EE 539B
Integrated Optics – From Micron Scale to Nanophotonics

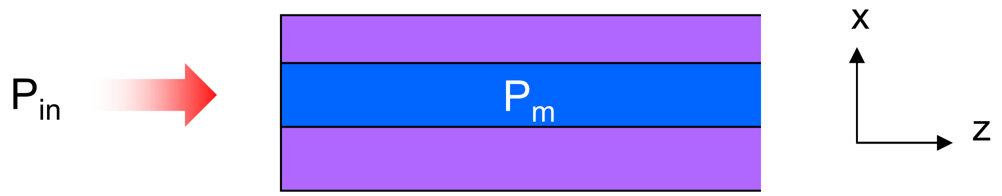
2 — Waveguide Coupling

2.1 Waveguide input and output coupling

2.2 Coupling Between Waveguides

"Integrated Optics — Theory and Technology," by R. G. Hunsperger, 5th ed., Ch. 7-8, Springer Verlag.

General Definitions for Coupling Loss



Coupling efficiency to the m-th mode

$$\eta_m = \frac{P_m}{P_{in}}$$

Q: A single mode optical beam is coupled into a waveguide with guiding core dimension a few times larger than the wavelength. What kind of modes will be generated in the waveguide?

Coupling loss (dB)

$$L = 10 \log \frac{P_{in}}{P_m}$$

Direct Focusing

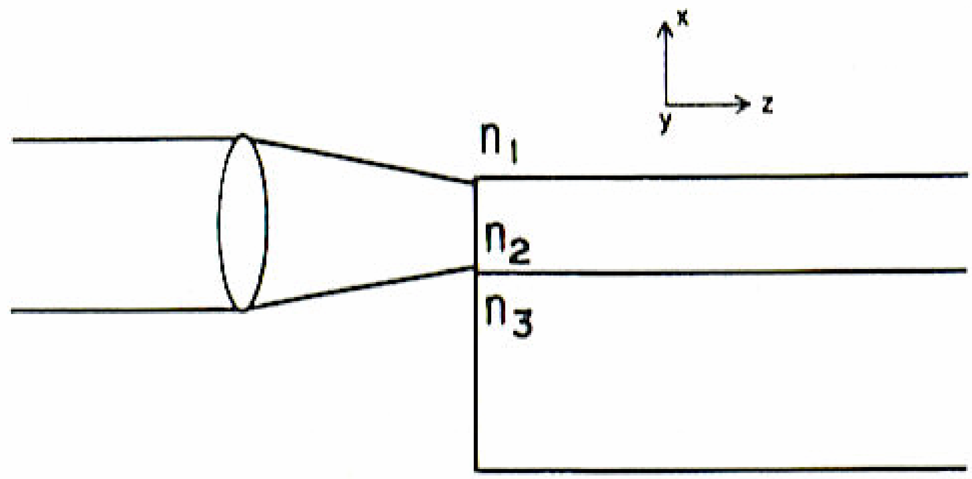


Fig. 7.1. The transverse coupling method, which is sometimes referred to as *end-fire* coupling

$$\eta_m = \frac{\left|\iint A(x,y) B_m^*(x,y)\,dxdy\right|^2}{\iint |A(x,y)|^2 dxdy \cdot \iint |B(x,y)|^2 dxdy}$$

$A(x,y)$: Field distribution of the incident beam

$B_m(x,y)$: Field distribution of the m-th mode

In most cases, *A*(*x*,*y*) can be represented by Gaussian beams.

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$TEM_{0,0}$ Gaussian Beam

Beam spreading

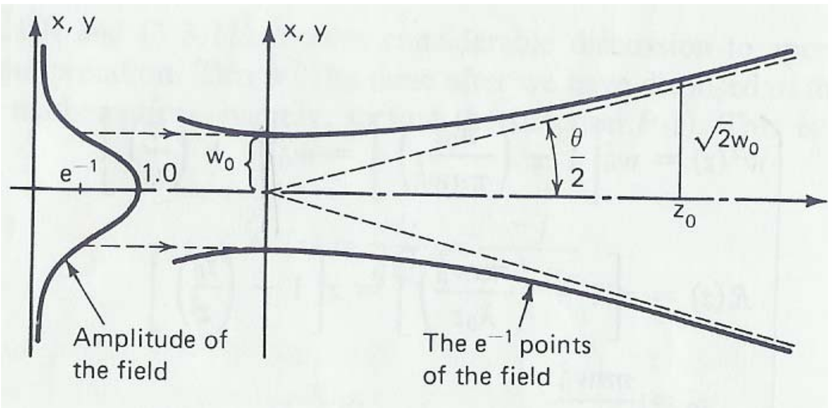
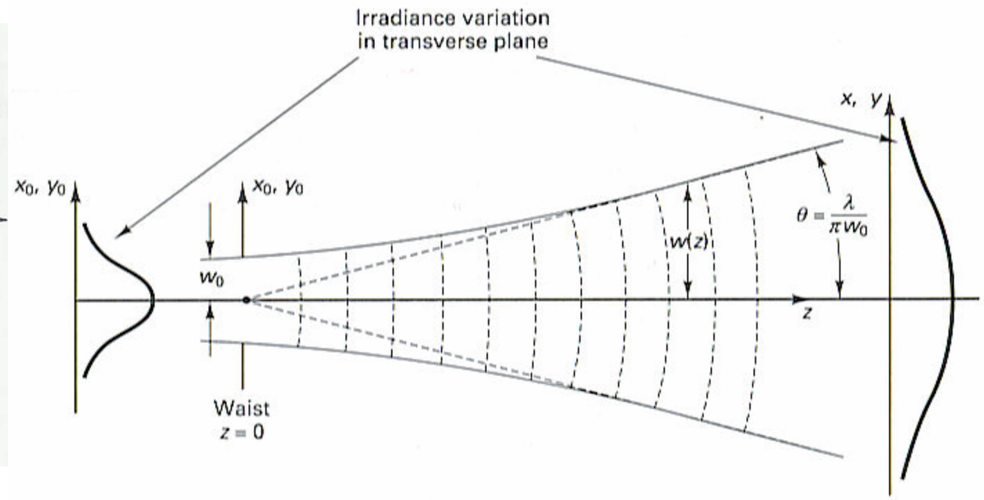


Figure 3.2 Spreading of a $TEM_{0,0}$ mode.

Wavefront change



$$A(\vec{r}) = A_0 \frac{W_0}{W(z)} \exp\left[-\frac{x^2+y^2}{W^2(z)}\right]$$ Amplitude factor

$$\times \exp\left\{-j\left[kz - \tan^{-1}\left(\frac{z}{z_0}\right)\right]\right\}$$ Longitudinal phase

$$\times \exp\left[-j\frac{k(x^2+y^2)}{2R(z)}\right]$$ Radial phase

$$W(z) = W_0\sqrt{1+\left(\frac{z}{z_0}\right)^2}$$ Beam radius

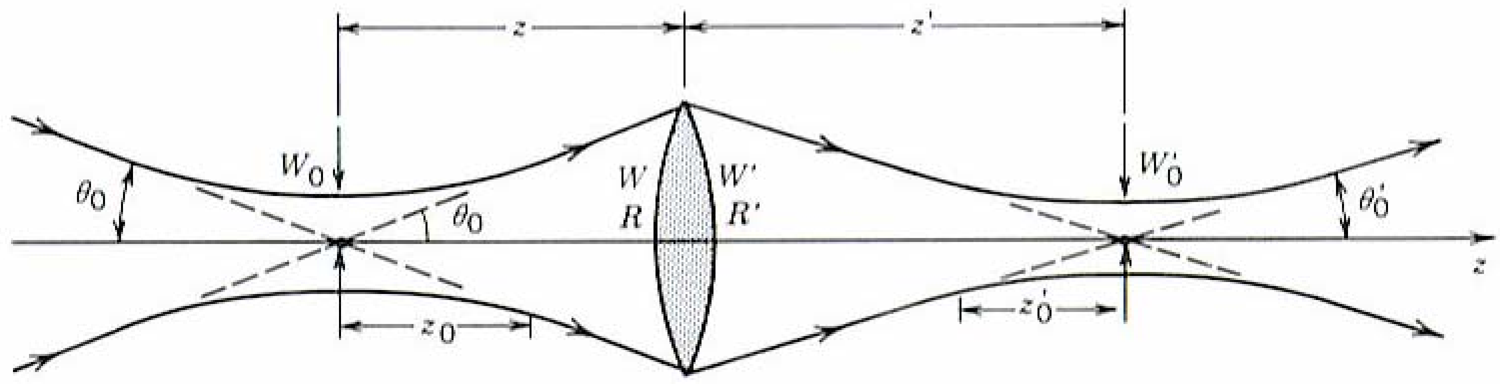
$$R(z) = z\left[1+\left(\frac{z_0}{z}\right)^2\right]$$ Radius of curvature of the wavefront

$$z_0 \equiv \frac{\pi W_0^2}{\lambda}$$ Rayleigh range

Q: Which factor affects the coupling most?

