Optimal Shape Design of an Electrostatic Comb Drive in Microelectromechanical Systems

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***Abstract*— Polynomial driving-force comb drives are synthesized using numerical simulation. The electrode shapes are obtained using the indirect boundary element method. Variable-gap comb drives that produce combinations of linear, quadratic, and cubic driving-force profiles are synthesized. This inverse problem is solved by an optimization procedure. Sensitivity analysis is carried out by the direct differentiation approach (DDA) in order to compute design sensitivity coefficients (DSC's) of force profiles with respect to parameters that define the shapes of the fingers of a comb drive. The DSC's are then used to drive iterative optimization procedures. Designs of variable-gap comb drives with linear, quadratic, and cubic driving force profiles are presented in this paper. [263]**

I. Introduction

ELECTROSTATIC actuators consisting of moving conductors and dielectrics have been proposed and used in macroscopic systems including electrostatic volt meters [1]. These capacitance-based actuators have been applied to microelectromechanical systems (MEMS) [2], [3] to move small structures. One important example of a microactuator is the comb-drive actuator consisting of interdigitated capacitors. In a typical comb drive, the capacitance is linear with displacement δ, resulting in an electrostatic driving force which is independent of the position of the moving fingers (relative to the fixed ones) except at the ends of the range of travel (Fig. 1). Electrostatic combs have been used for static actuation of friction test structures [4], microgrippers [5], and force-balanced accelerometers [6]. Potential applications of lateral resonators [7] include resonant accelerometers and gyroscopes, as well as resonant microactuators [7]. A detailed numerical simulation of a comb drive, using an indirect boundary element method, has been carried out in reference [8].

The standard comb drive in Fig. 1 has a uniform gap between the fixed and moving fingers and the driving force remains constant during most of the range of motion of the actuator. It is possible, by changing this gap profile, to obtain different force profiles $F(\delta)$. It is of interest in some applications to have force profiles such as linear, quadratic or cubic. One example is that, in many actuator applications, large displacement motion is highly desirable. However, the actuator springs exhibit nonlinear response for large displacements. The spring restoring force behaves as $R = k_1x + k_2x^3 + \cdots$, where x is the displacement and k_i are the spring constants. A large driving force is required in order to overcome the nonlinear restoring forces. Hence, a prohibitively large voltage must be applied on conventional comb actuators in order to achieve a large range of motion. It is therefore desirable to have comb drives with changing gap profiles such that the corresponding driving force profiles have similar nonlinear terms in x as does the restoring force, for a given applied voltage. Another example is related to tuning MEMS. A comb drive with linear, quadratic or cubic force profile can be used for electrostatic tuning. In many MEMS applications, micromechanical resonators play an important role. In such devices, independent tuning of linear or nonlinear stiffness coefficients is an important issue [9], especially in a device which has large displacement motions.

There are many MEMS simulation tools available in the literature. Among them, MEMCAD (http://www.memcad.com) from Microcosm Technologies, Inc. is an integrated package for mask layout, fabrication process description, geometric modeling, electromechanical simulation, and results visualization. It incorporates custom tools for mask layout, and for capacitance calculation using a boundary element method (BEM) code (called FASTCAP). Standard commercial packages are used for geometric modeling (SDRC's IDEAS) and structural analysis (ABAQUS). It also has CoSolve-EM for self-consistent coupled-domain electromechanical analysis [10]. Some work based on MEMCAD has already been reported recently [11], [12]. Another commercially available package is IntelliCAD from IntelliSense Corporation (http://www.intellis.com), which includes both commercial and custom tools and databases [13]. IntelliSense software products are also directed at providing MEMS modeling and simulation capability. However, these commercial CAD systems applicable to MEMS are primarily aimed at simulating and fabrication processes and electromechanical behavior of a given design. Parametric optimization of a design for specified requirements is not feasible except by iterating the simulation over many input data sets—which is computationally expensive and time consuming.

The present paper addresses the issues of simulation, and then design (inverse problem) of comb drives with variable-gap profiles. Two-dimensional (2-D) simulations of the exterior electrostatic field, and the resultant forces on the comb drive, are carried out with the exterior, indirect, boundary element method. Following direct simulation, sensitivity anal-

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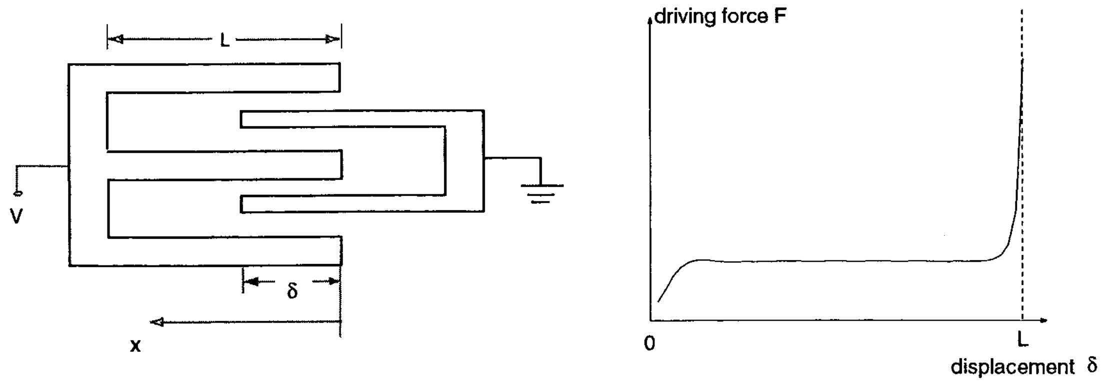


(a) (b)

Fig. 1. (a) A standard comb drive and (b) its force profile.

