Avon High School
AP Calculus AB

Name _____

AP Calculus AB – Semester I Review
Practice Packet #2

Calculators should only be used when you see a "C" next to the problem number.

1) $\lim_{x\to -5}\dfrac{2x+10}{x^2+2x-15}$ is

a) 0 b) $\dfrac{-1}{8}$ c) $\dfrac{-1}{4}$ d) ∞ e) none of these

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2) $\lim_{x\to 1}\dfrac{x^3-1}{x^2-1}$ is

a) $\dfrac{3}{2}$ b) $\dfrac{1}{2}$ c) 0 d) ∞ e) DNE

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3) $\lim_{x\to 4}\dfrac{-3x+1}{(x-4)^2}$ is

a) -11 b) -13 c) ∞ d) $-\infty$ e) DNE

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4) Let $f(x)=\begin{cases}\dfrac{4-x^2}{x-2} & \text{if } x \neq 2\\ 4 & \text{if } x = 2\end{cases}$ Which of the following statements I,II, and III are true?

I. $\lim_{x\to 2} f(x)$ exists II. $f(2)$ exists III. f is continuous at $x = 2$

a) only I b) only II c) I and II d) none of them e) all of them

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5) If $\begin{cases}f(x)=\dfrac{x^2-6x}{x} & \text{for } x \neq 0\\ f(0)=2k-1\end{cases}$

and if f is continuous at $x = 0$, then k =

a) -6 b) $\dfrac{-5}{2}$ c) 0 d) $\dfrac{5}{2}$ e) 6

6) If $f(x) = x^2 - 1$ and $g(x) = \frac{1}{x}$, which of the following statements is FALSE?

a) $f(x)$ is continuous everywhere
b) $g(x)$ is continuous except at $x = 0$
c) $f(g(x))$ is continuous except at $x = 0$
d) $g(f(x))$ is continuous except at $x = 1$
e) All are true

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7) The normal (perpendicular) line to the curve $y = \sqrt{8 - x^2}$ at (-2, 2) has slope

a) -2 b) $\frac{1}{2}$ c) $-\frac{1}{2}$ d) 1 e) -1

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8) If $f(x) = \frac{256}{\sqrt{x}} + 64\sqrt{x} + 3\sqrt[3]{x^2}$, then $f'(64) =$

a) 4.25 b) 8.75 c) 0.75 d) 10.25 e) 5.78

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9) If the graph of the second derivative of some function f is a line of slope -6, then f could be which type of function?

a) constant b) linear c) quadratic d) cubic e) quartic

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10) If $f(x) = \sin^2(3 - x)$ then $f'(0) =$

a) -2 cos 3 b) -2 sin 3 cos 3 c) 6 cos 3 d) 2 sin 3 cos 3 e) 6 sin 3 cos 3

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11) If $f(5) = 3$ and $f'(5) = -2$, find the derivative of $x^2 f(x)$ at $x = 5$.

a) 0 b) -18 c) -12 d) -20 e) -80

12) $y = -\dfrac{1}{\sqrt{x^2+1}}$, then $\dfrac{dy}{dx} =$

a) $\dfrac{x}{\left(x^2+1\right)^{1/2}}$ b) $\dfrac{x}{\left(x^2+1\right)^{3/2}}$ c) $\dfrac{-x}{\left(x^2+1\right)^{1/2}}$

d) $\dfrac{-x}{\left(x^2+1\right)^{3/2}}$ e) $\dfrac{x}{x^2+1}$

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13) If $y = \dfrac{3}{\sin x + \cos x}$, find $\dfrac{dy}{dx} =$

a) $3\sin x - 3\cos x$ b) $\dfrac{3}{\left(\sin x + \cos x\right)^2}$ c) $\dfrac{-3}{\left(\sin x + \cos x\right)^2}$

d) $\dfrac{3\left(\cos x - \sin x\right)}{\left(\sin x + \cos x\right)^2}$ e) $\dfrac{3\left(\sin x - \cos x\right)}{1 + 2\sin x \cos x}$

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14) If $x^2 + xy - y = 2$, find $\dfrac{dy}{dx}$

a) $\dfrac{2x+y}{1-x}$ b) $\dfrac{2x}{1-x}$ c) $\dfrac{2x-2}{1-x}$ d) $\dfrac{2-2x}{x}$ e) DNE

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15) If $\sin y = \cos x$, then find $\dfrac{dy}{dx}$ at the point $\left(\dfrac{\pi}{2}, \pi\right)$

a) -1 b) 0 c) 1 d) $\dfrac{\pi}{2}$ e) None of these

16) If $f(x) = x^3 - x$, then

a) $x = \frac{\sqrt{3}}{3}$ locates a relative maximum of f

b) $x = \frac{\sqrt{3}}{3}$ locates a relative minimum of f

c) $x = \sqrt{3}$ locates a relative maximum of f

d) $x = \sqrt{3}$ locates a relative minimum of f

e) $x = -\sqrt{3}$ locates a relative minimum of f

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17) $f(x) = x^n$, where n is a positive integer ≥ 2. The graph of $f(x)$ will have an inflection point when n is

a) even b) odd c) divisible by 3 d) for all values e) for no values

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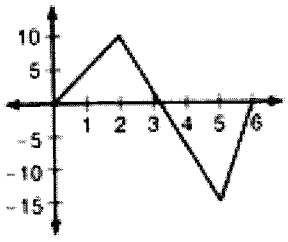
18) A particle is subjected to gravity according to the following graph of the particle's velocity.



