Exam 1

1) $y(n) = x(-n)$ Linear: Yes   Time Invariant : No   Static: No   Stable: Yes

   $y(n) = 2n^2 x(n) + nx(n+1)$ Linear: No   Time Invariant : No   Static: No   Stable: No

   $y(n) = \cos(2\pi x(n))$ Linear: Yes   Time Invariant : Yes   Static: Yes   Stable: Yes

1b) $x(n) = \cos(\pi \frac{\pi}{2}(2n))$ $x(t) = \cos(\Omega t)$ $F_s = 100 \, Hz$

   $x(t) = \cos(2\Omega \cdot 50 \, t)$ $\Omega \dfrac{\alpha}{100} = \dfrac{\pi}{2} \alpha$ $\boxed{\Omega = \dfrac{100\pi}{2}}$, $\dfrac{\Omega n}{100} = \dfrac{3\pi}{4}$

                                                    $\boxed{\Omega = \dfrac{300\pi}{4}}$

c) $f_s = 2 \, Hz$

d) $h(z) = (3z^0 + 1z^{-1})(6z^0 - 1z^{-1} + 2z^{-2} + z^{-3})$ $\boxed{h(n) = 0}$

e) $\displaystyle\sum_{k=0}^{\infty} A(0.7)^k B(\cos\pi(n-k))$ $\boxed{(i) \quad h, 0.7^n \cos(0.2\pi n + \theta)}$

2) $y(n) = \frac{2}{15} y(n-1) + \frac{4}{15} y(n-2) + x(n)$ $y(-1)=1$ $y(-2)=-1$

a) For impulse response $x(n) = \delta(n)$

$y(n) - \frac{2}{15} y(n-1) - \frac{4}{15} y(n-2) = 0$ $h(n) = (a(-\frac{4}{6})^n + b(\frac{4}{3})^n) u(n)$

$\lambda^n - \frac{2}{15}\lambda^{n-1} - \frac{4}{15}\lambda^{n-2} = 0$ $y(0) = 1$

$\lambda^{n-2}(\lambda^2 - \frac{2}{15}\lambda - \frac{4}{15}) = 0$ $\lambda = -\frac{4}{15}$ $y(1) = \frac{2}{15} y(0) + \frac{4}{15} y(-1) + \delta(1)$
$\qquad\qquad\qquad\qquad\qquad \lambda = \frac{4}{3}$

$\begin{cases} a + b = 1 \\ -\frac{4}{5}a + \frac{4}{3}b = \frac{2}{15} \end{cases}$ $\begin{matrix} a = 3/8 \\ b = 5/8 \end{matrix}$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad = \frac{2}{15}$

$$\boxed{h(n) = \left\{ \frac{3}{8} \left\{ -\frac{1}{5} \right\}^n + \frac{5}{8} \left\{ \frac{1}{3} \right\}^n \right\} u(n)}$$

b) System Stable, IIR System

c) $x(n) = u(n)$

$y_h(n) = h(n) = (a(-\frac{4}{6})^n + b(\frac{4}{3})^n) u(n)$ $y_p(n) = K(-1)^n u(n)$

$y(n) = \frac{2}{15} y(n-1) + \frac{4}{15} y(n-2) + x(n)$ $y(-1)=1$ $y(-2)=-1$

$K(-1)^n u(n) = K\frac{2}{15}(-1)^{n-1} u(n-1) + K\frac{4}{15}(-1)^{n-2} u(n-2) + u(n)$

$n = 2$ $K = \frac{2}{15}K + \frac{4}{15}K$ $y(0) = x(0) = 1$

$\qquad K = 64/15$ $y(1) = \frac{2}{15} y(0) + \frac{4}{15} y(-1) + x(1) = \frac{2}{15} + \frac{4}{15} + 1 = 6/5$

