MRS. G'S CALCULUS AB PRACTICE TEST!
SECTION I, Part A
Time- 60 minutes
Number of Questions- 30

NO CALCULATOR IS ALLOWED FOR THESE PROBLEMS

1. $\int_{-2}^{3}(x^2 + x - 3)dx =$

   (A) $\frac{5}{6}$ (B) $-\frac{5}{6}$ (C) $-\frac{1}{6}$ (D) $14\frac{1}{6}$

2. If $f(x) = e^{3x}(1 + x^2)$, then $f'(3) =$

   (A) $36e^9$ (B) $24e^9$ (C) $16e^9$ (D) $12e^9$

3. Let f be a differentiable function such that $f(2) = -7$ and $f'(2) = \frac{1}{5}$. What is the approximation for $f(2.1)$ found by using the line tangent to the graph of f at $x = 2$?

   (A) 6.98 (B) 7.02 (C) -6.98 (D) -7.02

4. Let g be defined by $g(x) = x^4 + 5x^3$. How many relative extrema does g have?

   (A) Zero (B) One (C) Two (D) Three

5. The velocity of a particle moving along the x-axis is given by $v(t) = x^2 - 2x$ for $t > 0$. What is the average velocity of the particle from the time $t = 1$ to the time $t = 4$?

   (A) 6 (B) 2 (C) 9 (D) 3

6. On a certain day, the rate at which money is deposited into an ATM is modelled by the function M, where $M(t)$ is measured in hundreds of dollars per hour and t is the number of hours since the bank opened. Using a trapezoidal sum with the three subintervals indicated by the data in the table, what is the approximate number of dollars (in hundreds of dollars) deposited in the first 9 hours since the center opened?

t (hours)	0	2	7	9
$M(t)$ (hundreds of dollars)	14	10	4	2

   (A) 58 (B) 65 (C) 29 (D) 44

7. What is the total area of the region between the curves $y = 3x^2 - 9$ and $y = -6x$ from $x = -3$ and $x = 1$?

   (A) 32 (B) 22 (C) 27 (D) 35

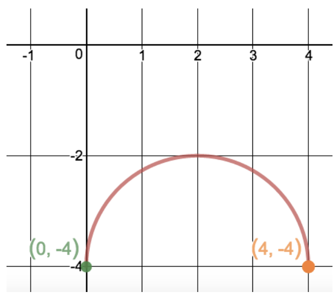
8. The function f is defined by $f(x) = x^2 + 3bx$, where b is a constant. If the line tangent to the graph of f at $x = -2$ is parallel to the line that contains the points (3,5) and (4,7), what is that value of b?

   (A) 6 (B) $\frac{3}{2}$ (C) 2 (D) $-\frac{1}{3}$

$$m(t) = \begin{cases} \frac{|t|}{t}, t \neq 0 \\ 0, t = 0 \end{cases}$$

9. The function m is defined above. The value of $\int_{-1}^{4} m(t)dt$ is
   (A) Nonexistent (B) 3 (C) 4 (D) -1

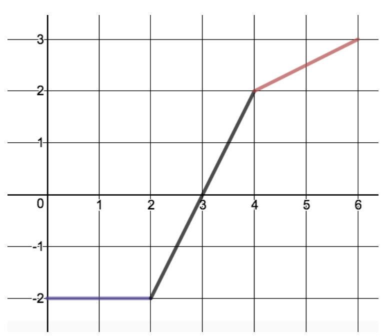
10. Let h be a continuous function. Using the substitution $u = 3x + 5$, the integral $\int_0^4 h(3x + 5)dx$ is equal to which of the following?
   (A) $\int_0^4 h(u)du$ (B) $\frac{1}{3}\int_0^4 h(u)du$ (C) $\int_5^{17} h(u)du$ (D) $\frac{1}{3}\int_5^{17} h(u)du$



11. The graph of $y = h(x)$ consists of a semicirclc with endpoints (0,-4) and (4,-4), as shown in the figure above. What is the value of $\int_0^4 h(x)dx$?
(A) $16 + 2\pi$ (B) $-16 + 2\pi$ (C) $16 - 2\pi$ (D) -2π

12. An object moves along a straight line so that at any time its acceleration is given by $a(t) = 4t$. At time t=0 the object's velocity is 12 and the object's position is 0. What is the object's position at time t=3?
(A) 18 (B) 36 (C) 63 (D) 54

