Midterm Submission

Tuesday, February 22, 2022 10:23 AM

ECE 466 Midterm 1

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- Don't forget to write your name.
- Open textbook.
- Read carefully and write legibly. For the problems with partial credit, show your work.
- For those of you who are remotely solving the exam:
  - You can solve your exam in a-4 sheets or on your tablet.
  - You need to send a scanned pdf or image until 11:45 AM, Tuesday 22nd, to sofuoglu@msu.edu. Otherwise, your exam will not be accepted.
  - Make sure your answers are legible from pdf or scanned image.

1. No partial points for the following.

(a) [15 Points] Check if the following systems fits the classifications on the columns.

System Equation	Linear	Time Invariant	Static	Causal	Stable
$y[n] = x[-n]$	✓	✓		✓	✓
$y[n] = 2n^2x[n] + nx[n+1]$	✓				✓
$y[n] = cos(2\pi x[n])$	✓	✓	✓	✓	✓

(b) [5 Points] The sequence $x[n] = \cos\left(\frac{\pi}{2}n\right)$ was obtained by sampling an analog signal $x(t) = \cos(\Omega t)$ at a sampling rate of $F_s = 100$ Hz. What are two possible values of Ω?

$x[n] = \cos\left(\frac{\pi}{2}n\right)$ $\qquad$ $x(t) = \cos(\Omega t)$

$\frac{\pi}{2} + 2\pi$ $\qquad$ $\frac{5\pi}{2} + 2\pi = \frac{9\pi}{2}$ $\qquad$ $\Omega = 2\pi f$

$\left(\frac{4\pi}{2}\right)$

$$\boxed{\Omega = \frac{5\pi}{2} \text{ or } \frac{9\pi}{2}}$$

(c) [5 Points] What is the ideal sampling frequency of $x(t) = u(t)$?

The unit step function cannot be accurately sampled since this would require infinite bandwidth. However, an approximation can be produced by using the highest sampling rate allowed by the ADC being used.

(d) [5 Points] The causal sequence $x[n] = \{\underline{3}, 1\}$ is input to a system with impulse response $h[n]$, producing the zero-state response $y[n] = \{\underline{6}, -1, 2, 1\}$. Determine $h[n]$.

FIR

$n=0: \; y[0] = h[0]\,x[0]$

$h[0] = \frac{y[0]}{x[0]} = \frac{3}{6} = \frac{1}{2}$

$n=1: \; y[1] = h[1]\,x[1]$

$h[1] = \frac{y[1]}{x[1]} = \frac{1}{-1} = -1$

$n=2: \; h[2] = \frac{y[2]}{x[2]} = \frac{0}{2} = 0$

$n=3: \; \frac{x[3]}{y[3]} = \frac{0}{1} = 0$

$$\boxed{\begin{array}{l} n=0: \; h[0] = \frac{1}{2} \\ n=1: \; h[1] = -1 \\ n=2: \; h[2] = 0 \\ n=3: \; h[3] = 0 \end{array}}$$

(e) The impulse response of a DT (Discrete Time)-LTI system is given by $h[n] = A(0.7)^n u[n]$. Suppose $x[n] = B\cos(0.2\pi n)u[n]$ is input to the system. Which of the following could be the output signal $y[n] = h[n] * x[n]$?

i. $K_1(0.7)^n \cos(0.2\pi n + \theta)u[n]$.

~~ii. $K_1(0.14)^n u[n] + K_1 \cos(0.14\pi n\theta)u[n]$~~.

iii. $K_1(0.7)^n u[n] + K_2 \cos(0.2\pi n + \theta)u[n]$.

iv. $K_1(0.7)^n u[-n] + K_2 \cos(0.2\pi n + \theta)u[n]$.

$y[n] = A(0.7)^n u[n] * B\cos(0.2\pi n)u[n]$

$y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$

$= \sum \left[B\cos(0.2\pi k)\,u[k]\right]\left[A(0.7)^{(n-k)}\,u[n-k]\right]$

$\boxed{\text{iii and iv}}$

2. [30 Points] Consider a causal LTI system described by the difference equation $y[n] = \frac{2}{15}y[n-1] + \frac{1}{15}y[n-2] + x[n]$ with $y[-1] = 1$, $y[-2] = -1$.

