10.5 PROBABILITIES OF INDEPENDENT AND DEPENDENT EVENTS

Probability of Independent Events:

Two events are independent if the occurance of one has no effect on the occurance of the other.

Ex: tossing a coin

If A and B are independent events, then the probability that both A and B occur is: P(A and B) = $P(A) \cdot P(B)$

1. In a survey at a football game, 50 of 75 male fans and 40 of 50 female fans said that they favor the new team mascot. If 1 male and 1 female fan are randomly selected, what is the probability that both favor the new mascot?

$P(A)$ = Male favors new mascot

$P(B)$ = Female favors new mascot

$P(A \text{ and } B) = P(A) \cdot P(B) = \frac{50}{75} \cdot \frac{40}{50} = \frac{8}{15}$

2. A survey found that 78% of students say that they read at least one book a month outside of school. If 3 students are selected at random, what is the probability that all 3 will say that they read at least on book a month outside of school?

$P(A)$ = 1st student reads

$P(B)$ = 2nd student reads

$P(C)$ = 3rd student reads

$P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B) \cdot P(C) = (.78)(.78)(.78) \approx 47.5\%$

3. During each of the 5 days of a special week, an employee is randomly given 1 of 10 prizes. All prizes are available each day, and one of the prizes is a $500 gift certificate. What is the probability that an employee receives the $500 prize at least once?

$P(\text{500 prize @ least once}) = 1 - P(\text{no one receives the 500 prize})$

$= 1 - \frac{9}{10} \cdot \frac{9}{10} \cdot \frac{9}{10} \cdot \frac{9}{10} \cdot \frac{9}{10} = 1 - \frac{9^5}{10^5} = 1 - \frac{59049}{100000} = 1 - 0.59049 = 0.40951$

Probability of Dependent Events:

Two events are dependent if the occurance of one effects the occurance of the other

The probability that B will occur given A has occurred is written $P(B|A)$ "prob. of b given a"

If A and B are dependent events, then the probability that both A and B occur is: P (A and B) = $P(A) \cdot P(B|A)$

4. The table shows the status of 200 registered college students. What is the probability that a randomly selected student

	Full Time	Part Time
Female	80	40
Male	60	20

a. is female?

$P(\text{female}) = \frac{\text{female}}{\text{total}} = \frac{120}{200} = \frac{3}{5}$ or 60%

b. if female, is a full time student?

$P(\text{full time} | \text{female}) = \frac{\text{full time female}}{\text{total female}} = \frac{80}{120} = \frac{2}{3}$ or $\approx 67\%$

5. You randomly select two cards from a standard deck of 52 cards. What is the probability that the first card is a heart and the second is a club if

   a. you replace the first card before selecting the second card? (independent)

   Let A = first card is a heart, let B = 2nd card is a club

   $P(A \text{ and } B) = P(A) \cdot P(B) = \frac{13}{52} \cdot \frac{13}{52} = \frac{1}{16}$ or 6.25%

   b. you do not replace the first card? (dependent)

   $P(A \text{ and } B) = P(A) \cdot P(B|A) = \frac{13}{52} \cdot \frac{13}{51} = \frac{13}{204}$ or $\approx 6.37\%$

Find the probability of drawing the given cards from a standard deck of 52 cards (a) with replacement and (b) without replacement.

6. A spade, then a club

   Let A = 1st card is a spade, B = 2nd card is a club

   a.) independent $P(A \& B) = P(A) \cdot P(B) = \frac{13}{52} \cdot \frac{13}{52} = \frac{1}{16}$ or 6.25%

   b.) dependent

   $P(A \text{ and } B) = P(A) \cdot P(B|A) = \frac{13}{52} \cdot \frac{13}{51} = \frac{13}{204}$ or $\approx 6.37\%$

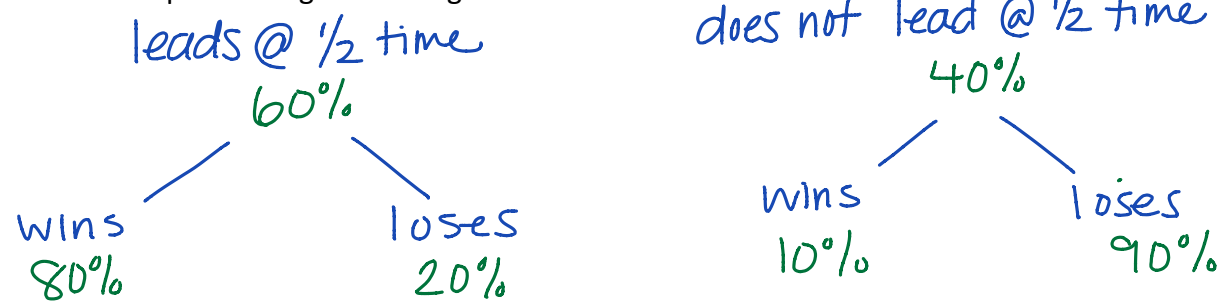
7. A jack, then another jack

   Let A = 1st card is a Jack, let B = 2nd card is a Jack

   a.) independent

   $P(A \& B) = P(A) \cdot P(B) = \frac{4}{52} \cdot \frac{4}{52} = \frac{1}{169}$ or $\approx 0.59\%$

8. A high school basketball team leads at halftime in 60% of the games in a season. The team wins 80% of the time when they have the halftime lead, but only 10% of the time when they do not. What is the probability that the team wins a particular game during the season?



P(win a game) = P(win by leading @ ½ time (A) or win by losing @ ½ time (B))

$= P(A) + P(B) = (.6)(.8) + (.4)(.1) = .48 + .04 = .52$ or 52%