Ref: Verdeyen, *Laser Electronics*, 3rd ed., Prentice Hall.

Gaussian Beam Through a Thin Lens



Waist radius	$W_0' = MW_0$
Waist location	$(z' - f) = M^2(z - f)$
Depth of focus	$2z_0' = M^2(2z_0)$
Divergence	$2\theta_0' = \dfrac{2\theta_0}{M}$
Magnification	$M = \dfrac{M_r}{(1 + r^2)^{1/2}}$
$r = \dfrac{z_0}{z - f}$,	$M_r = \left\lvert \dfrac{f}{z - f} \right\rvert$.

End-Butt Coupling

Exact coupling efficiency can be obtained by overlap integrals.

Approximation: (assuming all waveguide modes are well confined, and $t_g \le t_L$)

$$\eta_m = \frac{64}{(m+1)^2\pi^2}\cdot\frac{n_L n_g}{(n_L+n_g)^2}\cdot\cos^2\left(\frac{\pi t_g}{2t_L}\right)\cdot\frac{1}{\left[1-\left(\frac{t_g}{(m+1)t_L}\right)^2\right]^2}\cdot\frac{t_g}{t_L}\cdot\cos^2\left(\frac{m\pi}{2}\right)$$

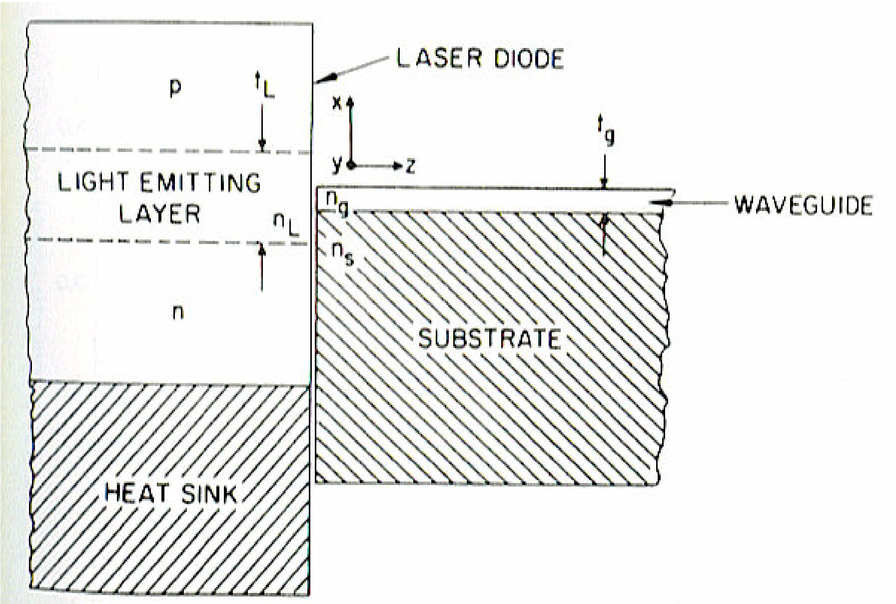


Fig. 7.2. Parallel end-butt coupling of a laser diode and thin-film waveguide

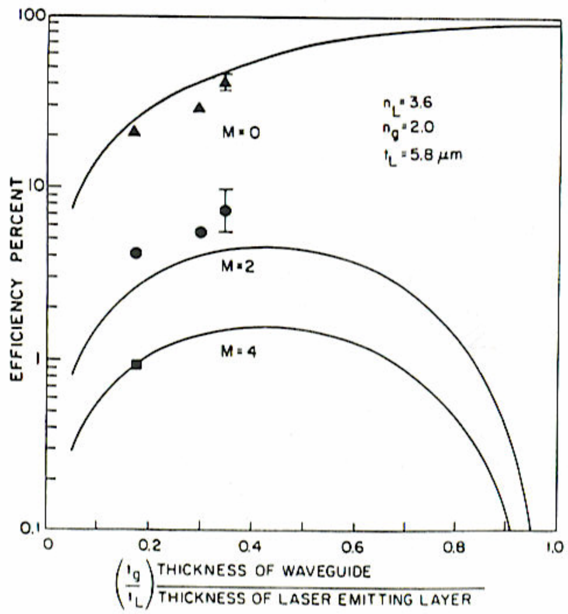


Fig. 7.3. Comparison of experimental coupling efficiency data with theoretical curves as a function of waveguide thickness [7.2]

Misalignment Effect

Lateral misalignment

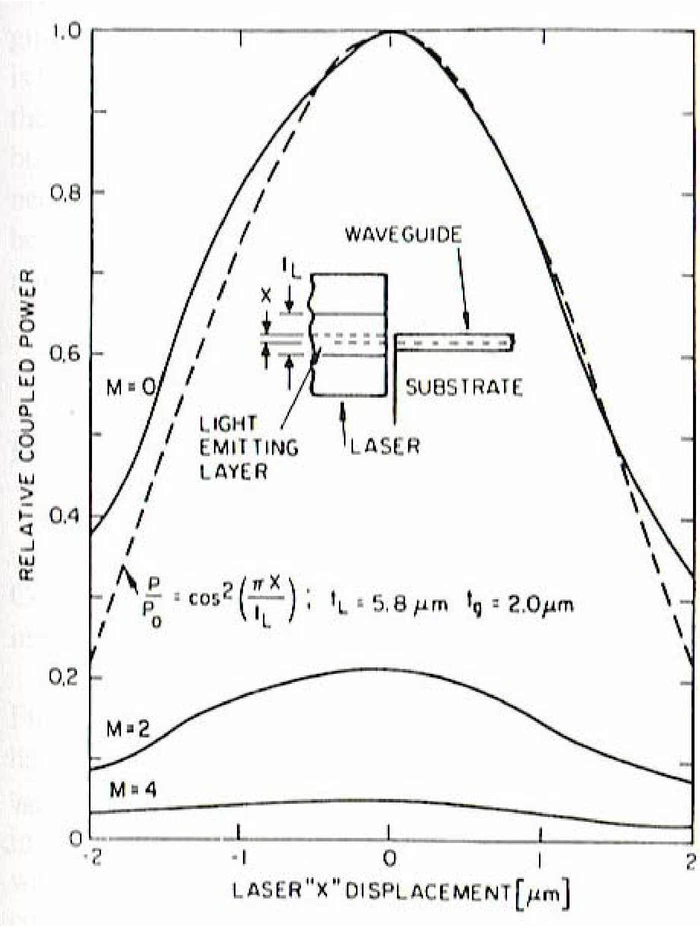


Fig. 7.4. Comparison of experimental coupling efficiency data (*solid line*) with theoretical curve (*dashed*) as a function of lateral misalignment of laser and waveguide [7.2]

Longitudinal misalignment

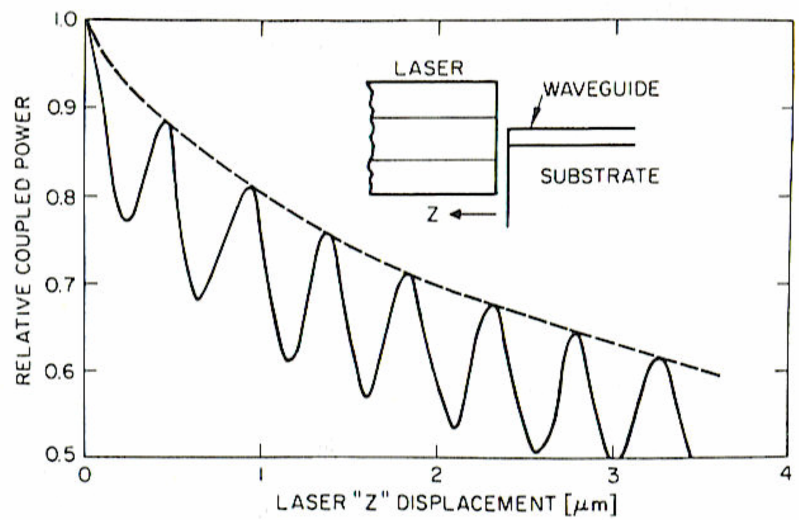


Fig. 7.5. Experimentally measured dependence of coupling efficiency on spacing between laser and waveguide [7.2]

Q: Why is it oscillating? Can we eliminate the oscillation?

$$\frac{P}{P_0} = \cos^2\left(\frac{\pi X}{t_L}\right) \quad \text{for } t_g < t_L,\ X \le \frac{t_L - t_g}{2}$$

Example

MathCAD program for fiber-waveguide coupling.

