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A bit more practice would come a long way.
Don't lose courage.

50

Stable & Unstable

Type

ECE 466

Exam 1

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1.)

a)

Sys. equation	Linear	Time Invariant	Static	Causal	Stable
$y[n] = x[-n]$	NO -1	NO	NO	NO	NO -1
$y[n] = 2n^2 x[n] + nx[n+1]$	NO -1	NO	NO	NO	YES -1
$y[n] = \cos(2\pi x[n])$	YES -1	YES	YES	NO -1	YES

All 1 column

$y_1[n] = x_1[-n]$

$y_2[n] = x_2[-n]$

$y'[n] = x_1[-n] + x_2[-n]$

$y''[n] = x_1[n] + x_2[n]$

$y[n] = x[-n]$

$y[n-4] = x[-n-4]$

$y[n] = 2(n-4)^2 x[n] + \underline{n-4}\, x[n+1]$

$y[n] = \cos(2\pi x[n])$

$\cos(2\pi x[n-4])$

$y_1[n] = 2n^2 x_1[n] + n x_1[n+1]$

$y_1[n] = \cos(2\pi x_1[n])$

$y_2[n] = \cos 2\pi x_2[n]$

$y'[n] = \cos(2\pi x_1[n])$

$+ \cos 2\pi x_2[n]$

$y''[n] = \cos(2\pi x_1[n] + x_2[n])$

$y[0] = 0 \cdot x[0] + 0 \cdot x[1]$

$y[1] = 2x[1] + x[2]$

future

$y[-1] = 2x[-1]$

past

b) $x[n] = \cos\left(\frac{\pi}{2} n\right)$ was obtained by sampling analog signal $x(t) = \cos(\Omega t)$ at $F_s = 100$ Hz

What are 2 possible values of Ω??

$t = \frac{n}{F_s}$ $t = \frac{n}{100}$

$w_1 = \frac{\pi}{2}(100t)$

$n = F_s t$

$n = 100t$

$= 50\pi t$

$\boxed{w_1 = \boxed{50\cancel{\pi t}}}$ π

$w_2 = \frac{\pi}{2} + \left(\frac{4\pi}{2}\right)(1$

$\Rightarrow \frac{5\pi}{2}(10$

$= 250$

$\boxed{w_2 = 250\pi}$

$\boxed{w_1 = 50\pi}$

c) What is the ideal sampling frequency of x

$$\boxed{F_N = \frac{w}{\pi}}$$

-4

d) The causal sequence $x[n] = \{3, 1\}$ is input to a system with impulse response $h[n]$, producing the zero-state response $y[n] = \{6, -1, 2, 1\}$
$\quad\quad\quad\quad 0 \;\; 1 \;\; 2 \;\; 3$

Determine $h[n]$

$$H(z) = \sum_{n=0}^{3} x[n] z^{-n}$$ → This is wrong

$$H(z) = x(0)z^{0} + x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3}$$

$$\Rightarrow 6 + (-z^{-1}) + 2z^{-2} + z^{-3}$$

$$\Rightarrow 6 - \frac{1}{z} + \frac{2}{z^2} + \frac{1}{z^3}$$

$$\Rightarrow 6$$

$x[n]$

Go with convolution if your input and output are short size?

-3

e) $\boxed{iii}$ ✓

2.) Consider causal LTI system described by

$y[n] = \frac{2}{15} y[n-1] + \frac{1}{15} y[n-2] + x[n]$ w/ $y[-1] = 1$

$y[-2] = -1$

a) Find impulse response $h[n]$

$y[n] - \frac{2}{15} y[n-1] - \frac{1}{15} y[n-2] = x[n]$

$y[z] - \frac{2}{15} y(z) z^{-1} - \frac{1}{15} y(z) z^{2}$

$y[z]\left(1 - \frac{2}{15} z^{-1} - \frac{1}{15} z^{-2}\right) = x(z)$

$= \frac{1}{1 - \frac{2}{15} z^{-1} + \frac{1}{15} z^{-2}} \Rightarrow \frac{15z^2}{-15z^2 - 2z + 1}$