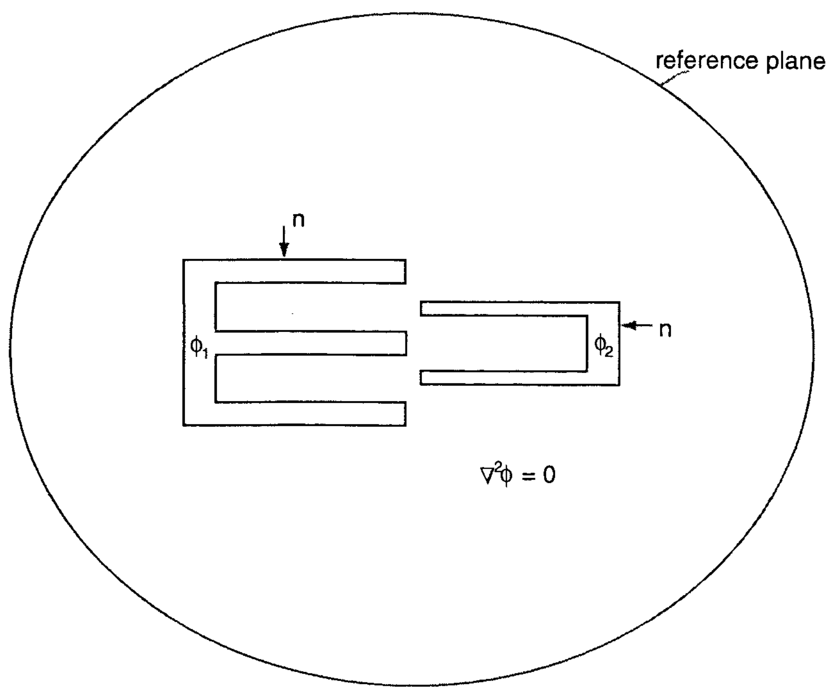


Fig. 2. A system of m-ideal conductors embedded in a uniform lossless dielectric medium.

ysis is carried out by the direct differentiation approach (DDA [14]–[16]). The variable of interest is the driving force while the design variables are parameters that determine the shape of the fixed fingers. (Initially, the widths of the moving fingers are assumed to remain uniform.) Next, an inverse problem is posed as follows: determine the width profile of the fixed fingers (and hence the gap profile) such that the driving force is a desired function of the displacement of the comb drive. Linear, quadratic, and cubic functions are considered in this work. The optimization code "dlcong" from the International Mathematics and Statistics Library (IMSL) package [17] is used for this phase of the work.

It is found that designs with uniform width moving fingers have certain shortcomings including a large size. An improved design is proposed in which both the fixed and moving fingers have variable width.

II. Mathematical Formulation

A. The Driving Force on a Comb Drive

A ideal comb drive can be modeled as a system of m conductors embedded in a uniform lossless dielectric medium (see Fig. 2). Each conductor has a constant electrostatic potential. The charge on each conductor is distributed on its surface and satisfies [18]

$$q_i(\boldsymbol{r}) = \epsilon \frac{\partial \phi_i(\boldsymbol{r})}{\partial \boldsymbol{n}} \tag{1}$$

where $q_i(\boldsymbol{r})$ is the surface charge density at point $\boldsymbol{r}$ on the surface of conductor i, ϵ is the dielectric constant of the medium, ϕ_i is the electrostatic potential of conductor i, and $\boldsymbol{n}$ is the inward normal to a conductor at point $\boldsymbol{r}$.

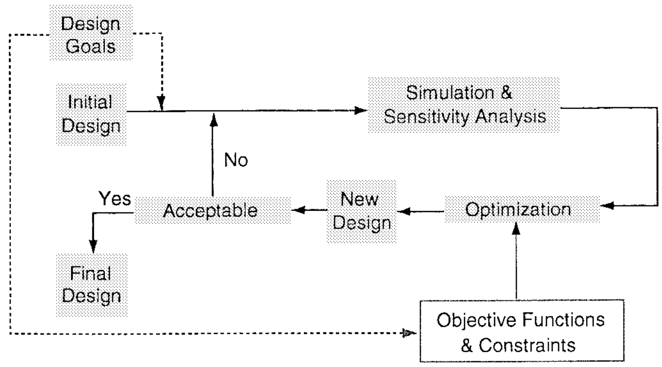


Fig. 3. Flow chart for optimal design.

The electrostatic potential ϕ in the region exterior to the conductors satisfies the Laplace equation

$$\nabla^2 \phi = 0 \tag{2}$$

with the boundary conditions $\phi = \phi_i$ on conductor $i, i = 1, 2, \cdots, m$, where m is the total number of conductors.

Using the indirect boundary element method formulation proposed by Shi *et al.* [19], the surface charge density $q_i(\boldsymbol{r})$ on conductor i can be obtained by solving (3) and (4) together with (1)

$$\phi_i = \sum_{j=1}^{m} \int_{\partial s_j} \frac{\partial \phi}{\partial n}(\boldsymbol{r}') G(\boldsymbol{r}, \boldsymbol{r}')\, ds(\boldsymbol{r}') + \hat{C} \tag{3}$$

$$Q = \sum_{j=1}^{m} \int_{\partial s_j} \frac{\partial \phi}{\partial n}(\boldsymbol{r}')\, ds(\boldsymbol{r}') \tag{4}$$

where $\boldsymbol{r}$ is the position vector of source point, $\boldsymbol{r}'$ is the position vector of field point, G is the Green's function, [which is equal to $(1/2\pi)\ \ln ||\boldsymbol{r} - \boldsymbol{r}'||$ in 2-D, $1/4\pi||\boldsymbol{r} - \boldsymbol{r}'||$ in three dimensions (3-D)], ∂s_j is the surface of conductor j, Q is the total charge of the system, which is zero in this work, and $\hat{C}$ is a constant.

The relationship between the electrostatic force f acting on the surface of a conductor and the charge density q of that conductor is

$$\boldsymbol{f} = -\frac{1}{2} \frac{q^2}{\epsilon} \boldsymbol{n}. \tag{5}$$

Thus, the driving force acting on the moving fingers along the x direction (see Fig. 1)

$$F = \int_{\Gamma} f_x\, ds \tag{6}$$

can be calculated from (5) if q is known. Here, f_x is the x component of force $\boldsymbol{f}$ and Γ is the surface of the moving fingers.

B. Sensitivity Analysis

Design sensitivity coefficients (DSC's) are the derivatives of physical quantities, for example force, stress, temperature etc., with respect to design variables such as geometrical parameters that determine the shape of a structure. In optimization problems, they are used as a guide to the best search direction in nonlinear programming algorithms. These algorithms typically iterate on the design variables along these directions until an optimal design is obtained. Accurate determination of the DSC's typically leads to fast convergence, and thus to more efficient design. There are several methods for computing DSC's. Among them, the finite difference method (FDM) is the easiest one. It calculates two functions from two slightly different design variables and takes the difference of these functions divided by the difference of the design variables as the DSC. This method is very easy to use but may not be accurate. In the present work, the DDA is used to find the DSC's. DSC's are calculated by differentiating (3) and (4) with respect to design variables and solving the resultant equations. One of the advantages of the DDA is high accuracy. The computed DSC's are typically obtained with the same accuracy as the physical quantities. The other advantage is computing efficiency. After discretizing the resultant integral equations, the linear system obtained has the same coefficient matrix A as the one obtained from the calculation of the physical quantities. Only the right-hand-side vector b must be recalculated.

In this work, the design variables are the shape parameters of a comb drive and the physical quantity is the driving force acting on the moving finger.