13. If $y = sinx - \ln(3x)$, then $\frac{d^3y}{dx^3} =$
(A) $-\cos x - \frac{2}{x^3}$
(B) $\cos x - \frac{2}{x^3}$
(C) $-\cos x - \frac{1}{x^3}$
(D) $\cos x - \frac{1}{x^3}$



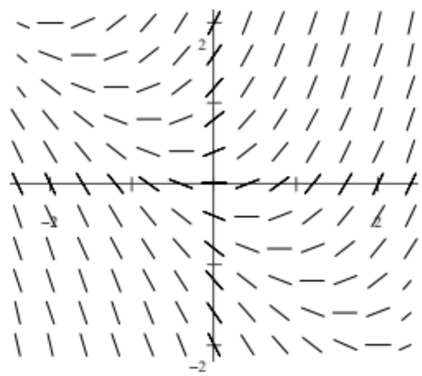
14. The graph of the function y, shown above, consists of three line segments. If the function k is the antiderivative of y such that $k(3) = 3$, for how many values of c, where $0 < c < 6$, does $k(c) = 4$?
   (A) 1 (B) 2 (C) 3 (D) 4

$$g(x) = \begin{cases} 3 + cx, x < 2 \\ 4\ln(x-1) - 2, x \geq 2 \end{cases}$$

15. Let g be the function defined above, where c is a constant. If g is continuous at $x = 2$, what is the value of c?

(A) $-\frac{5}{2}$ (B) 3 (C) -3 (D) $\frac{5}{2}$

16.



Shown above is the slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = 1 + x$ (B) $\frac{dy}{dx} = x^2$ (C) $\frac{dy}{dx} = x + y$ (D) $\frac{dy}{dx} = \frac{x}{y}$ (E) $\frac{dy}{dx} = \ln y$

17. The function $y = e^{7x} - 3x + 1$ is a solution to which of the following differential equations?

(A) $y'' + 7y' - 21 = 0$
(B) $y'' + y' + 21 = 0$
(C) $y'' - 7y' + 21 = 0$
(D) $y'' - 7y' - 21 = 0$

18. If $f(x) = sin^{-1}x$, then $f'(1) =$

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{\pi}{2}$ (D) undefined

19. $\lim_{x\to\pi} \frac{(x^5 - \ln x) - (\pi^5 - \ln \pi)}{x - \pi}$ is

(A) $5\pi^4 - \frac{1}{\pi}$ (B) 0 (C) $\pi^5 - \ln \pi$ (D) nonexistent

20. Let $y = f(x)$ be a twice-differentiable function such that $f(2) = -1$ and $\frac{dy}{dx} = y^4 + 5y$. Find $\frac{d^2y}{dx^2}$ at $x = 2$.

(A) 36 (B) -4 (C) 4 (D) 1

x	-1	1	3
$f(x)$	-.25	10	-1
$f'(x)$	7	0.5	0
$g(x)$	1	3	1
$g'(x)$	5	-2	2

21. The table above gives values of $f, f', f,$ and g' for selected values of x. If $d(x) = g(f(x))$, what is the value of $d'(3)$?

(A) 5 (B) 1 (C) 7 (D) 0

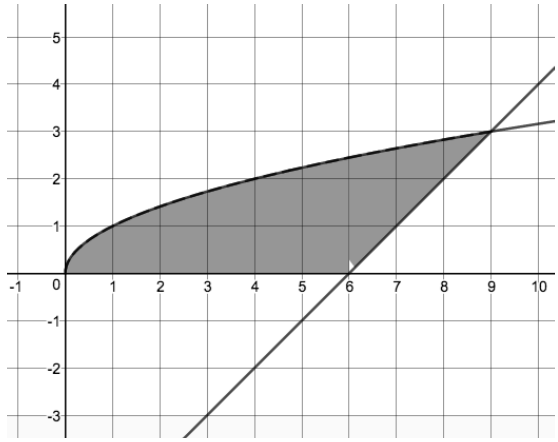
22. Let $y = h(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = \frac{x-3}{y}$ with the initial condition $h(1) = -3$. Which of the following is an expression for $h(x)$?

(A) $y = \sqrt{x^2 - 6x + 14}$

(B) $y = -\sqrt{x^2 - 6x + 7}$

(C) $y = -\sqrt{x^2 - 6x + 14}$

(D) $y = \sqrt{x^2 - 6x + 7}$



23. Let S be the shaded region in the first quadrant bounded by the graph of $y = \sqrt{x}$ and $y = x - 6$, as shown in the figure above. Which of the following gives the volume of the solid generated when S is revolved about the x axis?

