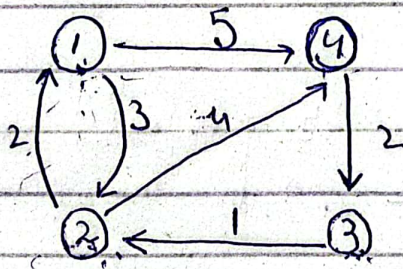


# Analysis of Algorithm.

## Dynamic programming.

- Divide & conquer vs Dynamic programming.
- Optimize solution.
- Dynamic programming Algorithms.
  - Floyd Warshall Algorithm.
  - Fibonacci series.
  - Binomial coefficient.
  - String Method.
  - Matrix chain Multiplication.

## \* Floyd Warshall Algorithm.



	1	2	3	4
1	0	3	$\infty$	5
2	2	0	$\infty$	4
3	$\infty$	1	0	$\infty$
4	$\infty$	$\infty$	2	0

In matrix 2 Row 1 of  $A_0$  & column 1 will become

	1	2	3	4
1	0	3	$\infty$	5
1,2	2	0	4	4
3	$\infty$	1	0	$\infty$
4	$\infty$	$\infty$	2	0

We choose the path  
passing through 1 or direct  
& choose minimum.



$$A_2 \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & 9 & 5 \\ 2 & 0 & 9 & 4 \\ 3 & 1 & 0 & 5 \\ \infty & \infty & 2 & 0 \end{bmatrix} \end{matrix}$$

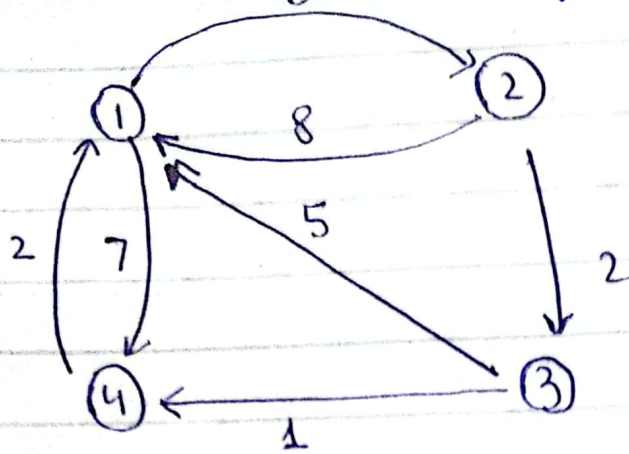
$$A_3 \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & 9 & 5 \\ 2 & 0 & 9 & 4 \\ 3 & 1 & 0 & 5 \\ 5 & 3 & 2 & 0 \end{bmatrix} \end{matrix}$$

$$A_4 \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & 7 & 5 \\ 2 & 0 & 6 & 4 \\ 3 & 1 & 0 & 5 \\ 5 & 3 & 2 & 0 \end{bmatrix} \end{matrix}$$

using previous matrix in last matrix if there is no direct or indirect path



Task:



Find shortest path

$$A_0 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 8 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix} \end{matrix}$$

$$A_1 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & 7 & 0 \end{bmatrix} \end{matrix}$$

$$A_2 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & 5 & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & \infty & 0 \end{bmatrix} \end{matrix}$$

$$A_3 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3 & 5 & 7 \\ 7 & 0 & 2 & 3 \\ 5 & 8 & 0 & 1 \\ 2 & \infty & \infty & 6 \end{bmatrix} \end{matrix}$$

$$A_4 = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & 3 \\ 3 & 7 & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix}$$



# Dynamic programming.

- Floyd Warshall algorithm.
- Fibonacci series.
- Binomial coefficient.

- Concept.
- Mathematical procedure.
- Binomial coefficient and Dynamic programming
- Example & exercise.