1) Gradients: Let c be one of the parameters that determine the shape of a finger (fixed or moving) of a comb drive. The precise shape parameters, used in this work, are defined later. From (6), the gradient of the driving force F with respect to c is

$$\overset{*}{F} = \int_{\Gamma} \overset{*}{f}_x\, ds + \int_{\Gamma} f_x\, d\overset{*}{s} \tag{7}$$

where $\overset{*}{(\cdot)} = \partial / \partial c$.

From (5), the sensitivity of f_x is

$$\overset{*}{f}_x = -\frac{q\overset{*}{q}}{\epsilon} n_x - \frac{1}{2} \frac{q^2}{\epsilon} \overset{*}{n}_x. \tag{8}$$

Please refer to, for example, [20] for formulae for $d\overset{*}{s}$ and $\overset{*}{n}_x$.

The sensitivity of the charge density q can be obtained by solving the following:

$$\begin{aligned} \sum_{j=1}^{n} \int_{\partial s_j} \overset{*}{q}(\boldsymbol{r}') G(\boldsymbol{r}, \boldsymbol{r}')\, ds(\boldsymbol{r}') + \overset{*}{C} \\ = -\sum_{j=1}^{n} \int_{\partial s_j} q(\boldsymbol{r}') \overset{*}{G}(\boldsymbol{r}, \boldsymbol{r}')\, ds(\boldsymbol{r}') \\ - \sum_{j=1}^{n} \int_{\partial s_j} q(\boldsymbol{r}') G(\boldsymbol{r}, \boldsymbol{r}')\, d\overset{*}{s}(\boldsymbol{r}') \end{aligned} \tag{9}$$

$$\sum_{j=1}^{n} \int_{\partial s_j} \overset{*}{q}(\boldsymbol{r}')\, ds(\boldsymbol{r}') = -\sum_{j=1}^{n} \int_{\partial s_j} q(\boldsymbol{r}')\, d\overset{*}{s}(\boldsymbol{r}'). \tag{10}$$

Here, $C = \epsilon \hat{C}$ is a constant.

Equations (9) and (10) are obtained by differentiating (3) and (4) with respect to c.

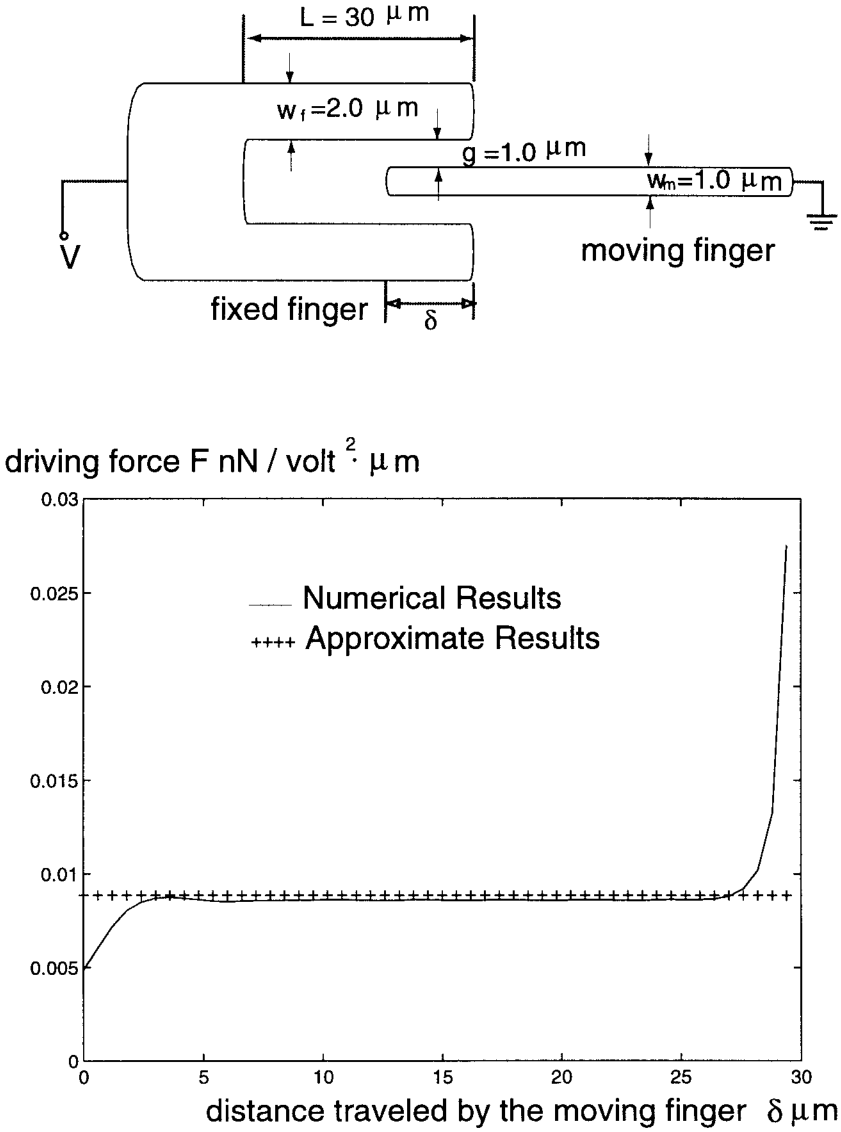


Fig. 4. A prototype comb drive with one set of straight fingers and its force profile.

C. The Inverse Problem

The design of a variable comb drive, with linear, quadratic or cubic driving force, can be posed as an optimal design problem. The goal of the optimization procedure is to minimize an objective function without violating the specified constraints.

The optimization problem is set up as

$$\min \phi(c_i)$$

subject to the constraints: $g_i(c_j) \leq 0$ where the objective function ϕ is chosen as the integral of the square of the difference between the actual and desired force profiles, over the range of operation of the comb drive, i.e.,

$$\phi(c_i) = \int_{\ell_1}^{\ell_2} (F(c_i,x) - h(x))^2 \, dx. \tag{11}$$

Here, c_i are the shape parameters of the comb drive, F is the driving force, x is the position of a moving finger, $h(x)$ is the desired force profile that can be linear, quadratic or cubic (such as the driving force needed to counteract the cubic restoring force of nonlinear actuator springs), ℓ_1, ℓ_2 are the initial and final positions of a moving finger, and g_i are the constraints imposed by practical design issues, such as the minimum gap between fingers, etc.

