotka-Sharpe's equation.
3. Define basic reproduction ratio.
4. Define diffusion process.
5. Define herd immunity.
6. Draw flow chart for the vaccination model.
7. Define M-matrix.
8. Define disease free equilibrium.

**Section—C**

3 each

**(Short Answer Type Questions)**

**Note :** Attempt all questions.

1. Describe an occupancy model assuming that the colonization rate in a patch depends only on the fraction of occupied patches, not on any properties of individual patches.
2. Describe model for a metapopulation with two patches and no internal patch dynamics.
3. Derive expression for herd immunity. A disease has  $R_0 \approx 5$ , find the fraction of population to be immunized to get herd immunity.

4. Write down assumptions and model equation for influenza model.
5. Draw flow chart for SEIR model. Write down assumptions and model equations.
6. Draw flow chart, assumptions and model equation for model with birth and death.
7. Using first generation matrix, obtain basic reproduction rate for the following model :

$$S'_U = -\beta S_U (I_U + \delta I_V)$$

$$S'_V = -\rho \beta S_V (I_U + \delta I_V)$$

$$I'_U = \beta S_U (I_U + \delta I_V) - \alpha_U I_U$$

$$I'_V = \sigma \beta S_V (I_U + \delta I_V) - \alpha_V I_V$$

The initial conditions prescribe  $S_U(0), S_V(0), I_U(0), I_V(0)$ , with

$$S_U(0) + I_U(0) = N_U$$

$$S_V(0) + I_V(0) = N_V.$$

8. Write a note on Interepidemic Period.

#### Section—D

5 each

#### (Long Answer Type Questions)

**Note :** Attempt all questions.

1. Discuss in detail mathematical model for age structured discrete continuous model.

*Or*

Explain method of characteristics for structured population model.

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2. Describe the metapopulation model with residence and travel.

*Or*

Find a solution of the diffusion equation :

$$u_t(x, t) = D u_{xx}(x, t)$$

for  $0 \leq x \leq L$ ,  $0 \leq t < \infty$ , subject to the boundary conditions  $u_x(0, t) = u_x(L, t) = 0$  and the initial condition  $u(x, 0) = f(x)$ ,  $0 \leq x \leq L$  with  $f$  a given smooth function.

3. Describe Quarantine-Isolation Model.

*Or*

Explain the role of disease as population control.

4. Answer the following for SIR epidemic model :

- (i) Write down assumptions and model equation.
- (ii) Show that long-term limit exists.
- (iii) Show that disease will always die out.
- (iv) Find out expression for maximum number of infected individuals.

*Or*

Explain SITR model.