(A) $\int_0^6 \sqrt{x}\,dx + \int_6^9 [\sqrt{x} - (x-6)]\,dx$

(B) $\pi \int_0^6 x\,dx + \pi \int_6^9 [\sqrt{x}^2 - (x-6)^2]\,dx$

(C) $\pi \int_0^9 [\sqrt{x} - (x-6)]^2\,dx$

(D) $\pi \int_0^3 (y + 6 - y^2)^2\,dy$

24. $\lim\limits_{x\to 0} \frac{\sin x}{e^{2x}-1}$

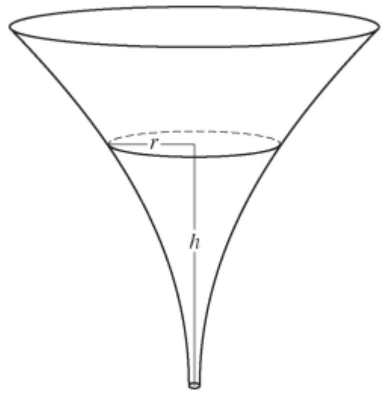
(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) nonexistent

25. Let f be a function with first derivative $f'(x) = 2x - 2$. It is known that $f(1) = 0$ and $f(3) = 4$. What value of x on the open interval (1,3) satisfies that conclusion of the Mean Value Theorem for f on the closed interval [1,3]?

(A) 2 (B) 1 (C) $\frac{2}{3}$ (D) -1

26. $\int_0^1 \frac{x^2-x-29}{x-6}\,dx$

(A) $\frac{11}{2} + \ln\frac{5}{6}$ (B) $\frac{29}{5}$ (C) $\frac{11}{2}$ (D) -145



27. The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}(3 + h^2)$, where $0 \le h \le 10$. The units of r and h are inches. The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is h=3 inches, the radius of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

(A) $-\frac{20}{15}$ in/sec (B)$-\frac{76}{500}$ in/sec (C) $\frac{2}{3}$ inch/sec (D) $-\frac{2}{3}$ in/second

28.

Consider all right circular cylinders for which the sum of the height and circumference is 30 centimeters. What is the radius of the one with maximum volume?

(A) 3 cm (B) 10 cm (C) 20 cm (D) $\frac{30}{\pi^2}$ cm (E) $\frac{10}{\pi}$ cm

29. The function f is defined by $f(x) = x^3 + 5x + 3$. If g is the inverse of f and $g(3) = 0$, what is the value of $g'(3)$?

(A) 5 (B) $\frac{1}{5}$ (C) 32 (D) $\frac{1}{32}$

30. Which of the following limits is equal to $\int_2^5 x^6 dx$?

A. $\lim_{n\to\infty} \sum_{k=1}^{n} \left(2+\frac{k}{n}\right)^6 \cdot \frac{3}{n}$

B. $\lim_{n\to\infty} \sum_{k=1}^{n} \left(2+\frac{k}{n}\right)^6 \cdot \frac{1}{n}$

C. $\lim_{n\to\infty} \sum_{k=1}^{n} \left(2+\frac{3k}{n}\right)^6 \cdot \frac{3}{n}$

D. $\lim_{n\to\infty} \sum_{k=1}^{n} \left(2+\frac{3k}{n}\right)^6 \cdot \frac{1}{n}$

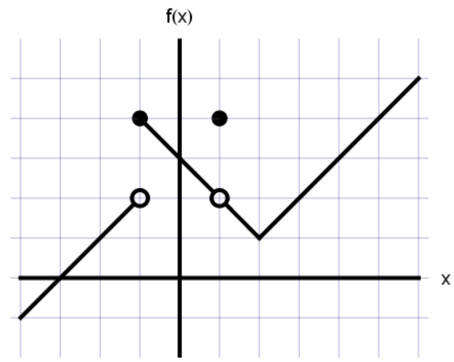
END OF PART A

SECTION I, Part B
Time- 45 minutes
Number of Questions- 15

A GRAPHING CALCULATOR IS REQUIRED FOR THESE PROBLEMS

76. Mrs. Gironda is adding points to your average at a rate of $r(t) = 31 + \sin(.2t)$ points per hour, where t is measured in hours since 2:30 pm. How many points has she added over the 3 hour interval from 4:30 pm to 7:30 pm?