At what time is the particle at its highest point?

a) 0 b) 2 c) 3 d) 5 e) 6

In questions 19-20, a particle moves along a horizontal line according to the formula

$$s = 2t^4 - 4t^3 + 2t^2 - 1$$

19) The particle is moving right when

a) $0 < t < \frac{1}{2}$ b) $t > 0$ c) $t > 1$ d) $0 < t < \frac{1}{2}, t > 1$ e) never

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C 20) The acceleration, a is increasing when

a) $t > 1$ b) $t > 0.5$ c) $t < 0.211$ or $t > 0.789$ d) $0 < t < 0.5$ e) $0 < t < 1$

21) The circumference of a circle is increasing at a rate of $\frac{2\pi}{5}$ inches per minute. When the radius is 5 inches, how fast is the area of the circle increasing in square inches per minute?

a) $\frac{1}{5}$ b) $\frac{\pi}{5}$ c) 2 d) 2π e) 25π

C 22) A conical tank has a height that is always 3 times its radius. If water is leaving the tank at the rate of 50 cubic feet per minute, how fast is the water level falling in feet per minute when the water is 3 feet high? Volume of a cone is $V = \frac{1}{3}\pi r^2 h$.

a) 1.000 b) 5.305 c) 15.915 d) 0.589 e) 1.768

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23) A function $f(x)$ is continuous for all x and has a local minimum at (1, 8). Which must be true?

a) $f'(2) = 0$ b) f' exists at $x = 2$ c) the graph is concave down at $x = 1$

d) $f'(x) < 0$ if $x < 1, f'(x) > 0$ if $x > 1$ e) $f'(x) > 0$ if $x < 1$, $f'(x) < 0$ if $x > 1$

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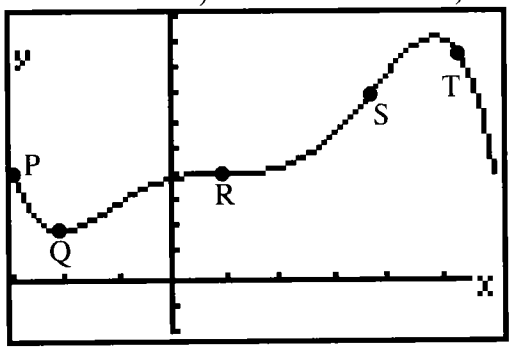
24) At what point on the graph $y = f(x)$ below are both $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ both less than zero?

a) P b) Q c) R d) S e) T



25) If $\lim_{x\to\infty}\frac{6x^2}{200-4x-kx^2}=\frac{1}{2}$, then $k=$

a) 3 b) -3 c. 12 d. -12 e. -3

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26) For the following piecewise function, determine the answer that best describes it:

$$f(x)=\begin{cases}x-\cos(x), x\geq 0\\ x^2+x-1, x<0\end{cases}$$

a) both continuous and differentiable b) neither continuous nor differentiable c) continuous only

d) differentiable only e) has a cusp point at $x=0$

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27) Let $f(x)$ be a polynomial function such that $f(3)=3$, $f'(3)=0$, and $f''(3)=-3$. What is the point (3,3) on the graph $y=f(x)$?

a) relative maximum b) relative minimum c) intercept

d) inflection point e) none of these

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28) Given that $f(x)=4+\frac{3}{x}$ find all values of c in the interval (1,3) that satisfy the mean value theorem.

a) 2 b) $\sqrt{2}$ c) $\sqrt{3}$ d) $\pm\sqrt{3}$ e) MVT doesn't apply

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29) $\int\left(x^2-4\sec x\tan x\right)dx=$

a) $2x-4\tan x+C$ b) $\frac{x^2}{3}-4\tan x+C$ c) $\frac{x^2}{3}-4\sec^2 x+C$

d) $\frac{x^2}{3}-4\sec x+C$ e) none of these

30) An equation of the line tangent to $y = \sin x + 2\cos x$ at $\left(\frac{\pi}{2}, 1\right)$ is

a) $2x - y = \pi - 1$ b) $2x + y = \pi + 1$ c) $2x - 2y = 2 - \pi$
d) $4x + 2y = 2 - \pi$ e) none of these

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31) $\int \sqrt{x}\left(\sqrt{x} + 1\right) dx$

a) $2\left(x^{3/2} + x\right) + C$ b) $\frac{x^2}{2} + x + C$ c) $\frac{1}{2}\left(\sqrt{x} + 1\right)^2 + C$
d) $\frac{x^2}{2} + \frac{2x^{3/2}}{3} + C$ e) $x + 2\sqrt{x} + C$

