Exam #1

ECE 466

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Question #1

a)

$y[n] = x[-n]$: Linear, time varying, Dynamic noncausal, BiBO stable

$y[n] = 2n^2 x[n] + n x[n+1]$: Time varying, Dynamic noncausal, BiBO unstable

$y[n] = \cos(2\pi x[n])$: time varying, static, causal BiBO stable

b)

$x(n) = \cos\left(\frac{\pi}{2} n\right) \longrightarrow \Omega = \frac{\pi}{2} \quad \Omega = \frac{-\pi}{2}$

because $\cos(\Omega n)$ at $\pm\frac{\pi}{2}$ results in similar signals

c) $\frac{100}{8} = 12.5\ \text{Hz}$

d

Question 2

a) $y[n] = \frac{2}{15} y[n-1] + \frac{1}{15} y[n-2] + x[n]$

$y[n] - \frac{2}{15} y[n-1] - \frac{1}{15} y[n-2] = x[n]$

$\lambda + \frac{2}{15}\lambda^{-1} - \frac{1}{15}\lambda^{-2} = 0$

$\frac{-\left(\frac{1}{15}\right) \pm \sqrt{\left(\frac{1}{15}\right)^2 - 4(1)\left(\frac{-1}{15}\right)}}{2\left(\frac{2}{15}\right)} \longrightarrow -\frac{1}{15} + \frac{\sqrt{14}i}{15}, \quad -\frac{1}{15} - \frac{\sqrt{14}i}{15}$

$y(0) = C_1(1) + C_2(-1)$ $\quad C_1 = 1, \quad C_2 = -1$

b) **FIR, Stable**

c) $y_n(n) = C_1(1)^n u[n] + C_2[-1]^n u[n]$

$y_p[n] = k(1)^n u[n]$

$k(-1)^n u_n + \left(-\frac{2}{15}\right)k - \left(\frac{1}{15}\right)k = 0$

$C_1 + C_2 = 1 \rightarrow C_1 = C_2 + 1 \longrightarrow C_1 = 4 + 1 = C_1 = 5$

$\frac{2C_1}{15} + \frac{1C_2}{15} = -1 \rightarrow \frac{2C_2 + 2}{15} + \frac{C_2}{15} = -15 \Rightarrow 3C_2 + 2 = 15$

$3C_2 = 12$

$\frac{3C_2}{3} = \frac{12}{3}$

$C_2 = 4$

$y_{ZSR}(n) = 5\left(\frac{2}{15}\right)^n + 4\left(\frac{1}{15}\right)^n u[n]$

d) $y[n] + \frac{1}{15} y[n-1] - \frac{2}{15} y[n-2] = 0$

$\frac{-2 \pm \sqrt{4-60}}{15} \rightarrow \frac{-1}{15} \pm \frac{\sqrt{14}i}{15}$

$y_n(n) = C_1\left(\frac{-1}{15} + \frac{\sqrt{14}i}{15}\right) + C_2\left(\frac{-1}{15} - \frac{\sqrt{14}i}{15}\right)$

$y(0) = \frac{\sqrt{14}i - 1}{15} +$ ·

undefined

$y_{zie}[n] = 1\left(\frac{-\sqrt{14}i}{15}\right) + \frac{\sqrt{14}i - 1}{15}$

e) There is no force response the response = 0 since the undefined aspect kills the input

Question 3

a)

$u_n \rightarrow \frac{1}{1-z^{-1}}$

$\alpha^n u[n] \rightarrow \frac{1}{1-\alpha z^{-1}}$

$u[n]$

$\frac{1+z^{-1}}{1-\frac{3}{5}z^{-1}+\frac{2}{25}z^{-2}}$

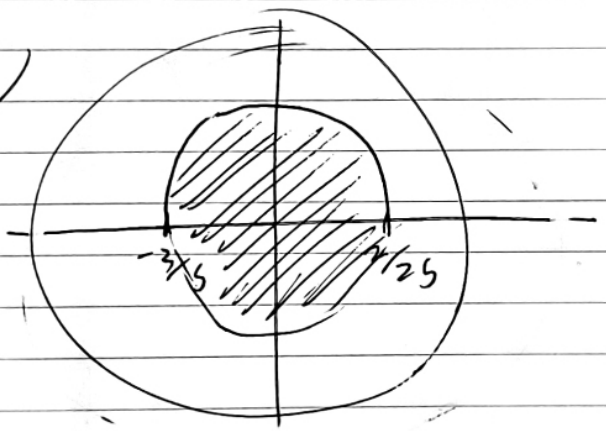
$-\frac{3}{5}\alpha^n u[n] \quad | \quad \frac{2}{25}\alpha^n[n]$

$$\frac{-3}{5}\alpha^{n-1}u[n] + \frac{2}{25}\alpha^{n-2}[n]$$

$u[n]$

b)

c)



d) $|z| > 1 \rightarrow u[n] \quad \alpha^n u[n] \rightarrow |\alpha| < |z|$

So

$$|\alpha| < |z| > 1$$