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1a i: Linear, Time Varying, Dynamic, non causal, Stable

ii: Nonlinear, Time Varying, Dynamic, non causal, non stable

iii Nonlinear, Time Invariant, static, causal, stable

1b. $W_s = 200\pi \frac{rad}{s}$ $\Omega = \frac{\pi}{2} = \frac{W_s}{2} + W_s n$ $\boxed{\Omega = 100\pi, 300\pi}$

1c. There is no ideal sampling frequency as $u(t)$ is non periodic so any sampling frequency will become a proper $u[n]$ except possibly shifted

1d. $x[n] = 3\delta[n] + \delta[n-1]$

$y[n] = 3h[n] + h[n-1]$

impulse response $= 0 \ \forall n < 0$

$h[0] = 6/3 = 2$

$h[1] = \frac{y[1] - h[0]}{3} = \frac{-3}{3} = -1$

$h[2] = \frac{y[2] - h[1]}{3} = \frac{2-(1)}{3} = 1$

$\boxed{h[n] = \{2, -1, 1\}}$

1e. $k_1 (.7)^n u[n] + k_2 \cos(.2\pi n + \theta)$ as in steady state the system will "track" the input

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2a. $y[n] - \frac{2}{15}y[n-1] - \frac{1}{15}y[n-2] = x[n]$

$\lambda^n - \frac{2}{15}\lambda^{n-1} - \frac{1}{15}\lambda^{n-2}$

$\lambda^{n-2}(\lambda^2 - \frac{2}{15}\lambda - \frac{1}{15})$

$\lambda = -\frac{1}{5}, \frac{1}{3}$

$\frac{\frac{2}{15} \pm \sqrt{\frac{4}{225} + \frac{4}{15}}}{2} \rightarrow \frac{1}{15} \pm \sqrt{\frac{1}{225} + \frac{15}{225}}$

$\lambda = \frac{1}{15} \pm \frac{4}{15} = \frac{1}{3}, -\frac{1}{5}$

$y[n] = A(-\frac{1}{5})^n + B(\frac{1}{3})^n$

Impulse Response → Zero state → $y[-2], y[-1] = 0$

$y[0] = 1$

$y[0] - 0 - 0 = 1$ $\quad A(-\frac{1}{5})^0 + B(\frac{1}{3})^0 = 1 \rightarrow A + B = 1 \rightarrow A = 1 - B$

$y[1] - \frac{2}{15} - 0 = 0$ $\quad y[1] = \frac{2}{15}$ $\quad A(-\frac{1}{5}) + B(\frac{1}{3}) \rightarrow -\frac{A}{5} + \frac{B}{3} = \frac{2}{15}$

$-\frac{1}{5} + \frac{B}{5} + \frac{B}{3} = \frac{2}{15}$ $\quad \frac{8}{15}B = \frac{1}{3}$ $\quad B = \frac{5}{8} \rightarrow A = \frac{3}{8}$

$$\boxed{h[n] = \frac{3}{8}\left(-\frac{1}{5}\right)^n + \frac{5}{8}\left(\frac{1}{3}\right)^n}$$

b. As $\forall a \neq 0 \nexists n, a^n = 0 \rightarrow h[n]$ is IIR as it will never reach 0

$\sum_{n=0}^{\infty} |h[n]| = \sum_{n=0}^{\infty} \frac{3}{8}(\frac{1}{5})^n + \sum_{n=0}^{\infty} \frac{5}{8}(\frac{1}{3})^n = \frac{3}{8}\left(\frac{1}{1-\frac{1}{5}}\right) + \frac{5}{8}\left(\frac{1}{1-\frac{1}{3}}\right)$

As this is $< \infty$, this system is stable

c. $y_h[n] = A(-1/5)^n + B(\frac{1}{3})^n$ from pt. A

$y_p = k u[n]$, $\quad k u[n] - \frac{2}{15} k u[n-1] - \frac{1}{15} k u[n-2] = u[n]$

$n \geq 2$, choose $n = 2$ $\quad k - \frac{2}{15}k - \frac{1}{15}k = 1$ $\quad \frac{12}{15}k = 1$ $\quad k = 1.25$

ZSR $\quad y[n] = A(-\frac{1}{5})^n + B(\frac{1}{3})^n + 1.25u[n]$

Sys $y[0] - 0 - 0 = x[0]$ $\quad y[0] = 1$ $\quad A + B + 1.25 = 1$ $\quad A + B = -.25$

$y[1] - \frac{2}{15}y[0] - 0 = x[1]$ $\quad y[1] = \frac{17}{15}$ $\quad -\frac{A}{15} + \frac{B}{3} + 1.25 = \frac{17}{15}$ $\quad -\frac{A}{15} + \frac{B}{3} = \frac{17}{15} - \frac{5}{4} =$

$-\frac{7}{60} \leftarrow \frac{68}{60} - \frac{75}{60}$

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$A+B=-.25$ $4A+4B=-1$ $24B=-8$ $B=-\frac{1}{3}$