Tapered Mode Size Converters

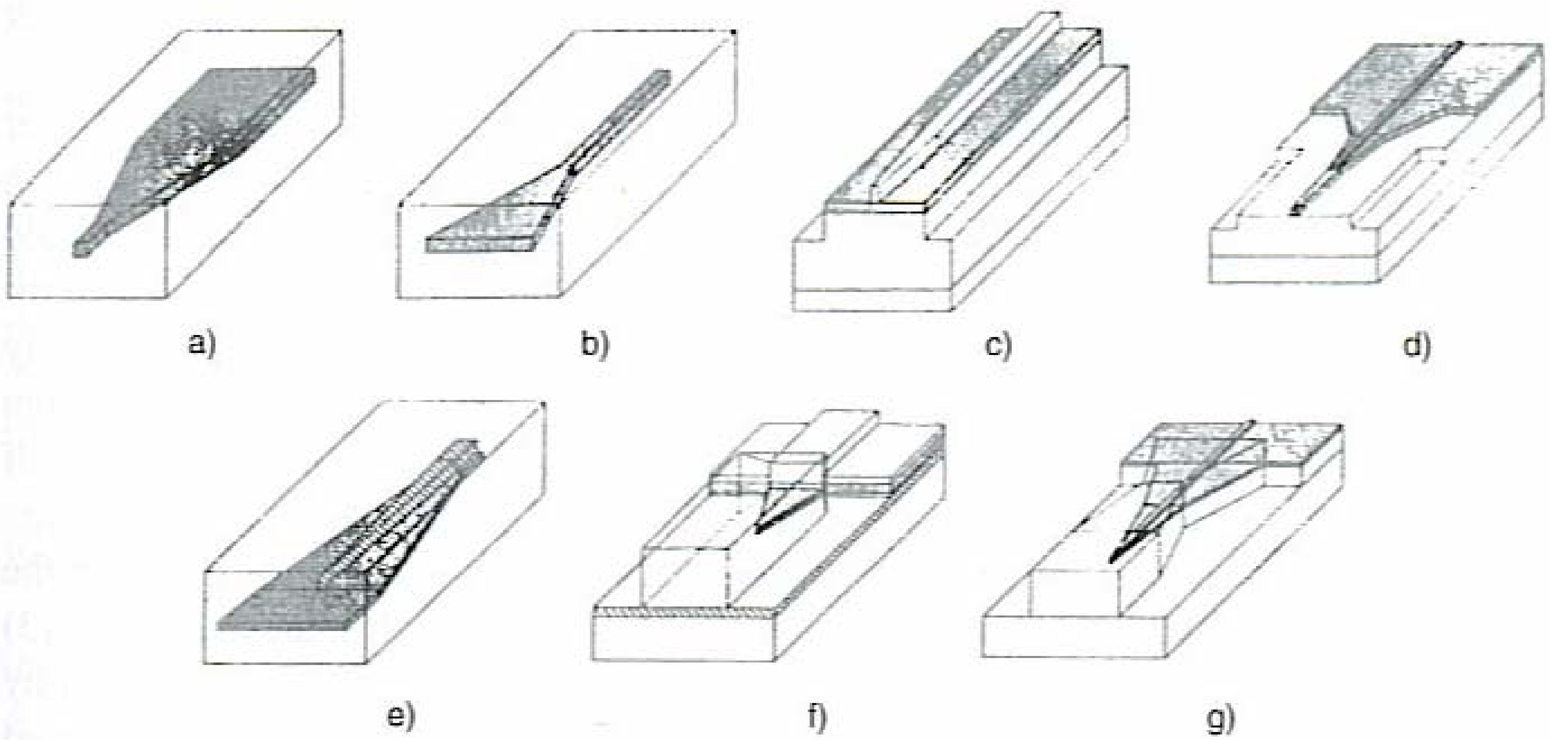


Fig. 7.12a–g. Lateral taper designs. **a** Lateral down-tapered buried waveguide. **b** Lateral up-tapered buried waveguide. **c** Single lateral taper transition from a ridge waveguide to a fiber-matched waveguide. **d** Multisection taper transition from a ridge waveguide to a fiber-matched waveguide. **e** Dual lateral overlapping buried waveguide taper. **f** Dual lateral overlapping ridge waveguide taper. **g** Nested waveguide taper transition from a ridge waveguide to a fiber-matched waveguide [7.25] ©1997 IEEE

Prism Couplers

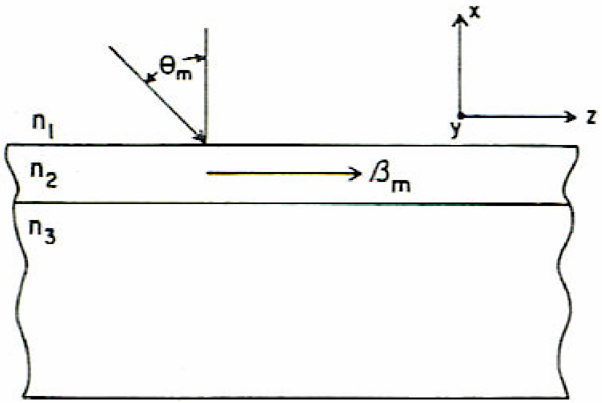


Fig. 7.6. Diagram of an attempt to obliquely couple light into a waveguide through its surface

Air-waveguide coupling

Phase-matching condition

$$\beta_m = kn_1 \sin\theta_m$$

cannot be satisfied.

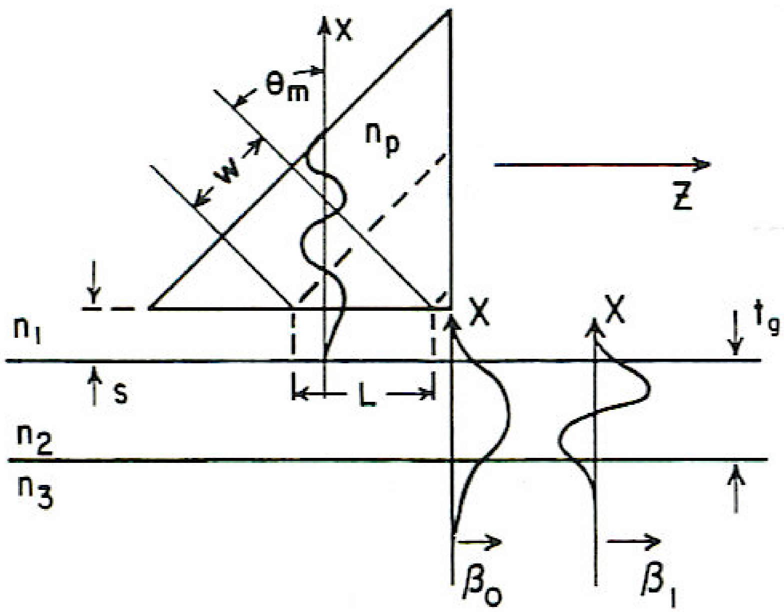


Fig. 7.7. Diagram of a prism coupler. The electric field distributions of the prism mode and the $m = 0$ and $m = 1$ waveguide modes in the x direction are shown

Prism-waveguide coupling

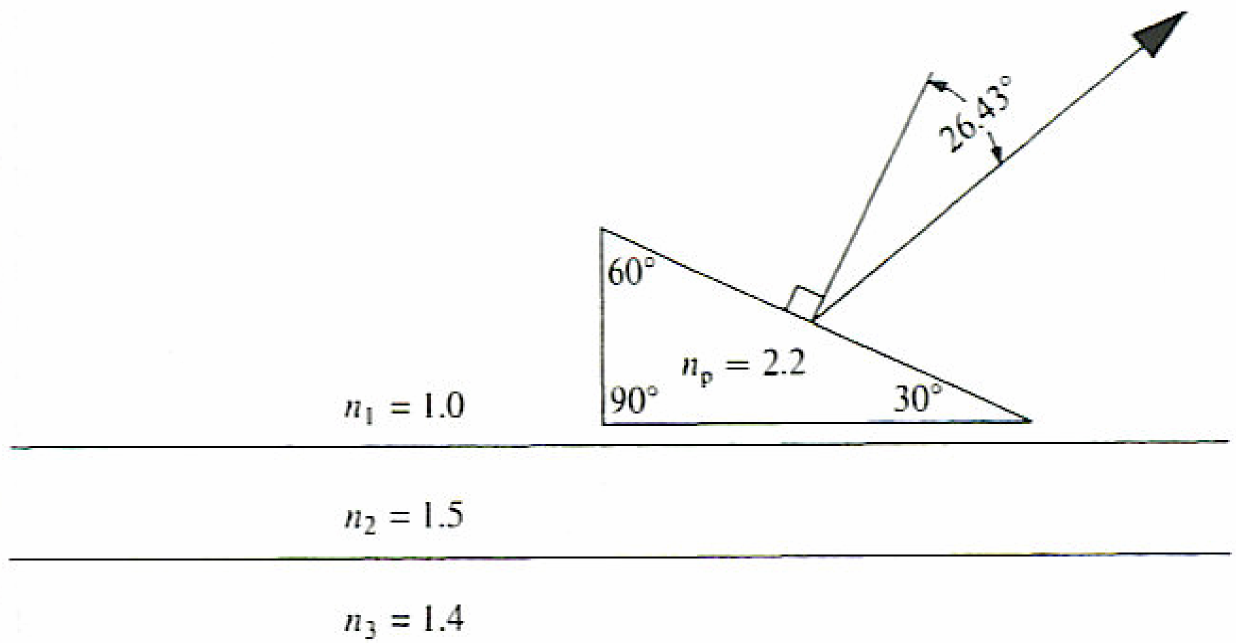
Phase-matching condition

$$\beta_m = kn_p \sin\theta_m$$

can be satisfied.
(Assuming normal incidence to the prism.)