The sensitivity of the objective function ϕ with respect to a design variable c is

$$\overset{*}{\phi} = \int_{\ell_1}^{\ell_2} 2(F(c_i,x) - h(x))\overset{*}{F} \, dx. \tag{12}$$

The design methodology adopted in this work is outlined in Fig. 3. Simulation and sensitivity analysis is carried out for an initial design. This information is supplied to an optimizer which produces a better design—one that reduces the value of the objective function without violating the constraints of the problem. Iterative improvements in designs continue until a preset stopping criterion is satisfied. This is the final design.

III. Numerical Implementation and Examples

A. The Driving Force

A prototype comb drive with one set of straight fingers is considered here, (see Fig. 4). The surface charge density q on

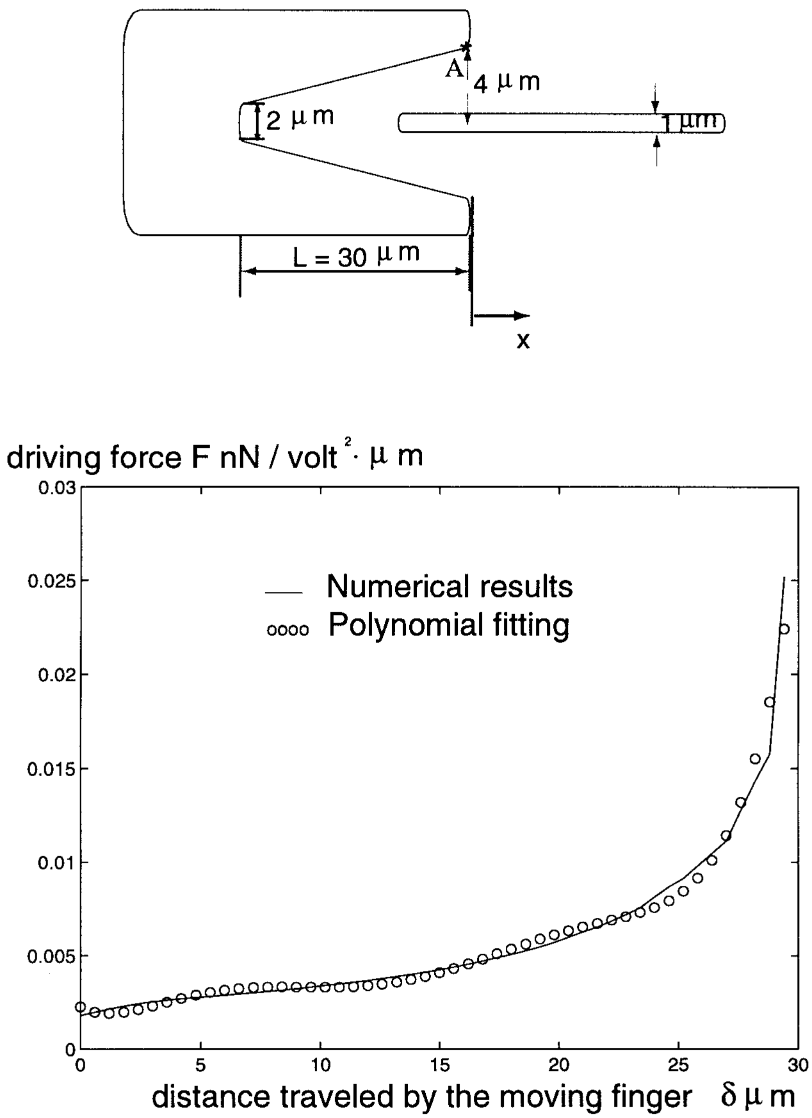


Fig. 5. A comb drive with a linear gap profile and its force profile.

the comb drive is calculated from (3) and (4). These integral equations are solved by the boundary element method. The boundaries are discretized using quadratic elements. A total of 152 elements are used here. It is known that the solution of an exterior Laplace equation becomes unbounded at a corner. In this work, the corners are "rounded off" by Hermitian curves in order to avoid these singularities.

Numerical results for the driving force as a function of the distance traveled by the moving finger are shown in Fig. 4. Also, the results from an approximate formula, which is widely used in practice, are shown there for comparison. This formula, based on a capacitance model [21], is

$$F = \epsilon \frac{hV^2}{g} \tag{13}$$

where F is the driving force acting on a moving finger, h is the height of the fingers (in a direction normal to Fig. 4), and V and g are the bias voltage and the gap between the fixed and moving fingers, respectively.

The results show that the driving force remains constant if the gap g and the height h remain constant. The difference between the two solutions is only 1.1%, which indicates that the capacitance model is a very good approximate formula for a standard comb drive with an uniform gap profile.

According to the formula (13), the driving force F can be a function of the distance δ traveled by a moving finger if the gap varies with δ. This provides the possibility of designing a variable-gap comb drive. The following numerical example further supports this possibility.

Consider a comb drive with its fixed fingers having a linear width profile, (see Fig. 5). The driving force of this "linear" comb drive, shown in Fig. 5, is a sixth-degree polynomial function of the displacement δ. Here, $\circ$'s denote the sixth-degree polynomial approximation to the actual numerical simulation.

B. Sensitivity Analysis

1) Design Variables: It is assumed that the fixed fingers of a comb drive are of variable width while the moving fingers are uniform. In view of the approximate formula (13), it is proposed that the gap profile between a fixed and a moving

