EK2360 - Calculation of electrostatic force in COMSOL

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November 4, 2015

There are two methods to calculate the electrostatic force in a COMSOL model: using Maxwell *surface stress tensor*, and *electric energy density*. The first method is highly dependent on the mesh density and symmetry, therefore, it is recommended to use the second method, *electric energy density*.

1 Maxwell Surface Stress Tensor

The force density that acts on the electrode of the actuator results from Maxwell's stress tensor:

$$\mathbf{F}_{\text{es}} = -\frac{1}{2}\left(\mathbf{E}\cdot\mathbf{D}\right)\mathbf{n} + \left(\mathbf{n}\cdot\mathbf{E}\right)\mathbf{D}, \tag{1}$$

where $\mathbf{E}$ and $\mathbf{D}$ are the electric field and electric displacement vectors, respectively, and $\mathbf{n}$ is the outward normal vector of the boundary. This force is always oriented along the normal of the boundary.

In order to use this method `Force calculation` has to be added to the COMSOL model before computation, since the parameter is needed in the post-processing.

1. Right click on `Electrostatics (es)` and add `Force Calculation` to the model.
2. Add the subdomain of one of the electrodes under `Domain Selection`.
3. In the `Force name` edit field, enter a name for the force, e.g. `Fes`.
4. `Compute` the model, so the `Fes` variable becomes available with the correct value.
5. For a 2D model right click on `Results --> Derived Values` and choose `Integration --> Line Integration`.
6. Add boundaries of the electrode used for the `Force calculation` under `Selection`.

   The force can be calculated by multiplying Maxwell surface stress tensor `es.nTy_Fes` and the device thickness `30e-6[m]`.
7. Therefore, in the `Expression` edit field, enter

   `es.nTy_Fes*30e-6[m]` (2)

   or the equivalent thickness parameter instead of `30e-6[m]`.

8. Click on `Evaluate`.

In a 3D model you need `Surface Integration` instead of `Line Integration` (and skip multiplying with the thickness parameter).

2 Electric energy density

The second method is more accurate in comparison to the Maxwell surface stress tensor which is highly dependent on the mesh density and symmetry. In this method, the electric energy density is computed for two different gaps between the electrodes to calculate

$$F = We/d \tag{3}$$

(see lecture notes). The force can be calculated by multiplying W_e/d and the device thickness (`30e-6[m]`). To use this method, follow steps provided below.

1. For a 2D model right click on `Results --> Derived Values` and choose `Integration --> Surface Integration`.
2. Add all subdomains which contain electric field energy under `Selection`.
3. In the `Expression` edit field, enter

$$\texttt{es.We} \tag{4}$$

4. Click on `Evaluate`. This computes the total electric energy for the present geometrical configuration (i.e. the set gap between the electrodes) which we call `We1`.
5. Apply a small change in the gap between the electrodes (e.g. from `5e-6[m]` to `5.1e-6[m]`).
6. `Compute` the model for the new gap.
7. Under `Results --> Derived Values --> Surface Integration`, click `Evaluate` to get the new electric energy which we call `We2`. Calculate the electrostatic force by

$$F_{es} = (We_2 - We_1) * thickness/d \tag{5}$$

For the provided comb drive model with 300 µm long electrodes and 100 fingers,

$$We_{1|gap=10.0\,\mu\mathrm{m}} = 1.117\,53 \times 10^{-5}\,\mathrm{J\,m^{-1}} \tag{6}$$

and

$$We_{2|gap=10.1\,\mu\mathrm{m}} = 1.124\,89 \times 10^{-5}\,\mathrm{J\,m^{-1}} \tag{7}$$

The computed electrostatic force is then

$$F_{es} = \left(1.124\,89 \times 10^{-5}\,\mathrm{J\,m^{-1}} - 1.117\,53 \times 10^{-5}\,\mathrm{J\,m^{-1}}\right) \cdot \frac{30\,\mu\mathrm{m}}{0.1\,\mu\mathrm{m}} = 22.08\,\mu\mathrm{N} \tag{8}$$

In a 3D model use `Volume Integration` instead of `Surface Integration` (and skip multiplying with the thickness parameter).