$-\frac{A}{15}+\frac{B}{3}=-\frac{7}{60}$ $-4A+20B=-7$ $4A+\frac{4}{3}=-1$ $4A=\frac{1}{3}$ $A=\frac{1}{12}$

$$\boxed{ZSR = Y_{ZSR}[n] = \frac{1}{12}\left(-\frac{1}{5}\right)^n - \frac{1}{3}\left(\frac{1}{3}\right)^n + 1.25u[n]}$$

d. $Y_h = A\left(-\frac{1}{5}\right)^n + B\left(\frac{1}{3}\right)^n$

$y[-1]=1$ $y[-2]=-1$

$y[0]-\frac{2}{15}(1)-\frac{1}{15}(-1)=0$ $y[0]=\frac{1}{15}$

$A+B=\frac{1}{15}$

$y[1]-\frac{2}{15}\left(\frac{1}{15}\right)-\frac{1}{15}(1)=0$ $y[1]-\frac{1}{15}\left(1+\frac{2}{15}\right)$ $y[1]=\frac{17}{225}$

$\left(-\frac{1}{5}\right)A+\left(\frac{1}{3}\right)B=\frac{17}{225}$

$$\begin{bmatrix}1 & 1\\ -\frac{1}{5} & \frac{1}{3}\end{bmatrix}^{-1}\begin{bmatrix}\frac{1}{15}\\ \frac{17}{225}\end{bmatrix}=\begin{bmatrix}-1/10\\ 1/6\end{bmatrix}$$

$$\boxed{ZIR = Y_{ZSR}[n] = -\frac{1}{10}\left(-\frac{1}{5}\right)^n + \left(\frac{1}{3}\right)^n \cdot \frac{1}{6}}$$

e. $Y_T[n] = \underbrace{\left(\frac{1}{12}-\frac{1}{10}\right)\left(-\frac{1}{5}\right)^n + \left(-\frac{1}{6}\right)\left(\frac{1}{3}\right)^n}_{\text{Transient}} + \underbrace{1.25u[n]}_{\text{steady state}}$

$Y_T = Y_{ZSR}+Y_{ZSR}$

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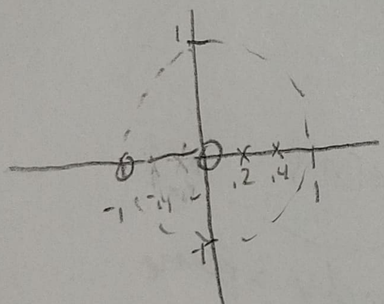
3a. $\dfrac{1+z^{-1}}{1+\frac{3}{5}z^{-1}+\frac{2}{25}z^{-2}} = H(z) = \dfrac{Y(z)}{X(z)}$ $\dfrac{z^2+z}{z^2+\frac{3}{5}z+\frac{2}{25}}$ $\dfrac{z(z+1)}{(z+.2)(z+.4)}$

$(1+z^{-1})(X(z)) = (1+\frac{3}{5}z^{-1}+\frac{2}{25})(Y(z))$

$\boxed{X[n]+X[n-1] = Y[n] - \frac{3}{5}Y[n-1] + \frac{2}{25}Y[n-2]}$

b. Gain = 1

c.



d. causal = $|z| > a$
it must be larger than its smallest pole
$|z| \geq .2$

e. this system is stable because both poles are in the unit circle

f.

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$$\boxed{ZSR = Y_{ZSR}[n] = \frac{1}{12}\left(-\frac{1}{5}\right)^n - \frac{1}{3}\left(\frac{1}{3}\right)^n + 1.25\,u[n]}$$

d. $Y_h = A\left(-\frac{1}{5}\right)^n + B\left(\frac{1}{3}\right)^n$

$Y[-1]=1$ $Y[-2]=-1$

$Y[0]-\frac{2}{15}(1)-\frac{1}{15}(-1)=0$ $Y[0]=\frac{1}{15}$

$A+B=\frac{1}{15}$

$Y[1]-\frac{2}{15}\left(\frac{1}{15}\right)-\frac{1}{15}(1)=0$ $Y[1]-\frac{1}{15}\left(1+\frac{2}{15}\right)$ $Y[1]=\frac{17}{225}$

$\left(-\frac{1}{5}\right)A+\left(\frac{1}{3}\right)B=\frac{17}{225}$

$$\begin{bmatrix}1 & 1\\ -\frac{1}{15} & \frac{1}{3}\end{bmatrix}^{-1}\begin{bmatrix}\frac{1}{15}\\ \frac{17}{225}\end{bmatrix}=\begin{bmatrix}-1/10\\ 1/6\end{bmatrix}$$

$$\boxed{ZIR = Y_{ZIR}[n] = -\frac{1}{10}\left(-\frac{1}{5}\right)^n + \left(\frac{1}{3}\right)^n\cdot\frac{1}{6}}$$

e. $Y_T[n] = \underbrace{\left(\frac{1}{12}-\frac{1}{10}\right)\left(-\frac{1}{5}\right)^n + \left(-\frac{1}{6}\right)\left(\frac{1}{3}\right)^n}_{\text{Transient}} + \underbrace{1.25\,u[n]}_{\text{steady state}}$

$Y_T = Y_{ZSR} + Y_{ZIR}$