

94.5

\therefore Non linear, Time Varying, Dynamic, non causal, non stable

$$\frac{1}{2} \cdot 200\pi$$

$$J_L = \frac{\pi}{2} = \frac{W_S}{2} + W_C n$$

$N = 100 \text{ H}, 300 \text{ H}$ -1

1d. $x[n] = 3\delta[n] + \delta[n-1]$

$$y[n] = 3h[n] + h[n-1]$$

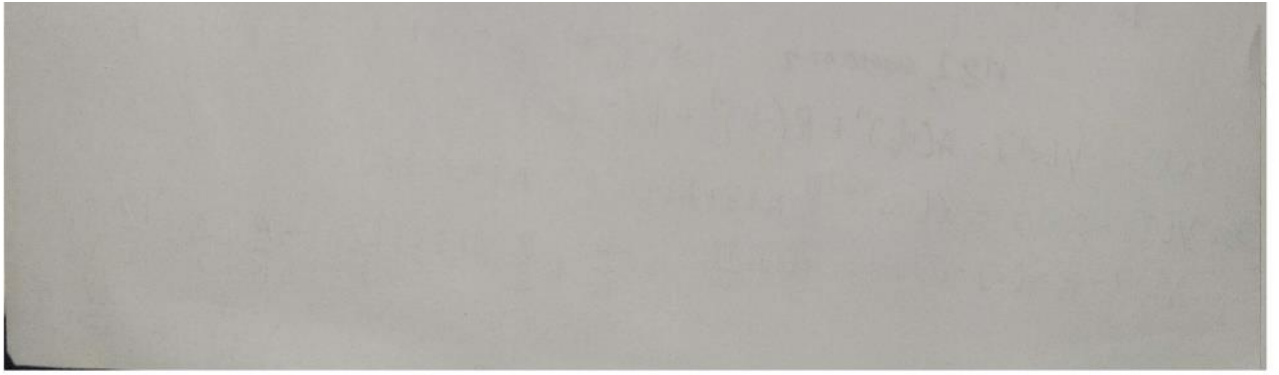
$y[n] = 3h[n] + h[n-1]$
 $h[0] = 6/3 = 2$ \rightarrow impulse response $= 0 \forall n < 0$
 $h[0] = \frac{-3}{1} = -1$

$$h[1] = \frac{4[1] - h[0]}{3} = \frac{-3}{3} = -1$$

$$h[2] = \frac{y[2] - h[1]}{3} = \frac{2 - (-1)}{3} = 1$$

$$h[n] = \{2, -1, 1\}$$

ie. $k_1 (0.7)^n u[n] + k_2 \cos(0.2\pi n + \theta)$ as in steady state the system will "track" the input ✓



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2a. $y[n] - \frac{2}{15}y[n-1] - \frac{1}{15}y[n-2] = x[n]$

$$\lambda^n - \frac{2}{15}\lambda^{n-1} - \frac{1}{15}\lambda^{n-2}$$

$$\lambda^{n-2}(\lambda^2 - \frac{2}{15}\lambda - \frac{1}{15})$$

$$\lambda = -\frac{1}{5}, \frac{1}{3}$$

$$\frac{\frac{2}{15} \pm \sqrt{\frac{4}{225} + \frac{4}{15}}}{2} \rightarrow \frac{1}{15} \pm \sqrt{\frac{1}{225} + \frac{15}{225}}$$

$$\lambda = \frac{1}{15} \pm \frac{4}{15} = \frac{1}{3}, -\frac{1}{5}$$

$$y[n] = A(-\frac{1}{5})^n + B(\frac{1}{3})^n$$

Impulse Response \rightarrow Zero state $\rightarrow y[-2], y[-1] = 0$ ✓

$$y[0] = 1$$

$$y[0] - 0 - 0 = 1 \quad A(-\frac{1}{5})^0 + B(\frac{1}{3})^0 = 1 \rightarrow A + B = 1 \rightarrow A = 1 - B$$

$$y[1] - \frac{2}{15} - 0 = 0 \quad y[1] = \frac{2}{15} \quad A(-\frac{1}{5}) + B(\frac{1}{3}) \rightarrow -\frac{A}{5} + \frac{B}{3} = \frac{2}{15}$$

$$-\frac{1}{5} + \frac{B}{5} + \frac{B}{3} = \frac{2}{15} \quad \frac{8}{15}B = \frac{1}{3} \quad B = \frac{5}{8} \rightarrow A = \frac{3}{8}$$

$$h[n] = \frac{3}{8}(-\frac{1}{5})^n + \frac{5}{8}(\frac{1}{3})^n$$
 ✓

b. As $\forall a \neq 0 \nexists n, a^n = 0 \rightarrow h[n]$ is IIR as it will never reach 0

$$\sum_{n=0}^{\infty} |h[n]| = \sum_{n=0}^{\infty} \frac{3}{8}(\frac{1}{5})^n + \sum_{n=0}^{\infty} \frac{5}{8}(\frac{1}{3})^n = \frac{3}{8}(\frac{1}{1-\frac{1}{5}}) + \frac{5}{8}(\frac{1}{1-\frac{1}{3}})$$

