Saad Hasan

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Not bad.
Need practice on time domain analysis (LCCDEs).

60

1.

Saad Hasan
A53038382

(a)

	Linear	Time-invariant	static	causal	stable
$y[n] = x[-n]$	yes	No	No	No	yes
$2n^2 x[n] + nx[n+1]$	No -1	yes -1	yes -1	yes -1	No
$\cos(2\pi x[n])$	No	yes	yes	yes	yes

(b) $x(t/T_s) = \cos(0.5\pi(100t)) = \cos(50\pi t)$

$\Omega = 50$ Hz Missing π

-2

(c) $F_s = 0$ Hz ∞

-3

It's the opposite.

(d) $H(z) = \frac{Y(z)}{X(z)} = \frac{3 + z^{-1}}{6 - z^{-1} + 2z^{-2} + z^{-3}} = \frac{3z+1}{6z^3 - z^2 + 2z + 1}$

-3

$h[n] = Z^{-1}\left[\frac{3z+1}{6z^3 - z^2 + 2z + 1}\right]$ ⇒ So?

Use convolution.

(e) $Y(z) = H(z)X(z) = \frac{1}{1-(0.7)z^{-1}}\left[\frac{B(1 - z^{-1}\cos(0.2\pi))}{1 - (2\cos(0.2\pi))z^{-1} + z^{-2}}\right]$ ⇒ This would mean both poles of $X(z)$ and $H(z)$ would be protected.

i $K_1(0.7)^n \cos(0.2\pi n + \Theta)u[n]$

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both poles of $X(z)$ and $H(z)$ would be protected.
Then, its a linear combination of exponentials.

-5

0 for impulse response.

2.

(a) $y[n] - 2/15\, y[n-1] - 1/15\, y[n-2] = \delta[n]$ $y[-1] = 1$ $y[-2] = -1$

−2

$\lambda^{n-2}(\lambda^2 - 2/15\,\lambda - 1/15) = 0$

$$\lambda_{1,2} = \frac{2/15 \pm \sqrt{4/225 + 60/225}}{2} = \frac{2/15 \pm 8/15}{2} = 1/3, -1/5$$

$y[0] - 2/15\, y[-1] - 1/15\, y[-2] = 1 \rightarrow y[0] = 1 + 2/15 - 1/15 = 16/15$

$y[1] - 2/15\, y[0] - 1/15\, y[-1] = 0 \rightarrow y[1] = (2/15)(16/15) + 1/15(1)$

$y[1] = 32/15 + 1/15 = 33/15$

$h[n] = c_1(1/3)^n + c_2(-1/5)^n$

$h[0] = c_1 + c_2 = 1$

$h[1] = 1/3\,c_1 - 1/5\,c_2 = 33/15$

$\rightarrow 1/5\,c_1 + 1/3\,c_1 = 1/5 + 33/15$

$\frac{3c_1}{15} + \frac{5c_1}{15} = \frac{36}{15} = 8c_1 = 36 \rightarrow c_1 = 9/2$

$c_2 = 1 - 9/2 = -7/2$

$h[n] = (9/2(1/3)^n - 7/2(-1/5)^n) u[n]$ −1

Infinite

(b) The system has finite impulse response and is stable ✓

because $\lambda_1, \lambda_2 < 1$

−2

$x[n] = u[n]$

(c) $y_p[n] = K u[n]$

$K u[n] - 2/15\, K u[n-1] - 1/15\, K u[n-2] = K u[n] \rightarrow K = 0$ −2

ZSR: $y[-1] = y[-2] = 0$ ✓

$y[0] = 1$

$y[1] - 2/15\, y[0] = 1 \rightarrow y[1] = 1 + 2/15 = 17/15$ ✓

$c_1(1/3)^n + c_2(-1/5)^n$

−2

$y[0] = c_1 + c_2 = 1$

$y[1] = 1/3\,c_1 - 1/5\,c_2 = 17/15$ (15)