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32) For $f(x) = x^{\frac{2}{3}}\left(x^2 - 4\right)$ on [-2 2] the "c" value that satisfies Rolle's Theorem is

a) 0 b) 2 c) ±2

d) There is no value for c because $f(0)$ does not exist
e) There is no value for c because $f(x)$ is not differentiable on (-2,2)

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33) If $f(x)$ is a continuous function with $f''(x) = -5x^2(2x-1)^2(3x+1)^3$, find the set of values of x for which $f(x)$ has an inflection point.

a) $\left\{0, -\frac{1}{3}, \frac{1}{2}\right\}$ b) $\left\{-\frac{1}{3}, \frac{1}{2}\right\}$ c) $\left\{-\frac{1}{3}\right\}$ d) $\left\{\frac{1}{2}\right\}$ e) no inflection points

34) The smallest slope of $f(x) = 6x^2 - x^3$ for $0 \le x \le 6$ occurs at $x =$

a) 0 b) 2 c) 3 d) 4 e) 6

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35) Find the equation of the line tangent to $y = \tan 2x$ at $x = \frac{\pi}{8}$.

a) $y - 1 = \sqrt{2}\left(x - \frac{\pi}{8}\right)$ b) $y - 1 = \frac{1}{2}\left(x - \frac{\pi}{8}\right)$ c) $y - 1 = \frac{1}{4}\left(x - \frac{\pi}{8}\right)$

d) $y - 1 = 2\left(x - \frac{\pi}{8}\right)$ e. $y - 1 = 4\left(x - \frac{\pi}{8}\right)$

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36) Given $f(x) = x^4(2x^2 - 15)$. On what interval(s) is the graph of f concave upwards?

a) $\left(0, \sqrt{3}\right)$ b) $\left(-\sqrt{3}, 0\right)$ c) $\left(-\sqrt{3}, 0\right) \cup \left(0, \sqrt{3}\right)$

d) $\left(-\sqrt{3}, \sqrt{3}\right)$ e) $\left(-\infty, -\sqrt{3}\right) \cup \left(\sqrt{3}, \infty\right)$

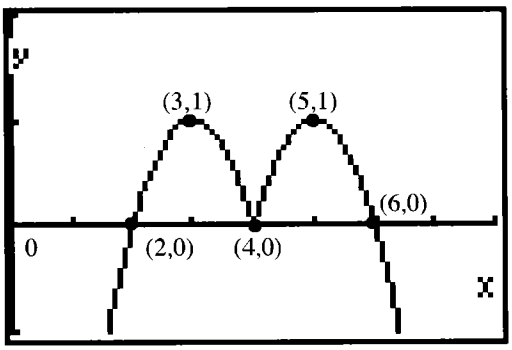
C 37) The graph of the function $y = x^5 - x^2 + \sin x$ has a point of inflection at $x =$

a) 0.324 b) 0.499 c) 0.506 d) 0.611 e) 0.704

C 38) If $f'(x) = 2(3x + 5)^4$, then the fifth derivative of $f(x)$ at $x = \frac{-5}{3}$ is

a) 0 b) 144 c) 1,296 d) 3,888 e) 7,776

Problems 39 and 40 refer to the graph below.



39) The figure shows the graph of $f'(x)$, the derivative of the function f. The domain of f is $0 \le x \le 8$. For what value(s) of x does the function have a relative minimum?

a) 2 b) 4 c) 6 d) 2 and 6 e) 3 and 5

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40) In what interval(s) is the graph of f concave up?

a) (3, 5) b) (2, 6) c) $[0,3) \cup (4,5)$ d) always e) never

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41) If $f'(x) = \sin x$ and $f(\pi) = 3$, then $f(x) =$

a) $\cos x + 4$ b) $\cos x + 3$ c) $-\cos x + 2$ d) $-\cos x - 2$ e) $-\cos x + 4$

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42) Let f be a function such that $\lim_{h \to 0} \frac{f(5+h) - f(5)}{h} = 4$. Which of the following must be true?

I. $f(5) = 4$

II. $f'(5) = 4$

III. f is continuous at $x = 5$

a) I only b) II only c) III only d) I and II only e) II and III only

C 43) If $y = 3x - 7, x > 0$, what is the minimum product of $x^2 y$?

a) -5.646 b) 0 c) 1.556 d) 2.813 e) 4.841

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44) Let f, g and their derivatives be defined by the table below. What is the derivative of $f\left(g\left(x\right)\right)$ at $x = 2$?

x	1	2	3	4
$f\left(x\right)$	3	0	1	2
$g\left(x\right)$	2	3	4	1
$f'\left(x\right)$	1	2	3	1
$g'\left(x\right)$	2	2	1	1

a) 6 b) 2 c) 8 d) 4 e) 0

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45) For how many values of x will the tangent lines to $y = 4\sin x$ and $y = \frac{x^2}{2}$ be parallel?

a) 0 b) 1 c) 3 d) 4 e) infinite