AP CALCULUS AB
DERIVATIVES EXAM I – REVIEW PACKET

For # 1 – 3, use the definition $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ to find the derivative of the given function at the indicated point.

1. $f(x) = 3 - x^2,\ a = -1$
2. $f(x) = \frac{1}{x},\ a = 2,$
3. $f(x) = \sqrt{x+1},\ a = 3$

For # 4 – 6, use the definition $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ to find the derivative of the function at the indicated point.

4. $f(x) = 2x + 3,\ a = -1$
5. $f(x) = \sqrt{x+1},\ a = 3$
6. $f(x) = x^3 + x,\ a = 0$

For # 7 – 9, use the definition $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to find the general derivative of the function given.

7. $f(x) = 3x - 12$
8. $f(x) = 4x^2 - 3x + 6$
9. $f(x) = \frac{1}{x}$

10. Find the equations of the line that is tangent and **normal** to the curve $y = x^3$ at the point (1, 1).

11. If $f(2) = 3$ and $f'(2) = 5$, find the equation of (**a**) the *tangent* line, and (**b**) the *normal* line to the graph of $y = f(x)$ at the point where ***x*** = 2.

For # 12 – 15, find all values for *x* for which the function is differentiable.

12. $f(x) = \frac{x^3 - 8}{x^2 - 4x - 5}$
13. $f(x) = 3\cos(|x|)$
14. $g(x) = \begin{cases} (x+1)^2 & x \le 0 \\ 2x+1 & 0 < x < 3 \\ (4-x)^2 & x \ge 3 \end{cases}$

15. $y = \sqrt[3]{3x-6} + 5$

16. Let f be the function defined as

$$f(x)=\begin{cases} 3-x & x<1 \\ ax^2+bx & x\ge 1 \end{cases}$$

A) If the function is continuous for all x, what is the relationship between a and b?

B) Find unique values for a and b that will make f both continuous and differentiable.

For # 17 – 19, determine if the statement is true or false.

17. If a function is continuous at a point, then it is differentiable at that point.

18. If a function has derivatives from both the left and the right at a point, then it is differentiable at that point.

19. If a function is differentiable at a point, then it is continuous at that point.

For # 20 – 25, write the expression as a sum of powers of x. (THIS INVOLVES ABSOLUTELY NO CALCULUS!!)

20. $(x^2-2)(x^{-1}+1)$

21. $\left(\dfrac{x}{x^2+1}\right)^{-1}$

22. $3x^2-\dfrac{2}{x}+\dfrac{5}{x^2}$

23. $\dfrac{3x^4-2x^3+4}{2x^2}$

24. $(x^{-1}+2)(x^{-2}+1)$

25. $\dfrac{x^{-1}+x^{-2}}{x^{-3}}$

For # 26 - 33, find the derivative of the function. Simplify fully.

26. $y=-x^2+3$

27. $y=\dfrac{x^3}{3}+\dfrac{x^2}{2}+x$

28. $f(x)=(x^2+1)(x^3+1)$

29. $y=\dfrac{(x-1)(x^2+x+1)}{x^3}$

30. $y=(1-x)(1+x^2)^{-1}$

31. $y=2\sqrt{x}-\dfrac{1}{\sqrt{x}}$

32. $y=3x+x\tan x$

33. $f(x)=\dfrac{x}{1+\sin x}$

34. Suppose *u* and *v* are functions of *x* that are differentiable at *x* = 0, and that $u(0) = 5, u'(0) = -3, v(0) = -1, v'(0) = 2.$ Find the values of the following derivatives at x = 0.

A) $\frac{d}{dx}(uv)$ B) $\frac{d}{dx}\left(\frac{u}{v}\right)$ C) $\frac{d}{dx}\left(\frac{v}{u}\right)$ D) $\frac{d}{dx}(3u - 2v)$

For # 35 – 36, find the horizontal tangents of the curve.

35. $y = x^3 - 2x^2 + x + 1$

36. $f(x) = x^4 - 7x^3 + 2x^2 + 15$

37. A particle moves along a line so that its position at any time $t \geq 0$ is given by the equation

$$s(t) = t^2 - 3t + 2$$

where *s* is measured in meters and *t* is measured in seconds.

A) Find the displacement during the first 5 seconds
B) Find the average velocity during the first 5 seconds
C) Find the instantaneous velocity when *t = 4*
D) Find the acceleration of the particle when *t = 4*
E) At what values of *t* does the particle change directions?
F) Where is the particle when *s* is a minimum?

38. Find the instantaneous rate of change of $f(x) = x^2 - \frac{2}{x} + 4$ at x = -1.

39. A particle moves along a straight line with the following position function:

$$s(t) = t^3 - 6t^2 + 8t + 2$$

A) Find the instantaneous velocity at time *t*
B) Find the instantaneous acceleration at time *t*
C) Determine when the particle is moving to the right.

40. Find $\lim_{h \to 0} \frac{\sin\left(\frac{\pi}{2} + h\right) - \sin\left(\frac{\pi}{2}\right)}{h}$

41. For the following function *f*, sketch a possible f'

42. State the Power Rule, Product Rule, and Quotient Rules.

43. Find the derivatives for the following set of functions:

A) $\sqrt{x}$ B) e^x C) $\frac{1}{x}$ D) sin x E) cos x F) tan x

G) cot x H) csc x I) sec x J) x K) 4

44. Complete the following trig properties / identities / double angle formulas:

sin (x + h) =

cos (x + h) =

sin 2x =

cos 2x =

45. Know the following limits:

A) $\lim_{x \to 0} \frac{\sin x}{x}$ B) $\lim_{x \to 0} \frac{\cos x - 1}{x}$

46. Suppose that the line tangent to the graph of $y = f(x)$ at *x* = *3* passes through the points (-2, 3) and (4, -1).

A) Find $f'(3)$

B) Find $f(3)$

C) What is the equation of the line tangent to *f* at 3?

47. If $f(x) = 3e^x + \sec x - \frac{4}{x^5}$, find $f'(2)$

48. Prove that $\frac{d}{dx}[\cos x] = -\sin x$