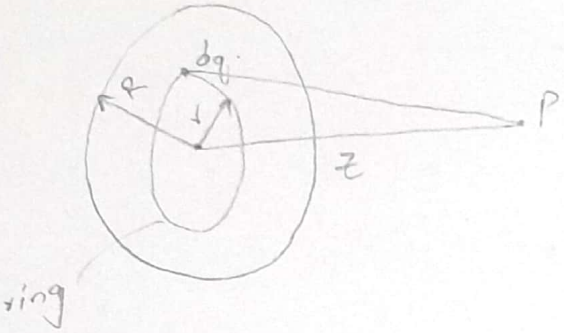
Electric field due To disk of charges:



where
r is the radius of ring
R is the radius of disc

Electric field due To ring of charges is

$$E_{ring} = \frac{kqz}{(z^2+R^2)^{3/2}}$$

for the above figure expression can be written as

$$E_{ring} = \frac{kdqz}{(z^2+r^2)^{3/2}} \longrightarrow (A)$$

Surface charge density is given by

$$\sigma = \frac{dq}{dA} \quad \text{OR} \quad dq = \sigma . dA \longrightarrow (1)$$

Area of circle is given by

$$A = \pi r^2$$

differentiate both sides.

$$dA = \frac{d}{dr}(\pi r^2).$$

$$dA = 2\pi r\, dr.$$

Substituting in equation (1)

$$dq = \sigma . 2\pi r dr$$

Subsituting the values of dq in equation (A)

$$d\epsilon = \frac{k \cdot \sigma 2\pi r dr \cdot z}{(z^2+r^2)^{3/2}}$$

Integrating both sides:

$$\int d\epsilon = \int_0^R \frac{k\sigma 2\pi r dr\, z}{(z^2+r^2)^{3/2}}$$

$$\epsilon = k \cdot \sigma 2\pi z \int_0^R \frac{r\,dr}{(z^2+r^2)^{3/2}} \longrightarrow (2)$$

$$\int_0^R \frac{r\,dr}{(z^2+r^2)^{3/2}}$$ (we use the subsilution method)

Let

$$z^2+r^2 = u$$

differentiating both sides.

$$du = 2r\,dr \Rightarrow \frac{du}{2} = r\,dr$$

So $$\int_0^R \frac{\frac{du}{2}}{(u)^{3/2}} \Rightarrow \frac{1}{2}\int_0^R u^{-3/2}\,du \Rightarrow \frac{1}{2}\int_0^R \frac{u^{-3/2+1}}{-\frac{3}{2}+1} \Rightarrow \frac{1}{2}\left/ \frac{u^{-1/2}}{-1/2}\right|_0^R$$

$$= -\frac{1}{2}\left[\frac{1}{u^{1/2}}\right]_0^R$$

So equation (2) becomes

$$\epsilon = k\sigma 2\pi z \left[-\frac{1}{u^{1/2}}\right]_0^R$$

Subsituting values of u

$$\epsilon_{disc} = k\sigma 2\pi z \left\{-\left[\frac{1}{(z^2+r^2)^{1/2}} - \frac{1}{(z^2+r^2)^{1/2}}\right]_0^R\right\}.$$

$$\epsilon_{disc} = k\sigma 2\pi z \left\{\frac{1}{z} - \frac{1}{(z^2+R^2)^{1/2}}\right] \Rightarrow$$

$$\epsilon = \frac{1}{4\pi\epsilon_0}\sigma 2\pi\left[\frac{z}{z} - \frac{z}{(z^2+R^2)^{1/2}}\right]$$

$$\boxed{\epsilon_{disc} = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]}$$

A disc of radius 3m contains a total charge of 480 nC.

a) what is the charge per unit area?

b) what is the electric field 25cm away from centre of disc?

Sol:- (a)

$$\sigma = \frac{Q}{A}$$

$$\sigma = \frac{Q}{\pi R^2}$$

$$\sigma = \frac{480 \times 10^{-9}}{\pi \times (3)^2}$$

$$\sigma = 16.97 \times 10^{-9} \ C/m^2$$

(b)

$$E = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$$

$$= \frac{16.97 \times 10^{-9}}{2 \times 8.85 \times 10^{-12}}\left[1 - \frac{0.25}{\sqrt{(0.25)^2 + 3^2}}\right]$$

$$= 880 \ N/C$$