## Binomial coefficient.

A binomial coefficient is an algorithm expression of the form  $a+b$  that we multiply or.

$$(a+b)^2 : a^2 + 2ab + b^2$$

$$(a+b)^3 : a^3 + 3a^2b + 3ab^2 + b^3$$

The Mathematical formula for Binomial

coefficient is.

$$C[n, k] = \frac{n!}{k!(n-k)!}$$

## 5C3 Using Dynamic programming.

n \ r →	0	1	2	3
0	1	X		
1	1	X	X	
2	1	2	1	X
3	1	3	3	1
4	1	4	6	4
5	1	5	10	10

$$\# C[i, j] = C[i-1, j] + C[i-1, j-1]$$

$$\# i=0 \text{ to } n$$

$$j=0 \text{ to } \min(i, r)$$

∴ whenever  $r > n$  place X there.

∴ whenever  $r \leq n$  blank the place.



# Dynamic programming:

→ Matrix chain Multiplication.

## Greedy algorithm:

Speed check

altern

- Concept
- Optimized solution.
- Application.

→ Kruskal's algorithm.

→ Greedy algorithm.

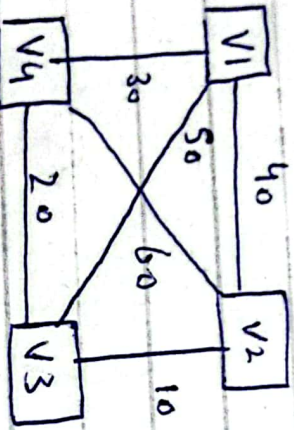
→ Prim's Algorithm

13-5-23

→ Finding MST using

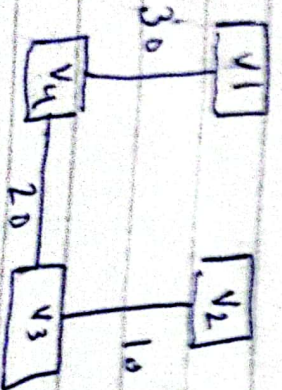
Kruskal Algorithm.

→ Example & exercises



Edges	weight.
(V2, V3)	10
(V3, V4)	20
(V1, V4)	30
(V1, V2)	40
(V1, V3)	50
(V2, V4)	60

Action	UV = { {V1}, {V2}, {V3}, {V4} }
Added	{ {V1}, {V2, V3}, {V4} }
Added	{ {V1}, {V2, V3, V4} }
Added	{ {V1, V2, V3, V4} }
Reject	Already added
Reject	Already added
Reject	Already added

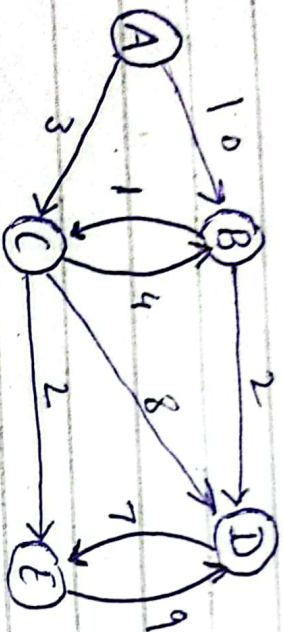


$$10 + 30 + 20 = 60$$



# Dijkstra Algorithm:-

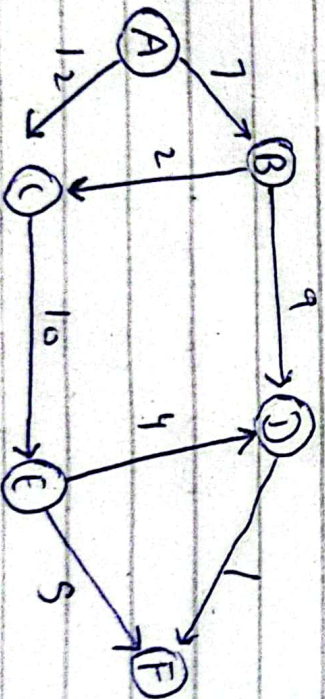
- Concept
- Shortest Path finder
- Example & exercise



$S = \{A, C, E, B, D\}$

Q	A	B	C	D	E
1	<div><div>∞</div><div>10</div></div>	<div><div>∞</div><div>3</div></div>	<div><div>∞</div><div>∞</div></div>	<div><div>∞</div><div>∞</div></div>	<div><div>∞</div><div>∞</div></div>
2	<div><div>7</div><div>7</div></div>	<div><div>11</div><div>11</div></div>	<div><div>11</div><div>11</div></div>	<div><div>11</div><div>11</div></div>	<div><div>11</div><div>11</div></div>

Example.



