1.

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(a)

	Linear	Time-invariant	static	causal	stable
$y[n] = x[-n]$	yes	NO	NO	NO	yes
$2n^2 x[n] + n x[n+1]$	NO	yes	yes	yes	NO
$\cos(2\pi x[n])$	NO	yes	yes	yes	yes

(b) $x(t/T_s) = \cos(0.5\pi(100t)) = \cos(50\pi t)$

$\Omega = 50\ Hz$

(c) $F_s = 0\ Hz$

(d)

$$H(z) = \frac{Y(z)}{X(z)} = \frac{3 + z^{-1}}{6 - z^{-1} + 2z^{-2} + z^{-3}} = \frac{3z+1}{6z^3 - z^2 + 2z + 1}$$

$$h[n] = Z^{-1}\left[\frac{3z+1}{6z^3 - z^2 + 2z + 1}\right]$$

(e) $$Y(z) = H(z)X(z) = \frac{1}{1-(0.7)z^{-1}}\left[\frac{B(1 - z^{-1}\cos(0.2\pi))}{1 - (2\cos(0.2\pi))z^{-1} + z^{-2}}\right]$$

i $K_1 (0.7)^n \cos(0.2\pi n + \theta) u[n]$

2.

(a) $y[n] - 2/15\, y[n-1] - 1/15\, y[n-2] = \delta[n]$ $\quad y[-1] = 1 \quad y[-2] = -1$

$\lambda^{n-2}(\lambda^2 - 2/15\lambda - 1/15) = 0$

$$\lambda_{1,2} = \frac{2/15 \pm \sqrt{4/225 + 60/225}}{2} = \frac{2/15 \pm 8/15}{2} = 1/3, -1/5$$

$y[0] - 2/15\, y[-1] - 1/15\, y[-2] = 1 \rightarrow y[0] = 1 + 2/15 - 1/15 = 16/15$

$y[1] - 2/15\, y[0] - 1/15\, y[-1] = 0 \rightarrow y[1] = (-2/15)(16/15) + 1/15(1)$

$y[1] = -32/15 + 1/15 = 33/15$

$h[n] = c_1(1/3)^n + c_2(-1/5)^n$

$h[0] = c_1 + c_2 = 1$

$h[1] = 1/3 c_1 - 1/5 c_2 = 33/15$

$\rightarrow 1/5 c_1 + 1/3 c_1 = 1/5 + 33/15$

$\frac{3c_1}{15} + \frac{5c_1}{15} = \frac{36}{15} = 8c_1 = 36 \rightarrow c_1 = 9/2$

$c_2 = 1 - 9/2 = -7/2$

$\underline{h[n] = 9/2(1/3)^n - 7/2(-1/5)^n}$

(b) The system has finite impulse response and is stable because $\lambda_1, \lambda_2 < 1$

(c) $y_p[n] = Ku[n]$

$Ku[n] - 2/15 Ku[n-1] - 1/15 Ku[n-2] = Ku[n] \rightarrow K = 0$

ZSR: $y[-1] = y[-2] = 0$

$y[0] = 1$

$y[1] - 2/15\, y[0] = 1 \rightarrow y[1] = 1 + 2/15 = 17/15$

$c_1(1/3)^n + c_2(-1/5)^n$

$y[0] = c_1 + c_2 = 1$ $\quad \rightarrow \frac{8c_1}{15} = \frac{22}{5} = \frac{66}{15} \rightarrow c_1 = \frac{33}{4} \quad c_2 = -\frac{29}{4}$

$y[1] = 1/3 c_1 - 1/5 c_2 = 17/5$

$\underline{y_{ZSR} = 33/4(1/3)^n - 29/4(-1/5)^n}$

2. (d)

$y[n] - 2/15\, y[n-1] - 1/15\, y[n-1] = 0$

$y[0] - 2/15\, y[-1] - 1/15\, y[-2] = 0$

$y[0] = 2/15(1) + 1/15(-1) = 1/15$

$y[1] - 2/15\, y[0] - 1/15\, y[-1] = 0 \Rightarrow y[1] = (2/15)(1/15) + 1/15 = 17/225$

$= c_1(1/3)^n + c_2(-1/5)^n$

$y[0] = c_1 + c_2 = 1/15$ $\rightarrow \frac{8c_1}{15} = \frac{1}{15} + \frac{17}{225} \rightarrow c_1 = \frac{4}{15}$

$y[1] = 1/3\, c_1 - 1/5\, c_2 = 17/225$ $\quad c_2 = \frac{-3}{15}$

$y_{ZIR}[n] = 4/15(1/3)^n - 3/15(-1/5)^n$

(e) $y_{total} = \underbrace{(4/15 + 33/4)(1/3)^n - (20/41 + 3/15)(-1/5)^n}_{\text{Transient}}$

no steady state response

3. (a) $H(z) = \frac{z+1}{z^2 - 3/5 z + 2/25}$ $\quad z = \frac{3/5 \pm \sqrt{9/25 - 8/25}}{2} = \frac{3/5 \pm 1/5}{2} = 2/5, 1/5$

$H(z) = \frac{z+1}{(z+2/5)(z-1/5)} = \frac{A}{(z+2/5)} + \frac{B}{(z-1/5)}$

$A(z-1/5) + B(z+2/5) = z+1$ $\quad A+B = 1$ $\quad 3/5\, B = 6/5 \rightarrow B = 2$

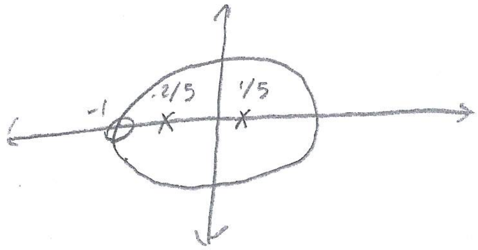
$z(A+B) + (-1/5 A + 2/5 B)$ $\quad -1/5 A + 2/5 B = 1$ $\quad A = -1$

$$\boxed{h[n] = -(-2/5)^n u[n] + 2(1/5)^n u[n]}$$

(b) $\frac{Y(z)}{X(z)} = H(z) = \frac{z+1}{z^2 - 3/5 z + 2/25}$

3.

(c) p: $-2/5,\ 1/5$ z: -1



(d) ROC: $|z| > 1/5$

(e) The system is stable because its poles are inside the unit circle

(f) $$Y(z) = H(z)X(z) = \frac{2}{z-2/5} - \frac{1}{z-1/5} = \frac{2(z-1/5)-(z-2/5)}{(z-2/5)(z-1/5)}$$

$$= \frac{z}{(z-2/5)(z-1/5)} \Rightarrow X(z) = \frac{\left(\frac{z}{(z-2/5)(z-1/5)}\right)}{\left(\frac{z+1}{(z+2/5)(z-1/5)}\right)}$$