$x_1 + x$

$\cos(2\pi x_1(n)) + \cos 2\pi x_2(n)$

$\cos(2\pi(x_1+x_2))$

$x_1(-t) + x_2(-t)$

$\rightarrow x_1(t)$

$(n-t)$

$n-t+1)$

$x_1(-2$

$y[n] =$

$2n^2 x_1[n] + n x_1[n+1]$

$+ \rightarrow x_2$

$2n^2(x_1+x_2) + n(x_1(n+1) + x_2(n+1))$

ECE 466 MidTerm 1

	Linear	Time Inv	Static	Causal	Stable
1) a) $y[n] = x[-n]$	✓	✗	✓	✗	✓
$y[n] = 2n^2 x[n] + nx[n+1]$	✓	✓	✗	✗	✗
$y[n] = \cos(2\pi x[n])$	✗	✓	✓	✓	✓

b) $x(n) = \cos\left(\frac{\pi}{2} n\right)$; $x(t) = \cos(\Omega t)$ $\quad F_s = 100\ Hz$

$x(n) = \cos\left(\frac{\Omega}{100}(n)\right)$ $\quad \Omega_1 = 50$

$\Omega_2 = 20$

$\cos(2\pi($

$-\frac{1}{2} < f_0 < \frac{1}{2}$

c) - Sampling a unit step function technically requires an infinite rate.
- Nyquist Frequency = $\frac{\text{Max Freq}}{2}$ but the Max frequency of unit step is infinity.
- We must choose a sampling rate which suits our intended design.

d) $x[n] = \{\underline{3}, 1\}$; $\boxed{h[n] = \{\underline{2}, -1, 0\}}$

$y[n] = \{\underline{6}, -1, 2, 1\}$

$x[n] * h[n] = y[n]$

$h[n] \rightarrow$ has $\underline{3}$ samples

e) $y[n] = \sum_{i=-\infty}^{\infty} x[i]h[n-i]$

$\sum_{i=-\infty}^{\infty} B\cos(0.2\pi i)\,u(i) \cdot A(0.7)^{n-i}u(n-i)$

$\sum_{i=0}^{\infty} B\cos(0.2\pi i) \cdot A(0.7)^{n-i}$

iii

2) a) $y[n] = \frac{2}{15}y[n-1] + \frac{1}{15}y[n-2] + x[n]$

$y[n] - \frac{2}{15}y[n-1] - \frac{1}{15}y[n-2] = 0$

$\lambda^n - \frac{2}{15}\lambda^{n-1} - \frac{1}{15}\lambda^{n-2} = 0$

$\lambda^2 - \frac{2}{15}\lambda - \frac{1}{15} = 0$

$\lambda_1 = -\frac{1}{5}$

$\lambda_2 = \frac{1}{3}$

$y_h(n) = c_1\lambda_1^n + c_2\lambda_2^n$

$= \left(-\frac{1}{5}\right)^n c_1 + \left(\frac{1}{3}\right)^n c_2$

$y(0) = 1 = c_1 + c_2$

$\cancel{y(1) = y(0)\,\frac{2}{15}\,y(0)}$

$y(1) - \frac{2}{15}y(0) = 0$

$y(1) = \frac{2}{15}$

$$\frac{2}{15} = c_1 \cdot \left(-\frac{1}{5}\right) + \frac{1}{3} c_2$$

$$c_1 = c_2 = 1$$

$$\boxed{h[n] = \left(\left(-\frac{1}{5}\right)^n + \left(\frac{1}{3}\right)^n\right) u(n)}$$

b) IIR, stable

c) ZSR, $x[n] = u[n]$

$$y_p[n] = k\,u(n) \qquad y_{total} = y_p(n) + y_h(n)$$

$$= k\,u(n) + \left(c_1 2^n + c_2 1^n\right) u(n)$$

$$y(0) = x(0) = 1$$
$$= k + c_1 + c_2$$

$$y(1) - \frac{2}{15} y(0) = x(1) = 1$$

$$y(1) = 1 + \frac{2}{15} = \frac{17}{15}$$

$$\frac{17}{15} = k + \left(-\frac{1}{5}\right) c_1 + \left(\frac{1}{3}\right) c_2$$

