

Not bad.

Need practice on  
z transform.

63.5

Exam 1

$$1) y(n) = x[-n] \quad \text{Linear: Yes} \quad \text{Time Invariant: No} \quad \text{Static: No} \quad \text{Stable: Yes}$$

$$y(n) = an^2 x(n) + nx[n+1] \quad \text{Linear: No} \quad \text{Time Invariant: No} \quad \text{Static: No} \quad \text{Stable: No}$$

$$y(n) = \cos(2\pi n) x(n) \quad \text{Linear: Yes} \quad \text{Time Invariant: Yes} \quad \text{Static: Yes} \quad \text{Stable: Yes}$$

$$1b) x(n) = \cos(\pi/2 n) \quad x(t) = \cos(\omega t) \quad f_s = 100 \text{ Hz}$$

$$x(t) = \cos(2\pi \cdot 50 t)$$

$$\omega \frac{T}{100} = \frac{\pi}{2} T$$

$$\omega = \frac{100\pi}{2}$$

$$\omega \frac{T}{100} = \frac{3\pi}{4}$$

$$\omega = \frac{300\pi}{4}$$

$$c) f_s = 2 \text{ Hz}$$

$$d) h(z) = (3z^0 + 1z^{-1})(6z^0 - 1z^{-1} + 2z^{-2} + z^{-3})$$

$$h[n] = 0$$

$$e) \sum_{k=0}^{\infty} A(\omega \cdot T)^k B(\omega \pi(n-k))$$

$$(i) H_1 \cdot z^n \cos(\omega \cdot 2\pi n + \theta)$$

Total = Particular + Homogeneous

 $\downarrow$   
 $\sim$  Input

 $\downarrow$   
 $\sim$  Impulse Response



$$2) y(n) = 0.15y(n-1) + 0.15y(n-2) + x(n) \quad y(-1)=1 \quad y(-2)=-1$$

a) for impulse response  $x(n] = \delta(n]$

$$y(n) - 0.15y(n-1) - 0.15y(n-2) = 0 \quad h(n) = (a(-1/6)^n + b(1/3)^n)u(n]$$

$$\lambda^n - 0.15\lambda^{n-1} - 0.15\lambda^{n-2} = 0 \quad \text{red } -1/5 \quad y(0) = 1$$

$$\lambda^2 - 0.15\lambda - 0.15 = 0 \quad \lambda = -0.15 \quad y(0) = 0.15y(0) + 0.15y(-1) + \delta(0]$$

$$\lambda = 1/3 \quad = 0.15$$

$$\begin{cases} a+b=1 \\ -0.15a + 0.15b = 0.15 \end{cases} \quad \begin{matrix} a=3/8 \\ b=5/8 \end{matrix}$$

$$h(n) = \left\{ \frac{3}{8} \left(-\frac{1}{6}\right)^n + \frac{5}{8} \left(\frac{1}{3}\right)^n \right\} u(n]$$

b) System Stable, IIR System ✓

c)  $x(n] = u(n]$

$$y_h(n) = h(n) = (a(-1/6)^n + b(1/3)^n)u(n] \quad y_p(n) = K(-1)^n u(n]$$

$$y(n) = 0.15y(n-1) + 0.15y(n-2) + x(n] \quad y(-1)=1 \quad y(-2)=-1$$

$$K(-1)^n u(n] = 0.15K(-1)^{n-1} u(n-1) + 0.15K(-1)^{n-2} u(n-2) + u(n]$$

$$n=0 \quad K = 0.15K + 0.15K \quad y(0) = x(0) = 1$$

$$K = 6/15$$

$$y(1) = 0.15y(0) + 0.15y(-1) + x(1) = 0.15 + 0.15(-1) + 1 = 1/5$$

$$\begin{cases} a+b=1 \\ -0.15a + 0.15b = 6/15 \end{cases} \quad \begin{matrix} a=-3/8 \\ b=11/8 \end{matrix}$$

$$y_{ZSR}(n) = \left\{ (-3/8)(-1/6)^n + \frac{11}{8} \left(\frac{1}{3}\right)^n \right\} u(n]$$

Where's particular? (2)

$$d) \quad h(n) = (a(-1/6)^n + b(1/3)^n) u(n)$$

$$y(n) = a_{115} y(n-1) + 1/15 y(n-2) + x(n)$$

$$y(0) = a_{115} y(-1) + 1/15 y(-2) = a_{115} + 1/15(-1) = a_{115} - 1/15 = 1/15 \quad y(0) = 1/15$$

$$y(1) = a_{115} y(0) + 1/15 y(-1) = a_{115} \cdot 1/15 + 1/15(-1) = 17/225$$

$$\begin{cases} a+b = 1/15 \\ -1/5a + 1/3b = 17/225 \end{cases} \quad a = -1/10 \quad b = 1/6$$

$$y_{ZSR} = ((-1/10)(-1/5)^n + 1/6(1/3)^n) u(n)$$

$$e) \quad y(n) = \left\{ (-1/8)(-1/5)^n + \frac{1}{8} \left( \frac{1}{3} \right)^n \right\} u(n) + \frac{6}{5} (-1)^n u(n)$$

$$y_{trans}(n) = y_{ZSR}(n) = \left\{ (-1/8)(-1/5)^n + \frac{1}{8} \left( \frac{1}{3} \right)^n \right\} u(n)$$

Steady State Response 0

$$3) \quad H(z) = \frac{1+z^{-1}}{1-\frac{3}{5}z^{-1}+\frac{2}{25}z^{-2}} \left\{ \frac{z^2}{z^2} \right\} = \frac{z^2+z}{z^2-\frac{3}{5}z+\frac{2}{25}} = \frac{1}{\frac{2}{15}z^2-\frac{3}{5}z+1} + \frac{1}{z+\frac{2}{15}z^{-1}-\frac{3}{5}}$$

use this directly.

$$\frac{2}{15} y(n-2) - \frac{3}{5} y(n-1) + x(n) + y(n) + \frac{2}{15} y(n-1) - \frac{3}{15} x(n) = 0$$

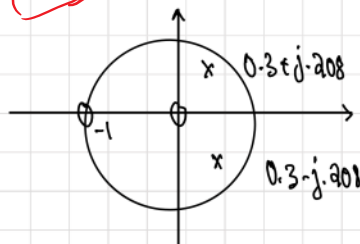
$$z^2 - z = 0 \quad z = -1, z = 0$$

$$z^2 - \frac{3}{5}z + \frac{2}{25} = 0$$

$$z = 0.3 \pm j0.208 \quad (-1)$$

b)

c)



$$d) \quad 0.3 - j0.208 < z < 0.3 + j0.208 = \text{ROC}$$

e) Stable, the ROC is in the unit circle ✓

$$f) \quad x(n) = 2\delta(n) - 1/15\delta(n-1)$$

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