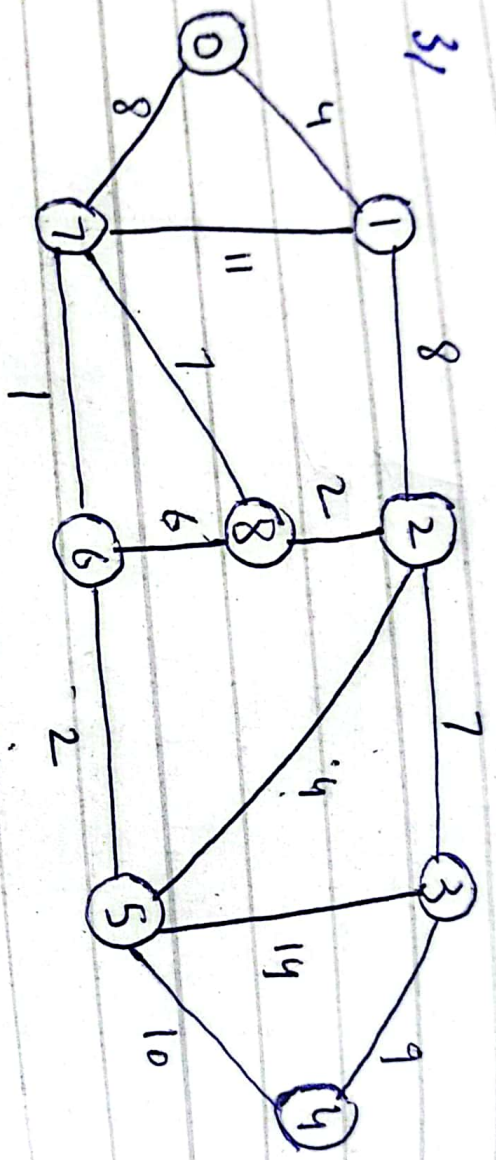
$S = \{A, B, C\}$

Q	A	B	C	D	E	F
1	<div><div>10</div><div>10</div></div>	<div><div>∞</div><div>∞</div></div>	<div><div>∞</div><div>∞</div></div>	<div><div>∞</div><div>∞</div></div>	<div><div>∞</div><div>∞</div></div>	<div><div>∞</div><div>∞</div></div>
2	<div><div>17</div><div>17</div></div>	<div><div>12</div><div>12</div></div>	<div><div>12</div><div>12</div></div>	<div><div>∞</div><div>∞</div></div>	<div><div>∞</div><div>∞</div></div>	<div><div>∞</div><div>∞</div></div>
3	<div><div>19</div><div>19</div></div>	<div><div>12</div><div>12</div></div>	<div><div>12</div><div>12</div></div>	<div><div>16</div><div>16</div></div>	<div><div>∞</div><div>∞</div></div>	<div><div>∞</div><div>∞</div></div>





Example 3/



$S = \{0, 1, 7, 6, 5, 2, 8, 3, 4\}$

Q	0	1	2	3	4	5	6	7	8
0	0	8	8	8	8	8	8	8	8
1	8	0	8	8	8	8	8	8	8
2	8	8	0	8	8	8	8	8	8
3	8	8	8	0	8	8	8	8	8
4	8	8	8	8	0	8	8	8	8
5	8	8	8	8	8	0	8	8	8
6	8	8	8	8	8	8	0	8	8
7	8	8	8	8	8	8	8	0	8
8	8	8	8	8	8	8	8	8	0