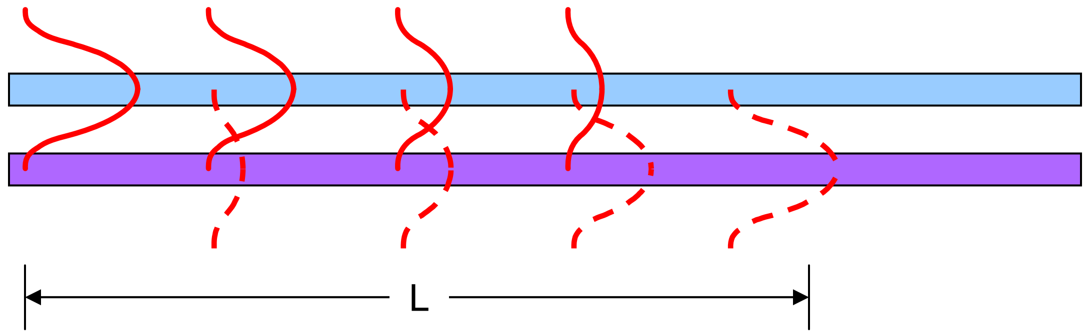
Example: Output Prism Coupler

A prism coupler with index n_p = 2.2 is used to observe the modes of a waveguide. The light source is a He-Ne laser with λ_0 = 632.8 nm. If the light from a particular mode is seen at an angle of 26.43° with the normal to the prism surface, what is the propagation constant β_m for that mode?



Q: What is the interaction length required to obtain complete coupling?

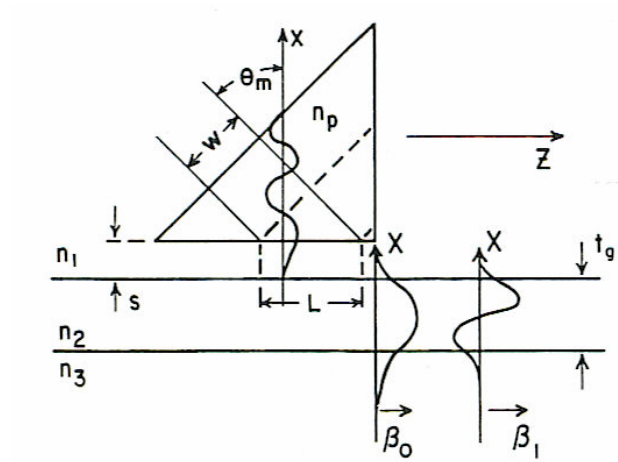
Coupled-Mode Theory



$$\kappa L = \frac{\pi}{2}$$

Q: What will happen if κL > π/2?

κ : Coupling coefficient (depending on overlap integral between the prism mode and the waveguide mode)



$$L = \frac{W}{\cos\theta_m} = \frac{\pi}{2\kappa}$$

For a given L, the coupling coefficient required for complete coupling:

$$\kappa = \frac{\pi\cos\theta_m}{2W}$$

Notes on Prism Coupling

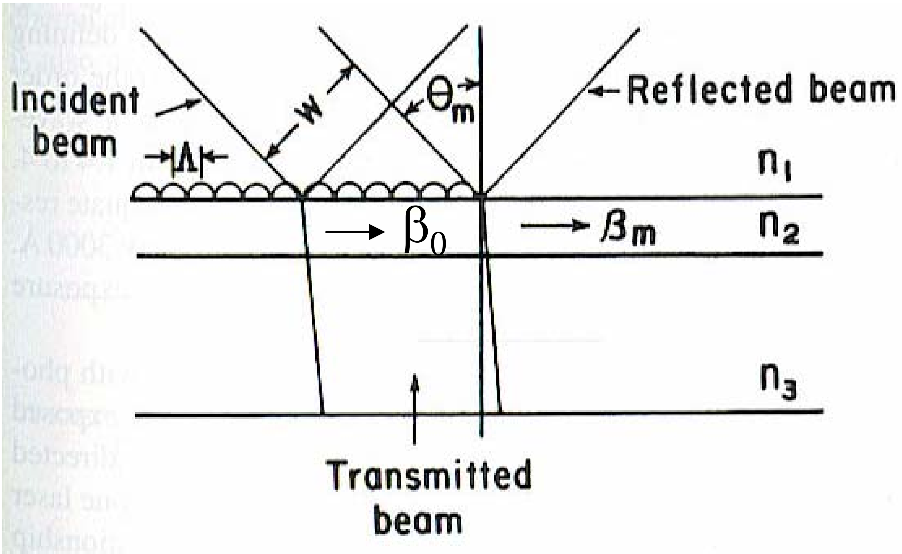
- In order to get 100% coupling with a uniform beam, the trailing edge of the beam must exactly intersect the right-angle corner of the prism.
- Disadvantages
  - For most semiconductor waveguides, $\beta_m \sim kn_2 \rightarrow$ Difficult to find prism materials

Table 7.1. Practical prism materials for beam couplers

Material	Approximate refractive index	Wavelength range
Strontium titanat	2.3	visible – near IR
Rutile	2.5	visible – near IR
Germaium	4.0	IR

  - Incident beam must be highly collimated
  - Coupling efficiency sensitive to the separation between the prism and the waveguide

Grating Coupler



Periodic structure of the grating perturbs the waveguide modes in the region underneath the grating.

$$\beta_\nu = \beta_0 + \frac{\nu 2\pi}{\Lambda}, \qquad \nu = 0,\ \pm 1,\ \pm 2,\ ...$$