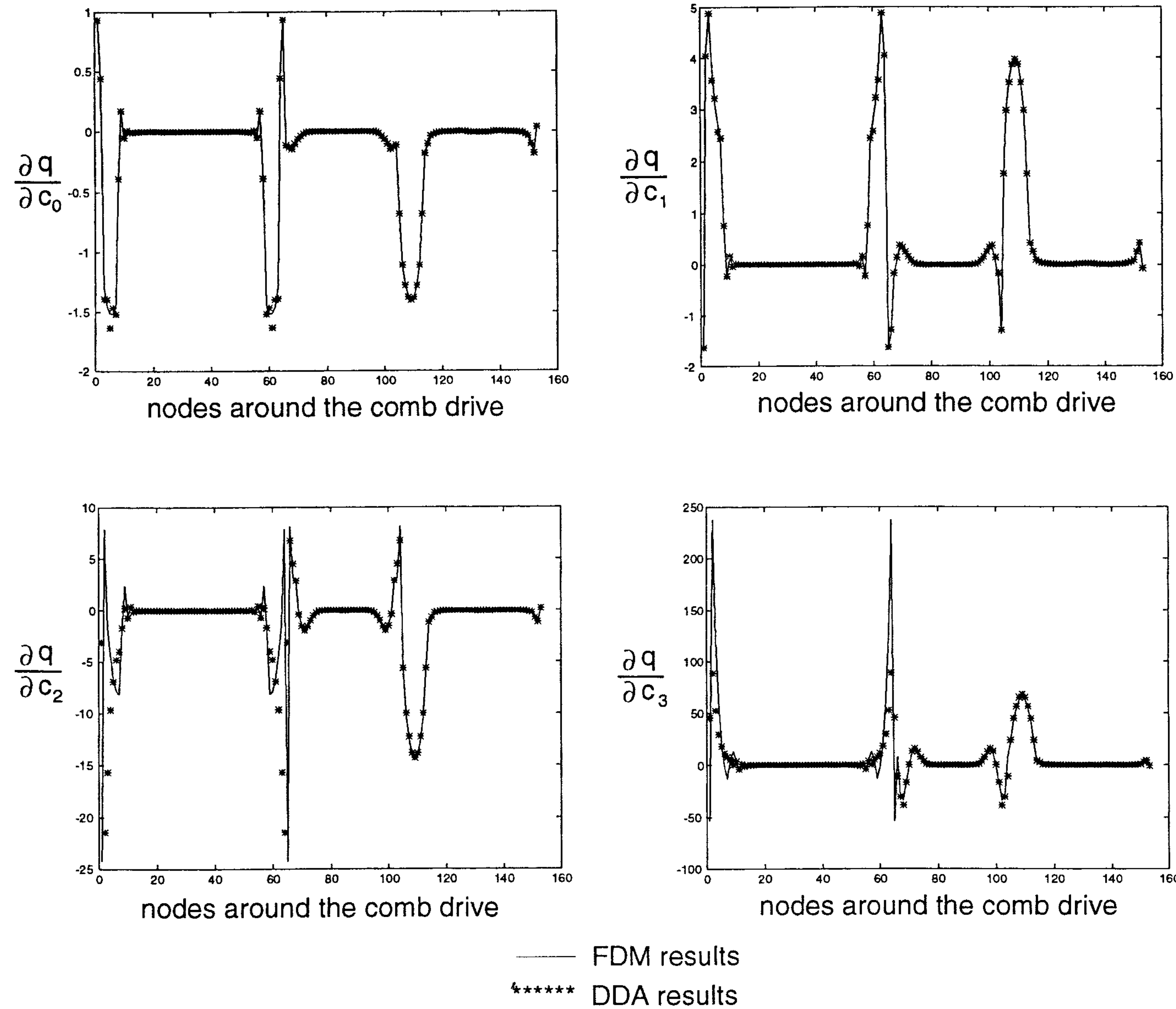


Fig. 6. Numerical results for the sensitivities of charge density with respect to shape parameters.

finger be an inverse polynomial

$$g(x) = \frac{1}{c_0 + c_1 x + c_2 x^2 + c_3 x^3}. \tag{14}$$

If $g(x)$ varies slowly, one would expect to have a driving force $F \approx \epsilon h V^2 (c_0 + c_1 x + c_2 x^2 + c_3 x^3)$, i.e., a combination of linear, quadratic and cubic functions.

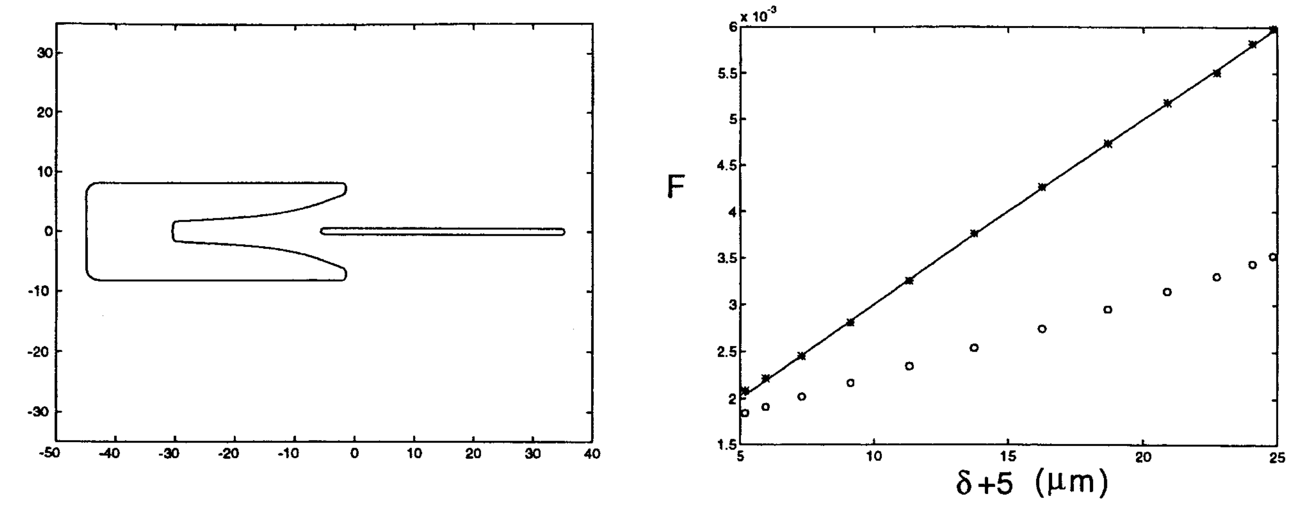
The design parameters are $c_i, i = 0, 1, 2, 3$. All the sensitivities of the physical quantities are calculated with respect to c_i.

2) Gradients: Numerical results for $\partial q / \partial c_i$ from the DDA of the BEM, are compared with those computed by the finite difference method (FDM) in Fig. 6. They agree very well when the gradients are not very large. When the value of q is very sensitive to those of c_i, bigger discrepancies between the results are observed. However, the forces in this problem are mostly determined by the fringe field so that the values of the sensitivities on the tip of the moving finger (nodes 105–115) are of primary interest. In this region, the results from the DDA and the FDM agree.