(A) 94.904 pts (B) 93.873 pts (C) 93.037 pts (D) 0 pts



77. The graph of the function f is shown above. For what vales of a does $\lim_{x \to a} f(x) = 2$?

(A) -1 (B) 1 (C) 2 (D) -1 and 1

78. The second derivative of a function is given by $f''(x) = \cos(2x) + \sin(x^2)$. How many points of inflection does the graph of f have on the interval $0 < x < 4$?

(A) 2 (B) 3 (C) 4 (D) 5

79. Over the time interval from $0 < t < 10$, where time is measured in seconds , the total distance travelled is 13 m and the total displacement if 3 m. For how many meters is the particle moving left?

(A) 8 m (B) 5 m (C) 10 m (D) 0 m

80. Mrs. Gironda's washing machine is leaking all over the basement! The rate at which the water is leaking is modelled by the differentiable function K, where $K(t)$ is measured in ounces per hour and time is measured in hours since 6 pm. Which of the following is the best interpretation of $\int_0^{12} K(t)dt$?

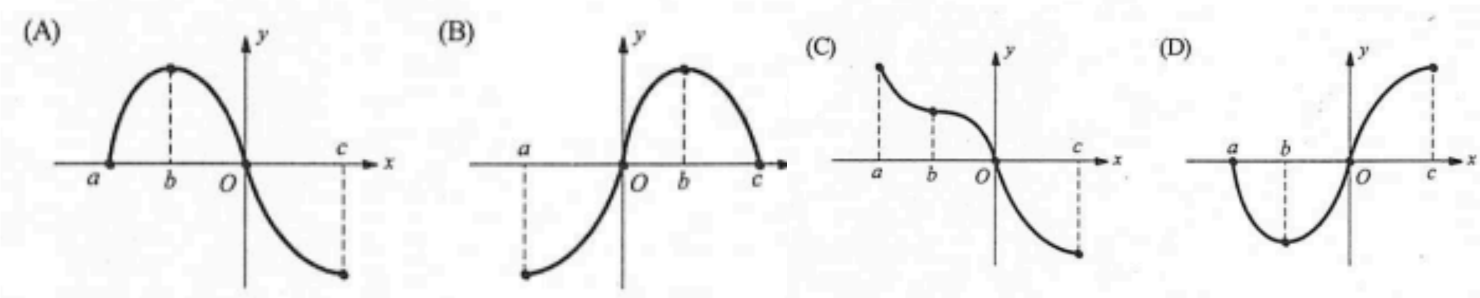
(A) The rate at which the water is leaking in ounces per minute between 6 pm and 6 am
(B) The average amount of water that has leaked in ounces per minute between 6 pm and 6 am
(C) The total amount of water that has leaked in ounces between 6 pm and 6 am
(D) The total amount of water that has leaked in ounces at 12 pm

81.

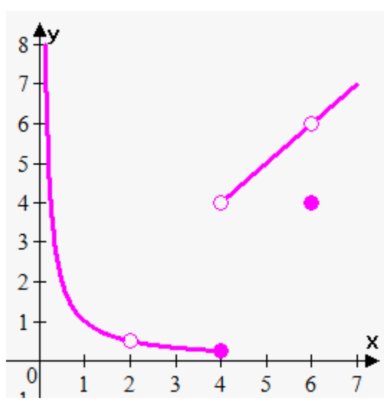
Let f be a function that is continuous on the closed interval $[a, c]$, such that the derivative of function f has the properties indicated on the table below.

x	$a<x<b$	b	$b<x<0$	0	$0<x<c$
$f'(x)$	$-$	0	$+$	3	$+$
$f''(x)$	$+$	$+$	$+$	0	$-$

Which of the following could be the graph of f ?



82. Let f be the function with derivative given by $f'(x) = \cos(x^2 - 4)$. At what values of x in the interval $0 < x < 4$ does f have a relative maximum?
(A) 2.36 and 3.443 (B)1.559, 2.36, 2.952, 3.443, and 3.872 (C) 1.559, 2.952, and 3.872 (D) 2



83. The graph of f is shown above. At what values of x does f have a jump discontinuity?

(A) 0 (B) 2 (C) 4 (D) 6

84. Let f be a differentiable function such that $f(1) = 3$ and $f'(x) = \sqrt{x^3 + 1}$. What is the value of $f(5)$?

(A) 25.485 (B) 22.485 (C) 11.225 (D) 9.811

85. Dogs are entering the dog park at a rate modelled by $f(t)$ dogs per hour and exiting the dog park at a rate modelled by $g(t)$ dogs per hour, where t is measured in hours. The functions f and g are nonnegative and differentiable for all times t. Which of the following inequalities indicates that the rate of change of the number of dogs in the park is decreasing at time t?

