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ECE 466 Midterm 1

Name:
PID:

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- Don't forget to write your name.
- Open textbook.
- Read carefully and write legibly. For the problems with partial credit, show your work.
- For those of you who are remotely solving the exam:
  - You can solve your exam in a-4 sheets or on your tablet.
  - You need to send a scanned pdf or image until 11:45 AM, Tuesday 22nd, to sofuoglu@msu.edu. Otherwise, your exam will not be accepted.
  - Make sure your answers are legible from pdf or scanned image.

1. No partial points for the following.

(a) [15 Points] Check if the following systems fits the classifications on the columns.

System Equation	Linear	Time Invariant	Static	Causal	Stable
$y[n] = x[-n]$	✓				✓
$y[n] = 2n^2x[n] + nx[n+1]$	✓				✓
$y[n] = cos(2\pi x[n])$			✓	✓	✓

(b) [5 Points] The sequence $x[n] = \cos\left(\frac{\pi}{2}n\right)$ was obtained by sampling an analog signal $x(t) = \cos(\Omega t)$ at a sampling rate of $F_s = 100$ Hz. What are two possible values of Ω?

$\cos(\Omega n T_s) = \cos\left(\frac{n\pi}{2}\right)$

$\frac{\Omega n}{100} = \frac{n\pi}{2}$

$\Omega = 50\pi$

$\frac{\Omega n T_s}{100} = \frac{\overset{4}{\cancel{2}} n\pi}{2} - \frac{n\pi}{2}$

$\frac{\Omega n}{100} = \frac{3n\pi}{2}$

$\Omega = 150\pi$

$\boxed{\begin{array}{l}\Omega = 50\pi \\ \Omega = 150\pi\end{array}}$

(c) [5 Points] What is the ideal sampling frequency of $x(t) = u(t)$?

$F_s = 100\ \text{Hz}$

(d) [5 Points] The causal sequence $x[n] = \{\underline{3}, 1\}$ is input to a system with impulse response $h[n]$, producing the zero-state response $y[n] = \{\underline{6}, -1, 2, 1\}$. Determine $h[n]$.

$$h[n] = 3 + \delta[n+1]$$

(e) The impulse response of a DT (Discrete Time)-LTI system is given by $h[n] = A(0.7)^n u[n]$. Suppose $x[n] = B\cos(0.2\pi n)u[n]$ is input to the system. Which of the following could be the output signal $y[n] = h[n] * x[n]$?

i. $K_1(0.7)^n \cos(0.2\pi n + \theta)u[n]$.
ii. $K_1(0.14)^n u[n] + K_1 \cos(0.14\pi n\theta)u[n]$.
iii. $K_1(0.7)^n u[n] + K_2 \cos(0.2\pi n + \theta)u[n]$.
iv. $K_1(0.7)^n u[-n] + K_2 \cos(0.2\pi n + \theta)u[n]$.

2. [30 Points] Consider a causal LTI system described by the difference equation $y[n] = \frac{2}{15}y[n-1] + \frac{1}{15}y[n-2] + x[n]$ with $y[-1] = 1$, $y[-2] = -1$.

(a) [6] Find the impulse response $h[n]$.
(b) [4] Determine if the system is (1) FIR or IIR, and (2) stable.
(c) [8] Find the zero state response for $x[n] = u[n]$. (*Decide on particular response's K first.*)
(d) [8] Find the zero input response.
(e) [4] Find the total response for $x[n] = u[n]$. Identify the steady state and transient responses.

$Y(z) - \frac{2}{15} z^{-1} Y(z) - \frac{1}{15} z^{-2} Y(z) = X(z) \Rightarrow \frac{1}{1 - \frac{2}{15} z^{-1} - \frac{1}{15} z^{-2}} = \frac{z^2}{z^2 - \frac{2}{15} z - \frac{1}{15}}$

a) $h(n) - \frac{2}{15} h(n-1) - \frac{1}{15} h(n-2) = \delta(n)$

b) system is FIR and Stable

c) $H(z) \; \frac{z}{z-1} \cdot \frac{z^2}{z^2 - \frac{2}{15} z - \frac{1}{15}} = \frac{Y(z)}{z} = \frac{z^2}{(z-1)(z^2 - \frac{2}{15} z - \frac{1}{15})} \Rightarrow$ — —

d) $y(z) - \frac{2}{15} z^{-1}(y(z) + z y(-1)) - \frac{1}{15}(y(z) + z y(-1) + z^{-2} y(-2)) = 0$

$y(z)[1 - \frac{2}{15} z^{-1} - \frac{1}{15} z^{-2}]$

e) $y(z)[1 - \frac{2}{15} z^{-1} - \frac{1}{15} z^{-2}]$

3. [30 points] A causal LTI system has a system function $H(z) = \frac{1+z^{-1}}{1-\frac{3}{5}z^{-1}+\frac{2}{25}z^{-2}}$.

(a) [5] Determine the difference equation that this system function describes.

(b) [2] What is the gain of the system?

(c) [5] Plot the pole-zero map.

(d) [5] Determine the region of convergence (ROC).

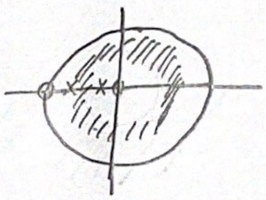
(e) [5] Is the system stable? Why?

(f) [8] Find the input signal $x[n]$ that will produce the output $y[n] = 2\left(\frac{2}{5}\right)^n u[n] - \left(\frac{1}{5}\right)^n u[n]$.

a) $y(n) - \frac{3}{5} y(n-1) + \frac{2}{25} y(n-2) = x(n) + x(n-1)$

b)

c) $\frac{z^2+z}{z^2-\frac{3}{5}z+\frac{2}{25}} = \frac{1}{25} \frac{z(z+1)}{(5x+1)(5x+2)}$ zero: 0, -1 poles: $-\frac{1}{5}$, $-\frac{2}{5}$



d) ROC $= -\frac{1}{5} < z < -\frac{2}{5}$

e) yes, ROC is not greater than 1

f) $x[n] = \frac{1}{28} \frac{n(n+1)}{(5n+1)(5n+2)}$