Avon High School **Name** _____
AP Calculus AB
SEMESTER 2
FINAL EXAM REVIEW

1. The function $G(x)=\begin{cases} x^2, & x<2 \\ 4-2x, & x>2 \end{cases}$ is not continuous at $x=2$ because:

A. $G(2)$ is not defined

B. $\lim_{x\to 2} G(x)$ does not exist

C. $\lim_{x\to 2} G(x) \neq G(2)$

D. All of the above

E. None of the above

2. $\lim_{x\to 0}\left(\frac{1}{x}+\frac{1}{x^2}\right)=$

3. $\lim_{x\to\infty}\frac{x^3-4x+1}{2x^3-5}=$

4. $\lim_{h\to 0}\frac{\arcsin(a+h)-\arcsin(a)}{h}=$

5. A ladder is 15 feet tall leans against a vertical wall of a home. If the bottom of the ladder is pulled away horizontally from the house at 4 $ft/\sec$, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 9 feet from the wall?

6. A cone (vertex down) with height 10 inches and radius 5 inches is being filled with water at a constant rate of 2 $in^3/\sec$. How fast is the surface of the water rising when the depth of the water is 6 inches?

7. Find the derivative of $y = e^{2x}$.

8. Find the derivative of $y = \ln(\sin x)$.

9. Find the derivative of $y = \dfrac{e^x - 1}{e^x + 1}$.

10. Find the derivative of $y = e^{\sin x}$.

11. Find the equation of the tangent line to the curve $\left(x^2 + y^2\right)^2 = 4x^2 y$ at the point $(1, 1)$.

12. Find the value of *c* guaranteed to exist by Rolle's Theorem for $f(x) = 2x^2 - 11x + 15$ in the interval $\left[\dfrac{5}{2}, 3\right]$.

13. Find the value of *c* guaranteed to exist by the Mean Value Theorem for $f(x) = (x-1)^3$ in the interval $[-1, 2]$.

14. $\int \left(\sec^2\theta - \sin\theta\right) d\theta =$

15. $\int \left(\cos 6x\right) dx =$

16. $\int_1^2 \left(\frac{3}{x^2} - 1\right) dx =$

17. $\int_{-1}^{1} x\left(x^2 + 1\right)\, dx =$

18. $\int \frac{e^{-x}}{1 + e^{-x}} dx =$

19. $\int 3^x\, dx =$

20. $\int_e^{e^2} \frac{1}{x \ln x} dx =$

21. Using the substitution $u = \sin(2x)$, $\int_{\pi/6}^{\pi/2} \sin^5(2x)\cos(2x)\,dx$ is equivalent to:

A. $-2\int_{1/2}^{1} u^5\,du$ **B.** $\frac{1}{2}\int_{1/2}^{1} u^5\,du$

C. $\frac{1}{2}\int_{0}^{\sqrt{3}/2} u^5\,du$ **D.** $\frac{1}{2}\int_{\sqrt{3}/2}^{0} u^5\,du$

E. $2\int_{\sqrt{3}/2}^{0} u^5\,du$

22. A particle moves along the *x*-axis with velocity given by $v(t) = 3t^2 - 4$ for time $t \geq 0$. If the particle is a position $x = -2$ at time $t = 0$, what is the position of the particle at time $t = 3$?

23. Find $\frac{dy}{dx}$ if $ye^x - x = y^2$.

24. Given $f(x) = x^3 + 2x + 1$, find $\left(f^{-1}\right)'(1)$.

25. Find the derivative of $y = \arctan\left(\frac{x}{2}\right)$.

26. Find the derivative of $y = arc\cot\left(\sqrt{x}\right)$.

27. Find the derivative of $y = \arccos(\cos x)$.

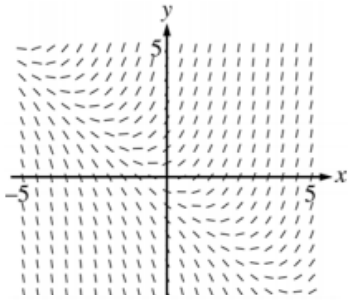
28. Find the derivative of $y = \ln(\arcsin x)$.

29. Find the particular solution to the differential equation $xyy' - \ln x = 0$ given the initial condition $y(1) = 0$.

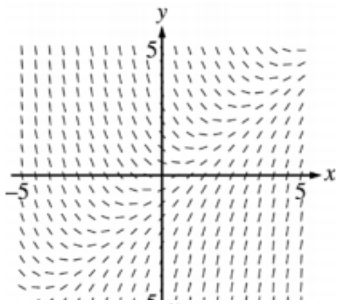
30. If $\frac{dy}{dx} = x + 2xy$ and $y(0) = 1$, find y as a function of x.

31. Which of the following is the slope field for the differential equation $y' = \frac{x}{y}$?

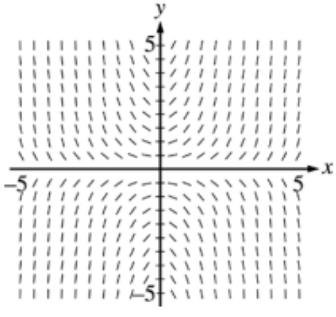
A.



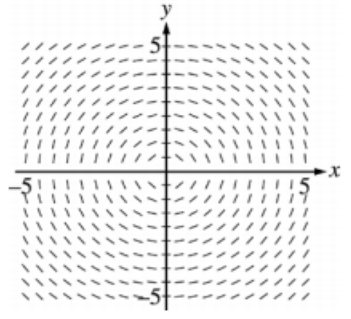
B.



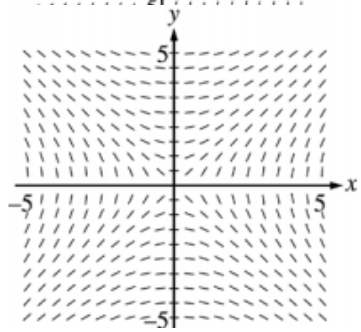
C.



D.



E.



32. Find the volume of the solid generated when the region in the first quadrant bounded by the graphs of $y = \cos x,\ y = \sin x$, the line $x = \frac{\pi}{4}$, and the y-axis is revolved about the x-axis.

33. Find the volume of the solid generated when the region in the first quadrant bounded by the graph of $y = (x+3)^3$, the line $x = 2$, and the x-axis is revolved about the line $y = -1$.

34. Let R be the region bounded by $y = e^x,\ y = 2,$ and $x = 0$. Find the volume of the solid whose base is bounded by the region R and the cross sections perpendicular to the x-axis are semicircles.

35. The base of a solid is the region in the first quadrant bounded by the line $y = -\frac{1}{2}x + 2$ and the coordinate axes. If every cross section perpendicular to the x-axis is a square, which of the following integrals represents volume of the solid?

A. $\int_0^2 \left[-\frac{1}{2}x+2\right] dx$ **B.** $\int_0^4 \left[-\frac{1}{2}x+2\right] dx$

C. $\int_0^2 \left[-\frac{1}{2}x+2\right]^2 dx$ **D.** $\int_0^4 \left[-\frac{1}{2}x+2\right]^2 dx$

E. None of these