(A) $f(t) < 0$ (B) $f'(t) < 0$ (C) $f(t) - g(t) < 0$ (D) $f'(t) - g'(t) < 0$

86. The velocity of a particle moving along the x axis is given by $v(t) = \sqrt{e^x} + \cos x$ for $t > 0$. Which of the following statements describes the motion of the particle at $t = 2$?

(A) The particle is moving left with positive acceleration
(B) The particle is moving right with positive acceleration
(C) The particle is moving left with negative acceleration
(D) The particle is moving right with negative acceleration

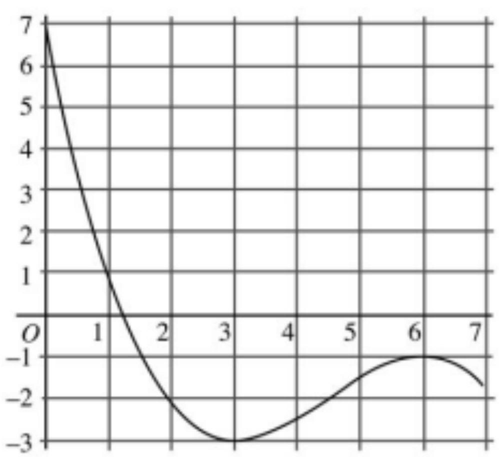
87. A balloon that is leaking air has an initial air pressure of 35 pounds per square inch (psi). The function $t = g(p)$ models the amount of time t, in hours, it takes for the air pressure of the tire to reach p psi. What are the units of $g'(p)$
(A) hours (B) psi (C) psi per hour (D) hours per psi

88. The first derivative of the function f is defined by $f'(x) = e^{3x} - \frac{x^2+3x-1}{x}$. On what intervals is f increasing?
(A) $-\infty < x < -3.303$ and $x > 0$
(B) $\infty > x > -3.303$ and $x < 0$
(C) $-\infty < x < \infty$
(D) There are no intervals on which f is increasing

x	0	2	4	7	10
$f(x)$	-1	3	5	-2	4

89. The table above shows selected values of a continuous function f. For $0 \le x \le 10$, what is the fewest possible number of times $f(x) = 2$?
(A) One (B) Two (C) Three (D) Four

90.



Graph of f

The graph of the function f shown in the figure above has horizontal tangents at $x = 3$ and $x = 6$. If $g(x) = \int_0^{2x} f(t)\,dt$, what is the value of $g'(3)$?

(A) 0 (B) −1 (C) −2 (D) −3 (E) −6

SECTION II, Part A
Time- 30 minutes
Number of problems- 2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE PROBLEMS.

1.

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

. Concert tickets went on sale at noon $(t = 0)$ and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

(a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. $(t = 5.5)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

(c) The number of people waiting in line on a different day is modelled by the function $D(t) = \ln x^2 + \frac{e^x}{x+1}$, where t is measured in minutes. Find the average number of people waiting in line for this day on the interval $0 \le x \le 9$. Is this value greater or less than the average number of people in line from part b? Give a reason for your answer.
(D) According to the model in part ©, is the number of people in line increasing or decreasing at time t=6? Give a reason for your answer.

2.

. A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + (t^2 + 3t)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

(a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

(d) A second particle R moves along the straight line so that its position at any time t is given by a differential function $x_R(t)$, where $x_R(1) = 5$ and $x_R(3) = 9$. Explain why there must a time t, for $1 < t < 3$, at which the velocity of particle R is 2.

END OF PART A

SECTION II, Part B
Time- 60 minutes
Number of problems- 4

NO CALCULATOR IS ALLOWED FOR THESE PROBLEMS

3.

The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

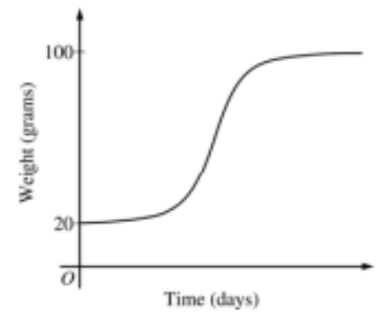
$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.

(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.