β_0 : Propagation constant of the m-th mode covered by the grating

$\beta_0 \sim \beta_m$

Phase-matching condition: $\beta_\nu = k n_1 \sin\theta_m$

can be satisfied even though $\beta_m > k n_1$

Example of Grating Coupler

Grating: Λ = 0.4 μm on a GaAs planar waveguide

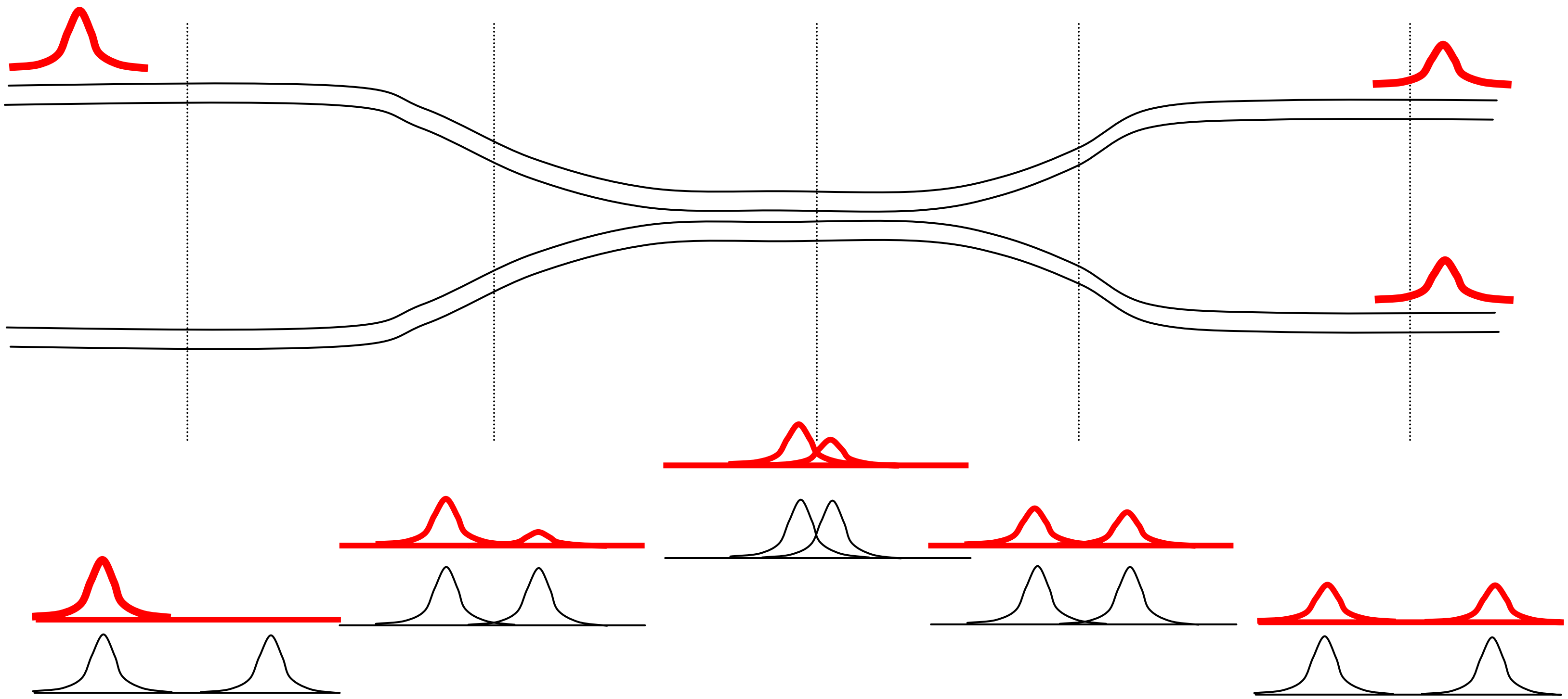
λ_0 = 1.15 μm

Propagation constant for the lowest-order mode in the waveguide: β_0 = 3.6k

Assume 1st-order coupling, $|\nu|$ = 1, what incident angle should the light make in order to couple to the lowest-order mode?

At what λ_0 do we start to need higher-order coupling?

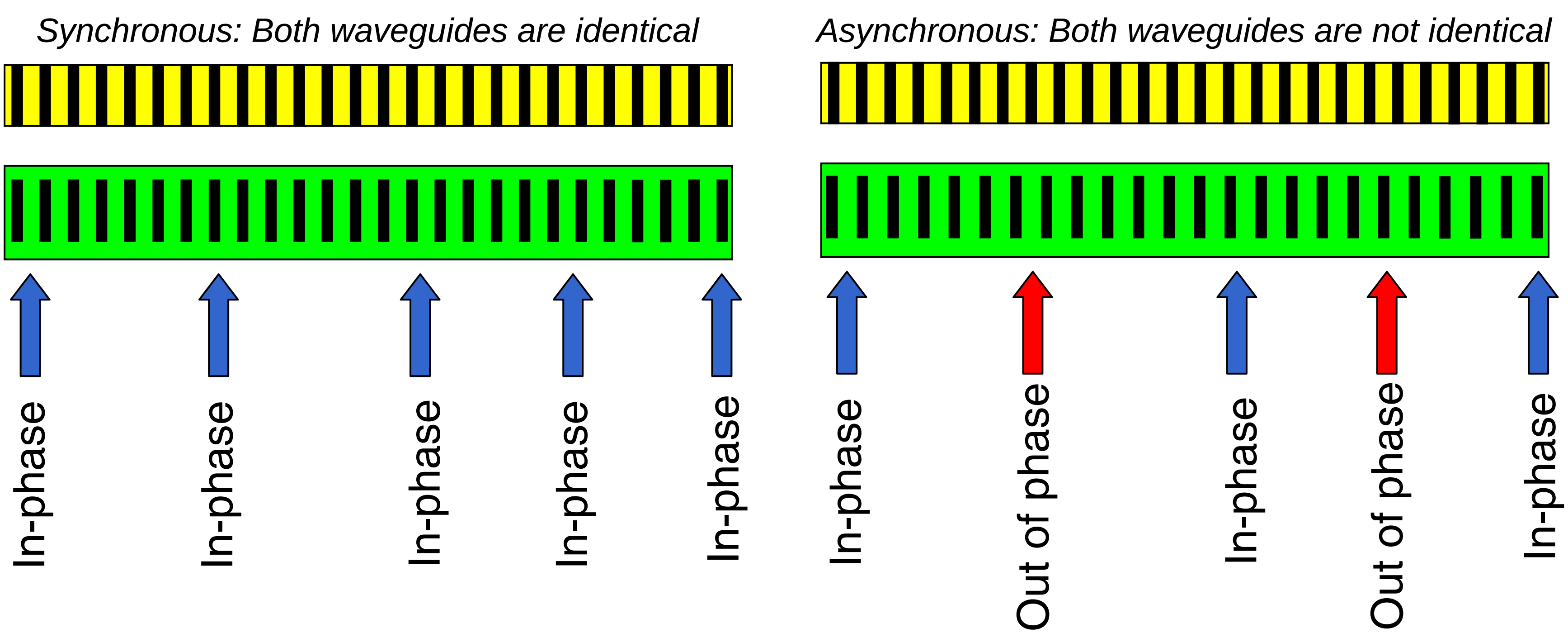
Directional Couplers



- Coupling: Mixing of two adjacent modes, exchanging power as they propagate along adjacent paths.

- Energy transfer in a coherent fashion. → Direction of propagation maintained.

Synchronous Versus Asynchronous



Power transfer continues all the time.
→ **Complete power transfer**

The power transfer that occurs while the waves are in phase is reversed when the waves are out of phase. → **Incomplete power transfer**

Multilayer Planar Waveguide Coupler

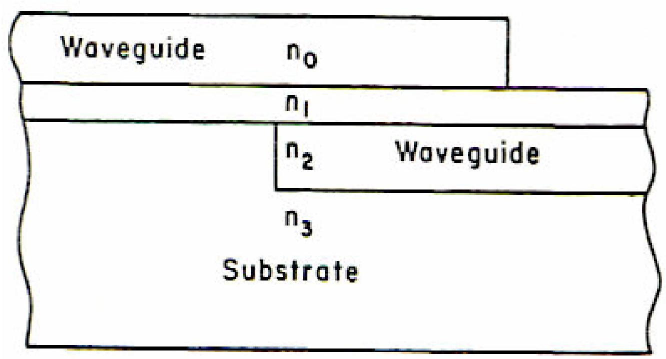


Fig. 8.1. Coupling between two planar waveguides by optical tunneling. Transfer of energy occurs by phase coherent synchronous coupling through the isolation layer with index n_1

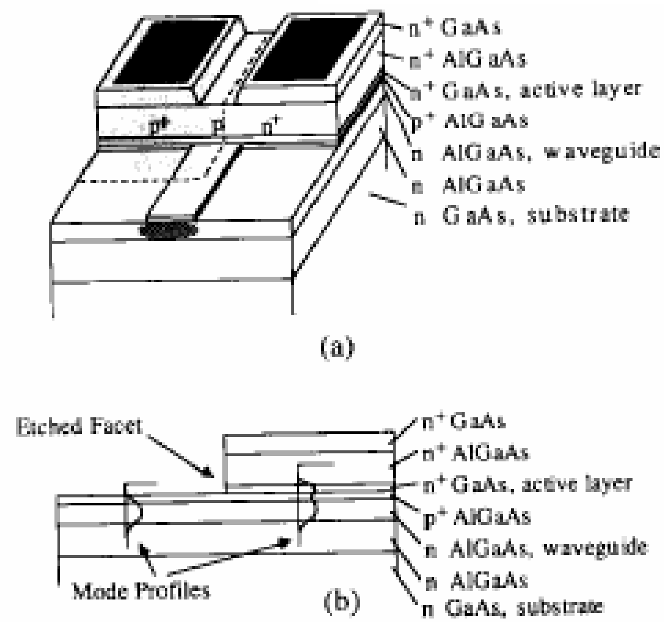


Fig. 1. (a) View of integrated TJS laser with rib waveguide as output. Zn diffused area is shown as shaded portion at top. Lower shaded spot is the optical output of the waveguide. (b) Cross section of the same device with approximate mode profiles of each region overlaid.

Ref: G. A. Vawter, J. L. Merz, and L. A. Coldren, "Monolithically integrated transverse-junction-strip laser with an external waveguide in GaAs/AlGaAs, J. Quantum Electronics, v. 25, no. 2, p. 154-162, 1989.