One can also calculate the sensitivity of the maximum driving force (F) with respect to the gap profile. For example, one can pick a specific point, say $x = -2\,\mu$m (which is point A in Fig. 5), and calculate the quantity $(|\Delta F|/F)/(|\Delta g|/g)$, keeping the slope g' and curvature g'' of g constant at that point. This has been done, and the result is

$$\frac{\frac{|\Delta F|}{F}}{\frac{|\Delta g|}{g}} = 0.7869. \tag{15}$$

Similarly, one can find the quantity $(|\Delta F|/F)/(|\Delta g''|/g'')$, keeping the gap g and slope g' constant at that point. This



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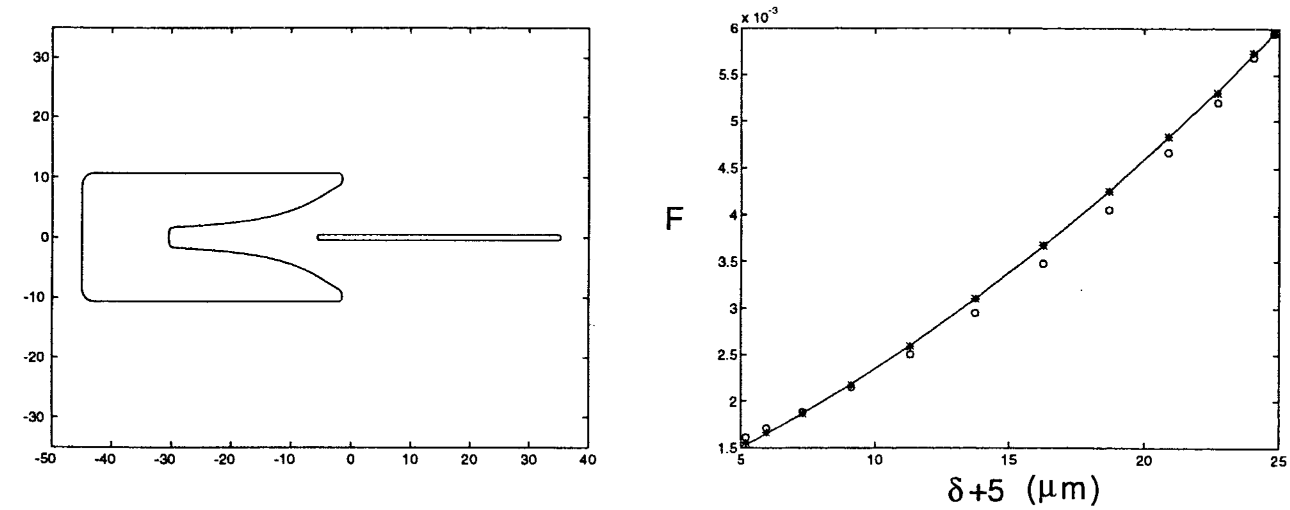

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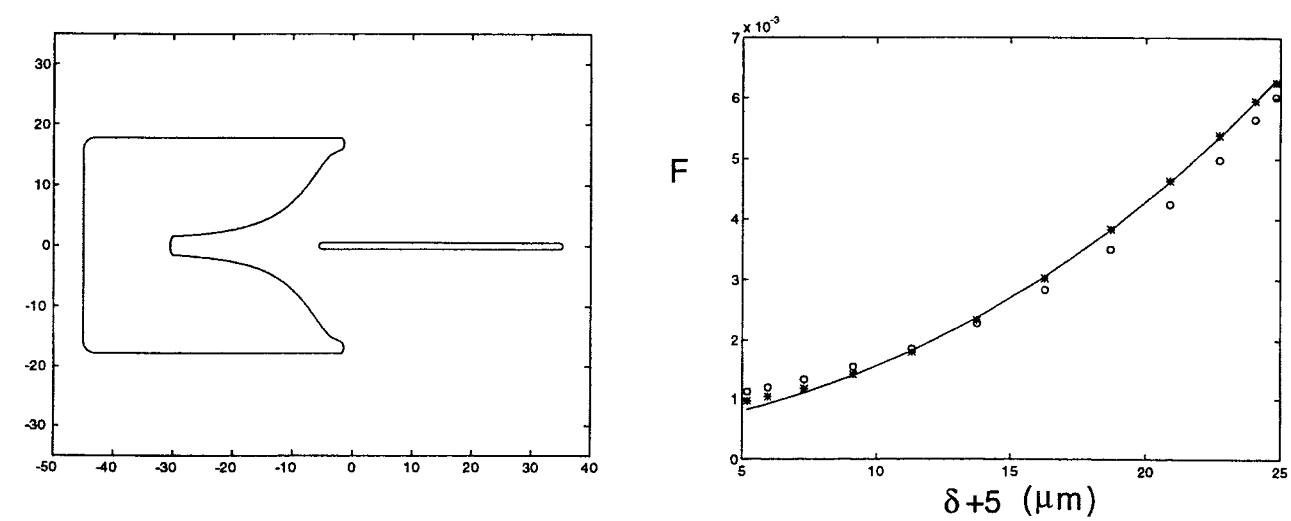


(c)

Fig. 7. Designs of variable comb drives with uniform moving fingers together with their force profiles. (Desired function ——, final force profile $* * * * *$, and initial force profile oooo.) (a) Linear, (b) quadratic, and (c) cubic motors.

TABLE I
DESIGN PARAMETERS AND DRIVING FORCES FOR VARIABLE COMB DRIVES WITH UNIFORM MOVING FINGERS

	Initial design parameters	Optimal design parameters	Driving force(nN/volt^2 $\times$ μm)
Linear Motor	$c_0 = 0.1522$ $c_1 = -0.7246 \times 10^{-2}$ $c_2 = 0.0$ $c_3 = 0.0$ $c_4 = 0.0$	$c_0 = 0.1507$ $c_1 = -0.3135 \times 10^{-2}$ $c_2 = 0.1679 \times 10^{-2}$ $c_3 = 0.8051 \times 10^{-4}$ $c_4 = 0.1244 \times 10^{-5}$	$F = 0.002 + 0.0002\delta$
Quadratic Motor	$c_0 = 0.1067$ $c_1 = -0.8581 \times 10^{-2}$ $c_2 = 0.2860 \times 10^{-3}$ $c_3 = 0.0$ $c_4 = 0.0$	$c_0 = 0.1048$ $c_1 = -0.2795 \times 10^{-2}$ $c_2 = 0.1324 \times 10^{-2}$ $c_3 = 0.4863 \times 10^{-4}$ $c_4 = 0.6654 \times 10^{-6}$	$F = 1.5 \times 10^{-3} + 1.5 \times 10^{-4}\delta$ $+0.375 \times 10^{-5}\delta^2$
Cubic Motor	$c_0 = 0.07249$ $c_1 = -0.4760 \times 10^{-2}$ $c_2 = 0.3173 \times 10^{-3}$ $c_3 = -0.7052 \times 10^{-5}$ $c_4 = 0.0$	$c_0 = 0.06897$ $c_1 = 0.5754 \times 10^{-2}$ $c_2 = 0.1419 \times 10^{-2}$ $c_3 = 0.1401 \times 10^{-4}$ $c_4 = -0.1870 \times 10^{-6}$	$F = 8 \times 10^{-4} + 1.2 \times 10^{-4}\delta$ $+0.6 \times 10^{-5}\delta^2 + 1.0 \times 10^{-7}\delta^3$