(a) [6] Find the impulse response $h[n]$.

(b) [4] Determine if the system is (1) FIR or IIR, and (2) stable.

(c) [8] Find the zero state response for $x[n] = u[n]$. (*Decide on particular response's K first.*)

(d) [8] Find the zero input response.

(e) [4] Find the total response for $x[n] = u[n]$. Identify the steady state and transient responses.

$y[n] = \frac{2}{15} y[n-1] + \frac{1}{15} y[n-2] + x(n)$ with $y(-1) = 1$, $y[-2] = -1$

a) $h[n] = C_1\left(\frac{2}{15}\right)^n u[n] + C_2\left(\frac{1}{15}\right)^n 2\,u[n] +$

$= \frac{2}{15}(1) + \frac{1}{15}(-1) + x(n)$

$= \frac{2}{15} - \frac{1}{15} + x(n) = \frac{1}{15}\, x(n)$

$\boxed{h[n] = \frac{1}{15}\, x[n]}$

b.) $\boxed{\begin{array}{l} \text{1.) FIR} \\ \text{2.) Stable} \end{array}}$

$\frac{1}{15} \sum |x(n)| \neq \infty$

c.) ZSR $x[n] = u[n]$

$$y[n] = \frac{2}{15} y[n-1] + \frac{1}{15} y[n-2] + x(n)$$

$$Y(z) = \frac{2}{15} z^{-1} + \frac{1}{15} z^{-2} + X(z)$$

$$H(z) = \frac{\frac{2}{15} z^{-1} + \frac{1}{15} z^{-2} + 1}{X(z)} \rightarrow \frac{\frac{2}{15} z^{-1} + \frac{1}{15} z^{-2} + 1}{u[n]}$$

d.) ZIR

Let,
$k_1 = \frac{3}{15}$
$k_2 = \frac{4}{5}$

$$Y_{ZIR}(n) = k_1\left(\frac{2}{15}\right)^n u(n) + k_2\left(\frac{1}{15}\right)^n u(n)$$

$$\boxed{Y_{ZIR}(n) = \frac{2}{15}^n u(n) + \frac{1}{15} u(n)}$$

e.) Total Response

$$\boxed{y(n) = \frac{3}{15}\left(\frac{2}{15}\right)^n u(n) + \frac{4}{5}\left(\frac{1}{15}\right)^n u(n)}$$

3. [30 points] A causal LTI system has a system function $H(z) = \frac{1+z^{-1}}{1-\frac{3}{5}z^{-1}+\frac{2}{25}z^{-2}}$.

(a) [5] Determine the difference equation that this system function describes.

(b) [2] What is the gain of the system?

(c) [5] Plot the pole-zero map.

(d) [5] Determine the region of convergence (ROC).

(e) [5] Is the system stable? Why?

(f) [8] Find the input signal $x[n]$ that will produce the output $y[n] = 2\left(\frac{2}{5}\right)^n u[n] - \left(\frac{1}{5}\right)^n u[n]$.

$$H(z) = \frac{1+z^{-1}}{1-\frac{3}{5}z^{-1}+\frac{2}{25}z^{-2}}$$

a.) $$\frac{Y(z)}{X(z)} = \frac{1+z^{-1}}{1-\frac{3}{5}z^{-1}+\frac{2}{25}z^{-2}}$$

$$Y(z)\left(1-\frac{3}{5}z^{-1}+\frac{2}{25}z^{-2}\right) = X(z)\left(1+z^{-1}\right)$$

$$y[n] - \frac{3}{5}y[n-1] + \frac{2}{25}y[n-2] = x[n] + x[n-1]$$

$$\boxed{y[n] = x[n] + x[n-1] + \frac{3}{5}y[n-1] - \frac{2}{25}y[n-2]}$$

b.)

c.) $$0 = 1 - \frac{3}{5}z^{-1} + \frac{2}{25}z^{-2}$$

$$0 = 1 - \frac{3}{5}\frac{1}{z} + \frac{2}{25}\frac{1}{z^2}$$

$$\frac{3}{5}\frac{1}{z} - \frac{2}{25}\frac{1}{z^2} = 1$$

$$\frac{1}{5}\frac{1}{z}\left(3 - \frac{2}{5}\frac{1}{z}\right) = 1$$

Extra page for Question 3