Not proper

$\frac{15z^2}{(3z-1)(-5z+1)}$

$\frac{1}{2}\frac{y(z)}{x(z)} = \frac{A}{3z-1} + \frac{B}{-5z-1} \rightarrow \frac{-15}{3z-1} + \frac{15}{-5z-1} \quad -15\frac{z}{3(z-1/3)} + 15\frac{z}{-5(z-1/5)}$

$-5\frac{z}{z-1/3} - 3\frac{z}{z-1/5}$

+

$h(n) = -5 \cdot \left(\frac{1}{3}\right)^n u(n)$

$-3 \cdot \left(\frac{1}{5}\right)^n u(n)$

$\Rightarrow -5\left(\frac{1}{3}\right)^n u(n) - 3\left(\frac{1}{5}\right)^n u(n)$

Close

-3

2b) System is $\boxed{\text{IIR}}$ ✓

& is $\boxed{\text{unstable}}$ X -3

Poles are inside unit circle and system is causal.

2c) $x[n] = u[n]$

$x(n) = \delta(n) + \delta(n-1) + - \delta(n-2)$ -7

$y(0) = h(0) = -8$

$y(1) = -\frac{5}{3} - \frac{3}{5} - 8$

$y(2) = -5(\frac{1}{3})^2 - 3(\frac{1}{3})^2 \left(-\frac{10}{3} - \frac{6}{5} - 16\right) - 8$

2d) Zero input response -7

$\Rightarrow y(0) = h(0) = \boxed{-8}$

2e) Total response $x[n] = u[n] =$ -3

$$\boxed{+ 40\left(\tfrac{1}{3}\right)^n u(n) + 24\left(\tfrac{1}{5}\right)^n u(n)}$$

3.) $H(z) = \dfrac{1+z^{-1}}{1-\frac{3}{5}z^{-1}+\frac{2}{25}z^{-2}}$

3a) Difference equation

$$\frac{Y(z)}{X(z)} = \frac{1+z^{-1}}{1-\frac{3}{5}z^{-1}+\frac{2}{25}z^{-2}}$$

$$1-\frac{3}{5}z^{-1}+\frac{2}{25}z^{-2}\ Y(z) = 1+z^{-1}\ X(z)$$

This is correct.

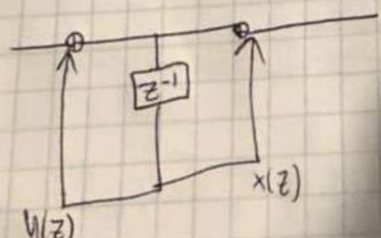
Inverse

$$y(0) + y(n) - \frac{3}{5}y(n-1) + \frac{2}{25}y(n-2) = x(0) + x(n-1)\ x(z)$$

$$y(n)\left[y(0) - \frac{3}{5}y(n-1) + \frac{2}{25}y(n-2)\right] = x(n)\left[x(0) + x(n-1)\right]$$

−2

3b) what is the gain of the system? −1

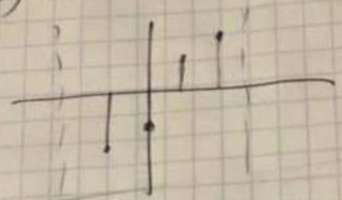


parallel or cascade $\Rightarrow$ $y(0) - \frac{3}{5}y(n-1) + \frac{2}{25}y(n-2)$

$\oplus$ $x(0) + x(n-1)$ = Gain $\frac{V_o}{V_s}$

cascade

3c) Pole-Zero map



−4

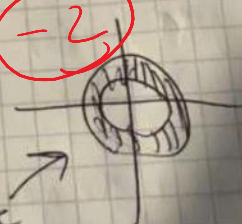
$\left\{ \begin{matrix} -5 & -3 \\ -1 & 0 \end{matrix} \right\}$

3d) ROC = entire z-plane except when z=0

−3

3e) The system would be stable ✓
because it has ~~bounded~~ input / output
region such as

−2



Stable system

3f) $\sum_{n=0}^{N} a^n = \frac{1-a^{N+1}}{1-a}$ $\quad \frac{2\left(\frac{2}{5}\right)^n u(n) - \left(\frac{1}{3}\right)^n u(n)}{1-2}$

$x[n] = -1 = -2\left(\frac{2}{5}\right)^n u(n) + \left(\frac{1}{5}\right)^n u(n)$

−4