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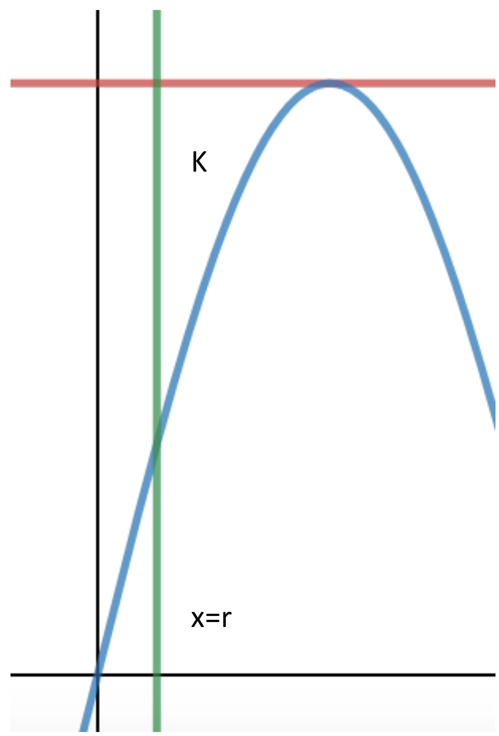
4. Let K be the region in the first quadrant bounded above by the horizontal line y=4, below by the function y=4sinx and on the left by x=r, where $r \in (0, \frac{\pi}{2})$.
   a) Find the area of K when r=$\frac{\pi}{4}$
   b) The area of K is a function of r. Find the rate of change of the area of K with respect to r when r=$\frac{\pi}{6}$
   c) Region S is revolved about the horizontal line y=6 to find a solid. Write, but do not evaluate, an integral expression involving one or more integrals that gives the volume of the solid when r=$\frac{\pi}{3}$



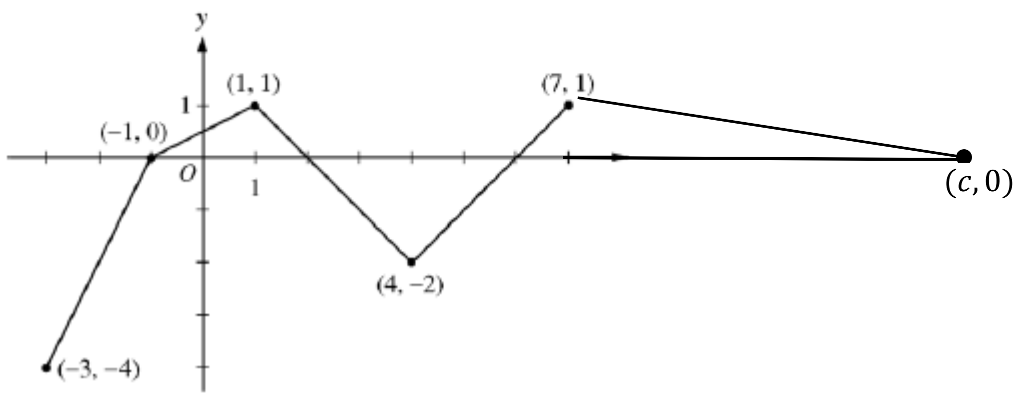
$$f(x) = \begin{cases} 8 - 3x - 3x^2 & \text{for } x \le 0 \\ 8 - e^{3x} & \text{for } x > 0 \end{cases}$$

5. Let f be the function defined above.

(a) Is f continuous at $x = 0$? Why or why not? Be sure to justify your answer.

(b) Find the global minimum value and global maximum value of f on the closed interval $-3 \le x \le 3$. Show the analysis that leads to your conclusion. (Hint: $e^9 \approx 8103$)

(c) Find the value of $\int_{-2}^{2} f(x)dx$.



Graph of f

6. The function f is defined on the interval $-3 \le x \le c$, where $c > 1$ and $f(c) = 0$. The graph of f, which consists of three line segments is shown above.

(a) Find the average rate of change of f over the interval $[-3,0]$. Show the computations that lead to your answer.

(b) For $-3 \le x \le c$, let g be the function defined by $g(x) = \int_{-1}^{x} f(t)\, dt$. Find the x-coordinate of each point of inflection of the graph of g. Justify your answer.

(c) Set up an equation that can be used to find the value of c when the average value of f over the interval $-3 \le x \le c$ is $\frac{1}{3}$.

(d) Assume $c > 2$. The function h is defined by $h(x) = f(2x)$. Find $h'(1)$ in terms of c.

(e) Find $\int_{-3}^{1} f(x)\, dx$.

(f) Find $\lim\limits_{x \to 2} \frac{g(x)}{e^{x-2} - 1}$