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Good job. 74

$x_1 + x$

$x_1(-t) + x_2(-t)$

$\to x_1(+)$

$\cos(2\pi x_1(n)) + \cos 2\pi x_2(n)$

$\cos(2\pi(x_1+x_2))$

$(n-t)$ $(n-t+1)$

$x_1(-2$

$y[n] =$

$2n^2 x_1[n] + n x_1[n+1]$

$+ \ldots x_2$

$2n^2(x_1+x_2) + n(x_1(n+1) + x_2(n+1))$

ECE 466 MidTerm 1

	Linear	Time Inv	Static	Causal	Stable
1) a) $y[n] = x[-n]$	✓	✗	~~✗~~ ✓ -1	✗	✓
$y[n] = 2n^2 x[n] + n x[n+1]$	✓	✓ -1	✗	✗	✗
$y[n] = \cos(2\pi x[n])$	✗	✓	✓	✓	✓

b) $x(n) = \cos(\frac{\pi}{2} n)$; $x(t) = \cos(\Omega t)$ $F_s = 100\ Hz$

$x(n) = \cos(\frac{\Omega}{100} n)$

$\cos(2\pi($

-3

$\Omega_1 = 50 \to$ missing

$\Omega_2 = 20 \to$ wrong

$-\frac{1}{2} < f_0 < \frac{1}{2}$

c) - Sampling a unit step function technically requires an infinite rate. ✓

- Nyquist Frequency $= \frac{\text{Max Freq}}{2}$ but the Max frequency of unit step is infinity. ✓

- We must choose a sampling rate which suits our intended design.

d) $x[n] = \{\underline{3}, 1\}$; $\boxed{h[n] = \{\underline{2}, -1, 0\}}$ 1

$y[n] = \{\underline{6}, -1, 2, 1\}$

$x[n] * h[n] = y[n]$ -1

$x[n] * h[n] = y[n]$ -1

$h[n] \to$ has 3 samples

e) $y[n] = \sum_{i=-\infty}^{\infty} x[i]h[n-i]$

$$\sum_{i=-\infty}^{\infty} B\cos(0.2\pi i)u(i) \cdot A(0.7)^{n-i}u(n-i)$$

$$\sum_{i=0}^{\infty} B\cos(0.2\pi i) \cdot A(0.7)^{n-i}$$

iii

2) a) $y[n] - \frac{2}{15}y[n-1] + \frac{1}{15}y[n-2] + x[n]$

$$y[n] - \frac{2}{15}y[n-1] - \frac{1}{15}y[n-2] = 0$$

$$\lambda^n - \frac{2}{15}\lambda^{n-1} - \frac{1}{15}\lambda^{n-2} = 0$$

$$\lambda^2 - \frac{2}{15}\lambda - \frac{1}{15} = 0$$

$$\lambda_1 = -\frac{1}{5}$$

$$\lambda_2 = \frac{1}{3}$$

$$y_h(n) = c_1\lambda_1^n + c_2\lambda_2^n$$

$$= \left(-\frac{1}{5}\right)^n c_1 + \left(\frac{1}{3}\right)^n c_2$$

$$y(0) = 1 = c_1 + c_2$$

$$y(1) - \frac{2}{15}y(0) = 0$$

$$y(1) = \frac{2}{15}$$

$$\frac{2}{15} = c_1 \cdot \left(\frac{-1}{5}\right) + \frac{1}{3} c_2$$

$c_1 = c_2 = 1$ → Not really (-2)

$$\boxed{h[n] = \left(\left(\frac{-1}{5}\right)^n + \left(\frac{1}{3}\right)^n\right) u(n)}$$

b) IIR, Stable

c) ZSR, $x[n] = u[n]$

→ What's K? (-2)

$y_p[n] = k\,u(n)$

$y_{total} = y_p(n) + y_h(n)$

$= k\,u(n) + (c_1 2^n + c_2 1^n) u(n)$

$y(0) = x(0) = 1$

$= k + c_1 + c_2$

$y(1) - \frac{2}{15} y(0) = x(1) = 1$

$$y(1) = 1 + \frac{2}{15} = \frac{17}{15}$$

$$\frac{17}{15} = k + \left(\frac{-1}{5}\right)c_1 + \left(\frac{1}{3}\right)c_2$$