TABLE II
DESIGN PARAMETERS OF THE FIXED FINGERS IN THE COMB DRIVES IN FIG. 8

	Design parameters
Linear motor	$c_0 = 0.3188$, $c_1 = -0.7246 \times 10^{-2}$, $c_2 = 0.0$ $c_3 = 0.0$, $c_4 = 0.0$
Quadratic motor	$c_0 = 0.2378$, $c_1 = -0.5721 \times 10^{-2}$, $c_2 = 0.1907 \times 10^{-3}$ $c_3 = 0.0$, $c_4 = 0.0$
Cubic motor	$c_0 = 0.1522$, $c_1 = -0.7262 \times 10^{-2}$, $c_2 = 0.7469 \times 10^{-3}$ $c_3 = 0.1893 \times 10^{-4}$, $c_4 = 0.0$

time, one gets

$$\frac{\frac{|\Delta F|}{F}}{\frac{|\Delta g''|}{g''}} = 0.146. \tag{16}$$

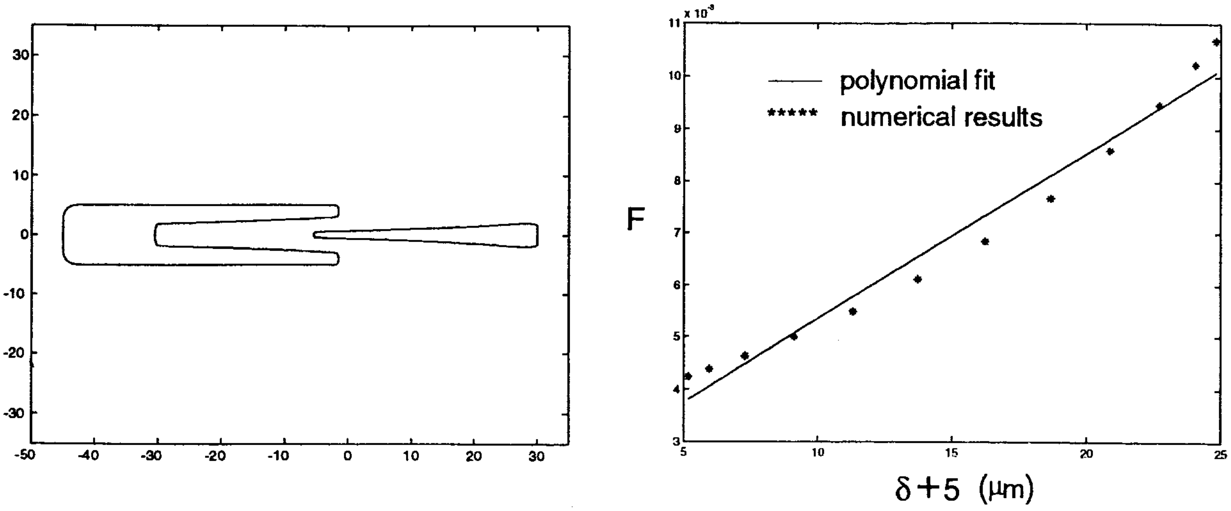
C. The Inverse Problem

The goal here is to design three variable comb drives, which have linear, quadratic, or cubic driving force profiles as functions of the distance traveled by the movable fingers. However, a comb drive with purely linear, quadratic, or cubic driving force profile usually occupies a large area. For example, for a comb drive with driving force $F(x) = cx^3$, according to the simple formula (13), the gap $g(x)$ will be roughly proportional to $1/x^3$. If x, the position of the tip of the moving finger, varies between 5–25 μm, the gaps at the two ends will have a ratio of 125, i.e., if $g(x)$ at $x = 25\,\mu$m is 1 μm, then $g(x)$ at $x = 5\,\mu$m will be 125 μm. This will result in an unacceptably wide comb drive. Due to this practical design consideration, the range of x in the desired function $h(x)$ is shifted by x_0. Instead of having $h(x)$ proportional to x, x^2, or x^3, it is taken to be proportional to $x+x_0$, $(x+x_0)^2$, or $(x+x_0)^3$. By choosing suitable values of x_0, the opening between the fixed fingers can be controllebd. Also, in these examples, the design space is enlarged by adding another term c_4x^4 in the denominator of the expression for $g(x)$ in (14).

The new objective function ϕ for the inverse problem is

$$\phi(c_i) = \int_{\ell_1}^{\ell_2} (F(c_i, x) - h(x+x_0))^2\,dx. \tag{17}$$

This problem has been solved by using the optimization code "dlcong" from the IMSL package. This code is based on M. J. D. Powell's TOLMIN, which solves linearly constrained optimization problems. The algorithm starts by checking the equality constraints for inconsistency and redundancy. If the equality constraints are consistent, the method will revise the initial guess provided by the user to satisfy the equality constraints. Then, it is adjusted to satisfy the simple bounds and inequality constraints. This is done by solving a sequence of quadratic programming subproblems to minimize the sum of the constraint or bound violations. Now, for each iteration with a feasible point (values of design parameters), a quadratic programming problem is solved in order to obtain the search direction and the Lagrange multipliers. After the search direction is obtained, a line search is performed to locate a better point. The iterations continue until the stopping criterion (which is $|\partial\phi/\partial c_i| \leq 0.001$ in this problem) is satisfied. This assures that a local minimum of the objective function is achieved. This is usually sufficient from a practical design point of view.



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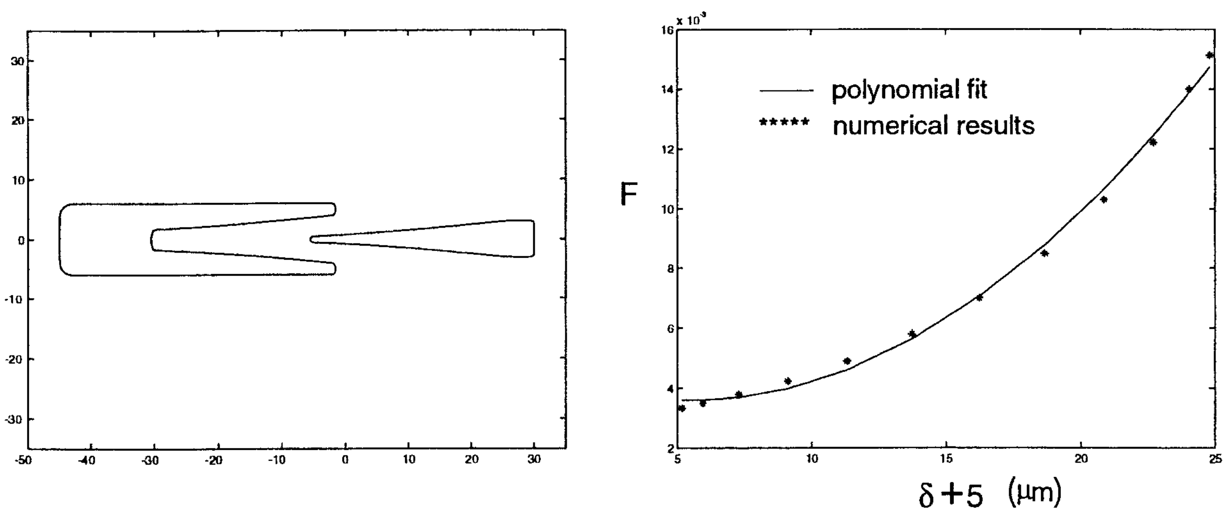