Dual-Channel Directional Coupler

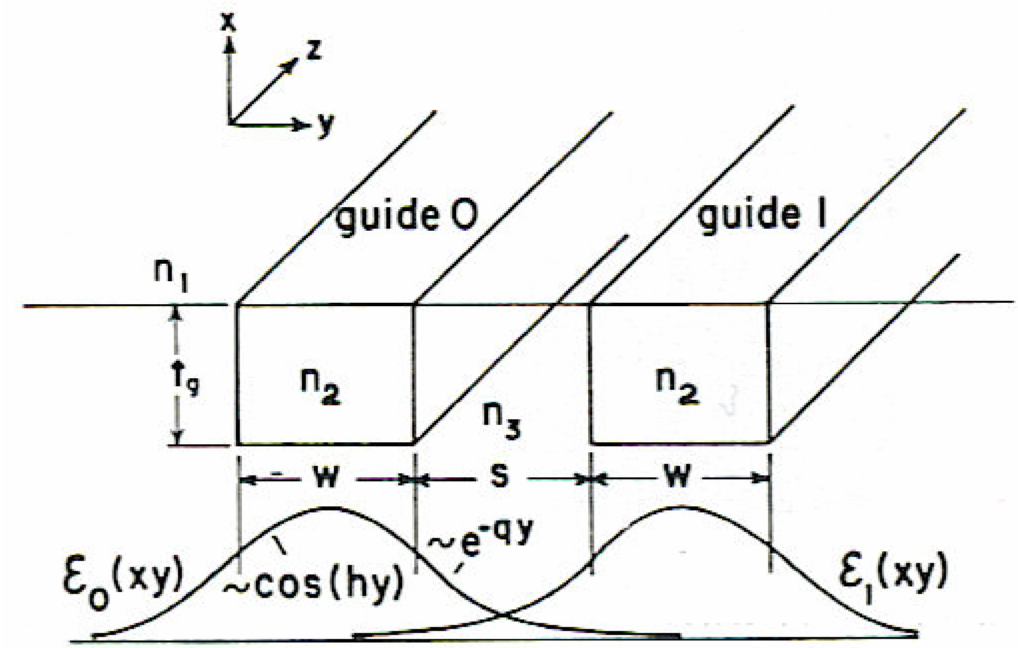
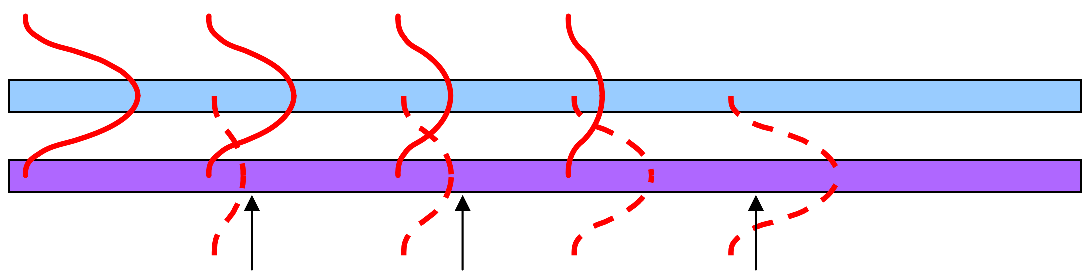


Fig. 8.2. Diagram of dual-channel directional coupler. The amplitudes of the electric field distributions in the guides are shown below them

- Fraction of the power coupled per unit length determined by overlaps of the modes.
- Determine the amount of transmitted power by bending away the secondary channel at proper point.



Transmission Characteristics

3dB directional coupler, interaction length = 1mm

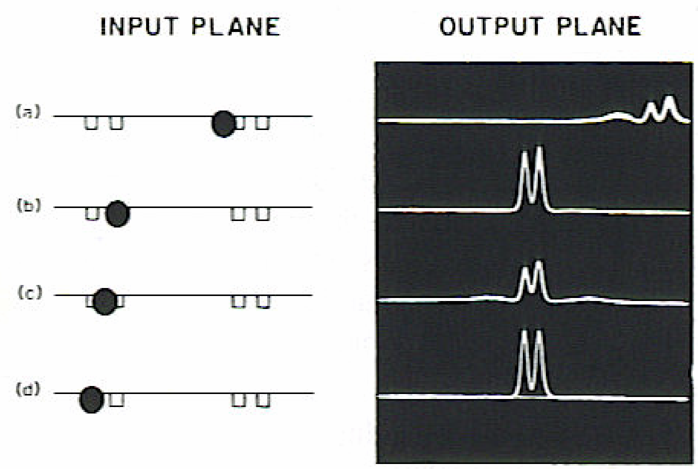


Fig. 8.3. Optical power distribution at the output of a 3 dB dual-channel directional coupler, for various input conditions (as explained in the text). The oscillographs of output power were made using a scanning system like that shown in Fig. 2.3. The waveguides, which were formed by proton bombardment of the GaAs substrate, had 3 µm × 3 µm cross-section and were separated by 3 µm. The interaction length was 1 mm

100% directional coupler, interaction length = 2.1mm

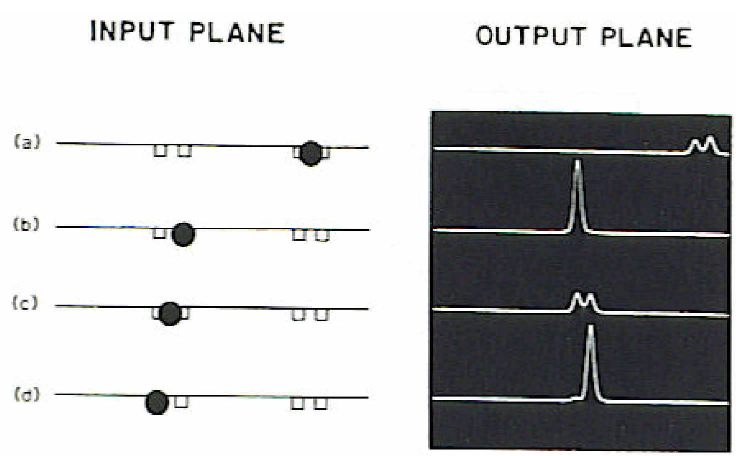
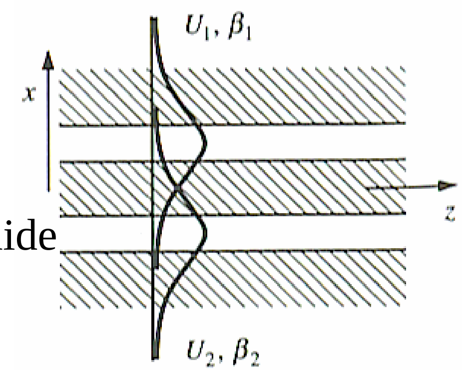


Fig. 8.4. Optical power distribution at the output of a 100% dual-channel directional coupler for various input conditions. The waveguides were like those of Fig. 8.3, except the interaction length was 2.1 mm

Coupled-Mode Theory
— Synchronous Coupling

Electric field of the propagating mode in the waveguide:



$$E(x,y,z) = E_1(z)\mathbf{U}_1(x,y)e^{-j\beta_1 z} + E_2(z)\mathbf{U}_2(x,y)e^{-j\beta_2 z}$$
$$= A_1(z)\mathbf{U}_1(x,y) + A_2(z)\mathbf{U}_2(x,y)$$

$\mathbf{U}_{1,2}(x,y)$: Normalized field distribution in an unperturbed waveguide

Power flow in the waveguides: $P_{1,2}(z) = |A_{1,2}(z)|^2$

Coupled-mode equations:

$$\frac{dA_1(z)}{dz} = -j\beta A_1(z) - j\kappa A_2(z)$$
$$\frac{dA_2(z)}{dz} = -j\beta A_2(z) - j\kappa A_1(z)$$

κ: Coupling coefficient

Initial condition:

$$A_1(0) = 1$$
$$A_2(0) = 0$$

Solutions:

$$A_1(z) = \cos(\kappa z)\exp(-j\beta z)$$
$$A_2(z) = -j\sin(\kappa z)\exp(-j\beta z)$$
$$\beta = \beta_r - j\,\alpha/2$$

α: Loss coefficient

Power Transfer in Synchronous Coupling

Power flow:

$$P_1(z) = |A_1(z)|^2 = \cos^2(\kappa z)\exp(-\alpha z)$$

$$P_2(z) = |A_2(z)|^2 = \sin^2(\kappa z)\exp(-\alpha z)$$

Phase in the driven guide always lag 90º behind the phase of the driving guide.

Q: What's the consequence of this?

