10.2 COMBINATIONS and the BINOMIAL THEOREM

It is not always important to count all of the different orders that a group of objects can be arranged. A combination is a selection of r objects from a group of n objects where the order is not important.

Combinations of n Objects Taken r at a Time The number of combinations of r objects taken from a group of n distinct objects is denoted by ${}_{n}C_{r}$ and is given by the formula:

$$\frac{n!}{r!(n-r)!}$$

1. A standard deck of 52 playing cards has 4 suits with 13 different cards in each suit. If the order in which the cards are dealt is not important, how many different 5-card hands are possible?

choosing 5 out of 52 cards
$$n = 52$$
, $r = 5$
 $62(s = \frac{52!}{5!(52-5)!} = 2,598,960$

hands

2. A teacher is choosing 3 representatives from their homeroom class. If there are 28 students in the class, how many combinations of students are possible?

$$38C_3 = \frac{28!}{3!(28-3)!} = 3276$$
 combinations of 3 students

Multiple Events

- When finding the number of ways both an event A and an event B can occur, you need to multiply.
- When finding the number of ways that event A or event B can occur, you add.
- 3. How many 5 card hands are possible where all 5 cards are the same color? choosing 1 out of 2 colors and 5 out of 26 in that color:
 - aC1: 26C5 = 131,560 hands
- 5. How many 5 cards hands are there that have either all spades or all diamonds?
 - choosing 5 out of 13 spades or 5 out of 13 diamonds

$$13C_5 + 13C_5 = 2574$$
 hands

- 4. How many 5 card hands are possible where all 5 cards are the same suit? 4 Swits and 5 out of 13 in that swit: 4C1.13(5 = 5148 hands
- 6. How many 5 card hands have 3 clubs?
 - 3 out of 13 dubs and but of 39 other cards

$$13(3 \cdot 39(2 = 286.74)$$

= 211926
hands

7. You are going to toss 10 different coins. How many different ways will at least 4 of the coins show heads?

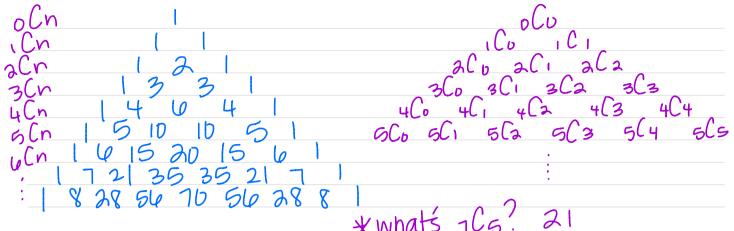
at least 4 show heads means 4 heads or 5 heads or 6 easier way:

subtract possibilities we don't want from the total # of possibilities

 $2^{10} - {}_{10}C_0 + {}_{10}C_1 + {}_{10}C_2 + {}_{10}C_3 = 848$

Pascal's Triangle

Many of the relationships among combinations can be seen in the array of numbers known as Pascal's Triangle.



8. You're getting t-shirts made but the print shop can only print 2 colors on a shirt. How many combinations of 2 colors can you make from a choice of 6?

Binomial Expansions – We'll now explore the connection between Pascal's triangle and binomial expansions.

$$(x+y)^0 =$$

$$(x+y)^1 = X + y$$

$$(x+y)^2 = (X+y)(X+y) = X^2 + 2xy + y^2$$

$$(x+y)^{3} = (x^{2} + 2xy + y^{2})(x+y) = x^{3} + x^{2}y + 2x^{2}y + 2xy^{2} + y^{2}x + y^{3}$$

$$= x^{3} + 3x^{2}y + 3xy^{2} + y^{3}$$

$$(x+y)^{4} = x^{4} + 4x^{3}y + 6x^{2}y^{2} + 4xy^{3} + y^{4}$$

$$= x^{4} + 4x^{3}y + 6x^{2}y^{2} + 4xy^{3} + y^{4}$$

$$= x^{4} + 4x^{3}y + 6x^{2}y^{2} + 4xy^{3} + y^{4}$$

$$= x^{4} + 4x^{3}y + 6x^{2}y^{2} + 4xy^{3} + y^{4}$$

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$$= x^{4} + 4x^{3}y + 6x^{2}y^{2} + 4xy^{3} + y^{4}$$

Binomial Theorem

For any positive integer n, the binomial expansion of $(a+b)^n$ is:

$$nCoa^{n}b^{o} + nC_{1}a^{n-1}b^{1} + nC_{2}a^{n-2}b^{2} + nC_{3}a^{n-3}b^{3} + ... + nC_{n}a^{o}b^{n}$$

1. Expand
$$(x+y)^5 = X^5 + 5X^4y + 10X^3y^2 + 10X^2y^3 + 5Xy^4 + y^5$$

2. Expand
$$(3x-2)^4$$
 $|(3x)^4(-3)^4+4(3x)^3(-3)^4+6(3x)^2(-3)^2+4(3x)^4(-3)^3+1(3x)^4(-3)^4$
 $=81x^4+4(27x^3(-3)+16(9x^2)(4)+4(3x)(-8)+(16)$
 $=81x^4-216x^3+216x^2-96x+16$

3. Expand
$$(5x^2)^3 (6x)^3 (-2y)^6 + 3(5x)^2 (-2y)^1 + 3(5x)^2 (-2y)^2 + 1(5x)^2 (-2y)^3 + 3(5x)^2 (-2y)^2 + 3(5x)^2 (-2y)^2 + (-8y^3)^3$$

$$= (35x^3 - 150x^2y + 60xy^2 - 8y^3)$$