$y(2) - \frac{2}{15}(y(1)) + y(0) = 1$

(-4)

$$y(2) = 10$$

d) ZIR is when $x(n)=0$

$$\frac{2}{15}E^{-1} + \frac{1}{15}E^{-2} + E\,x(n) = E$$

$$\lambda_1 = -\frac{1}{5}$$

$$\lambda_2 = \frac{1}{3}$$

$$y_0[k] = c_1\left(-\frac{1}{5}\right)^k + c_2\left(\frac{1}{3}\right)^k$$

-1

Can't use initial conditions like this. They affect the system, are not caused by it.

$$k=-1: \quad 1 = c_1\left(-\frac{1}{5}\right)^{-1} + c_2\left(\frac{1}{3}\right)^{-1}$$

$$k=-2: \quad -1 = c_1\left(-\frac{1}{5}\right)^{-2} + c_2\left(\frac{1}{3}\right)^{-2}$$

$$c_1(-5) + 3c_2 = 1$$
$$c_1(+25) + 9c_2 = -1$$

$\downarrow$

solve

$$c_1 = -\frac{1}{10} \qquad c_2 = \frac{1}{6}$$

$$y_0[k] = -\frac{1}{10}\left(-\frac{1}{5}\right)^k + \left(\frac{1}{6}\right)\left(\frac{1}{3}\right)^k$$

e) $y_{total} = \underbrace{y_p(n) + y_k(n)}_{y_{ZSR} + y_{ZIR}} = k\,u(n) + \left(-\frac{1}{10}\right)\left(-\frac{1}{5}\right)^k + \left(\frac{1}{6}\right)\left(\frac{1}{3}\right)^k u(n)$

-3

3) a) $H(z) = \dfrac{1+z^{-1}}{1-\frac{3}{5}z^{-1}+\frac{2}{25}z^{-2}}$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{1+z^{-1}}{1-\frac{3}{5}z^{-1}+\frac{2}{25}z^{-2}}$$

$$Y(z)\left(1-\frac{3}{5}z^{-1}+\frac{2}{25}z^{-2}\right) = X(z)\left(1+z^{-1}\right)$$

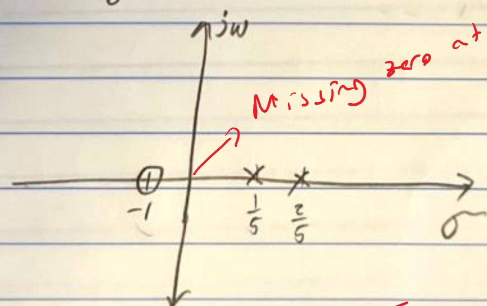
$$\boxed{y(n) - \frac{3}{5}y(n-1) + \frac{2}{25}y(n-2) = x(n) + x(n-1)}$$ ✓

b) $H(1) = \dfrac{1+1}{1-\frac{3}{5}+\frac{2}{25}}$ = Gain ✗ −1

$= \dfrac{25}{6}$

c)

Missing zero at origin −1



Poles at = $\frac{1}{5}$ and $\frac{2}{5}$

Zero at = −1

e) Stable. The roots are < 1 ✓

d) ROC: $\frac{1}{5} < |z| < \frac{2}{5}$ ✗ Causal system −3

f) $x[n] * H(n) = y[n]$

$$y[n] = 2\left(\frac{2}{5}\right)^n u(n) - \left(\frac{1}{5}\right)^n u(n)$$

$$Y[z] = 2\left(\frac{z}{z-\frac{2}{5}}\right) - \frac{z}{z-\frac{1}{5}}$$

$$X[z] = \frac{Y[z]}{H[z]}$$ Correct approach

$$= \boxed{\frac{2\left(\frac{z}{z-\frac{2}{5}}\right) - \frac{z}{z-\frac{1}{5}}}{\frac{1+z^{-1}}{1-\frac{3}{5}z^{-1}+\frac{2}{25}z^{-2}}}}$$

Add this up

Separate into parts

Inv Z transform

$x(n)$

$$h(z) = 1+z^{-1} \cdot 1 - \frac{5z}{3} + \frac{25z^2}{2}$$

-3

$x(n)$