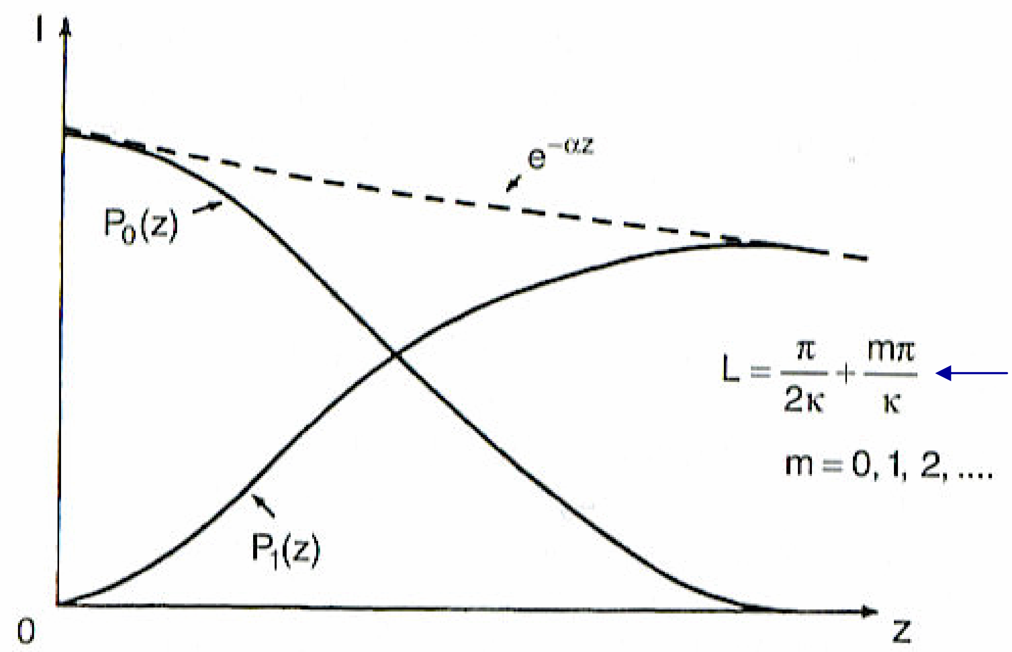


Fig. 8.5. Theoretically calculated power distribution curves for a dual-channel directional coupler. The initial condition of $P_0(0) = 1$ and $P_1(0) = 0$ has been assumed

Length necessary for complete transfer of power from one waveguide to the other.

Coupled-Mode Theory
— Asynchronous Coupling

Asynchronous → $\beta_1 \neq \beta_2$

Coupled-mode equations:

$$\frac{dA_1(z)}{dz} = -j\beta_1 A_1(z) - j\kappa_{12}A_2(z)$$

$$\frac{dA_2(z)}{dz} = -j\beta_2 A_2(z) - j\kappa_{21}A_1(z)$$

Define:

$$\begin{bmatrix} A_1(z) \\ A_2(z) \end{bmatrix} = \begin{bmatrix} a_1(z) \\ a_2(z) \end{bmatrix} \exp(-j\beta z)$$

β : Coupled propagation constant

Condition for non-trivial solutions results in:

$$\beta = \bar{\beta} \pm g$$

$$\bar{\beta} \equiv \frac{\beta_1 + \beta_2}{2}$$

$$g^2 \equiv \kappa_{12}\kappa_{21} + \left(\frac{\Delta\beta}{2}\right)^2$$

Initial condition:

$$A_1(0) = 1$$

$$A_2(0) = 0$$

Power Transfer in Asynchronous Coupling

Power flow:

$$P_1(z) = \cos^2(gz)e^{-\alpha z} + \left(\frac{\Delta\beta}{2}\right)^2 \frac{\sin^2(gz)}{g^2} e^{-\alpha z}$$

$$P_2(z) = |A_1(z)|^2 = \frac{\kappa_{12}\kappa_{21}}{g^2}\sin^2(gz)e^{-\alpha z}$$

Ψ= g (= κ for synchronous coupling)
Assuming lossless for the figures.

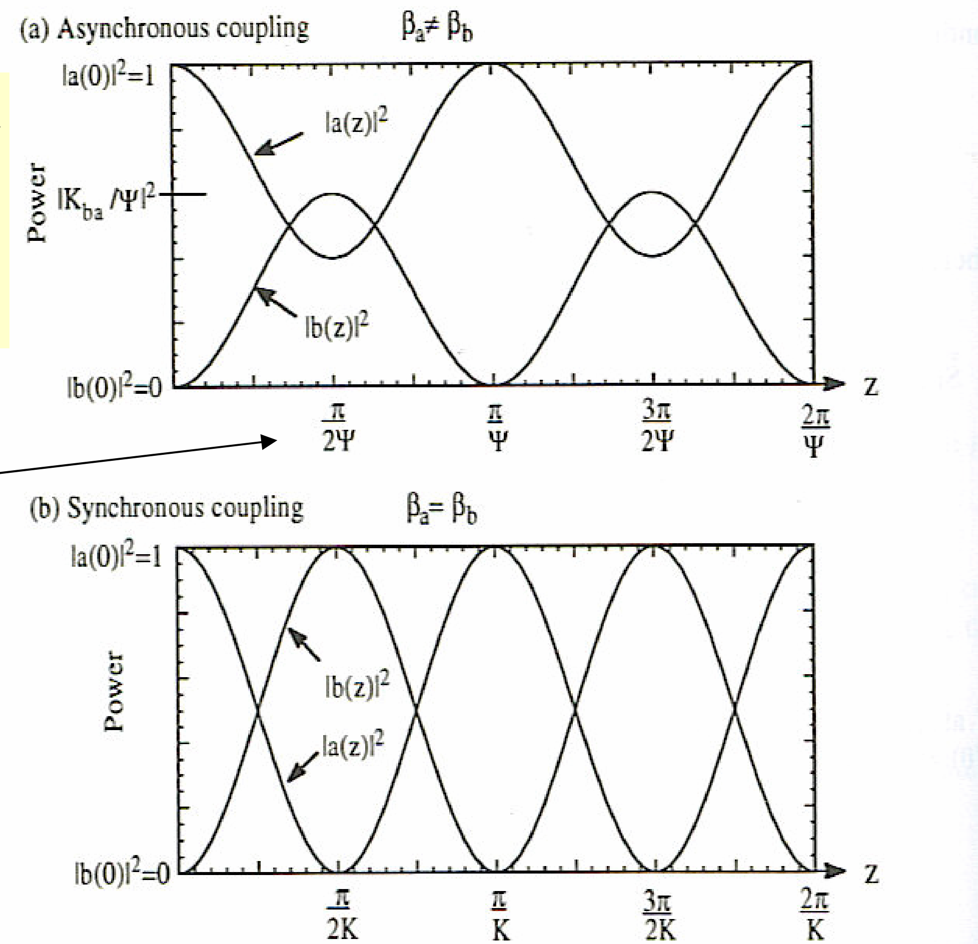


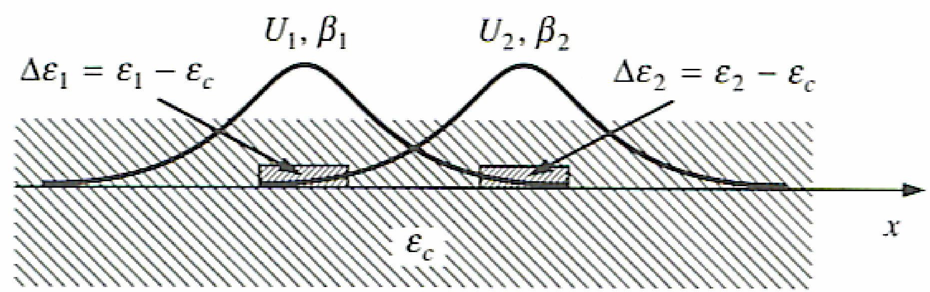
Figure 8.13. Guided powers $|a(z)|^2$ and $|b(z)|^2$ vs. the coupling distance z: (a) asynchronous coupling $\beta_a \neq \beta_b$, (b) synchronous coupling $\beta_a = \beta_b$.

What is κ, κ_{12}, and κ_{21}?

Exact solution:

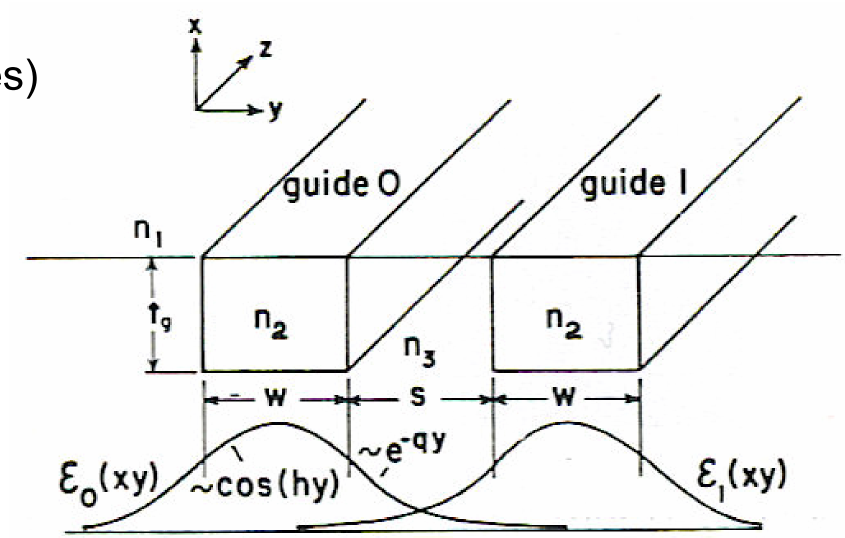
$$\kappa_{12} = \frac{k_0^2}{2\beta_1}\frac{\int(\varepsilon_1-\varepsilon_c)\mathbf{U}_1^*\cdot\mathbf{U}_2 dA}{\int|\mathbf{U}_1|^2\, dA}$$

$$\kappa_{21} = \frac{k_0^2}{2\beta_2}\frac{\int(\varepsilon_2-\varepsilon_c)\mathbf{U}_2^*\cdot\mathbf{U}_1 dA}{\int|\mathbf{U}_2|^2\, dA}$$