$\rightarrow \frac{8c_1}{15} = \frac{22}{5} = \frac{66}{15} \rightarrow c_1 = \frac{33}{4}$ $c_2 = \frac{-29}{4}$

This is wrong

$c_2 - \frac{1}{5} c_2 \neq 0$

$y_{ZSR} = 33/4(1/3)^n - 29/4(-1/5)^n$

2. (d)

$y[n] - 2/15\, y[n-1] - 1/15\, y[n-1] = 0$

$y[0] - 2/15\, y[-1] - 1/15\, y[-2] = 0$

$y[0] = 2/15(1) + 1/15(-1) = 1/15$ ✓

$y[1] - 2/15\, y[0] - 1/15\, y[-1] = 0 \Rightarrow y[1] = (2/15)(1/15) + 1/15 = 17/225$ ✓

$c_2 - \frac{1}{5}c_2 \neq 0$ −1

$c_1(1/3)^n + c_2(-1/5)^n$

$y[0] = c_1 + c_2 = 1/15$ $\quad \rightarrow \frac{8c_1}{15} = \frac{1}{15} + \frac{17}{225} \rightarrow c_1 = \frac{4}{15}$

$y[1] = 1/3\, c_1 - 1/5\, c_2 = 17/225$ $\quad c_2 = \frac{-3}{15}$

$y_{ZIR}[n] = 4/15(1/3)^n - 3/15(-1/5)^n$

(e) $y_{total} = \underbrace{(4/15 + 33/4)(1/3)^n - (20/4 + 3/15)(-1/5)^n}_{\text{Transient}}$ ✓

no steady state response ✓

3. (a) $H(z) = \frac{z+1}{z^2 - 3/5 z + 2/25}$ missing z −1 $\quad z = \frac{3/5 \pm \sqrt{9/25 - 8/25}}{2} = \frac{3/5 \pm 1/5}{2} = 2/5, 1/5$

$H(z) = \frac{z+1}{(z+2/5)(z-1/5)} = \frac{A}{(z+2/5)} + \frac{B}{(z-1/5)}$

$A + \frac{1}{5}A \neq 0$ −1

$3/5 B = 6/5 \rightarrow B = 2$

$A = -1$

$A(z-1/5) + B(z+2/5) = z+1 \qquad A+B=1$

$z(A+B) + (-1/5 A + 2/5 B) \qquad -1/5A + 2/5B = 1$

$\boxed{h[n] = -(-2/5)^n u[n] + 2(1/5)^n u[n]}$

Use Transfer Function $\leftrightarrow$ LCCDE
z Transform
−2

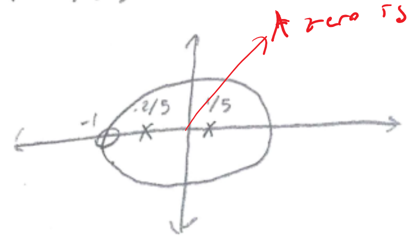
(b) $\frac{Y(z)}{X(z)} = H(z) = \frac{z+1}{z^2 - 3/5 z + 2/25}$

? −1

3.

(c) p: $-2/5, 1/5$ z: -1

A zero is missing (-1)



(d) ROC: $|z| > 1/5$ X $|z| > \frac{2}{5}$ (-2)

(e) The system is stable because its poles are inside the unit circle ✓

(f) $Y(z) = H(z)X(z) = \frac{z}{z-2/5} - \frac{1}{z-1/5} = \frac{z(z-1/5) - (z-2/5)}{(z-2/5)(z-1/5)}$ (-1)

$= \frac{z}{(z-2/5)(z-1/5)} \Rightarrow X(z) = \frac{\left(\frac{z^2}{(z-2/5)(z-1/5)}\right)}{\left(\frac{z+1}{(z+2/5)(z-1/5)}\right)}$

(-4)