MATH 3353-001/2, Fall 2016
Lab # 7: **Extra Credit: DUE 12/10/16, 11:59 pm**

To receive credit for this lab, you must turn in your diary that demonstrates you running these commands. Please name it as follows: `<Your_last_name>_lab7_diary.txt` (for example, `Barreiro_lab7_diary.txt`). There will be a place on Canvas for you to submit your file. I recommend that you save your commands in a `.m` file (for example, `myscript.m`): this will allow you to work out any mistakes before you run it and save your diary. Then run:

```
>> diary Barreiro_lab7_diary.txt
>> myscript
>> diary off
```

and you're done! **Don't forget to erase your diary before you do your submission run**

In this lab, we will use least-squares approximation on real data.

Background: The basic cell in your nervous system is called a *neuron*. By controlling the flow of ions (charged particles) across its membrane, neurons maintain a voltage potential across their cell membranes of about -70 mV at rest. This transmembrane voltage potential is a primary observable in experiments, and will be affected by electrical currents from neighboring cells (or the experimenter!). When this potential difference gets close enough to 0 (maybe ≈ -50 mV, depending on the specific cell type), a rapid deflection of the potential occurs called a *spike* or *action potential*: this causes the neuron to communicate to other neurons.

One quantitative model of this process proposes that the behavior of V during the build-up to a spike can be modeled by a differential equation

$$\frac{dV}{dt} = F(V) + I \tag{1}$$

The fact that $F(V)$ is a function of voltage reflects the fact that the excitability of the cell membrane is governed by membrane ion channels whose probability of being open depends on the voltage itself. $I(t)$ is the applied current and is a function of time.

Suppose that we have a neuron with unknown electrical properties. However, we can perform an experiment in which we measure the voltage, $V(t)$, in response to an applied current $I(t)$. With that information, we can compute $\frac{dV}{dt}$ and rewrite our expression above as:

$$\frac{dV}{dt} - I = F(V) \tag{2}$$

The left hand side is known ($\frac{dV}{dt}$ is measured from the experiment, $I(t)$ was determined by the experimenter): our task is to determine the function $F(V)$.

But how do we choose out of all possible functions $F(V)$?? Least-squares fitting (covered in §6.5 of our text) offers a powerful method to find the $F(V)$ that best fits the data, out of a set of candidate functions. Such a set of functions must be expressed as a linear combination of known quantities: for example, when we looked for a best-fit line, we looked at functions of the form $F(V) = C + DV$.

1. Download the data file, `lab7_data.mat`, to Matlab's working directory and read it into Matlab using the `load` command:

```
>>  load lab7_data.mat
```

Use the `whos` command to examine your workspace: you should see two vectors, `V` and `Fv`. The vector `V` contains voltage values, in units of mV; the vector `Fv` contains the left-hand side of Eqn. (2) that were measured at the same times. **Use the `size` command to print the dimensions of `V` and `Fv` to your diary.**

2. Use the normal equations ($A^T A\hat{\mathbf{x}} = A^T \mathbf{b}$) to fit the data to a line: i.e. find C, D such that

$$F(V) \;=\; C + DV \tag{3}$$

Print C and D to your diary.

3. Your fitted curve, $C + DV$, will not exactly go through all of the data points. Report the average error as follows:

```
>>  E_Linear = sum((Fv - yLin).^2)/M      % This computes the error defined in 6.5,
>>                                       % then divides by the number of data points
```

Here `yLin` is the vector of points that lie along the line you fit; if your over-determined linear system is $A\mathbf{x} = \mathbf{b}$, then `yLin` $= \mathrm{proj}_{\mathrm{Col}\,A}\mathbf{b}$. M is the total number of data points.
Print `E_Linear` to your diary.

4. The *quadratic integrate-and-fire* equation is a simplified model of how the membrane potential responds to electrical currents:

$$F(V) \;=\; a(V - V_{rest})(V - V_{thr}) \tag{4}$$

where V_{rest} is a resting voltage and V_{thr} is the threshold voltage (if $I = 0$ and $V > V_{thr}$, then V will continue to increase very rapidly: which we interpret as a spike). V_{thr}, V_{rest}, and a are constants.

The left hand side is known (measured from the experiment): our task is to determine the constants a, V_{rest}, and V_{thr}.

(a) Use the normal equations ($A^T A\hat{\mathbf{x}} = A^T \mathbf{b}$) to fit the data to a parabola: i.e. find C, D and E s.t.

$$F(V) \;\approx\; C + DV + EV^2 \tag{5}$$

Print C, D and E to your diary.

(b) Solve for a, V_{thr}, and V_{rest} in terms of C, D, and E. That is, find coefficients to equate the two polynomials:

$$C + DV + EV^2 = a(V - V_{rest})(V - V_{thr})$$

Print a, V_{thr} and V_{rest} to your diary.

(c) Your fitted curve, $a(V - V_{rest})(V - V_{thr})$, will not exactly go through all of the data points. Report the average error as follows:

```
>>  E_Quad = sum((Fv - yQuad).^2)/M
```

Here `yQuad` is the vector of points that lie along the parabola you fit, and M is the total number of data points.
Print `E_Quad` to your diary.

5. Maybe we could do better with a different polynomial approximation. Use least-squares to fit the data to a cubic function:

$$F(V) \approx C + DV + EV^2 + GV^3 \tag{6}$$

Print C, D, E and G to your diary. Report the average error, as above, named `E_Cubic`. Did you get any improvement?

6. **Suggested:** You may want to look at the data using the `plot` command:

```
>>   plot(V, Fv,'.');
>>   axis([-80,-30,-10,5]);        % to choose the scale of your plot
```

Does it look like this should be well-modeled by a quadratic function?

You can also plot your best-fit curves to see how they compare; follow the previous command with

```
>>   hold on;                  % To plot multiple curves on the same axis
>>   plot(V, yLin,'c.');       % Use a different color (see "help plot" for options)
```

and/or

```
>>   hold on;                  % To plot multiple curves on the same axis
>>   plot(V, yQuad,'y.');      % Use a different color (see "help plot" for options)
```