# *10.1 THE COUNTING PRINCIPLE and PERMUTATIONS*

1. A restaurant is running a special 'You Pick 3' meal deal. You have your choice of one of three appetizers (mozzarella sticks, chicken fingers, or chips and salsa), 4 entrées (pasta, chicken, beef, or chicken Caesar salad), and 2 desserts (ice cream brownie sundae or apple crisp). How many meal choices do you have?

   24

**Fundamental Counting Principle**

*Two events*: If one event can occur in *m* ways and another event can occur in *n* ways, then the number of ways that *both* events can occur is $m \cdot n$.

*Three or more events:* The fundamental counting principle can be extended to three or more events. For example, if three events can occur in *m, n,* and *p* ways, then the number of ways that *all* three events can occur is $m \cdot n \cdot p$.

2. At a used book sale, you are interested in 5 novels, 3 books of nonfiction, and 7 comic books. If you buy one of each kind, how many different choices do you have?

   $5 \cdot 3 \cdot 7 = 105$

3. The digits 0, 1, 2, 3, and 4 are used to generate 4-digit customer codes. How many different codes are possible if digits There are 5 digits

   a. can be repeated?

   $5 \cdot 5 \cdot 5 \cdot 5 \cdot = 625$

   b. cannot be repeated?

   $5 \cdot 4 \cdot 3 \cdot 2 = 120$

4. A standard California license plate is a number, followed by 3 letters, followed by 3 numbers.

   a. How many different license plate configurations are possible?

   $10 \cdot 26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 175{,}760{,}000$

   b. How many different license plate configurations are possible of no letters or numbers can be repeated?

   $10 \cdot 26 \cdot 25 \cdot 24 \cdot 9 \cdot 8 \cdot 7 = 78{,}624{,}000$

**Permutations**

An ordering of *n* objects is a *permutation* of the objects. For instance, how many permutations of the letters A, B, and C are there? ABC ACB BAC BCA CAB CBA 6

How could we arrive at the number 6 without having to generate a list of the possible orders?

The number of **permutations** of n objects is **n!.**
**n!** is read as 'n **factorial'** and is equivalent to *n ·(n−1) ·(n−2) ·... ·3 ·2 ·1.*

$3! = 3 \cdot 2 \cdot 1 = 6$

5. There are 8 groups trying out for Battle of the Bands. How many different ways can their performances be ordered?

   $8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40{,}320$

6. How many ways can 3 groups place 1st, 2nd, and 3rd place out of the 8 to go to the final round of Battle of the Bands?

8 choices for 1st, 7 for 2nd, 6 for third : $8 \cdot 7 \cdot 6 = 336$

The answer to #6 can be found by taking a permutation of a certain number of objects from a group.

**Permutations of *n* Objects Taken *r* at a Time**

The number of permutations of *r* objects taken from a group of *n* distinct objects is denoted ${}_nP_r$ and is given by this formula:

$$\frac{n!}{(n-r)!}$$

7. The school is selecting a president and a vice-president out of 5 students. How many possible outcomes are there?

$n=5$, $r=2$

$${}_nP_r = \frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = 20$$

Permutations with Repetition - Consider the letters **S,** O, S. If you consider **S** and S to be distinct, then there are 6 permutations,

**S**OS  **S**SO  O**S**S  OS**S**  SO**S**  S**S**O

But, if the two occurrences of S are considered to be interchangeable, then there are only 3 distinguishable permutations:

How could we arrive at this answer without having to generate all of the permutations?

**Permutations with Repetition**

The number of distinguishable permutations of *n* objects where one object is repeated $s_1$ times, another is repeated $s_2$ times, and so on is:

$$\frac{n!}{s_1! \cdot s_2! \cdots}$$

8. Find the number of distinguishable permutations of the letters in SWIMMING?

8 letters $\Rightarrow n=8$

M repeats twice $\Rightarrow s_1=2$, I repeats twice $\Rightarrow s_2=2$

$$\frac{8!}{2! \cdot 2!} = 10{,}080$$

PUT IT ALL TOGETHER…

9. Determine how many different 5-digit postal zip codes are possible if digits can be repeated and the zip code must begin with a 9.

$\underline{1}\ \underline{10}\ \underline{10}\ \underline{10}\ \underline{10} = 10{,}000$

10. How many distinguishable permutations are there of the letters of the word HEDGEHOG?

$$\frac{8!}{2!2!2!} = 5040$$

11. In a dog show, how many ways can four Pomeranians, five golden retrievers, two Great Pyrenees, and six English terriers line up in front of the judges if the dogs of the same breed are considered identical?

$$\frac{17!}{4!5!2!6!} = 85{,}765{,}680$$

In how many different ways can three dogs win first, second, and third place?

$${}_{17}P_3 = \frac{17!}{(17-3)!} = 4080$$