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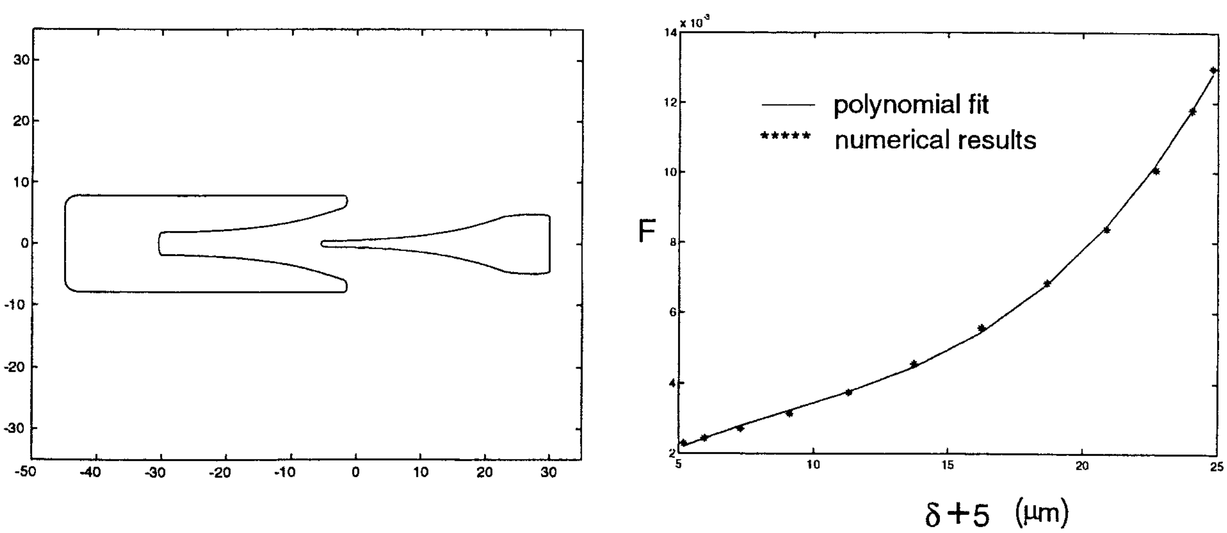

(c)

Fig. 8. Designs of variable comb drives with variable moving fingers together with their force profiles. (a) Linear motor: $F(\delta) = 0.000\,32\delta + 0.0038$ nN/V^2 μm. (b) Quadratic motor: $F(\delta) = 3.0 \times 10^{-5}\delta^2 - 3.20 \times 10^{-5}\delta + 0.0036$ nN/V^2 μm. (c) Cubic motor: $F(\delta) = 1.57 \times 10^{-6}\delta^3 - 2.00 \times 10^{-5}\delta^2 + 3.2 \times 10^{-4}\delta + 2.14 \times 10^{-3}$ nN/V^2 μm.

TABLE III
FORCE RANGES OF DIFFERENT COMB DRIVES

	force profile	force range (nN / volt^2 × μm)
Standard comb drive	Constant	0.0059
Comb drive with uniform moving fingers	Linear	(0.002, 0.006)
	Quadratic	(0.0015, 0.006)
	Cubic	(0.0010, 0.0063)
Comb drive with variable moving finger	Linear	(0.0042, 0.011)
	Quadratic	(0.0033, 0.014)
	Cubic	(0.0023, 0.013)

The optimizer uses the function F and sensitivity $\overset{*}{F}$ [(6), (7)] from the comb drive simulations that have been presented in this paper. The second derivatives are approximated in "dlcong" by the Broyden, Fletcher, Goldfarb, and Shanno (BFGS) formula, developed by Broyden *et al*. Three designs are shown in Fig. 7, along with their force profiles. All the shape parameters and the expressions for the driving forces are given in Table I. These three, plus the usual one with constant gap, serve as the fundamental comb drives.

A problem for the above designs, however, is that they produce relatively small driving forces at the maximum gap, especially for the cubic motor. Since the moving finger is very flexible, it becomes unstable for large voltages. In order to overcome this drawback, the moving finger is also shaped. The shape of the moving finger is chosen such that when it is fully inserted, the gap profile becomes uniform. The shape parameters for the fixed fingers are shown in Table II. The choice of those parameters is guided by consideration of the minimum gap and maximum opening of the comb drive. For example, the linear motor in Table II has a minimum gap of 1.5 μm and a maximum opening of 3 μm. The resultant comb drives have several advantages. First, the maximum driving force increases dramatically. Second, the structure is more stable and therefore large voltages can be applied. Finally, the area occupied by a comb drive is much smaller. The new designs with variable widths of both the fixed and the moving fingers, and their force profiles, are shown in Fig. 8. Table III gives a comparison of a straight standard comb drive with the two kinds of variable shape. The minimum gap in each comb drive is 1.5 μm, and the maximum displacement of the moving fingers is 20 μm. These fundamental comb drives can be arranged in parallel, with suitable bias voltages applied to each comb drive, in order to obtain any desired polynomial (up to cubic) force profile.

IV. CONCLUSIONS

A comb actuator is a basic actuation device of MEMS. The range of operation of an usual comb drive is limited by its nonlinear restoring spring force. It is shown in this paper that by solving an appropriate inverse problem, comb drives with variable-gap profiles can be designed that will deliver desired driving force profiles. In principle, therefore, nonlinear (e.g., cubic) spring forces can be cancelled by appropriate nonlinear driving forces, thereby greatly increasing the range of operation of a comb actuator. Also, tuning of the linear or nonlinear stiffness coefficients can be carried out conveniently with shape motors.

Transverse stability of the actuator-suspension structure is another important issue in comb drive design. The designs proposed in Fig. 8 are more stable than standard comb drives. Formal inclusion of stability criteria in the optimal design process is recommended for future research.

Fabrication of such comb actuators is currently underway at the Cornell Nanofabrication Facility.

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