Approximation: (For well-confined modes)

$$\kappa = \frac{2h^2 q e^{-qs}}{\beta W(q^2+h^2)}$$

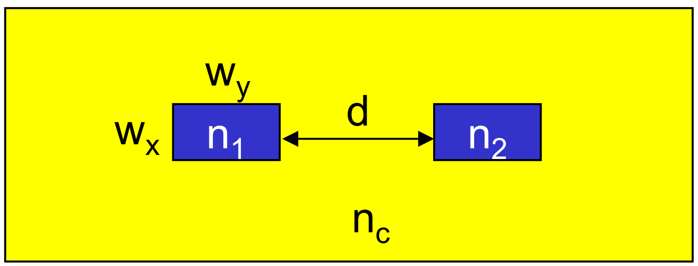


Applications: Modulators and Switches

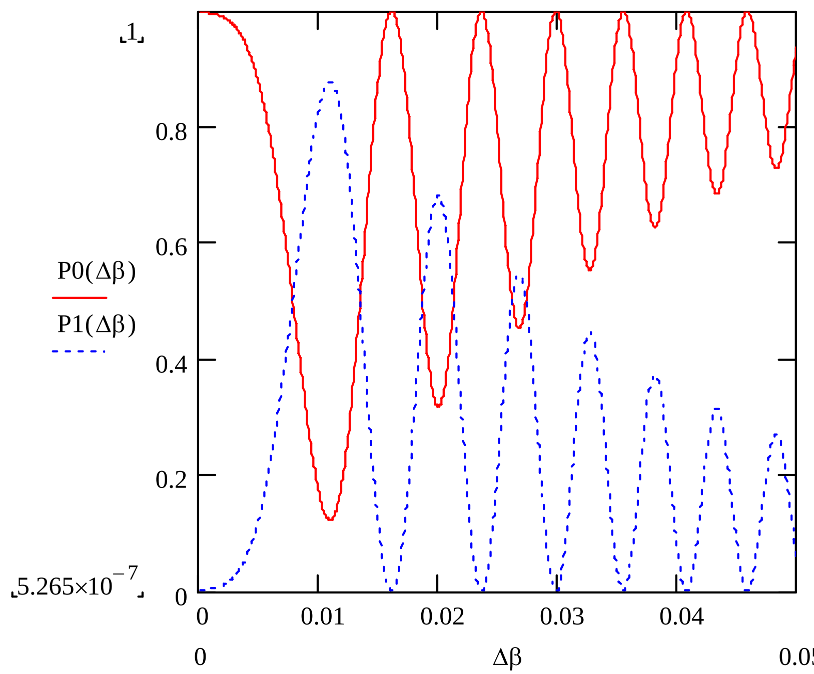
Exercise:
Let's design a modulator using directional coupler

MathCAD program

Control Δβ electrically



Waveguide	n_s	n_f	n_c	a_E	a_M
GaAlAs, double heterostructure	3.55	3.6	3.55	0	0
Sputtered glass	1.515	1.62	1	3.9	27.1
Ti-diffused $LiNbO_3$	2.214	2.234	1	43.9	1093
Outdiffused $LiNbO_3$	2.214	2.215	1	881	21206



Lose coupling

Q1: How to choose the waveguide length?
Q2: What is the best Δβ range?
Q3: How to control Δβ electrically?

Directional Couplers as Modulators

Control $\Delta\beta$ electrically

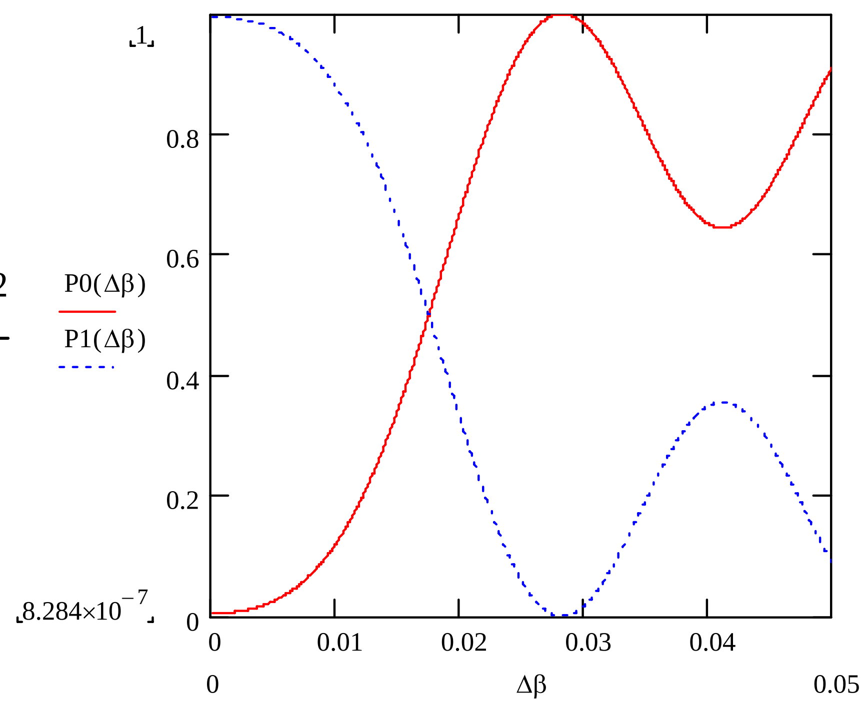
Synchronous coupling, $\kappa_{12} = \kappa_{21} = 0.015$

$z := 300$ Coupling length, in μm

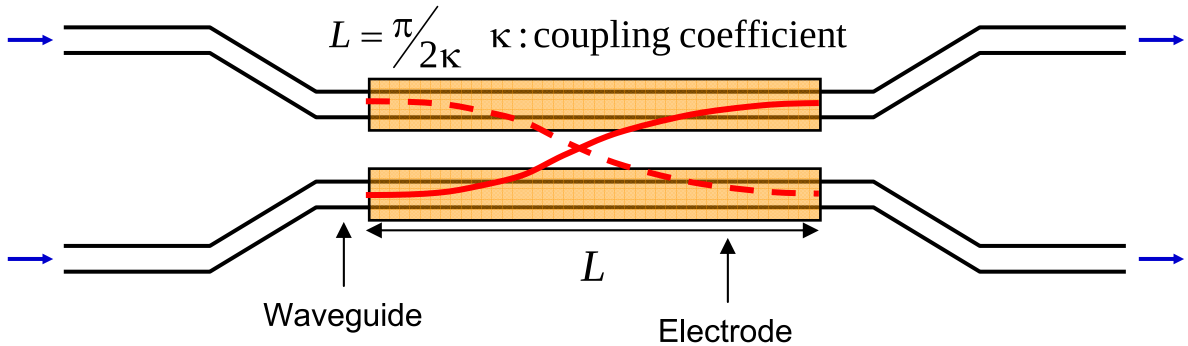
$$g(\Delta\beta) := \sqrt{\kappa 12 \cdot \kappa 21 + \left(\frac{\Delta\beta}{2}\right)^2}$$

$$P0(\Delta\beta) := \cos\left(g(\Delta\beta)\cdot z\right)^2 + \left(\frac{\Delta\beta}{2}\right)^2 \cdot \frac{\sin\left(g(\Delta\beta)\cdot z\right)^2}{g(\Delta\beta)^2}$$

$$P1(\Delta\beta) := \left(\frac{\kappa 12 \cdot \kappa 21}{g(\Delta\beta)^2}\right) \cdot \sin\left(g(\Delta\beta)\cdot z\right)^2$$



Directional Couplers as Switches



Synchronous coupling → Cross state.
When the effective indices, and therefore propagation constants, in the two waveguides are sufficiently different by applying bias, no coupling will occur → Bar-state

$z := 300$ Coupling length, in μm

$$g(\Delta\beta) := \sqrt{\kappa 12 \cdot \kappa 21 + \left(\frac{\Delta\beta}{2}\right)^2}$$

$$P0(\Delta\beta) := \cos\left(g(\Delta\beta)\cdot z\right)^2 + \left(\frac{\Delta\beta}{2}\right)^2 \cdot \frac{\sin\left(g(\Delta\beta)\cdot z\right)^2}{g(\Delta\beta)^2}$$

$$P1(\Delta\beta) := \left(\frac{\kappa 12\cdot\kappa 21}{g(\Delta\beta)^2}\right)\cdot\sin\left(g(\Delta\beta)\cdot z\right)^2$$