$\begin{cases} a + b = 1 \\ -\frac{4}{5}a + \frac{4}{3}b = 6/5 \end{cases}$ $\begin{matrix} a = -13/8 \\ b = 24/8 \end{matrix}$ $$\boxed{y_{2sr}(n) = \left\{ (-13/8)(-4/5)^n + \frac{24}{8}\left\{\frac{1}{3}\right\}^n \right\} u(n)}$$

d) $h(n) = (a(-\tfrac{4}{6})^n + b(\tfrac{4}{3})^n)u(n)$

$y(n) = \tfrac{2}{15}y(n-1) + \tfrac{4}{15}y(n-2) + x(n)$

$y(0) = \tfrac{2}{15}y(-1) + \tfrac{4}{15}y(-2) = \tfrac{2}{15} + \tfrac{1}{15}(-1) = \tfrac{2}{15} - \tfrac{4}{15} = \tfrac{4}{15}$ $y(0) = \tfrac{4}{15}$

$y(1) = \tfrac{2}{15}y(0) + \tfrac{4}{15}y(-1) = \tfrac{2}{15} \cdot \tfrac{4}{15} + \tfrac{4}{15}(1) = \tfrac{17}{225}$

$\begin{cases} a + b = \tfrac{4}{15} \\ -\tfrac{4}{15}a + \tfrac{4}{3}b = \tfrac{17}{225} \end{cases}$ $a = -\tfrac{1}{10}$ $b = \tfrac{4}{6}$

$$\boxed{y_{ZIR} = ((-\tfrac{1}{10})(-\tfrac{4}{5})^n + \tfrac{4}{6}\{\tfrac{4}{3})^n)u(n)}$$

e) $\boxed{y(n) = \{(-\tfrac{13}{8})(-\tfrac{4}{5})^n + \tfrac{2}{8}\{\tfrac{1}{3}\}n\}u(n) + \tfrac{6}{5}(-1)^n u(n)}$

$\boxed{y_{trans}(n) = y_{ZSR}(n) = \{(-\tfrac{13}{8})(-\tfrac{4}{5})^n + \tfrac{2}{8}\{\tfrac{1}{3}\}n\}u(n)}$

$\boxed{\text{Steady State Response } 0}$

3) $H(z) = \dfrac{1 + z^{-1}}{1 - \tfrac{3}{5}z^{-1} + \tfrac{2}{25}z^{-2}} \left\{ \dfrac{z^2}{z^2} \right\} = \dfrac{z^2 + z}{z^2 - \tfrac{3}{5}z + \tfrac{2}{25}z} = \dfrac{1}{\tfrac{2}{15}z^{-2} - \tfrac{3}{5}z^{-1} + 1} + \dfrac{1}{z + \tfrac{2}{15}z^{-1} - \tfrac{3}{5}}$

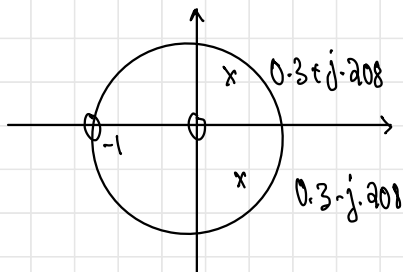$\boxed{\tfrac{2}{15}y(n-2) - \tfrac{3}{5}y(n-1) + x(n) + y(n) + \tfrac{2}{15}y(n-1) - \tfrac{3}{15}x(n) = 0}$

$z^2 - z = 0$ $z = -1, z = 0$

$z^2 - \tfrac{3}{5}z + \tfrac{2}{25}z = 0$

$z = 0.3 \pm j\, 0.208$

b)

c)

x $0.3 + j \cdot 208$

-1

x $0.3 - j \cdot 208$

d) $0.3 - j \cdot 208 \angle z \angle 0.3 + j \cdot 208 = ROC$

e) Stable, the ROC is in the unit circle

5) $x(n) = 2\,\delta(n) - \tfrac{4}{15}\delta(n-1)$