$$y(2) - \frac{2}{15}(y(1)) + y(0) = 1$$

$$y(2) = 10$$

d) ZIR is when $x(n)=0$

$$\frac{2}{15}E^{-1}+\frac{1}{15}E^{-2}+E\,x(n)=E$$

$$\lambda_1=-\frac{1}{5}$$

$$\lambda_2=\frac{1}{3}$$

$$y_0[k]=c_1\left(-\frac{1}{5}\right)^k+c_2\left(\frac{1}{3}\right)^k$$

$$k=-1: \quad 1=c_1\left(-\frac{1}{5}\right)^{-1}+c_2\left(\frac{1}{3}\right)^{-1}$$

$$k=-2: \quad -1=c_1\left(-\frac{1}{5}\right)^{-2}+c_2\left(\frac{1}{3}\right)^{-2}$$

$$c_1(-5)+3c_2=1$$
$$c_1(+25)+9c_2=-1$$

$\downarrow$

solve

$$c_1=-\frac{1}{10} \qquad c_2=\frac{1}{6}$$

$$y_0[k]=-\frac{1}{10}\left(-\frac{1}{5}\right)^k+\left(\frac{1}{6}\right)\left(\frac{1}{3}\right)^k$$

e) $y_{total}=y_p(n)+y_k(n)=Ku(n)+\left(-\frac{1}{10}\right)\left(-\frac{1}{5}\right)^k+\left(\frac{1}{6}\right)\left(\frac{1}{3}\right)^k u(n)$

3) a) $H(z) = \dfrac{1+z^{-1}}{1-\frac{3}{5}z^{-1}+\frac{2}{25}z^{-2}}$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{1+z^{-1}}{1-\frac{3}{5}z^{-1}+\frac{2}{25}z^{-2}}$$

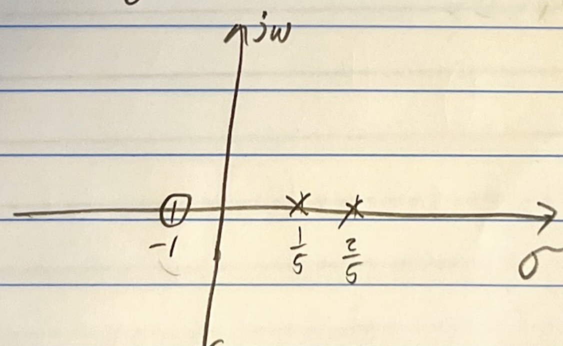
$$Y(z)\left(1-\frac{3}{5}z^{-1}+\frac{2}{25}z^{-2}\right) = X(z)(1+z^{-1})$$

$$\boxed{y(n) - \frac{3}{5}y(n-1) + \frac{2}{25}y(n-2) = x(n) + x(n-1)}$$

b) $H(1) = \dfrac{1+1}{1-\frac{3}{5}+\frac{2}{25}} = \text{Gain}$

$= \dfrac{25}{6}$

c)



Poles at $= \frac{1}{5}$ and $\frac{2}{5}$

Zero at $= -1$

e) Stable. The roots are < 1

d) ROC: $\frac{1}{5} < |z| < \frac{2}{5}$

$\cancel{f)}$ $x[n] * H(n) = y[n]$

$$y[n] = 2\left(\frac{2}{5}\right)^n u(n) - \left(\frac{1}{5}\right)^n u(n)$$

$$Y[z] = 2\left(\frac{z}{z - \frac{2}{5}}\right) - \frac{z}{z - \frac{1}{5}}$$

$$X[z] = \frac{Y[z]}{H[z]}$$

$$= \boxed{\dfrac{2\left(\dfrac{z}{z-\frac{2}{5}}\right) - \dfrac{z}{z-\frac{1}{5}}}{\dfrac{1+z^{-1}}{1-\frac{3}{5}z^{-1}+\frac{2}{25}z^{-2}}}}$$

Inv Z transform

$x(n)$

$$h(z) = 1 + z^{-1} \cdot 1 - \frac{5z}{3} + \frac{25z^2}{2}$$

$h(n)$ =