As this is $< \infty$, this system is stable ✓

c. $y_h[n] = A(-\frac{1}{5})^n + B(\frac{1}{3})^n$ from pt. a

$$y_p = k u[n], \quad k u[n] - \frac{2}{15}k u[n-1] - \frac{1}{15}k u[n-2] = u[n]$$
 ✓

$$n \geq 2, \text{ choose } n=2 \quad k - \frac{2}{15}k - \frac{1}{15}k = 1 \quad \frac{12}{15}k = 1 \quad k = 1.25$$

$$y[n] = A(-\frac{1}{5})^n + B(\frac{1}{3})^n + 1.25u[n]$$

2b. $y[0] - 0 - 0 = x[0] \quad y[0] = 1 \quad A + B + 1.25 = 1 \quad A + B = -0.25$

$$y[1] - \frac{2}{15}y[0] - 0 = x[1] \quad y[1] = \frac{17}{15} \quad -\frac{A}{5} + \frac{B}{3} + 1.25 = \frac{17}{15} \quad -\frac{A}{5} + \frac{B}{3} = \frac{17}{15} - \frac{5}{4} = \frac{7}{60}$$

$$M + 15 = -0.5$$

$$-\frac{7}{60} \leftarrow \frac{68}{60} - \frac{75}{60}$$

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$$A+B = -25$$

$$-\frac{A}{15} + \frac{B}{3} = -\frac{7}{60}$$

$$4A + 4B = -1$$

$$-4A + 20B = -7$$

$$24B = -8 \quad B = -\frac{1}{3}$$

$$4A - \frac{4}{3} = -1$$

$$4A = \frac{1}{3} \quad A = \frac{1}{12}$$

$$ZSR = Y_{ZSR}[n] = \frac{1}{12} \left(-\frac{1}{5}\right)^n - \frac{1}{3} \left(\frac{1}{3}\right)^n + 1.25 u[n]$$

$$d. Y_n = A \left(-\frac{1}{5}\right)^n + B \left(\frac{1}{3}\right)^n$$

$$Y[-1] = 1 \quad Y[-2] = -1$$

$$Y[0] - \frac{2}{15}(1) - \frac{1}{15}(-1) = 0 \quad Y[0] = \frac{1}{15}$$

$$A+B = \frac{1}{15}$$

$$Y[1] - \frac{2}{15} \left(\frac{1}{15}\right) - \frac{1}{15}(1) = 0 \quad Y[1] = \frac{1}{15} \left(1 + \frac{2}{15}\right) \quad Y[1] = \frac{17}{225}$$

$$\left(-\frac{1}{5}\right)A + \left(\frac{1}{3}\right)B = \frac{17}{225}$$

$$\begin{bmatrix} 1 & 1 \\ -\frac{1}{5} & \frac{1}{3} \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{15} \\ \frac{17}{225} \end{bmatrix} = \begin{bmatrix} -\frac{1}{10} \\ \frac{1}{6} \end{bmatrix}$$

$$ZIR = Y_{ZIR}[n] = -\frac{1}{10} \left(-\frac{1}{5}\right)^n + \left(\frac{1}{3}\right)^n \cdot \frac{1}{6}$$

$$e. Y_T[n] = \left(\frac{1}{12} - \frac{1}{10}\right) \left(-\frac{1}{5}\right)^n + \left(-\frac{1}{6}\right) \left(\frac{1}{3}\right)^n + 1.25 u[n]$$

$$Y_T = Y_{ZSR} + Y_{ZIR} \quad \text{Transient}$$

steady state

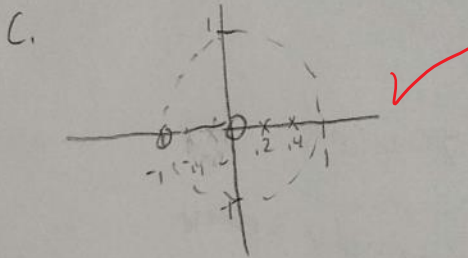
Perfect! Thank you.

3 a. $\frac{1+z^{-1}}{1+\frac{3}{5}z^{-1}+\frac{2}{25}z^{-2}} = H(z) \frac{Y(z)}{X(z)} \frac{z^2+z}{z^2+\frac{3}{5}z+\frac{2}{25}} \frac{z(z+1)}{(z+.2)(z+.4)}$

$$(1+z^{-1})(X(z)) = (1+\frac{3}{5}z^{-1}+\frac{2}{25}z^{-2})(Y(z))$$

$$X[n] + X[n-1] = Y[n] - \frac{3}{5}Y[n-1] + \frac{2}{25}Y[n-2]$$

b. Gain = 1 ✓



d. causal = $|z| > a$
 it must be larger than its smallest pole
 $|z| \geq .2$ -1 greatest, outer most

e. This system is stable because both poles are in the unit circle ✓

f.

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$$f. Y(z) = z \sum_{n=0}^{\infty} \frac{z}{5^n} - \sum_{n=0}^{\infty} \frac{1}{5^n} = \frac{z}{1 - \frac{z}{5}} - \frac{1}{1 - \frac{1}{5}} = \frac{z}{z - \frac{z}{5}} - \frac{1}{z - \frac{1}{5}}$$

$$\frac{1}{z} Y(z) = \frac{z(z - \frac{1}{5}) - z(z - \frac{z}{5})}{(z - \frac{z}{5})(z - \frac{1}{5})} = \frac{z^2 - \frac{z}{5} - z^2 + \frac{z^2}{5}}{(z - \frac{z}{5})(z - \frac{1}{5})}$$

$$Y(z) = \frac{z^2}{(z - \frac{z}{5})(z - \frac{1}{5})}$$

$$X(z) = \frac{Y(z)}{H(z)} = \frac{\frac{z^2}{(z - \frac{z}{5})(z - \frac{1}{5})}}{\frac{z^2 + 2}{(z - \frac{z}{5})(z - \frac{1}{5})}}$$

$$\rightarrow X(z) = \frac{z^2}{z^2 + 2} = \frac{z}{z + 1} \quad (-2)$$

$\Rightarrow x[n] = (-1)^n u[n]$

$$X[n] = z^{-1} \left(\frac{z^2}{z^2 + 2} \right)$$

Almost had it.